Entanglement Holography

Robert Myers

with de Boer, Heller & Neiman

arXiv:1509.00113

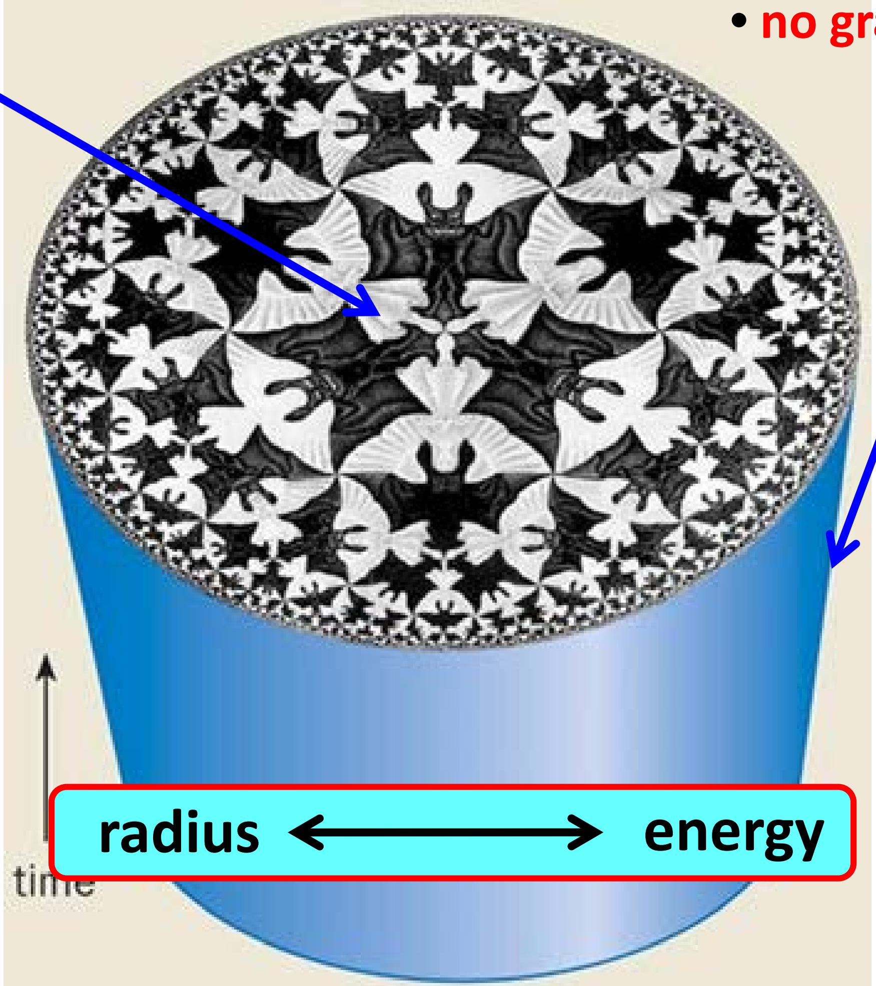
AdS/CFT Correspondence:

Bulk:
- quantum gravity
- negative cosmological constant
- **d+1** spacetime dimensions

Holography ⟷

Boundary:
- quantum field theory
- no scale (at quantum level)
- **d** spacetime dimensions
- **no gravity!**

anti-de Sitter space

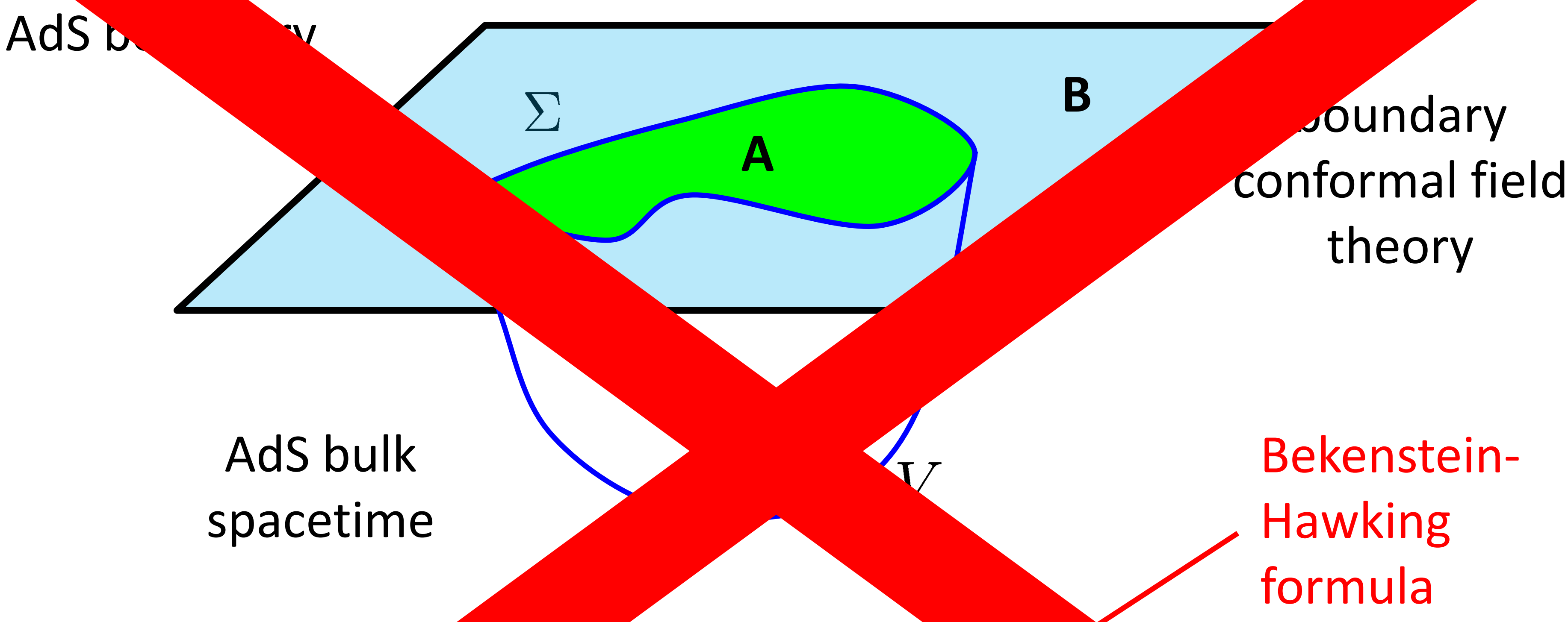


conformal field theory

(Maldacena '97)

H…graphic Entanglement Entropy:

(Ryu & Takaya… i)



$$\ldots A) = \underset{V \sim A}{\text{ext}} \; \frac{\ldots}{4G_N}$$

• 2006 c… ure ⟶ many detailed consistency tests…

(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Fa… .)

• … 3 proof (for static geometries) (Maldacena & Lewko…)

…S/CFT Correspondence:

Bulk:
- quant… …ty
- negative c… …ical constant
- **d+1** spacetime… …ions

Holography ⟷

Boundary:
- quantu… …eory
- no s… …quantum level)
- … …me dimensions

…avity!

anti-de Sitter space

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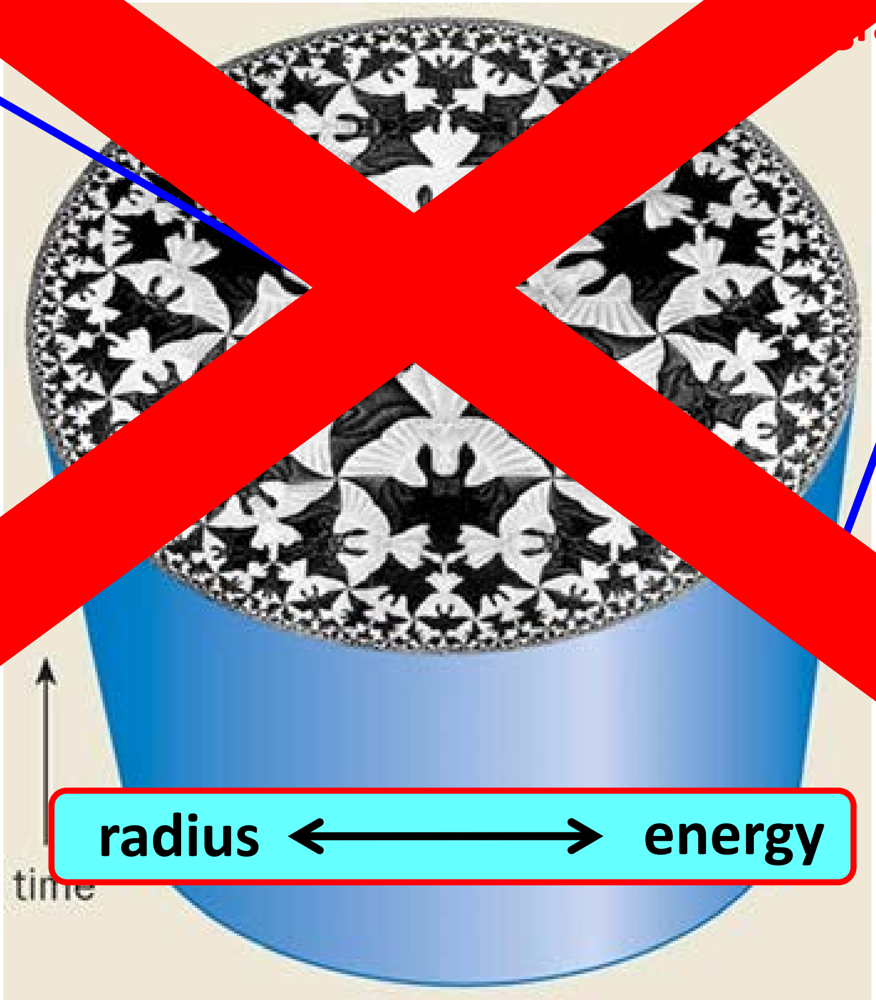


(Maldace… …7)

First Law of Entanglement

(Blanco, Casini, Hung & RM
Bhattacharya, Nozaki, Takayanagi & Ugajin)

- relative entropy: $S(\rho_1|\rho_0) = \mathrm{tr}(\rho_1 \log \rho_1) - \mathrm{tr}(\rho_1 \log \rho_0)$

- provides a distance measure between states:

ρ_0 = reference state; ρ_1 = perturbed state

= exp(i H A) ⟵ modular (or entanglement) Hamiltonian

- $S(\rho_1|\rho_0) \geq 0$ and hence linear $d\lambda$ term vanishes

$$\longrightarrow \quad \delta S_A = \delta \langle H_A \rangle$$

"1st law" of entanglement entropy (also $\mathrm{Tr}\,(\delta\rho) = 0$)

$$\delta S_A = \delta\langle H_A\rangle$$

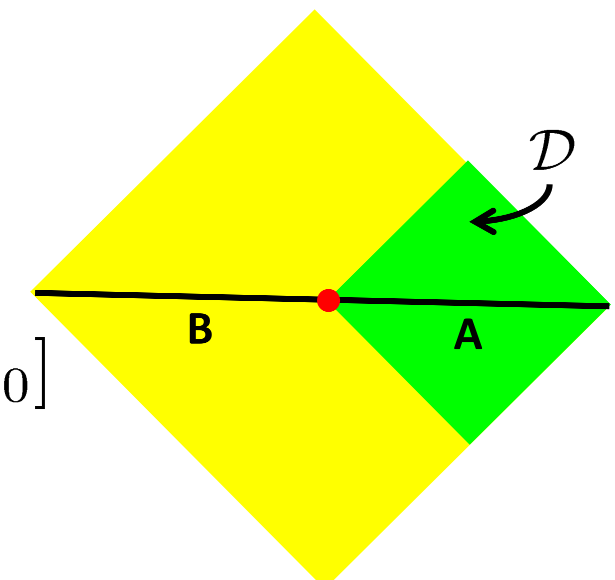
"1st law" of entanglement entropy

- generally H_A is "**nonlocal mess**" and flow is nonlocal/**not geometric**

$$H_A = d^{d-1}x\, \beta_1^{\mu\nu}(x)\, T_{\mu\nu} + d^{d-1}x \quad d^{d-1}y\, \beta_2^{\mu\nu;\lambda\sigma}(x;y)\, T_{\mu\nu} T_{\lambda\sigma} + \cdots$$

⟶ hence usefulness of first law is very limited, in general

- famous exception: Rindler wedge



H_A = boost generator

$$= 2\pi K = -2\pi \int_{A(x>0)} d^{d-2}y\, dx\, [x\, T_{00}]$$

(by causality, ρ_A and H_A describe physics throughout causal domain $\mathcal{D}$)

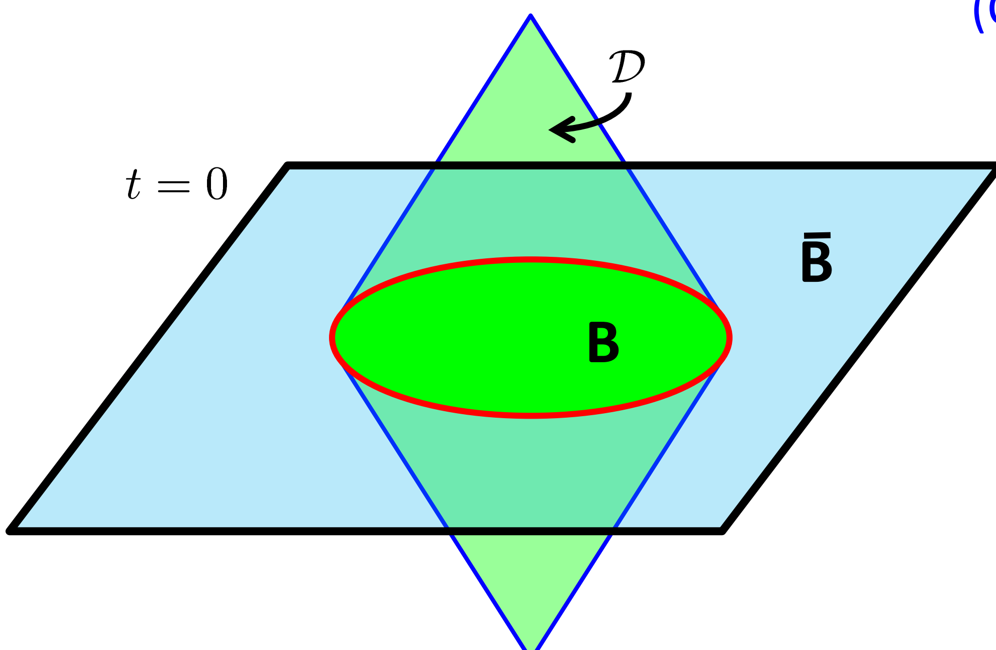
$$\delta S_A = \delta\langle H_A \rangle$$

“1st law” of entanglement entropy

- **another exception**: CFT in vacuum of d-dim. flat space and entangling surface which is S^{d-2} with radius R

$$H_B = 2\pi \int_B d^{d-1}y \, \frac{R^2 - |\vec{y}|^2}{2R} T_{tt}(\vec{y}) + c'$$

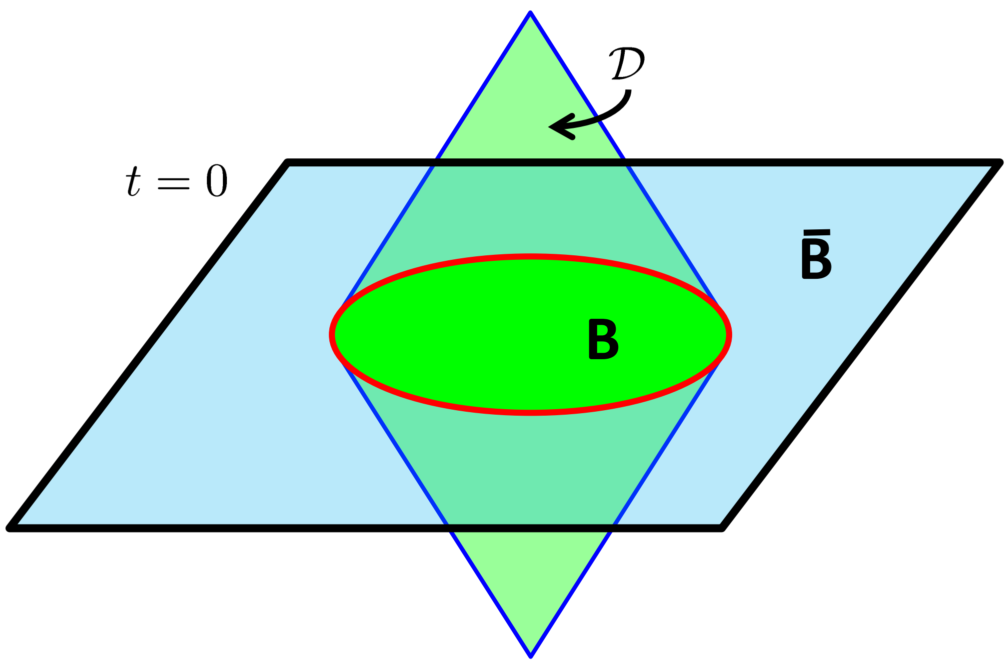
(Casini, Huerta & RM)



“1st law” of entanglement entropy:

- small excitations of CFT vacuum in d-dim. flat space and entangling surface which is S^{d-2} with radius R:

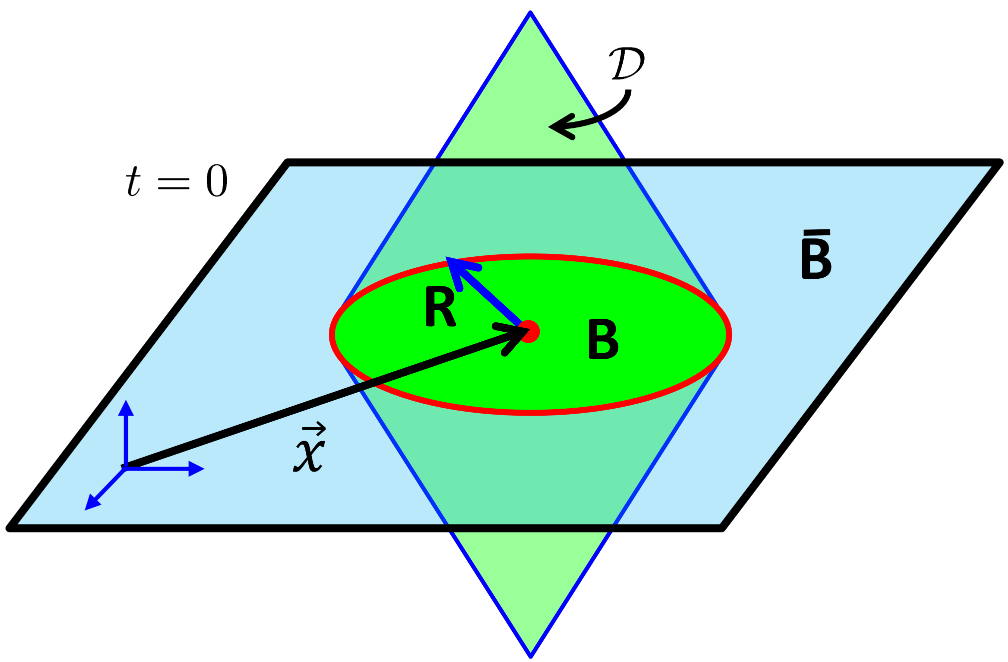
$$\delta S = \delta\langle H_B\rangle = 2\pi \int_B d^{d-1}y\, \frac{R^2 - |\vec{y}|^2}{2R}\, \langle T_{tt}(\vec{y})\rangle$$



“1st law” of entanglement entropy:

- small excitations of CFT vacuum in d-dim. flat space and entangling surface which is S^{d-2} with radius R:

$$\delta S(R, \vec{x}) = 2\pi \int_B d^{d-1}y \, \frac{R^2 - |\vec{y} - \vec{x}|^2}{2R} \langle T_{tt}(\vec{y}) \rangle$$



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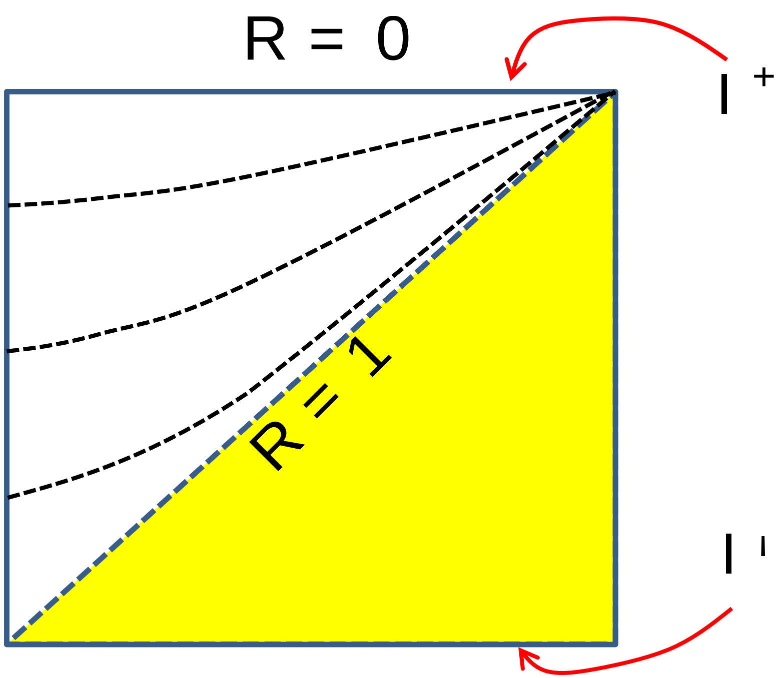
- **boundary-to-bulk propagator in d-dim de Sitter space!**

(eg, see: Xiao 1402.7080)



$$ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$$

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- **boundary-to-bulk propagator in d-dim de Sitter space!**

(eg, see: Xiao 1402.7080)

$$ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$$

- straightforward to show δS satisfies wave equation in dS_d

$$\left(\nabla^2_{dS} - m^2\right)\delta S = 0 \qquad \text{with} \qquad m^2 L^2 = -d$$

- $\langle T_{tt}\rangle$ sets δS at very small R and EE perturbations at larger scales determined by the local Lorentzian propagation into dS geometry

Compare and Contrast: begin with d-dim. CFT

- **Entanglement Holography:**

undetermined constant? → Lorentzian

$$ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$$

holo. coordinate = scale (radius of ball)

spatial coordinates in (d–1)-dim. time slice

- two-derivative wave equation relies only on first law of entanglement
  → appropriate states in any CFT in any number of dimensions

- **AdS/CFT correspondence:**

fixed by ℓ_P → Euclidean

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)$$

holo. coordinate = scale (roughly)

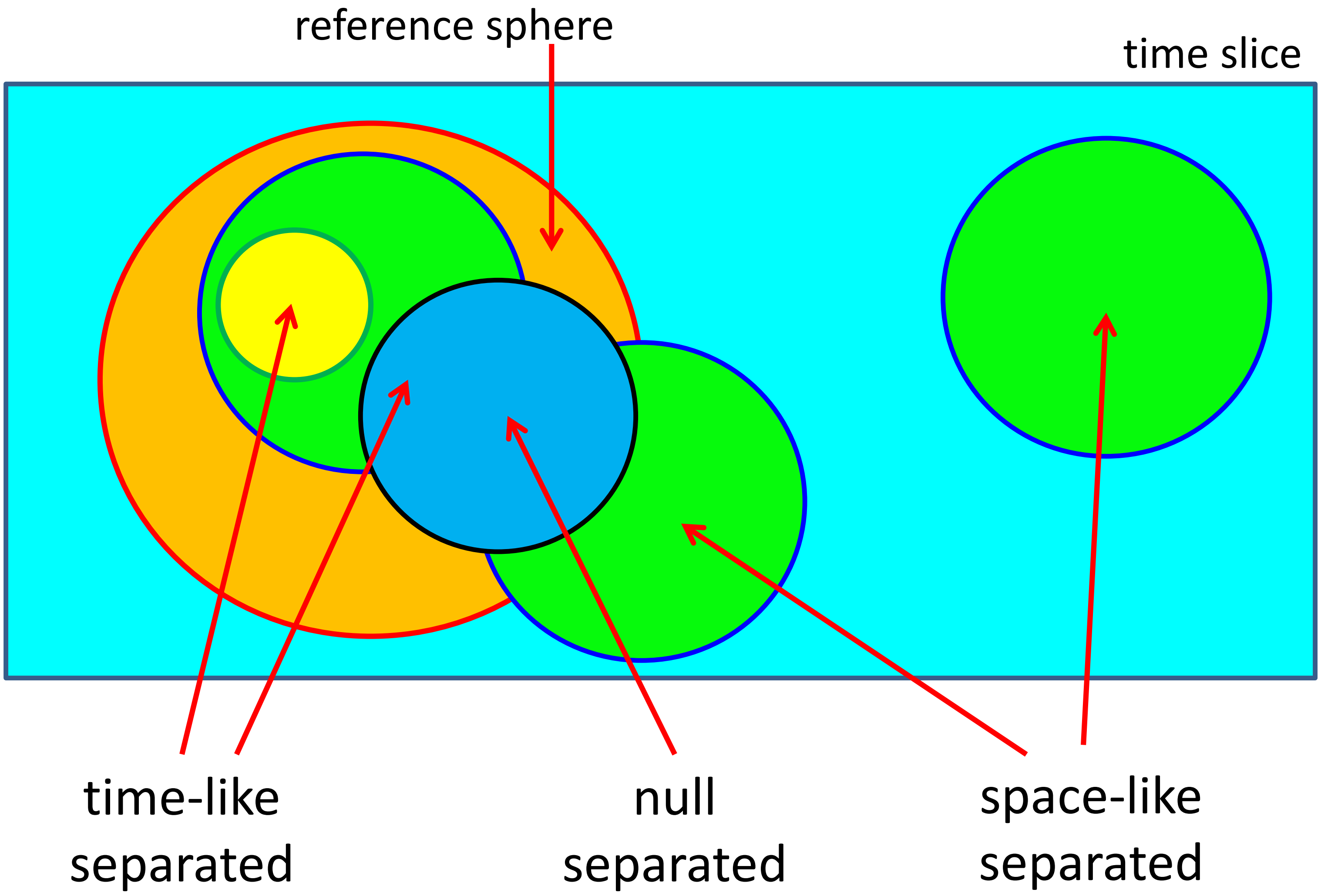
spacetime coordinates for d-dim. CFT

- two-derivative bulk theory relies on weak curvature and weak coupling
  → holographic CFT requires strong coupling and large # of d.o.f.

Why is scale time-like?

- geometry naturally gives partial ordering of spheres

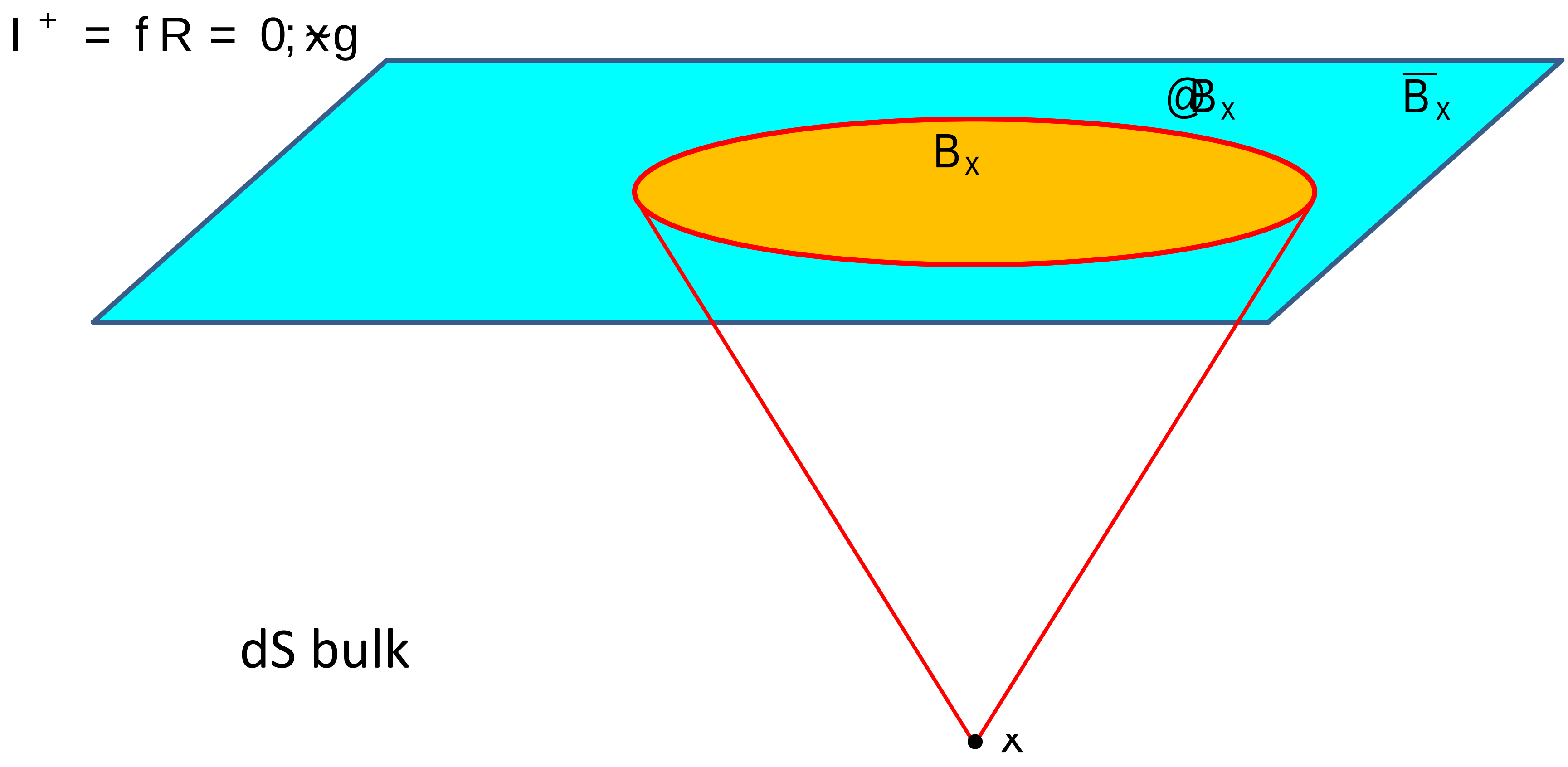
→ suggests holographic geometry should be Lorentzian



(ordering of intervals for d=2 discussed by Czech, Lamprou, McCandlish & Sully)

Mapping deSitter ↔ Balls?

- choose one of asymptotic boundaries of dS (eg, $\mathcal{I}^+$) ↔ time slice
- for any point x in bulk and send out future light cone to $\mathcal{I}^+$
- intersects $\mathcal{I}^+$ on a sphere and interior uniquely defines `dual' ball B_x



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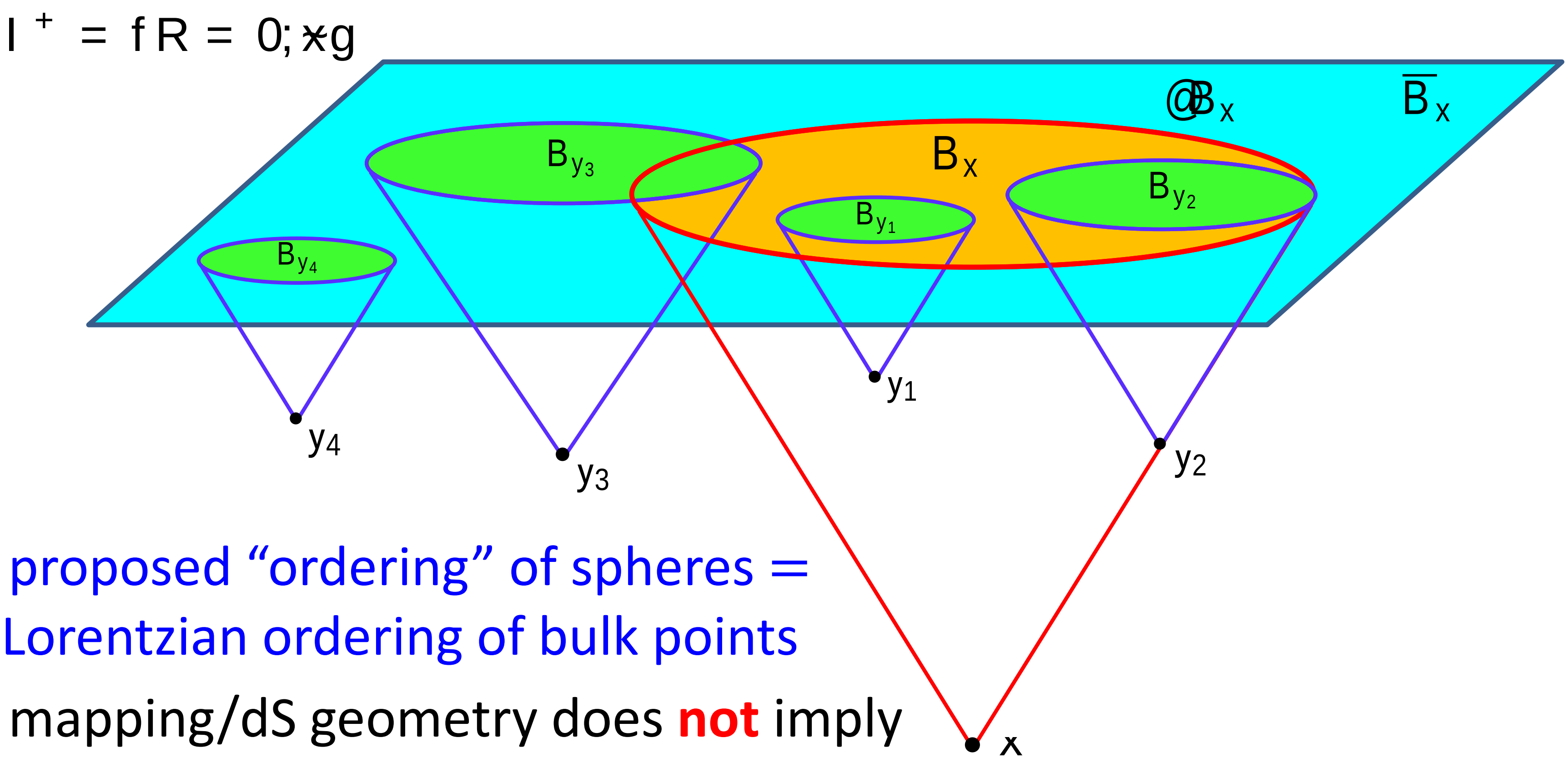

- proposed "ordering" of spheres $=$ Lorentzian ordering of bulk points
- mapping/dS geometry does **not** imply local dynamics respecting this structure

Example:

$$\delta S(R,\vec{x}) = 2\pi \int_B d^{d-1}y \, \frac{R^2 - |\vec{y}-\vec{x}|^2}{2R} \langle T_{tt}(\vec{y})\rangle$$

- consider state: $|A\rangle = |0\rangle + \epsilon\, T_{tt}(t_0 + i\tau, \vec{x}_0)|0\rangle$

small expansion parameter $\epsilon/\tau^d \ll 1$

regulate UV divergences

- expectation value is fixed by 2-pt function $\langle 0|T_{tt}(t,\vec{x})\, T_{tt}(0,\vec{0})|0\rangle$

$$\langle \Psi|T_{tt}(t,x)|\Psi\rangle = \epsilon\, C_T \left[\frac{1}{(\Delta x^2 - (\Delta t + i\tau)^2)^d}\left(\frac{(\Delta x^2 + (\Delta t + i\tau)^2)^2}{(\Delta x^2 - (\Delta t + i\tau)^2)^2} - \frac{1}{d}\right) + c.c.\right] + O(\epsilon^2)$$

with $\Delta x^2 = |\vec{x}-\vec{x}_0|^2$ and $\Delta t^2 = |t - t_0|^2$

Example: $$\delta S(R;\vec{x}) = 2\pi \int_B d^{d-1}y \, \frac{R^2 - |\vec{y} - \vec{x}|^2}{2R} \langle T_{tt}(\vec{y}) \rangle$$

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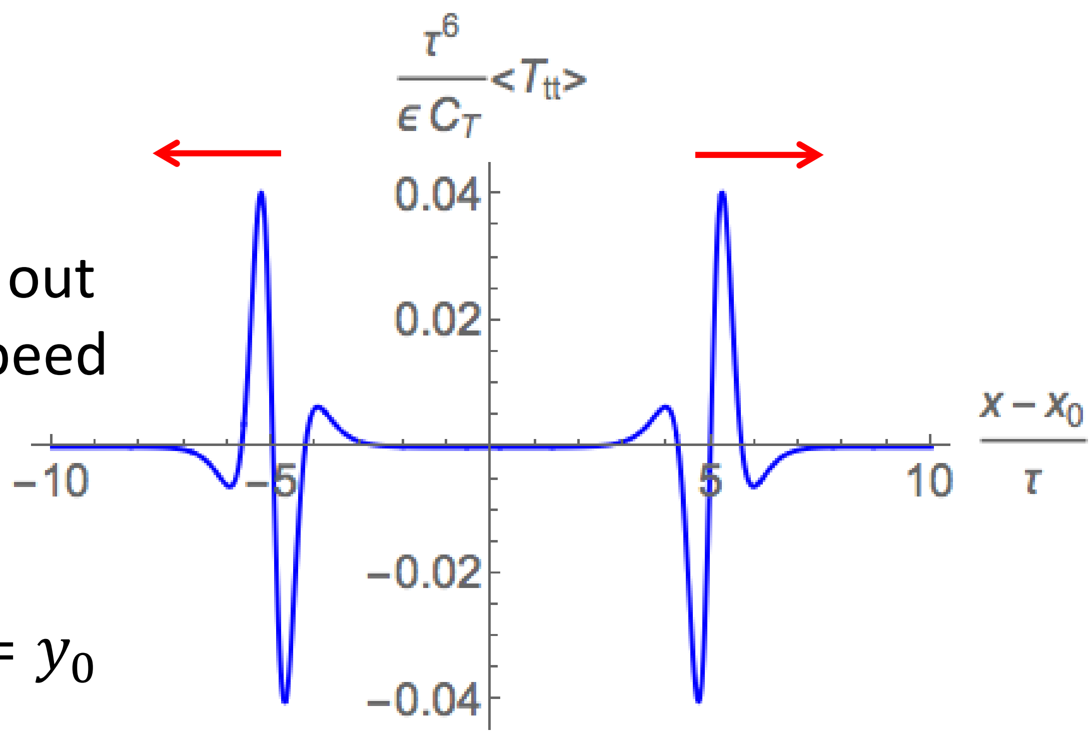


- sphere expanding out from $(t_0, \vec{x}_0)$ at speed of light

$d = 3$: $t = 5, y = y_0$

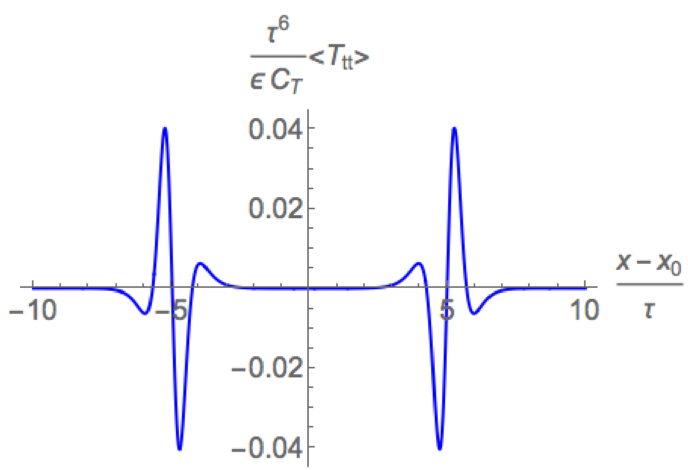
Example:

$$\pm S(R;x) = 2¼ \int_B d^{d¡1}y \; \frac{R^2 ¡ |y ¡ x|^2}{2R} \, hT_{tt}(y)i$$

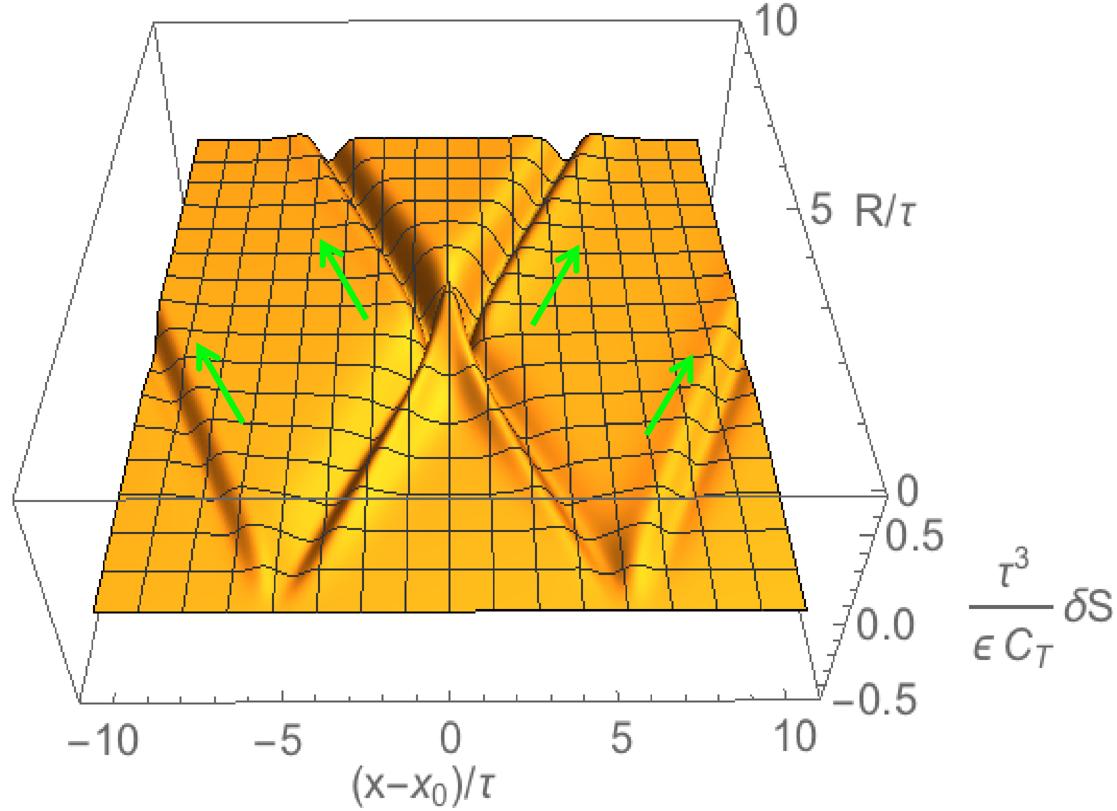
- consider state: $|Ai = |0i + ^2\, T_{tt}(t_0 + i¿; x_0)|0i$

small expansion parameter $^2 = ¿^d ¿ 1$

regulate UV divergences



$\mathrm{d} = 3{:}\; t = 5, y = y_0$



Comment:

- same wave equation derived from AdS/CFT correspondence

Nozaki, Numasawa, Prudenziati& Takayanagi: arXiv:1304.7100
Bhattacharya, Takayanagi: arXiv:1308.3792

- Eg, linearized Einstein eqs in AdS_4 implied for holographic EE

$$\left[\frac{\partial^2}{\partial R^2} - \frac{1}{R}\frac{\partial}{\partial R} - \frac{3}{R^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right]\delta S(t;x;y;R) = 0$$

- can be recast as d=3 deSitter wave equation:

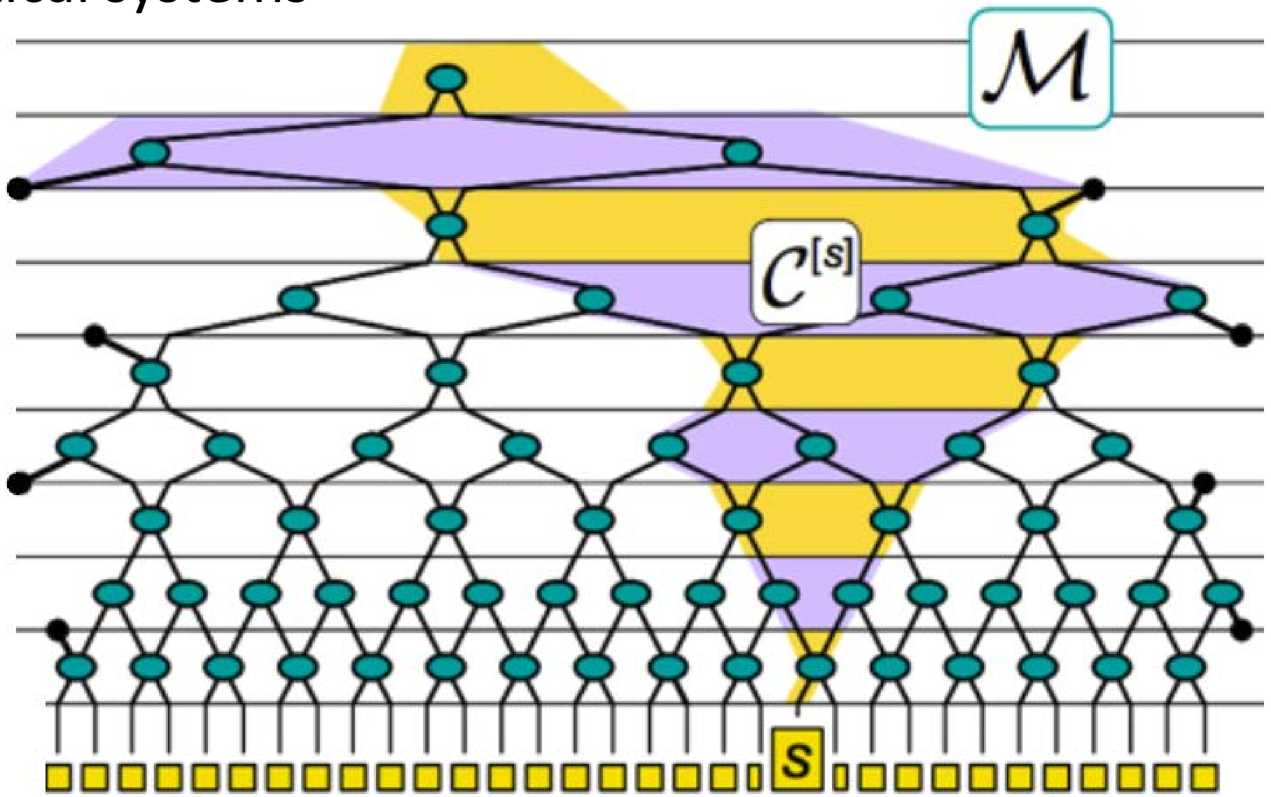
$$\left[-\frac{R^3}{L^2}\frac{\partial}{\partial R}\left[\frac{1}{R}\frac{\partial}{\partial R}\right] + \frac{R^2}{L^2}\frac{\partial^2}{\partial x^2} + \frac{R^2}{L^2}\frac{\partial^2}{\partial y^2} + \frac{3}{L^2}\right]\delta S(t;x;y;R) = 0$$

d'Alembertian on dS_3 — mass term

- here, we see equation readily extends to any d and follows purely from underlying conformal symmetry

Comment:

- MERA (Multi-scale Entanglement Renormalization Ansatz) provides efficient tensor network representation of ground-state wave-function in d=2 critical systems (Vidal)



- has been argued that MERA has (Lorentzian) causal structure with coarse-graining direction being time-like! (Beny; Czech etal)

Comment:

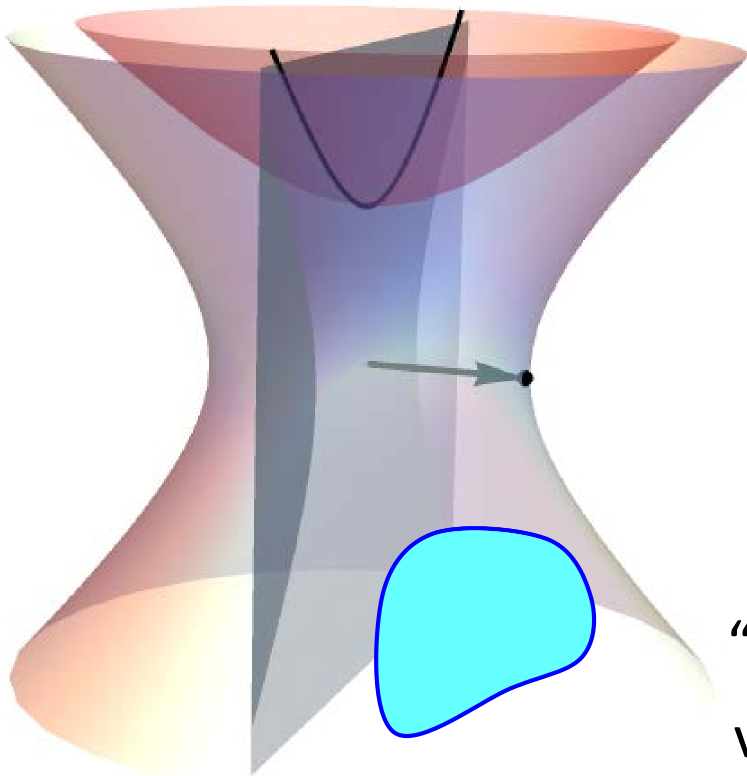
- deSitter geometry appears in recent discussions of integral geometry and the interpretation of MERA in terms of AdS_3/CFT_2

(Czech, Lamprou, McCandlish & Sully: arXiv:1505.05515)

- consider space of intervals u<x<v on time slice of 2d CFT

⟷ space of geodesics on 2d slice of AdS_3 ⟷ pts in 2d de Sitter

AdS/CFT



$$ds^2 = L^2 \frac{du\,dv}{(v-u)^2}$$

dS scale? ↗

motivate the choice: $L^2 = \frac{c}{3}$

⟶ $ds^2 = \partial_u \partial_v S_0 \, du\, dv$

with $S_0 = \frac{c}{3} \log \frac{v-u}{\delta}$

"hole-ography":

volume in dS_2 = length in AdS_3 slice

Boundary data:

- recall deSitter metric: $ds^2 = \frac{L^2}{R^2}\left(-dR^2 + d\vec{x}^2\right)$
- wave equation $\left(\nabla^2_{dS} - m^2\right)\delta S = 0$ is singular as $R \to 0$

⟶ 2 independent sol's: $\delta S \overset{R\to 0}{=} F(\vec{x})/R + f(\vec{x})\, R^d + \cdots$

$\Delta = -1$ $\qquad\qquad$ $\Delta = d$

- "1st law" solution: $\delta S(R,\vec{x}) = 2\pi \int_B d^{d-1}y \, \frac{R^2 - |\vec{y} - \vec{x}|^2}{2R} \langle T_{tt}(\vec{y})\rangle$

⟶ $F(\vec{x}) = 0\,; \qquad f(\vec{x}) = \frac{\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+3}{2}\right)} \cdot \langle T_{tt}(\vec{x})\rangle$

- $\langle T_{tt}\rangle$ sets δS at very small R and EE perturbations at larger scales determined by the **local Lorentzian propagation** into dS geometry
- $m^2 L^2 = -d$: mass tachyonic! → above precisely removes the `non-normalizable' or unstable modes

Alternate conformal frames:

- wave equation $'r^{2}_{dS}$ ¡ m^{2} ` $\pm S = 0$ is covariant

  ⟶ can use any coordinate system on dS geometry

- coord transformation in bulk corresponds to conformal transformation in boundary theory ⟶ new holographic construction extends to CFT in any conformally flat background

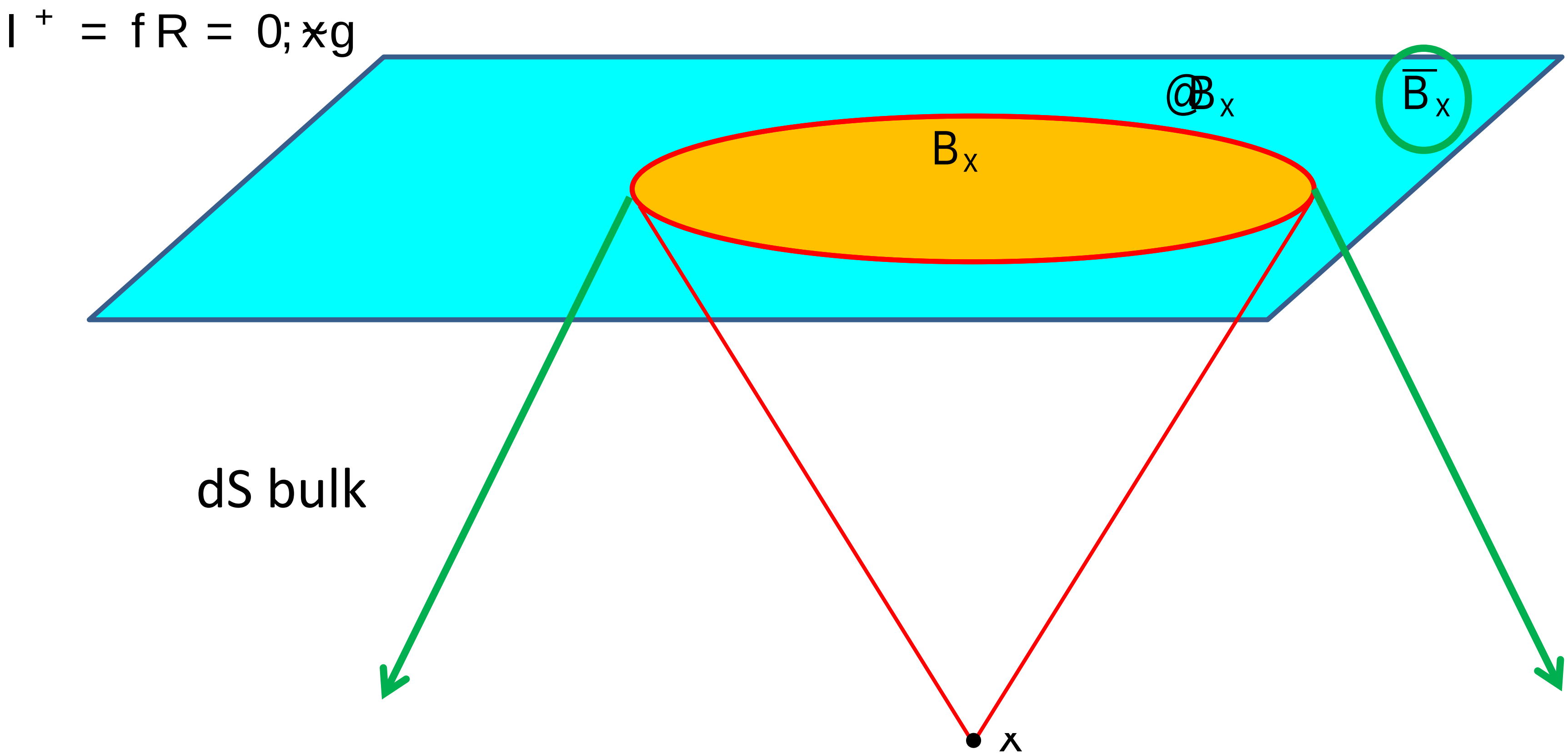
- for example, in cylindrical bkgd $R \times S^{d-1}$, time slice is S^{d-1}

  ⟶ wave equation in global coord's

$$ds^2 = L^2($$ ¡ d¿^{2} + cosh(¿)^{2}d- $_{d¡\ 1}$)

Alternate conformal frames:

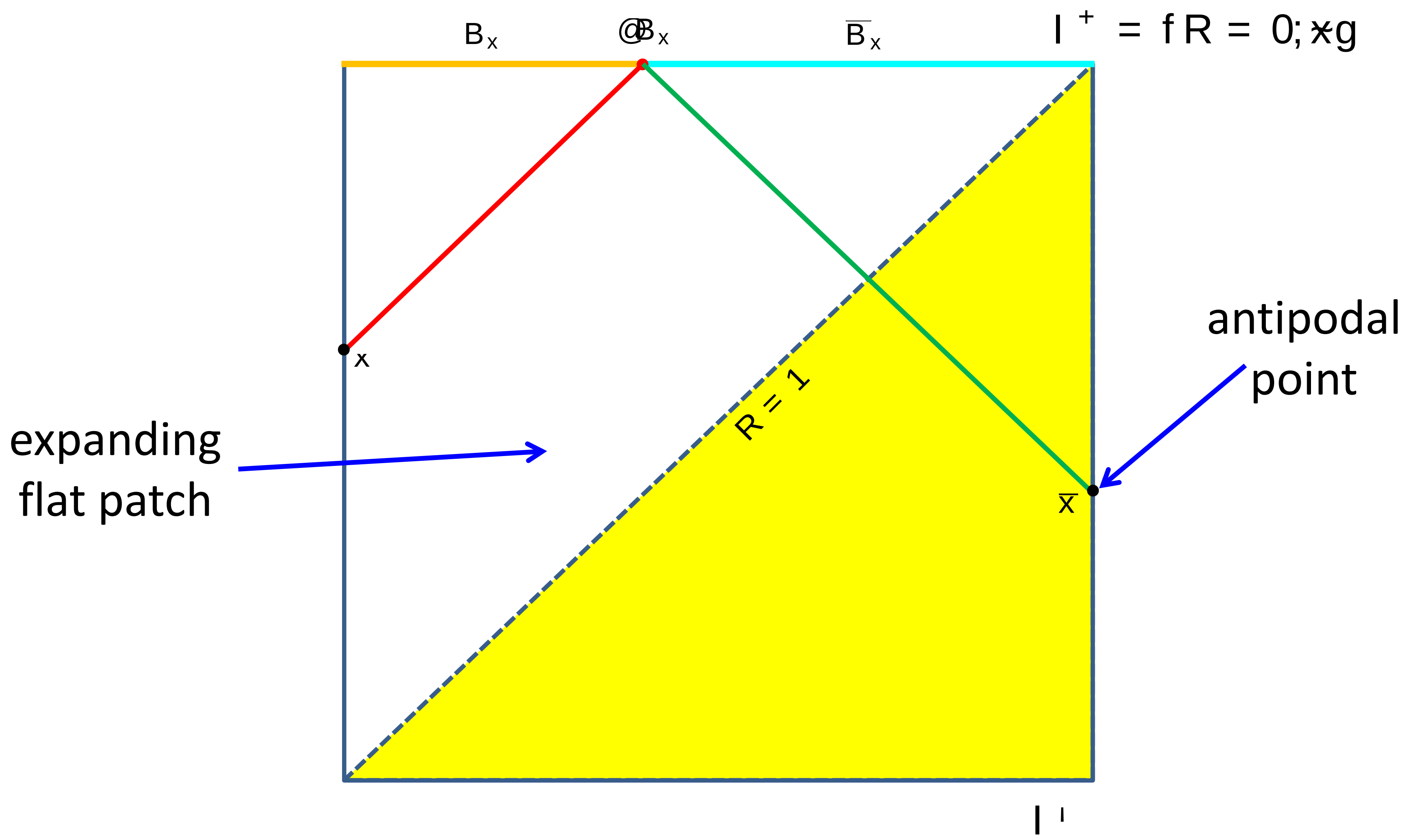
- recall to mapping deSitter ↔ balls



- with flat bkgd, exterior region is infinite and outward light-sheet `expands’ forever but $\overline{B}_x$ becomes complementary sphere on S^{d-1} and outward light-sheet converges on antipodal point $\overline{x}$

Alternate conformal frames:

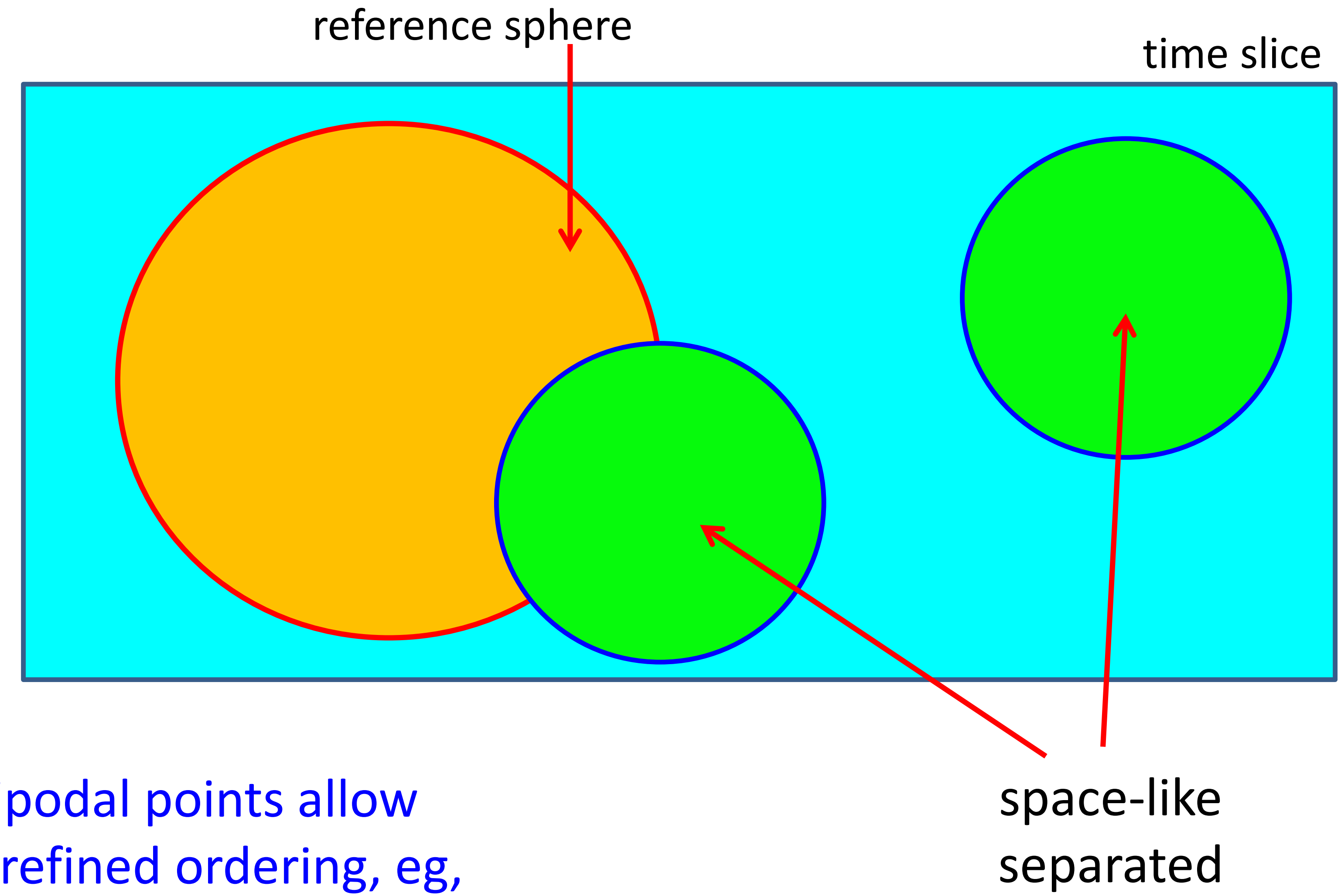
- consider mapping deSitter ↔ balls on Penrose diagram



Why is scale time-like?

- geometry naturally gives partial ordering of spheres

  → holographic geometry should be Lorentzian



- antipodal points allow for refined ordering, eg,

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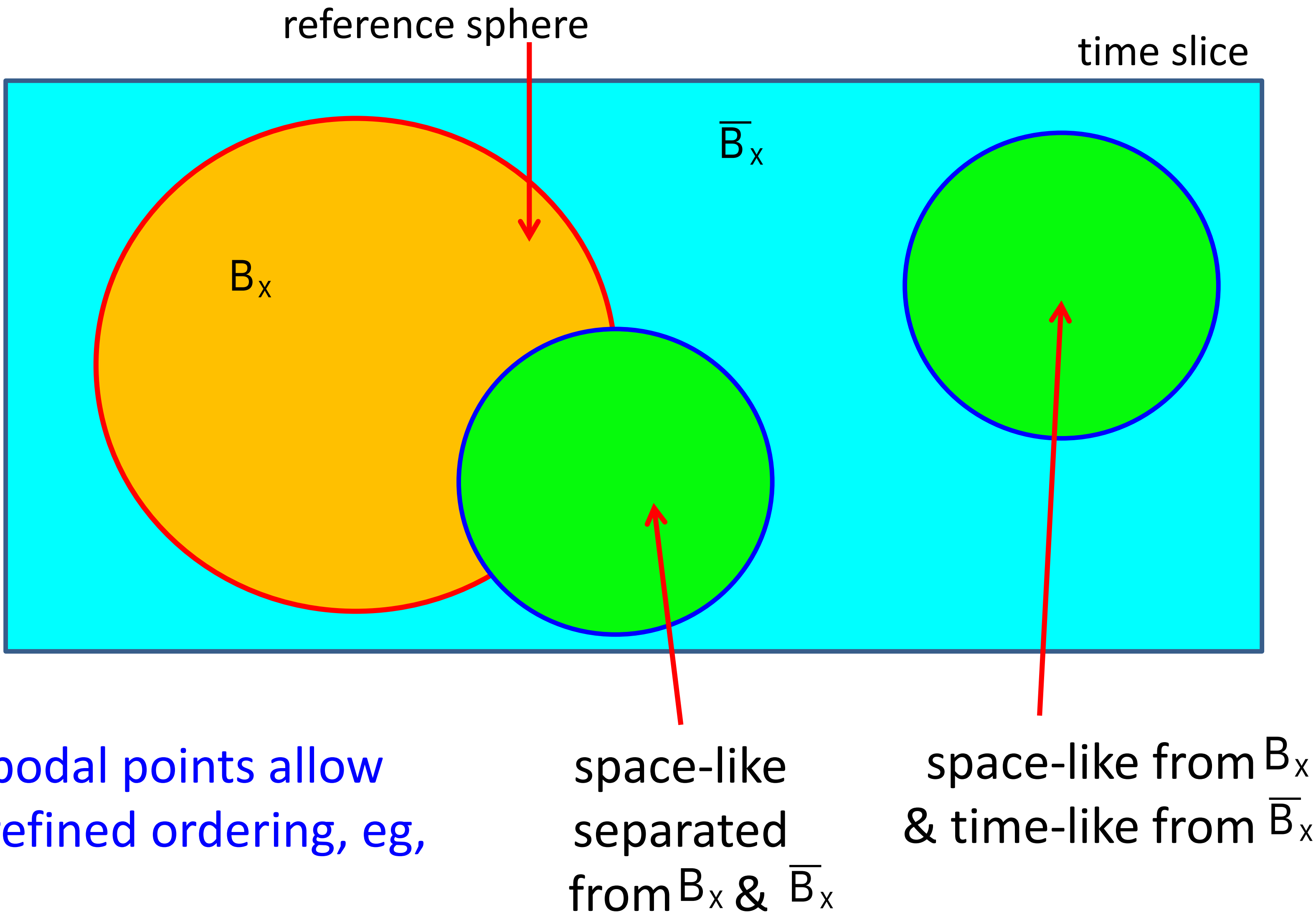


- antipodal points allow for refined ordering, eg,

Antipodal symmetry:

- in pure state, $S(V) = S(\bar{V})$ $\longrightarrow$ $\delta S(B) = \delta S(\bar{B})$
- on auxiliary dS geometry, **antipodally even** solutions: $\delta S(x) = \delta S(\bar{x})$
- antipodal symmetry restricts allowed boundary data:

$$\frac{2\pi^{\frac{d-1}{2}}}{-\Gamma\left(\frac{d+3}{2}\right)}\cdot F(\vec{x}) = \int_{I^+} d^{d-1}y\, |\vec{x}-\vec{y}|^2\, f(\vec{y})$$

- however, recall: $F(\vec{x}) = 0\,;\quad f(\vec{x}) \propto \langle T_{tt}(\vec{x})\rangle$

$$\longrightarrow \quad \int d^{d-1}y\, |\vec{x}-\vec{y}|^2 \langle T_{tt}(\vec{y})\rangle = 0 \quad \text{for all } \vec{x}$$

- independent constraints:

$$\int d^{d-1}x\, \langle T_{tt}(\vec{x})\rangle = 0 = \int d^{d-1}x\, \vec{x}\, \langle T_{tt}(\vec{x})\rangle = \int d^{d-1}x\, |\vec{x}|^2 \langle T_{tt}(\vec{x})\rangle$$

total energy | boost gen's | temporal gen of special conformal trans

Antipodal symmetry:

- on auxiliary dS geometry, **antipodally even** solutions: $\pm S(x) = \pm S(\bar{x})$
- independent constraints:

$$d^{d-1}x\, \langle T_{tt}(\vec{x})\rangle = 0 = \quad d^{d-1}x\, \vec{x}\, \langle T_{tt}(\vec{x})\rangle = \quad d^{d-1}x\, |\vec{x}|^2\, \langle T_{tt}(\vec{x})\rangle$$

total energy — boost gen's — temporal gen of special conformal trans

- vanishing total energy? ⟶ only to first order in expansion
- some mixed states will also satisfy antipodal symmetry
- on spherical time slice:

$$d^{d-1}n\, \langle T_{tt}(n)\rangle = 0 = \quad d^{d-1}n\; n^I\, \langle T_{tt}(n)\rangle$$

where S^{d-1} parameterized by d-dimensional unit vectors n^I

⟶ zero'th and first moments of energy density vanish

Recall example:

- consider state: $|A\rangle = |0\rangle + \varepsilon\, T_{tt}(t_0 + i\tau, \vec{x}_0)|0\rangle$

small expansion parameter → ε ; τ → regulate UV divergences

- expectation value is fixed by 2-pt function $\langle 0|T_{tt}(t,\vec{x})\, T_{tt}(0,\vec{0})|0\rangle$

$$\langle \psi|T_{tt}(t,x)|\psi\rangle = \varepsilon\, C_T \frac{1}{(\Delta x^2 - (\Delta t + i\tau)^2)^d} \frac{(\Delta x^2 + (\Delta t + i\tau)^2)^2}{(\Delta x^2 - (\Delta t + i\tau)^2)^2} - \frac{1}{d} + c.c. + O(\varepsilon^2)$$

with $\Delta x^2 = |\vec{x} - \vec{x}_0|^2$ and $\Delta t^2 = |t - t_0|^2$

- can verify above satisfies constraints ⟶ δS antipodally symmetric

$$d^{d-1}x\, \langle T_{tt}(\vec{x})\rangle = 0 = d^{d-1}x\, \vec{x}\, \langle T_{tt}(\vec{x})\rangle = d^{d-1}x\, |\vec{x}|^2 \langle T_{tt}(\vec{x})\rangle$$

- for any pure state $|\psi\rangle = |0\rangle + \epsilon|\phi\rangle$ ⟶ $\langle T_{tt}(\vec{x})\rangle \simeq \varepsilon \langle 0|T_{tt}(\vec{x})|A\rangle + h.c.$

⟶ constraints satisfied by conformal invariance of $|0\rangle$

Second example:

- consider mixed state on spherical time slice (with radius r):

$$\rho = |0\rangle\langle 0| + \eta\, |E\rangle\langle E|$$

small expansion parameter (→ η)

energy eigenstate with constant $\langle T_{tt}(x)\rangle$ (→ $|E\rangle$)

- for ball of angular width θ_0, "1st law" becomes: (Herzog)

$$\delta S = 2\pi \int_0^{\theta_0} \underbrace{r^{d-1}\Omega_{d-2}\sin^{d-2}\theta\, d\theta}_{\text{volume element}}\; \underbrace{r\,\frac{\cos\theta - \cos\theta_0}{\sin\theta_0}}_{\text{bulk-to-bdry propagator}}\; \underbrace{\frac{\eta\, E}{r^{d-1}\Omega_{d-1}}}_{\text{energy density}}$$

- "antipodal constraints": $\int d^{d-1}n\, \langle T_{tt}(n)\rangle \neq 0\,;\qquad \int d^{d-1}n\; n^I \langle T_{tt}(n)\rangle = 0$

- scale set by θ_0; problem at $\theta_0 = \pi$! (where bulk pts reach I ')

⟶ dS propagation breaks down where $\eta\, r\, E \sim \theta_0$

Extension to Higher Spin Charges:

- CFT with conserved symmetric traceless currents $T_{\mu_1\cdots\mu_s}$ with $s \geq 1$
- modular Hamiltonian is flux of $J_\mu^{(2)} = T_{\mu\nu} K^\nu$ through B where K^ν is conformal Killing vector that leaves ∂B invariant

$$\longrightarrow \quad H_B = \int d\Sigma^\mu \, J_\mu^{(2)}$$

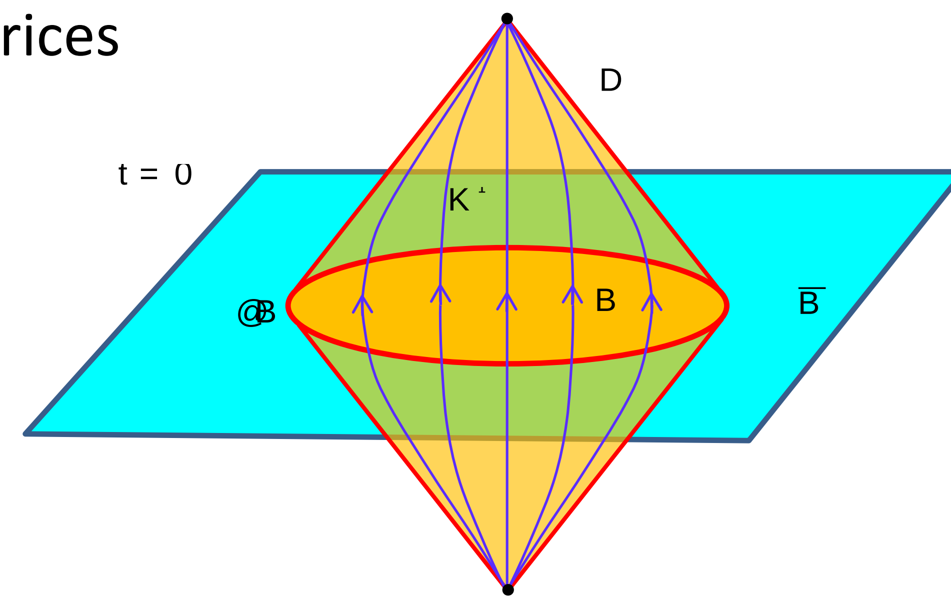
- extends to higher spin charges:

$$Q^{(s)} = \int d\Sigma^\mu \, J_\mu^{(s)} \quad \text{with} \quad J_\mu^{(s)} = T_{\mu\mu_2\cdots\mu_s} K^{\mu_2} \cdots K^{\mu_s}$$

- appeared in modified density matrices

$$\rho_B \sim \exp\left(-\sum \mu_s Q^{(s)}\right)$$

(s=1: Belin, Hung etal;
s≥3: Hijano & Kraus)



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$$Q^{(s)} = \int d\Sigma^{\mu} J_{\mu}^{(s)} \quad \text{with} \quad J_{\mu}^{(s)} = T_{\mu \mu_2 \cdots \mu_s} K^{\mu_2} \cdots K^{\mu_s}$$

- on *t=0* slice, yields:

$$Q^{(s)} = (2\pi)^{s-1} \int_B d^{d-1}y \left(\frac{R^2 - |\vec{x} - \vec{y}|^2}{2R} \right)^{s-1} T_{tt\ldots t}(\vec{y})$$

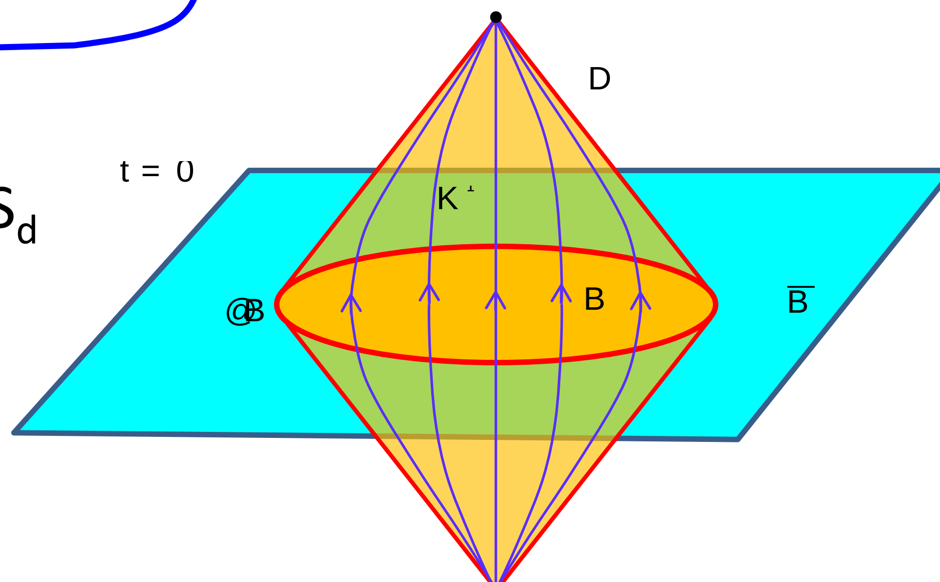
bdry-to-bulk propagator for deSitter

- $Q^{(s)}$ satisfies wave equation in dS_d

$$\left(\nabla_{dS}^2 - m^2 \right) Q^{(s)} = 0$$

with

$$m^2 L^2 = -(s-1)(d+s-2)$$



Conclusions & Outlook:

- EE of excitations of CFT vacuum arranged in novel holographic manner
- δS satisfies wave equation in dS_d where scale plays the role of time

$$\left(\nabla^2_{dS} - m^2\right)\delta S = 0 \quad \text{with} \quad m^2 L^2 = -d$$

- $\langle T_{tt} \rangle$ sets δS at very small R and EE perturbations at larger scales determined by the local Lorentzian propagation into dS geometry
- for CFTs with higher spin currents, additional dynamical fields on dS_d

⟶ applies for any CFT in any d; relies only on the 1st law of entanglement; does **not** require strong coupling or large # dof

Question: Is there a full description of CFT in terms of a local theory of interacting fields propagating in dS spacetime?

(dS/CFT correspondence with unitary boundary CFT?)

Question: How is curvature scale in dS geometry fixed?

- in AdS/CFT, AdS scale set by coupling to gravity, ie, $\left({}^{L}/_{\ell_P}\right)^{d-2} \sim C_T$

⟶ need to understand dynamics of dS geometry(?)

Question: How to new construction extend beyond CFT vacuum?

- how is holographic geometry modified for perturbations of EE around excited states?
- how is holographic geometry modified for perturbations of EE around CFT deformed by relevant operator?

⟶ in AdS/CFT, wave equation acquires source terms

' r $^{2}_{\text{dS}}$ i m^{2} ˇ ±S ' \ hOi hOi "

Bhattacharya, Takayanagi: arXiv:1308.3792

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→ integral geometry proposal has some answers for d=2

(Czech, Lamprou, McCandlish & Sully: arXiv:1505.05515)

Comment:

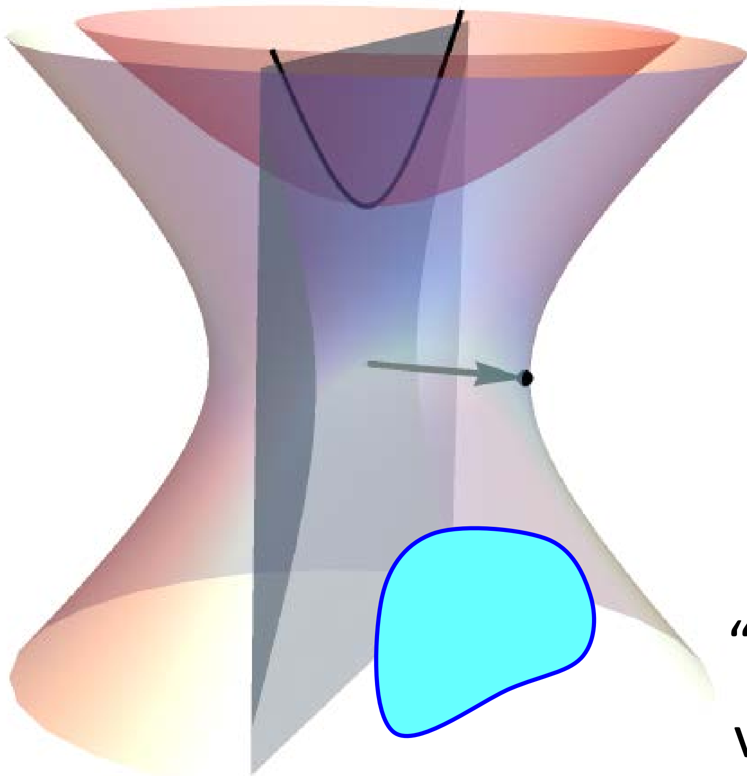
- deSitter geometry appears in recent discussions of integral geometry and the interpretation of MERA in terms of AdS_3/CFT_2

(Czech, Lamprou, McCandlish & Sully: arXiv:1505.05515)

- consider space of intervals u<x<v on time slice of 2d CFT

⟷ space of geodesics on 2d slice of AdS_3 ⟷ pts in 2d de Sitter

AdS/CFT



$$ds^2 = L^2 \frac{du\,dv}{(v-u)^2}$$

dS scale? ↗

motivate the choice: $L^2 = \frac{c}{3}$

⟶ $ds^2 = \partial_u \partial_v S_0 \, du\,dv$

with $S_0 = \frac{c}{3} \log \frac{v-u}{\delta}$

"hole-ography":

volume in dS_2 = length in AdS_3 slice

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CFT vacuum: $L^2 = c{=}3\,;$ excited states: $ds^2 = @_u @_v S\, du\, dv$

⟶ motivated by connections to "hole-ography"

(Czech, Lamprou, McCandlish & Sully: arXiv:1505.05515)

Question: Interacting fields in dS spacetime?

- description with free field propagation breaks down for large/IR scales in mixed state example

→ need to go beyond 1^{st} law!!

Second example:

- consider mixed state on spherical time slice (with radius r):

$$\rho = |0\rangle\langle 0| + \eta\, |E\rangle\langle E|$$

small expansion parameter → η; energy eigenstate with constant $\langle T_{tt}(x)\rangle$ → $|E\rangle$

- for ball of angular width θ_0, "1st law" becomes:

$$\delta S = 2\pi \int_0^{\theta_0} \underbrace{r^{d-1}\,\Omega_{d-2}\,\sin^{d-2}\theta\, d\theta}_{\text{volume element}}\;\; \underbrace{r\,\frac{\cos\theta - \cos\theta_0}{\sin\theta_0}}_{\text{bulk-to-bdry propagator}}\;\; \underbrace{\frac{\eta\, E}{r^{d-1}\,\Omega_{d-1}}}_{\text{energy density}}$$

- "antipodal constraints": $d^{d-1}n\, \langle T_{tt}(n)\rangle \neq 0\,;\qquad d^{d-1}n\; n^i\, \langle T_{tt}(n)\rangle = 0$

- scale set by θ_0; problem at $\theta_0 = \pi$! (where bulk pts reach $\mathcal{I}^+$)

→ dS propagation breaks down where $\eta\, r\, E \sim \theta_0$

Question: Interacting fields in dS spacetime?

- description with free field propagation breaks down for large/IR scales in mixed state example

  → need to go beyond 1st law!!

- preliminary investigation of HEE in AdS_3/CFT_2 including terms which are second order in the stress tensor, find interacting equation:

$$\left(\nabla^2_{dS} - m^2\right)\delta S = g\,\delta S^2 + \cdots$$

(with van Raamsdonk)

- do higher spin charges interact in local way in dS geometry?

Question: What about time dependence in CFT?

- so far focused on single time slice; natural to consider perturbations of EE for all spheres throughout spacetime on any time slice & any frame

- adopt group theoretic perspective of wave equation:

→ background for spheres on fixed time slice:

$$SO(1,d)/SO(1,d-1) \simeq \quad \text{d-dim. deSitter space}$$

→ background for spheres throughout spacetime:

$$SO(2,d)/[SO(1,d-1) \times SO(1,1)]$$

→ $2d$-dimensional space

→ signature: (d,d) ← too many times?!?!

(seek guidance from AdS/CFT)

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Lots to explore!!