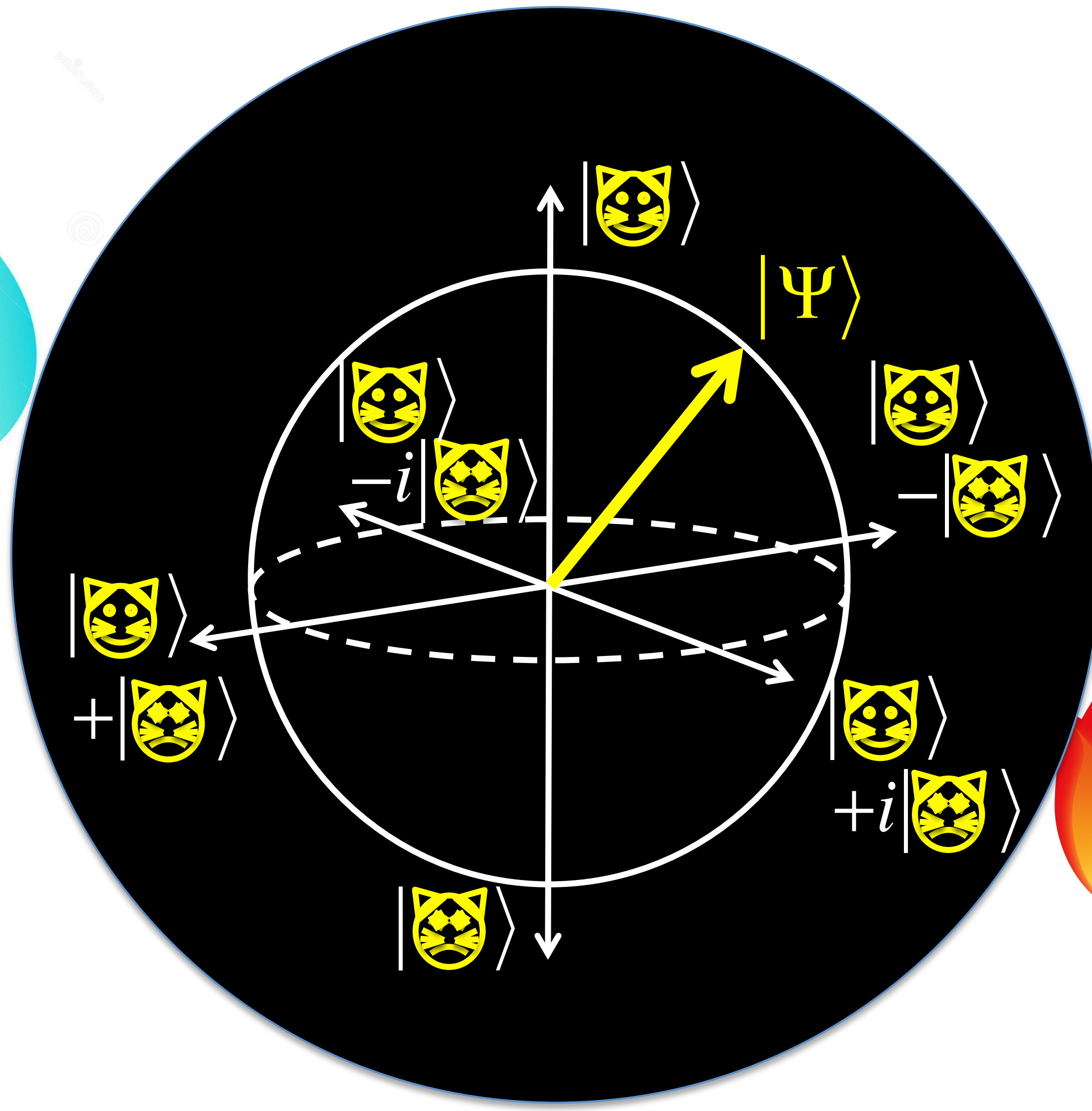
Black Hole Information:
From Thermodynamics to Firewalls



SPRINGER BRIEFS IN PHYSICS

Robert B. Mann

Black Holes: Thermodynamics, Information, and Firewalls

Springer

A Brief History of Black Hole Information

- 1783: John Michell proposes the idea of a "dark star"
- 1916: General Relativity Formulated
- 1917: Schwarzschild solution obtained
- 1930: Chandresekar computes upper bounds for masses to avoid gravitational collapse
- 1939: Oppenheimer-Snyder collapse solution yields "frozen stars"
- 1962: First definition of gravitational energy (ADM)
- 1967: 'Black Hole' applied to Schwarzschild solution (Wheeler)
- 1972: Beckenstein points out area/entropy relationship
- 1974: Hawking establishes BH temperature
- 1974: Laws of gravitational thermodynamics ←→ Laws of BH Mechanics
- 1976: Unruh effect discovered
- 1977: Gibbons-Hawking effect – causal horizons have temperature

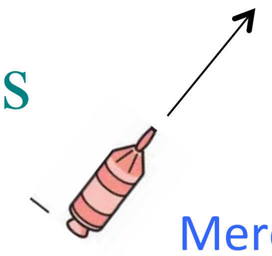
A Brief History of Black Hole Information

- 1982: AdS-BH Phase transitions
- 1983: Hartle-Hawking no-boundary proposal
- 1984: Brick-wall Model
- 1992: 2D black hole radiation with back-reaction
- 1993: BHEntropy as Noether Charge of Diffeomorphisms; Quasi-local Methods developed; Black Hole Complementarity Proposed
- 1994: BH pair-production rates calculated
- 1996: Theoretical arguments given for counting black hole states
- 2004: BH radiation understood as tunneling
- 2005: Entanglement found to be observer-dependent
- 2009: Pressure-Volume Terms introduced into BH Thermodynamics
- 2012: Firewall Paradox
- 2013: Re-entrant Phase Transitions, BH Triple-points discovered

What is a Black Hole?

How fast must a rocket be launched to fully escape earth's gravity?

$$v = 11.2 \text{ km/s}$$



Sun - 617.7 km/s - 55x that of Earth

Mercury - 4.25 km/s

Venus - 10.46 km/s

Earth - 11.186 km/s

Moon - 2.38 km/s

Mars - 5.027 km/s

Jupiter - 59.5 km/s

Saturn - 35.5 km/s

Uranus - 21.3 km/s

Neptune - 23.5 km/s

Pluto - 1.27 km/s

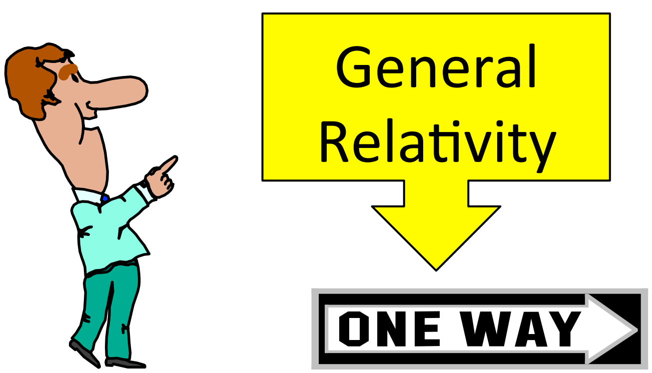
Q: What would a star look like if its escape speed was faster than light could travel?

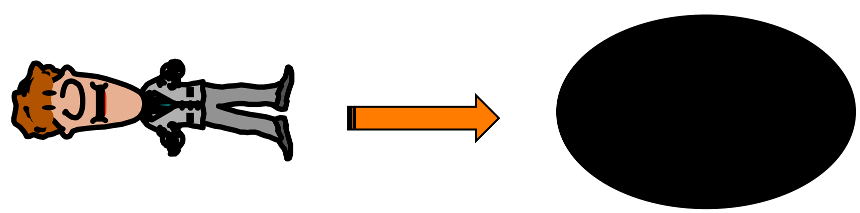
A: It would be dark – light wouldn't shine from it because it couldn't escape

Dark Stars → Black Holes

If the semi-diameter of a sphaere of the same density with the sun were to exceed that of the sun in the proportion 500 to 1, a body falling from an infinite height towards it, would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its vis inertiae, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity.

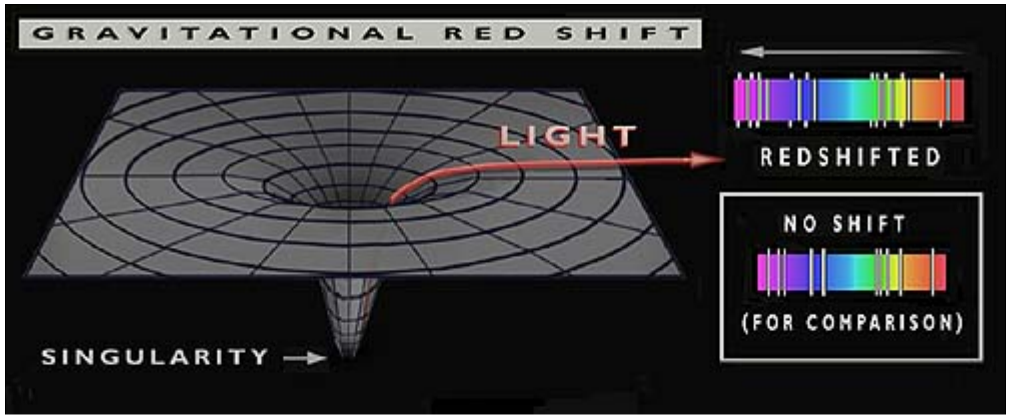
Rev. John Michell (1724-1793)





Dark Star (1783): A region of space for which the escape velocity is greater than the speed of light

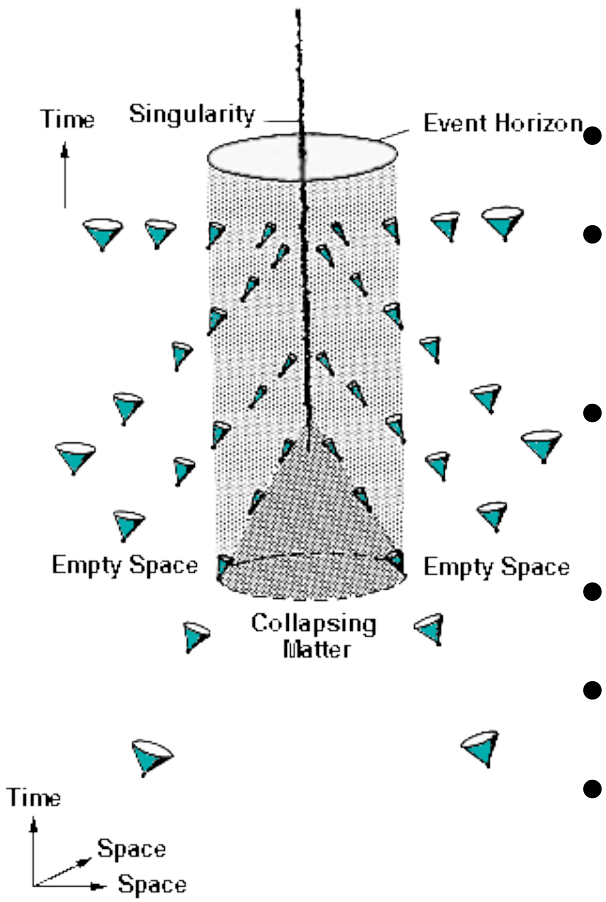
Information can be Trapped



- Black Hole (1783): A region of space for which the escape velocity is greater than the speed of light
- Black Hole (2015): A region of space bounded by a trapped surface, for which both ingoing & outgoing light rays have negative expansion

Information is trapped

The Strange Properties of Black Holes



- Inevitable result of gravitational collapse (gravity always wins!)
- Can be mined for energy (if they spin)
- A singularity at the core where time and space no longer exist
- Can be produced in pairs in the early universe
- Behave as thermodynamic objects
- A one-way flow of classical information
- A paradox for quantum information

The 4 Laws of Black Hole Mechanics

Bardeen/Carter/Hawking
CMP **31** (1973) 161

- 0th Law $\kappa = \text{constant}$
  - surface Gravity is constant over the event horizon
- 1st Law $dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ + \cdots$
  - differences in mass between nearby solutions are equal to differences in area times the surface gravity plus additional work terms
- 2nd Law $dA \geq 0$

  Bekenstein PRD **7** (1973) 2333
  - area of the event horizon never decreases in any physical process
- 3rd law $\kappa_n > \kappa_{n+1} > 0 \quad n < \infty$

  Israel PRL **57** (1986) 397
  - No procedure can reduce the surface gravity to 0 in a finite number of steps

Black Hole Thermodynamics

Thermodynamics	Gravity

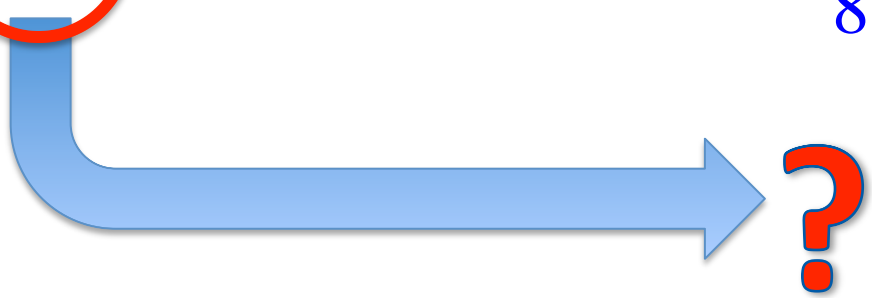
$$\text{Energy} \quad E \leftrightarrow M \quad \text{Mass}$$

$$\text{Temperature} \quad T \leftrightarrow \frac{\hbar\kappa}{2\pi} \quad \text{Surface gravity}$$

$$\text{Entropy} \quad S \leftrightarrow \frac{A}{4\hbar} \quad \text{Horizon Area}$$

$$dE = TdS + VdP + \text{work terms} \leftrightarrow dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ$$

First Law	First Law



Smarr Formula

L. Smarr PRL 30, 71 (1973)
[Err. 30, 521 (1973)].

$$ds^2 = -Vdt^2 + \frac{dr^2}{V} + r^2 d\Omega_2^2$$

Schwarzschild Black hole $V = 1 - \frac{2M}{r}$

$$E = M = \frac{r_+}{2} \qquad T = \frac{1}{4\pi r_+} \qquad S = \pi r_+^2 \quad \Rightarrow \quad M = 2TS$$

Smarr ✓

Schwarzschild-AdS Black hole $V = 1 - \frac{2M}{r} + \frac{r^2}{l^2}$

$$E = M = \frac{l^2 + r_+^2}{2l^2} r_+ \qquad T = \frac{l^2 + 3r_+^2}{4\pi r_+ l^2}$$

$$S = \pi r_+^2 \quad \Rightarrow \quad M \neq 2TS$$

~~Smarr~~ ?

Scaling Arguments

Caldarelli/Cognola/Klemm, CQG 17, 399 (2000)

Suppose

$$f\left(\alpha^p x, \alpha^q y\right) = \alpha^r f\left(x, y\right) \Rightarrow r f\left(x, y\right) = p \frac{\partial f}{\partial x} x + q \frac{\partial f}{\partial y} y$$

S-AdS Black Hole $\quad M \propto L^{D-3} \quad A \propto L^{D-2} \quad \Lambda \propto L^{-2}$

$$M = M\left(A, \Lambda\right) \Rightarrow \left(D-3\right) M = \left(D-2\right) \frac{\partial M}{\partial A} A - 2 \frac{\partial M}{\partial \Lambda} \Lambda$$

$$S = \frac{A}{4G} \qquad T = \frac{\kappa}{2\pi} = 4G \frac{\partial M}{\partial A} \qquad P = -\frac{\Lambda}{8\pi} = \frac{\left(D-2\right)\left(D-1\right)}{16\pi l^2}$$

$$\Rightarrow M = \frac{\left(D-2\right)}{\left(D-3\right)} TS - \frac{2}{\left(D-3\right)} VP \qquad V = -8\pi \frac{\partial M}{\partial \Lambda}$$

Pressure from the Vacuum?

J. Creighton and R.B. Mann, PRD 53 (1995) 4569
T. Padmanabhan, CQG 19 (2002) 5387
Dolan CQG 28 (2011) 125020; 235017

Schwarzschild-AdS Black hole

$$E = M = \frac{l^2 + r_+^2}{2l^2} r_+ \qquad T = \frac{l^2 + 3r_+^2}{4\pi r_+ l^2} \qquad S = \pi r_+^2 \qquad (D = 4)$$

$$M = 2(TS - VP)$$

Smarr

$$dE = TdS + VdP$$

First Law

Provided

$$P = -\frac{1}{8\pi}\Lambda = \frac{3}{8\pi}\frac{1}{l^2}$$

Thermodynamic Pressure

$$V = -8\pi \frac{\partial M}{\partial \Lambda} = \frac{4\pi}{3} r_+^3$$

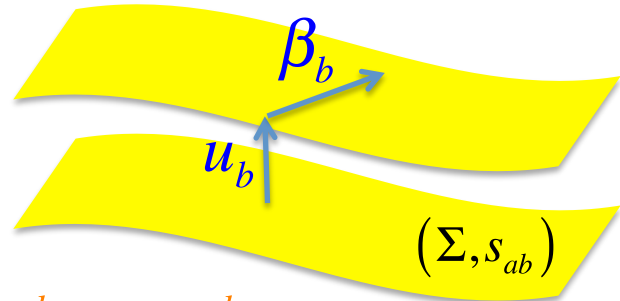
Thermodynamic Volume

Black Hole Mechanics Revisited

$$g_{ab} = s_{ab} - u_a u_b$$

$$\xi^a = N u^a + \beta^a$$

$$N = -\xi \bullet u$$



$$\pi^{ab} = -\sqrt{s}\,(K^{ab} - K s^{ab})$$

Constraint Equations

$$H \equiv -2G_{ab}u^a u^b = -R^{(D-1)} + \frac{1}{|s|}\left(\frac{\pi^2}{D-2} - \pi^{ab}\pi_{ab}\right) = -2\Lambda$$

$$H_b \equiv -2G_{ac}u^a s^c_b = -2D_a\left(|s|^{-\frac{1}{2}}\pi^{ab}\right) = 0$$

Kastor/Ray/Traschen
CQG 26 195011
(2009)

$$D_a B^a = N\delta H + \beta^a \delta H_a = -2N\delta\Lambda \Rightarrow D_a\left(B^a - 2\delta\Lambda\omega^{ab}u_b\right) = 0$$

$$B^a[\xi] = N(D^a\delta s - D_b\delta s^{ab}) - \delta s D^a N + \delta s^{ab} D_b N$$
$$+ \frac{1}{\sqrt{|s|}}\beta^b\left(\pi^{cd}\delta s_{cd} s^a_b - 2\pi^{ac}\delta s_{bc} - 2\delta\pi^a_b\right)$$

$$\xi^b = \nabla_c \omega^{cb}$$

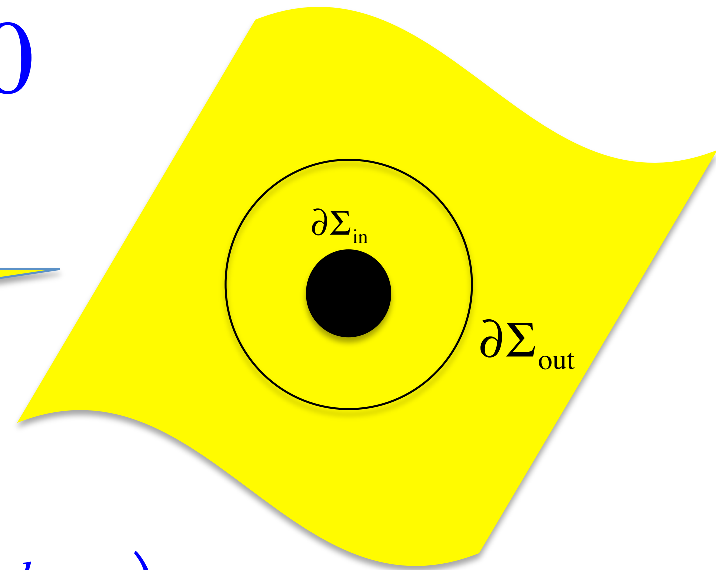
Killing Potential

$$D_a\left(B^a - 2\delta\Lambda\omega^{ab}u_b\right) = 0$$

$$\int_\Sigma dV u^a D_a\left(B^a - 2\delta\Lambda\omega^{ab}u_b\right) = 0$$

$$\int_{\partial\Sigma_{out}} dS r_c \left(B^c[\xi] - 2\delta\Lambda\omega^{cb}u_b\right)$$

$$= \int_{\partial\Sigma_{in}} dS r_c \left(B^c[\xi] - 2\delta\Lambda\omega^{cb}u_b\right)$$



$$16\pi\delta M \equiv -\int_\infty da r_c B^c[\partial/\partial t] \quad 16\pi\delta J^i \equiv \int_\infty da r_c B^c[\partial/\partial\varphi^i]$$

$$\xi = \partial/\partial t + \Omega^i \partial/\partial\varphi^i$$

$$\delta M = T_h \delta S_h + \sum_i (\Omega_h^i - \Omega_\infty^i)\delta J^i + V_h \delta P$$

$$V_h = \int_\infty dS r_c u_d (\omega^{cd} - \omega_{AdS}^{cd}) - \int_{BH} dS r_c u_d \omega^{cd}$$

Thermodynamic Volume

$$V_h = \int_\infty dS r_c u_d (\omega^{cd} - \omega^{cd}_{AdS}) - \int_{BH} dS r_c u_d \omega^{cd}$$

First Law

$$\delta M = T_h \delta S_h + \sum_i (\Omega^i_h - \Omega^i_\infty)\delta J^i + V_h \delta P$$

Integrate

Smarr Relation

$$\frac{D-3}{D-2} M = T_h S_h + \sum_i (\Omega^i_h - \Omega^i_\infty) J^i - \frac{2}{D-2} P V_h$$

D-dim'l Schwarzschild-AdS Black hole

$$ds^2 = -V dt^2 + \frac{dr^2}{V} + r^2 d\Omega^2_{D-2} \qquad V = 1 - \frac{\tilde{M}}{r^{D-3}} + \frac{r^2}{l^2} \qquad \omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}$$

$$M = (D-2)\omega_{D-2} \frac{l^2 + r_+^2}{16\pi l^2} r_+^{D-3} \quad S = \frac{\omega_{D-2}}{4} r_+^{D-2} \quad J^i = 0$$

First Law ✓

Smarr ✓

$$T = \frac{(D-3)l^2 + (D-1)r_+^2}{4\pi r_+ l^2} \quad P = \frac{(D-2)(D-1)}{16\pi l^2} \quad V = \frac{\omega_{D-2} r_+^{D-1}}{(D-1)}$$

The Chemistry of Black Holes

Include gauge charges:

First Law

$$\delta M = T_h \delta S_h + \sum_i (\Omega_h^i - \Omega_\infty^i)\delta J^i + \Phi_h \delta Q + V_h \delta P$$

Smarr Relation

$$\frac{D-3}{D-2} M = T_h S_h + \sum_i (\Omega_h^i - \Omega_\infty^i) J^i + \frac{D-3}{D-2}\Phi_h Q - \frac{2}{D-2} P V_h$$

Thermodynamic Potential: Gibbs Free Energy

$$G = M - TS = G(T, P, J_i, Q)$$

Kubiznak/Mann
JHEP **1207** (2012) 033

- Equilibrium: Global minimum of Gibbs Free Energy
- Local Stability: Positivity of the Specific Heat

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_{P, J_i, Q} > 0$$

Mass as Enthalpy

Thermodynamics | Gravity

Enthalpy $H \leftrightarrow M$ Mass

Temperature $T \leftrightarrow \frac{\hbar\kappa}{2\pi}$ Surface gravity

Entropy $S \leftrightarrow \frac{A}{4\hbar}$ Horizon Area

$$dH = TdS + VdP + \cdots \leftrightarrow dM = \frac{\kappa}{8\pi} dA + VdP + \cdots$$

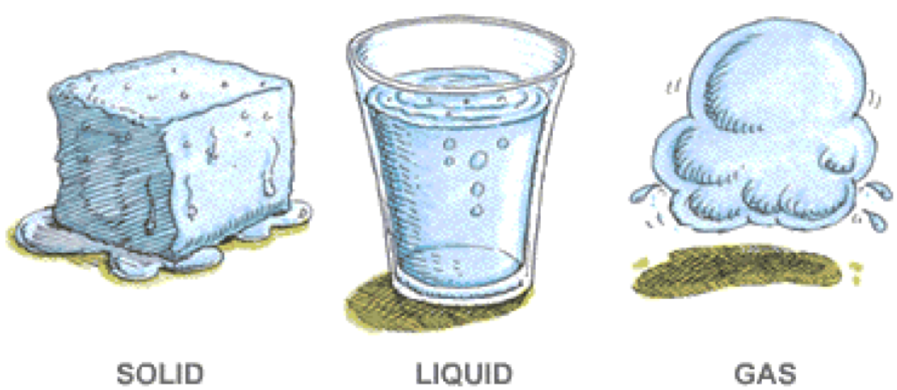
First Law | First Law

$$H = E + PV + \cdots \leftrightarrow M = E - \rho V$$

Mass
= Total Energy
– Vacuum
Contribution
(infinite)

Everyday AdS Black Hole Thermodynamics

- Hawking Page Transition
- Van der Waals Fluid and Charged AdS Black Holes
- Reentrant Phase Transitions
- Black Hole Triple Points ←→ Solid/Liquid/Gas



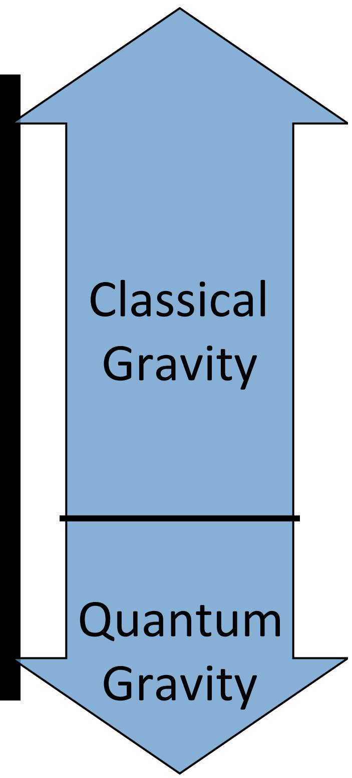
Altamirano/Kubiznak/
Mann/Sherkatgenad
Galaxies **2** (2014) 89

Kubiznak/Mann
CJP **93** (2015) 999

Hot Black Holes?

- Semi-classical QFT in curved spacetime indicates that black holes behave like hot objects that are maximally disordered (they have maximal entropy)
- Temperature increases with decreasing mass

Milky Way BH: $T = 1.43\times10^{-14}\ {}^{\circ}K$	$R = 1.27\times10^{7}$ km
Sun: $T = 6.18\times10^{-8}\ {}^{\circ}K$	$R = 2.948$ km
Mercury : $T = 2.57\ {}^{\circ}K$	$R = .049$ cm
Saturn's Rhea: $T = 330\ {}^{\circ}K$	$R = 3.43\ \mu$m
Mt. Everest: $T = 7.70\times10^{8}\ {}^{\circ}K$	$R = 1.5\times10^{-12}$ m
Proton: $T = 4.61\times10^{50}\ {}^{\circ}K$	$R = 2.5\times10^{-54}$ m



The Arrow of Time

Why is the entropy of the present day universe so low?

$S_{\text{matter}} = 10^{88}$ ← Early universe (all known matter)

$S_{\text{Milky Way BH}} = 10^{90}$ ← Galactic Black Hole

$S_{\text{observable U}} = 10^{100}$ ← Known universe 10^{11} Black Holes

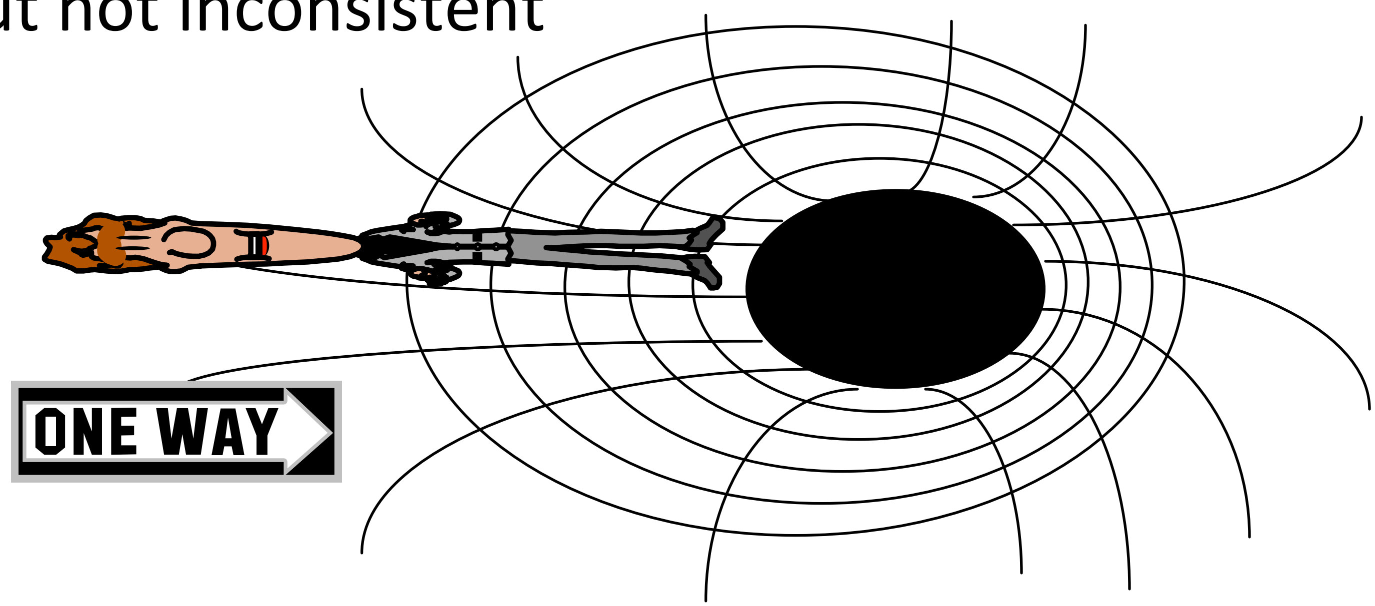
$S_{\max} = 10^{122}$ ← Entire Universe is a Black Hole

Is this the cosmological arrow of time?

The Black Hole Information Conundrum

Classical Black Holes

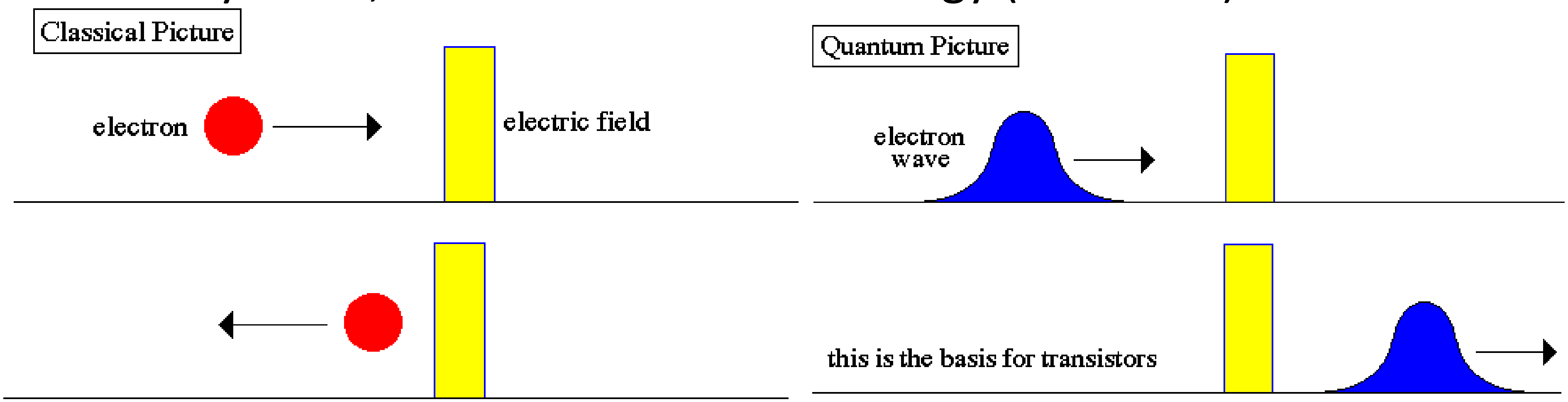
- escape velocity greater than the speed of light
- infinite redshift of light emitted from collapsing object
- typically contain a spacetime singularity
- all information absorbed -- nothing emitted
- peculiar, but not inconsistent



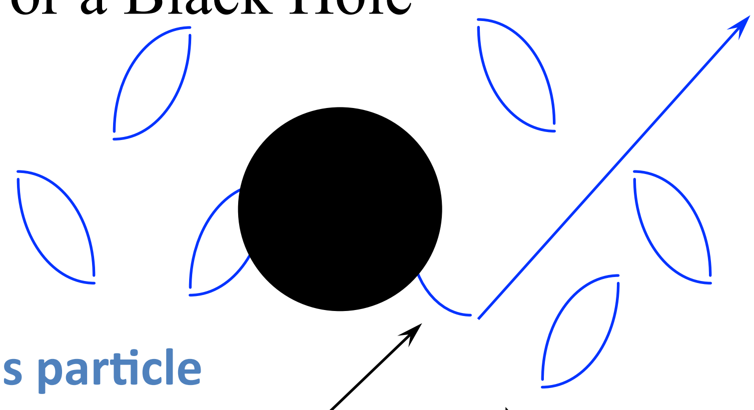
Quantum Black Holes

Parikh/Wilczek
PRL **85** (2000) 5042
Vanzo et.al.
JHEP **0505** (2005) 014
Kerner/Mann
PRD **73** (2006) 104010

- Quantum effects permit particles to tunnel out of the gravitational potential well
- as they do so, the black hole loses energy (and mass)



For a Black Hole



this particle escapes out to infinity causing the black hole to lose energy

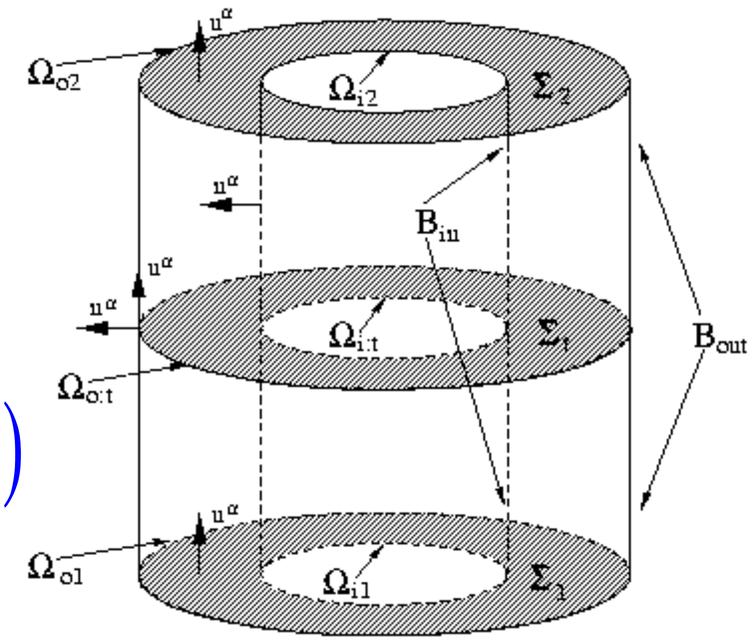
Quantum particles tunnel out of the horizon

Kerner/Mann
CQG **25** (2008) 095014

this particle falls in the hole

A black hole is a hot particle radiator!

Entropy from Semi-Classical Quantum Gravity



Consider an ensemble of Euclidean spacetimes of the form $t \rightarrow i\tau$

$$ds^2 = N^2 d\tau^2 + h_{ij}\left(dx^i + V^i d\tau\right)\left(dx^j + V^j d\tau\right)$$

Partition Function

$$Z = Tr\left[e^{-\beta H}\right]$$

Must integrate over all metrics and matter fields satisfying the requisite Euclidean periodicity conditions at infinity

Path Integration

$$Z = \int D[g]D[\Psi]\exp[-I(g,\Psi)]$$

$$\approx \exp[-I_{\text{cl}}]$$

Thermodynamics

$$\log Z = S - \beta H_\infty$$

$$S = \beta H_\infty - I_{\text{cl}}$$

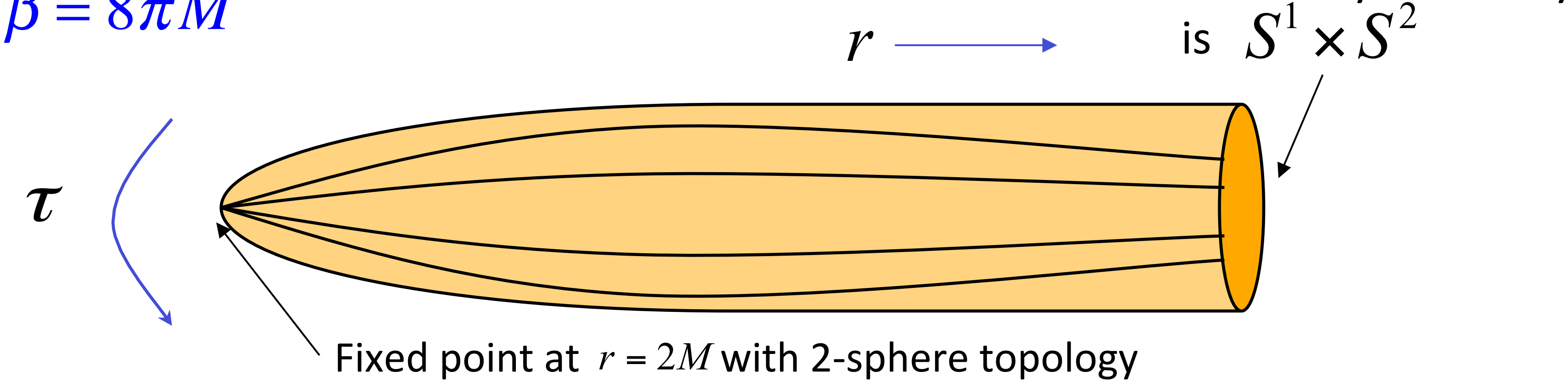
Gravitational Entropy

e.g. Schwarzchild

$t \to i\tau$

$$ds^2 = (1 - \tfrac{2M}{r})d\tau^2 + \frac{dr^2}{(1 - \tfrac{2M}{r})} + r^2 d\Omega^2$$

$$\Delta\tau = \beta = 8\pi M$$



$$E_{\text{schw}} = -R + 2M + O(\frac{M^2}{R}) \qquad E_{\text{flat}} = -R + M + O(\frac{M^2}{R})$$

$$I_{\text{schw}} = 8\pi M\left(\frac{3}{2}M - R + O(\frac{M^2}{R})\right) \qquad I_{\text{flat}} = 8\pi M\left(M - R + O(\frac{M^2}{R})\right)$$

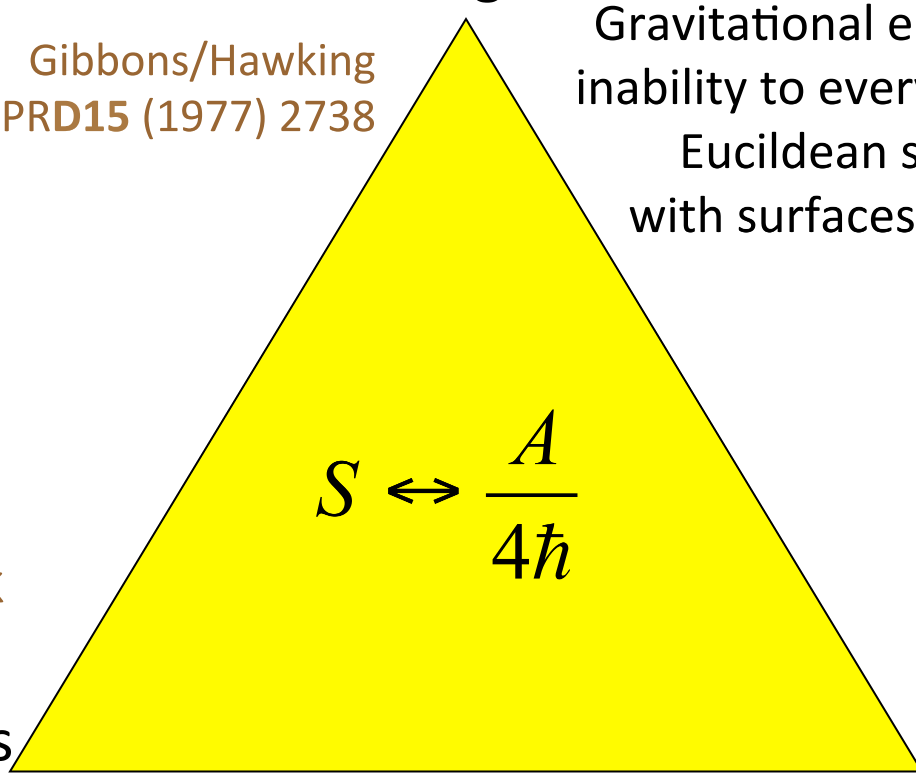
$$\Delta S = \beta\Delta E - \Delta I = 4\pi M^2$$

Gibbons/Hawking

Euclidean
Path-Integration

Gibbons/Hawking
PR**D15** (1977) 2738

Gravitational entropy due to
inability to everywhere foliate
Eucildean spacetime
with surfaces of constant τ



$$S \leftrightarrow \frac{A}{4\hbar}$$

Brown/York

Quasilocal
Thermodynamics

Gravitational entropy is the difference
between the total energy and the
ree energy divided by the temperature

Brown/York PR**D47** (1993) 1407

Wald/Francaviglia

Noether Charge

Gravitational entropy is the
Noether Charge of diffeomorphisms

Wald PR**D48** (1993) 3427
Fatibene/Ferraris/Francaviglia
JMP **35** (1994) 1644

Is Gravitational Entropy "real"?

- Is entropy ← → area a coincidence? Or is it actually related to some underlying degrees of freedom?
- Consider pair production: vacuum energy can be unstable to pair production of black holes
  - negative potential energy of created pair balances their positive rest-mass energy
  - Background field provides necessary force to accelerate the black holes
- Various sources have been explored:

  constant electromagnetic field

  cosmological vacuum energy

  cosmic strings, domain walls

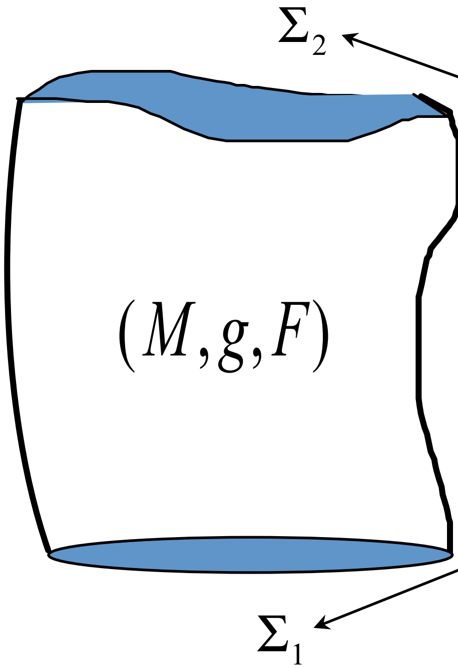
  Booth Bousso Brown Caldwell Chamblin Dowker Eardley Emparan Garfinkle Gauntlett Gibbons Giddings Hawking Horowitz Kastor Mann Ross Strominger Traschen Wu
- Number of states ~ Production rate ~ exp(Entropy)

- 3 stage procedure
  - Find appropriate solution to Einstein-Maxwell eqns
    - charged/rotating black hole pair
    - Cosmological C-metric with rotation ⟶ KNdS metric
  - Construct appropriate instantons that mediate the creation process
  - Calcuate the instanton action to obtain the production rate $P \propto \exp(-2I_i)$

In all cases: $I_{bh} = -\sum_{horizons} \frac{\mathcal{A}_h}{8}$ $\quad$ $\mathbf{P}_{\text{relative}} = \exp\left(2I_{dS} - 2I_{bh}\right)$

Suggests that gravitational entropy really does count degrees of freedom associated with a black hole!

Pair Production and Path Integrals



$$\mathcal{H} \equiv {}^{(3)}R(\Sigma,h) + K^2 - K^{ij}K_{ij} - 2(E^2 + B^2) = 0$$ Hamiltonian constraint

$$\mathcal{H}_i \equiv D_j K_i^j - D_i K - 2\varepsilon_{ijk} E^j B^k = 0$$ Momentum constraint

$$\mathcal{G}_{el} \equiv D_j E^j = 0 \qquad \mathcal{G}_{mg} \equiv D_j B^j = 0$$

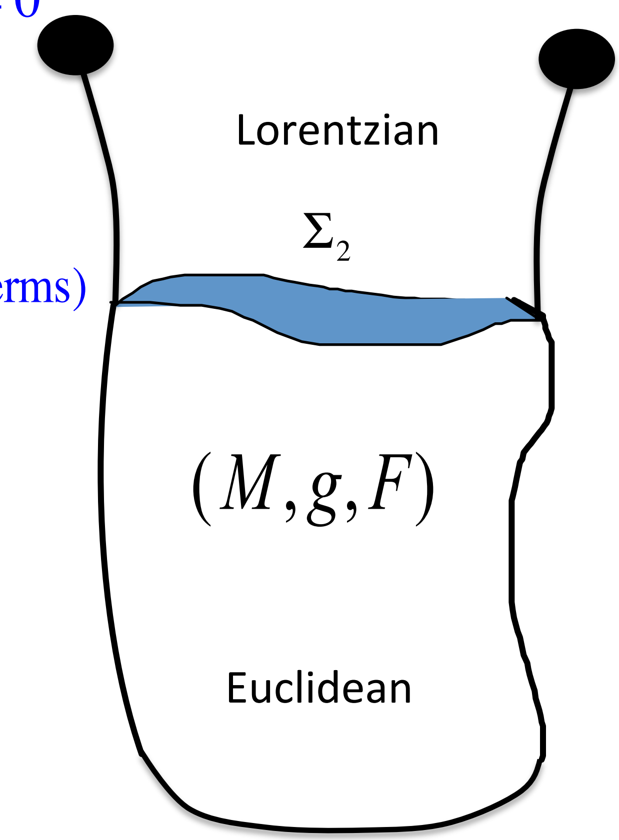
Gauge constraints

$$I[M_{\Sigma_2-\Sigma_1}, g, F] = \int_{\Sigma_2-\Sigma_1} d^4x\left(\sqrt{-g}L(g,F)\right) + (\text{boundary terms})$$

$$P(\Sigma_1 \to \Sigma_2) = |\Psi_{12}|^2$$

instanton approximation

$$\Psi_{12} = \int d[M]d[g]d[A]e^{-iI[M,g,F]} \approx e^{-I_i}$$



Pair Production: Eliminate Σ_1 and smoothly match (M,g,F) to Σ_2 where Σ_2 matches onto the Lorentzian BH pair

Instanton Construction

Analytically continue so that the matching quantities on the hypersurface Σ remain real

Brown/ Martinez/York
PRL **66** (1991) 2281

$N \to i\tilde{N}$

$V^j \to i\tilde{V}^j$

$F_{jt} \to i\tilde{F}_{jt}$

(like $t \to it$)

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + V^i dt\right)\left(dx^j + V^j dt\right)$$

$$\Rightarrow \quad ds^2 = \left(\tilde{N}^2 - h_{ij}\tilde{V}^i\tilde{V}^j\right)dt^2 + 2ih_{ij}\tilde{V}^i dx^j dt + h_{ij}dx^i dx^j$$

Thermal Equilibrium

KNdS: $T_{bh} = T_{ch}$

$\Rightarrow$
- $\left(E_0^2 + G_0^2 + a^2\chi^2\right)\chi^2 = M^2$ — lukewarm
- $r_{bh} \to r_{ch}$ — Nariai

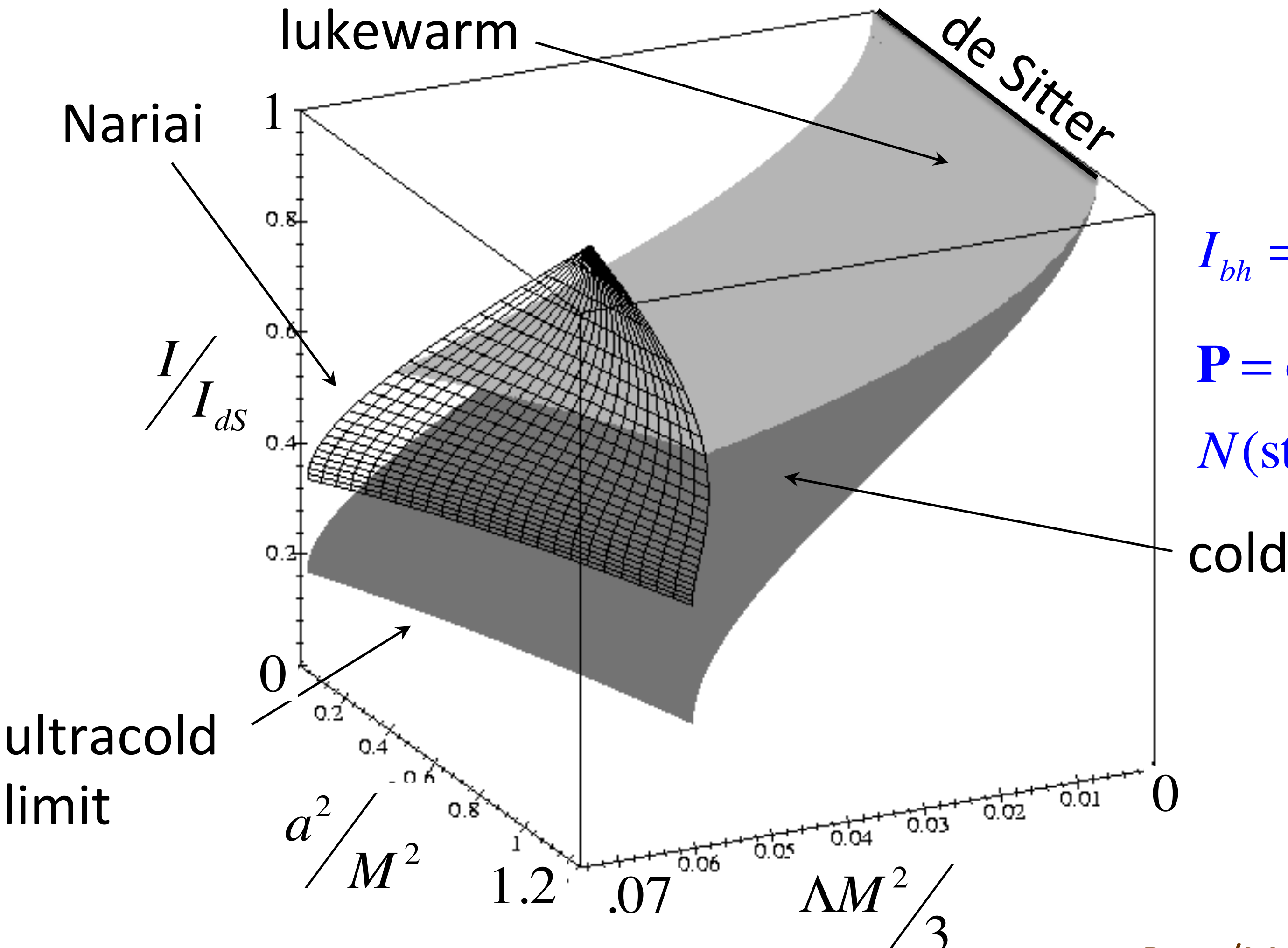
$\Rightarrow$
- $r_{in} = r_{bh}$ — cold
- $r_{in} = r_{bh} \to r_{ch}$ — ultracold (2)

- non-Lorentzian metric is complex
- matching quantities ($h_{ij}, K_{ij}, E_i = e_i^\alpha F_{\alpha\beta}u^\beta, B_i = -\frac{1}{2}e_i^\alpha g_{\alpha\beta}\varepsilon^{\beta\gamma\mu\nu}F_{\mu\nu}u_\gamma$) all remain real, as do energy, angular momentum and charge
- dynamical equations of motion, horizon structure and ergosurface all preserved
- reduces to Euclidean instanton when $V^j = 0$

KNdS Metric

$$ds^2 = -\frac{Q}{G\chi^4}\left(dt - a\sin^2\theta\, d\phi\right)^2 + \frac{G}{Q}dr^2 + \frac{G}{H}d\theta^2 + \frac{H\sin^2\theta}{G\chi^4}\left(adt - [r^2 + a^2]d\phi\right)^2$$

$$A = \frac{E_0 r}{G\chi^2}\left(dt - a\sin^2\theta\, d\phi\right) + \frac{G_0\cos\theta}{G\chi^2}\left(adt - [r^2 + a^2]d\phi\right)$$



In all cases:

$$I_{bh} = -\sum_{horizons} \frac{\mathcal{A}_h}{8}$$

$$\mathbf{P} = \exp\left(2I_{dS} - 2I_{bh}\right)$$

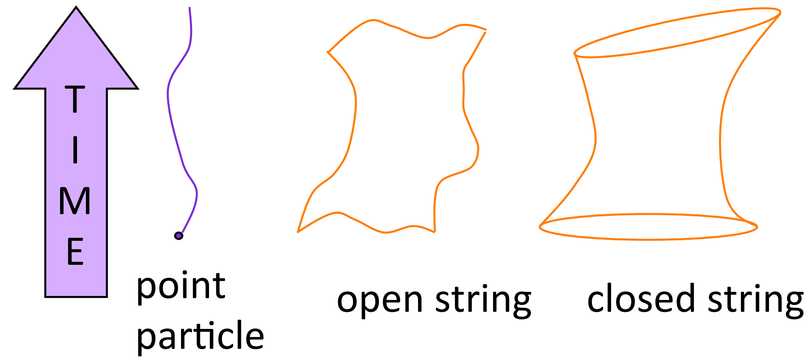
$$N(\text{states}) \sim \exp(\text{Entropy})$$

dS spacetime has maximal entropy!

Ross/Mann PRD **52**:2254 (1995)

Booth/Mann PRL **81** 5052 (1998)

And the degrees of freedom are…?

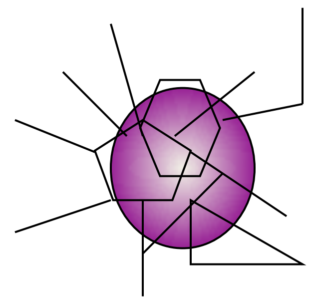


String Theory: Strominger+…

• stringy excitations of D-branes that are dual to the black hole

⇨ but only works for extremal and near-extremal SUSY black holes

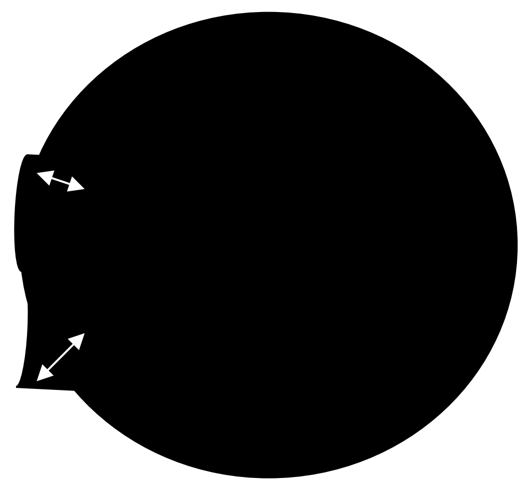
Loop Gravity: Ashtekar/Rovelli



• piercings of event horizon by spin-network structure of loop quantum gravity

⇨ but contains an arbitrary parameter

Boundary Diffeomorphisms: Carlip



• diffeomorphisms at the horizon which are conformal field theoretic

⇨ obscures the underlying theory

Horizon Supertranslations? Hawking/Perry/Strominger

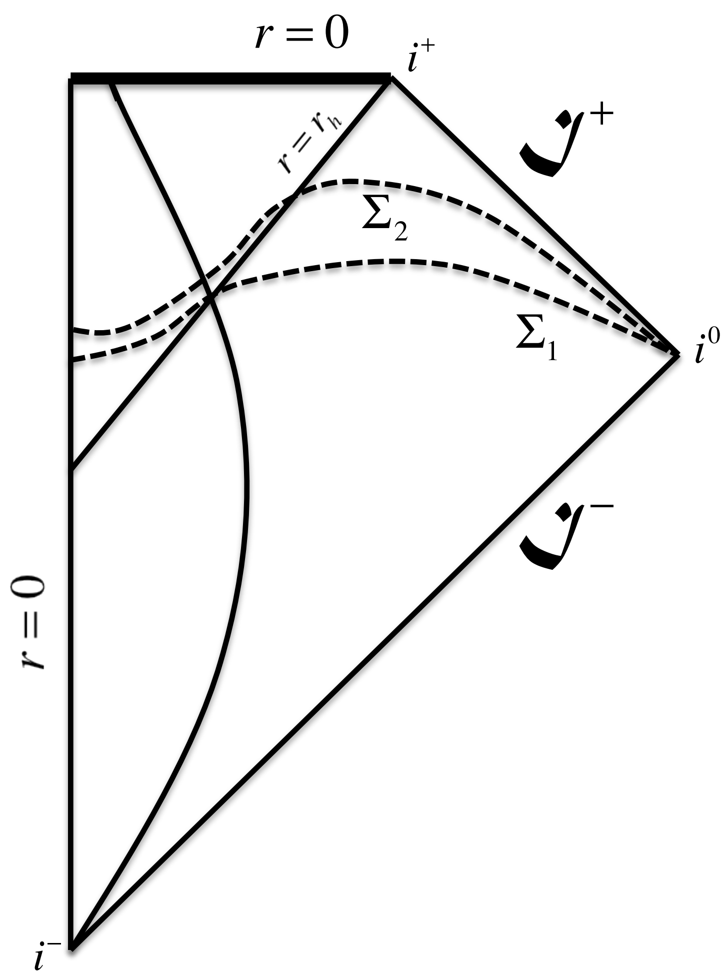
QFT in Curved Spacetime

- All quantum states defined on a spacelike slice of (4d) spacetime $\left|R^{(\Sigma)}_{abcd}\right| << 1/l_p^2$ $\left|K^{\Sigma}_{ab}\right| << 1/l_p^2$
- Full spacetime curvature must be small in a neighbourhood of the slice $\left|R_{abcd}\right| << 1/l_p^2$
- Wavelength of any quanta are large $\lambda_{\text{quanta}} >> l_p$
- Positive Energy conditions hold
- Stress-energy densities less than Planck density
- Slice evolves "smoothly" for some finite interval of proper time

$$|\, dN/d\tau \,| \ll 1/l_p \qquad |\, dN^a/d\tau \,| \ll 1/l_p$$

The "Niceness" Conditions

Mathur CQG **26** (2009) 224001



$$\left|R^{(\Sigma)}_{abcd}\right| << 1/l_p^2$$

$$\left|K^{\Sigma}_{ab}\right| << 1/l_p^2$$

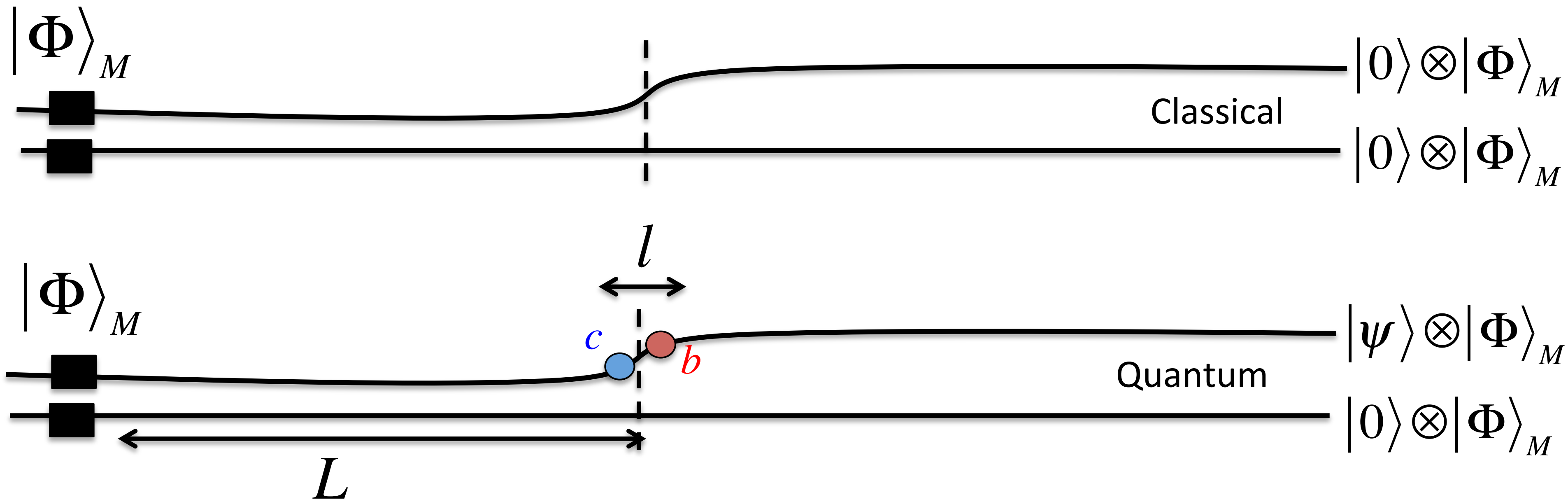
$$\left|R_{abcd}\right| << 1/l_p^2$$

$$\lambda_{\text{quanta}} >> l_p$$

$$|dN/d\tau| \ll 1/l_p$$

$$|dN^a/d\tau| \ll 1/l_p$$

Particle Pairs from Distorted Spacetime



$$|\psi\rangle = \psi_0 e^{\sigma c^\dagger b^\dagger} |0\rangle_c |0\rangle_b \quad = \left(\alpha\, |0\rangle_c |0\rangle_b + \beta\, |1\rangle_c |1\rangle_b\right) + \cdots$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$|\Psi\rangle \simeq |\psi\rangle \otimes |\Phi\rangle_M + O\left(\frac{l}{L}\right)$$

$$|\alpha| = |\beta|$$

$$S_{\text{ent}} = -\text{Tr}_{c,M}[\rho \log \rho] = -\left(|\alpha|^2 \log|\alpha|^2 + |\beta|^2 \log|\beta|^2\right) = \log 2$$

For a black hole, get maximal entanglement → maximal entropy

Possible Deviations?

$$|\alpha|^2+|\beta|^2=1$$

$$|\psi\rangle=\psi_0 e^{\sigma c^\dagger b^\dagger}|0\rangle_c|0\rangle_b \quad =\left(\alpha\,|0\rangle_c|0\rangle_b+\beta\,|1\rangle_c|1\rangle_b\right)+\cdots$$

$$|\Psi\rangle\approx\left(\tilde{\alpha}|\Phi_0\rangle_M+\tilde{\beta}|\Phi_1\rangle_M\right)\otimes\left((\alpha+\epsilon)|0\rangle_c|0\rangle_b+(\beta-\epsilon)|1\rangle_c|1\rangle_b\right)$$

$$S_{\text{ent}}=-Tr_{c,M}[\rho\ln\rho]$$

$$=-\left(|\alpha+\epsilon|^2\log|\alpha+\epsilon|^2+|\beta-\epsilon|^2\log|\beta-\epsilon|^2\right)$$

$$=-\left(|\alpha|^2\log|\alpha|^2+|\beta|^2\log|\beta|^2\right)$$

$$+2\epsilon\left(|\beta|\log(2|\beta|^2)-|\alpha|\log(2|\alpha|^2)\right)+\cdots$$

Permitted by locality

$$\left|S_{\text{ent}}-S_0\right|<<S_0$$

BUT

$$|\Psi\rangle\approx\left((\tilde{\alpha}+\epsilon)|\Phi_0\rangle_M|0\rangle_c+(\tilde{\beta}-\epsilon)|\Phi_1\rangle_M|1\rangle_c\right)\otimes\left(\alpha\,|0\rangle_b+\beta|1\rangle_b\right)$$

$$S_{\text{ent}}=-Tr_{c,M}[\rho\ln\rho]=0$$

Forbidden by locality

Normal Radiation

Excited states

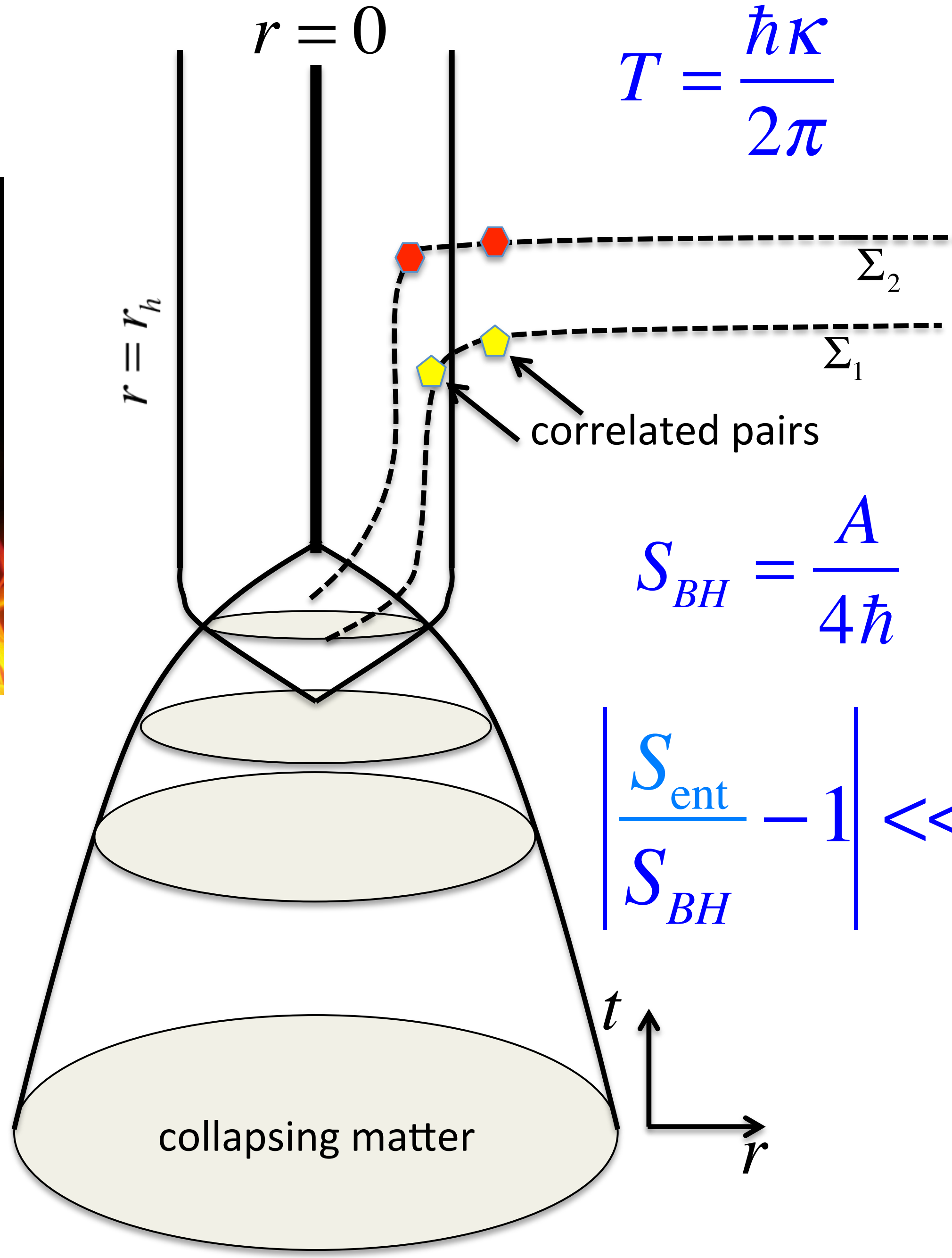
→ less-excited states

by emitting quanta

Black Hole Radiation

Vacuum

→ Pairs of quanta due to spacetime distortion



$$T = \frac{\hbar\kappa}{2\pi}$$

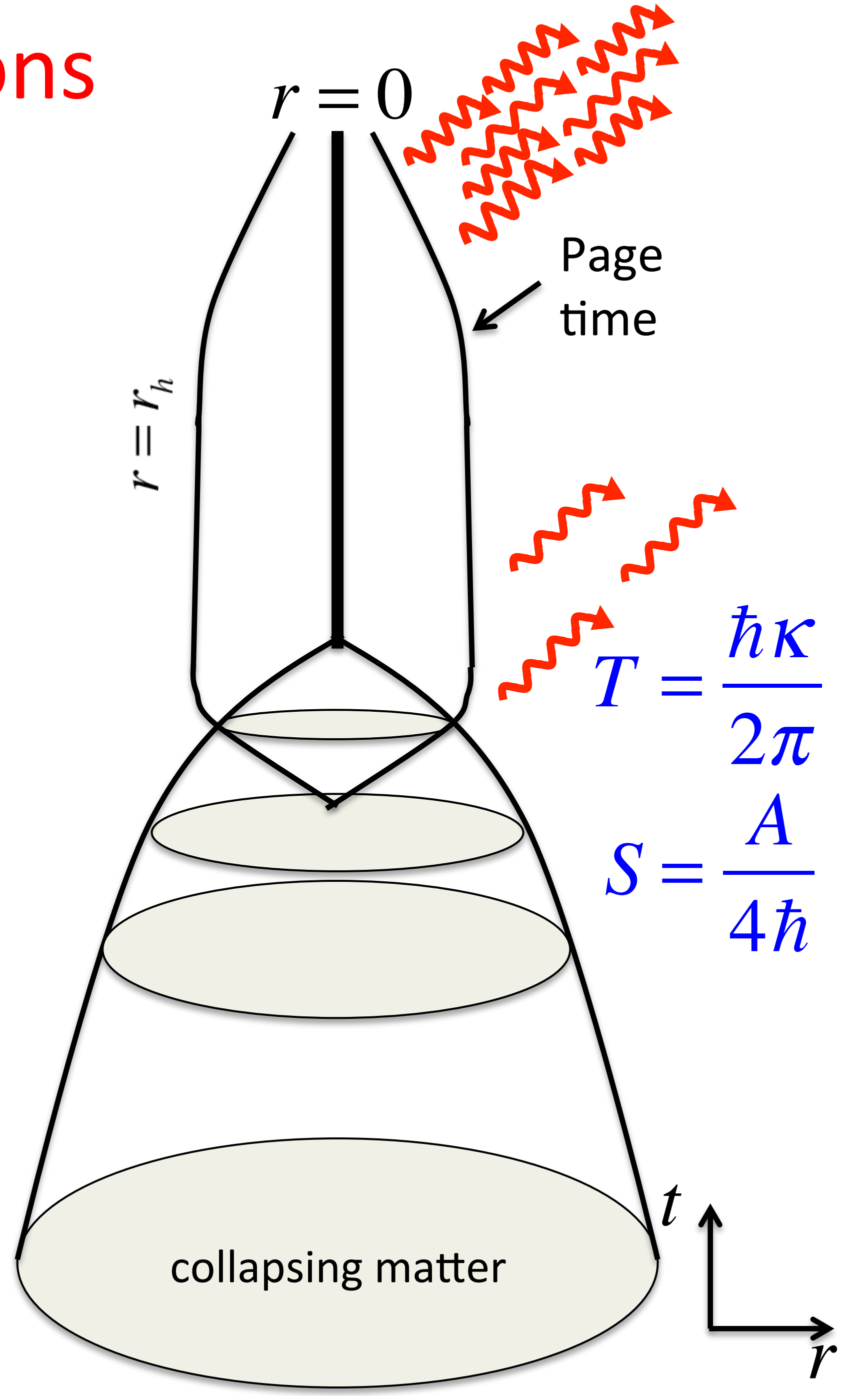
$$S_{BH} = \frac{A}{4\hbar}$$

$$\left| \frac{S_{\text{ent}}}{S_{BH}} - 1 \right| << 1$$

Crucial Implicit Assumptions

- Quantum state is regular (Hadamard) at the horizon
- Local QFT applies at the horizon (“no drama”)
- Black hole loses mass as it radiates, but slowly enough to retain niceness conditions

$$t_{\rm evap} \sim (M/M_P)^3$$



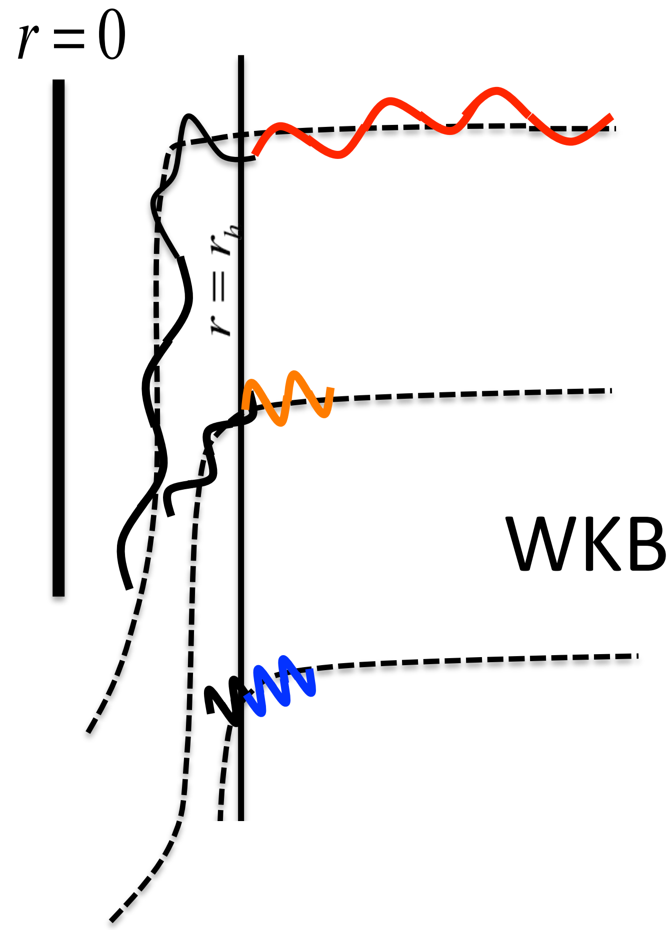
Trans-Planckian Problem

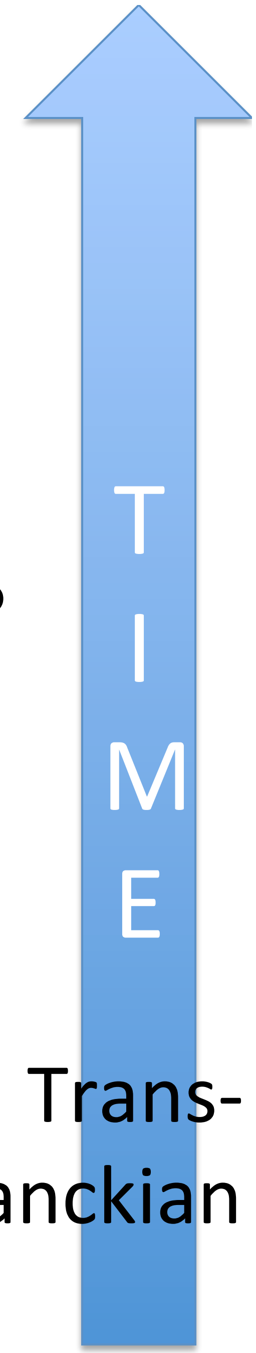
Barcelo/Liberati/Visser, Liv.Rev.Rel. **8** (2005) 12

- Finite-energy quanta emitted near horizon will redshift to zero energy
- Hence observation of finite energy quanta implies emission at energies

$$E \gg E_{Pl} \qquad \omega \gg \omega_P = 10^{43}\,\mathrm{s}^{-1}$$

- Violates original assumptions





Resolution?

$$\omega_P \gg \omega_{WKB} \gg (kM)^{-1}$$

- Hawking radiation is a low-energy phenomenon
- Pairs ripped apart when WKB approximation holds

Schutzhold/Unruh PRD **88** (2013) 124009

Information Paradox

Hawking PR**D14** (1976) 2460

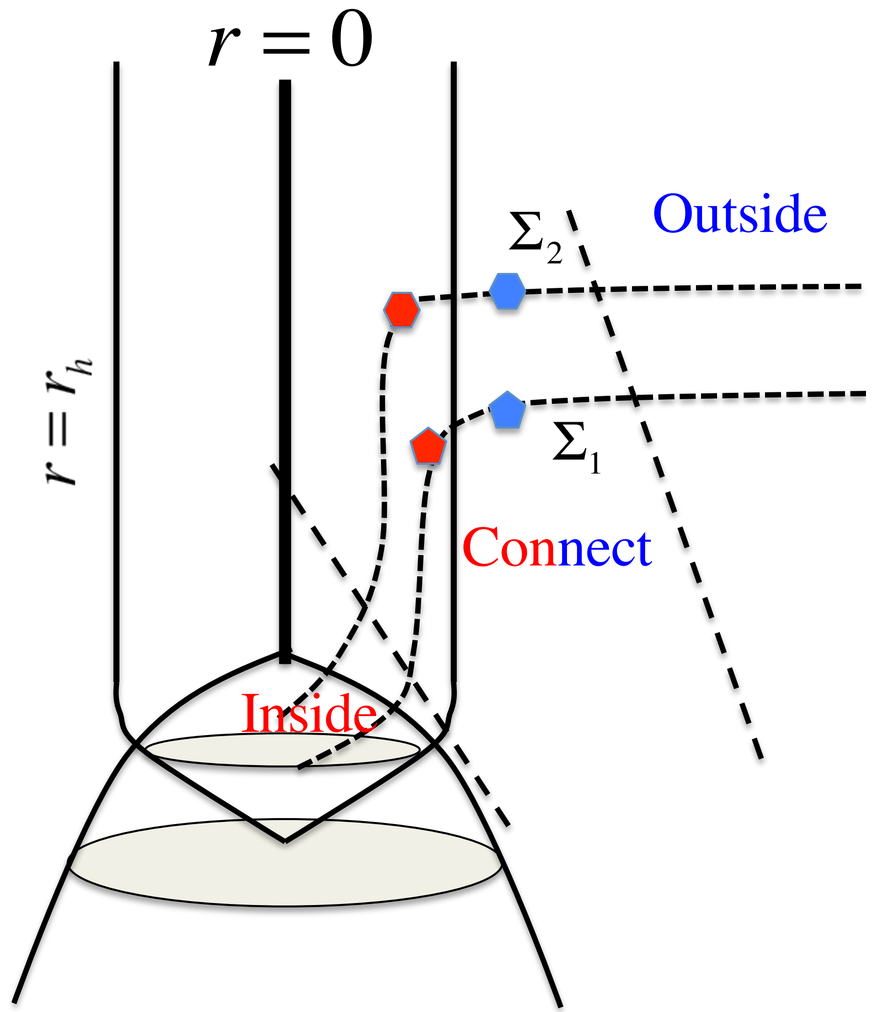
Outside: t=constant
Inside: r=constant

$$|\Psi\rangle_1 \simeq \frac{1}{\sqrt{2}}|\Phi\rangle_{I1} \otimes \left(|0_k\rangle_{I1}|0_{-k}\rangle_{O1} + |1_k\rangle_{I1}|1_{-k}\rangle_{O1}\right)$$

$$\rho_{O_1} = Tr_I\left[|\Psi\rangle\langle\Psi|\right] = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$S_{ent}(1) = -\mathrm{Tr}\left[\rho_{O_1}\log\rho_{O_1}\right]$$

$$= 2\times\frac{1}{2}\log 2 = 2\log 2$$



Next emission

$$|\Psi\rangle_2 \approx \frac{1}{2}|\Phi\rangle_{I2} \otimes \left(|0_k\rangle_{I1}|0_{-k}\rangle_{O1} + |1_k\rangle_{I1}|1_{-k}\rangle_{O1}\right) \otimes \left(|0_k\rangle_{I2}|0_{-k}\rangle_{O2} + |1_k\rangle_{I2}|1_{-k}\rangle_{O2}\right)$$

$$\rho_{O_1} = Tr_I\left[|\Psi\rangle\langle\Psi|\right] = \mathrm{diag}\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}\right) \quad S_{ent}(2) = -\mathrm{Tr}\left[\rho_{O2}\ln\rho_{O2}\right] = 4\ln 2$$

*n*th emission

$$|\Psi\rangle_n \approx \frac{1}{2^{n/2}}|\Phi\rangle_{In} \prod_{m=1}^{n} \otimes \left(|0_k\rangle_{Im}|0_{-k}\rangle_{Om} + |1_k\rangle_{Im}|1_{-k}\rangle_{Om}\right)$$

$$\rho_{O_n} = Tr_I\left[|\Psi\rangle\langle\Psi|\right] = \mathrm{diag}\left(2^{-n},2^{-n},\ldots,2^{-n}\right)$$

$$S_{ent}(n) = -\mathrm{Tr}\left[\rho_{On}\ln\rho_{On}\right] = 2^n \ln 2$$

Entropy grows unboundedly!

$$n \to \infty$$

$$n = \sigma^{-1}\left(\frac{M}{M_p}\right)^2$$

$M = M_\odot$ ⇒

$$n \simeq 10^{76}$$

Solutions?

- Remnants
  - Something terminates evolution once $M = M_r \geq M_{Pl}$
  - Remnant must be *n*-fold degenerate since its entanglement with radiation is *n log2*
  - Each remnant state gives finite loop correction to scattering processes → sum over *n* is divergent unless its couplings vanish
- Mixedness
  - Black hole evaporates leaving radiation with entanglement entropy *n log2* but unentangled with any quantum state
  - Initial pure state evolves to mixed state → violates unitarity
- Bleaching
  - Information can never enter the black hole
  - Some strange process decouples the information of a state from its energy and momentum
  - Initial state should never have formed the black hole in the first place

What about Small Corrections?

Recall

$$|\Psi\rangle_j \approx \frac{1}{2^{n/2}}\left[|\Phi\rangle_{Ij}\prod_{m=1}^{j-1}\otimes\left(|0_k\rangle_{Im}|0_{-k}\rangle_{Om}+|1_k\rangle_{Im}|1_{-k}\rangle_{Om}\right)\right]\left(|0_k\rangle_{Ij}|0_{-k}\rangle_{Oj}+|1_k\rangle_{Ij}|1_{-k}\rangle_{Oj}\right)$$

Change to

$$|\tilde{\Psi}\rangle_j = |\tilde{\Psi}\rangle_{j-1}^{+}|\Xi\rangle_j^{+} + |\tilde{\Psi}\rangle_{j-1}^{-}|\Xi\rangle_j^{-}$$

$$|\Xi\rangle_j^{\pm} = \left(|0_k\rangle_{Ij}|0_{-k}\rangle_{Oj} \pm |1_k\rangle_{Ij}|1_{-k}\rangle_{Oj}\right)$$

$$|\tilde{\Psi}\rangle_{j-1}^{\pm} = \sum_{l,m}\alpha_{l,m}|\tilde{\psi}_l^{\pm}(\Phi,\mathrm{I})\rangle|\chi_m(\mathrm{O})\rangle$$

Density Matrix for Created Pair

$${}_{j-1}^{-}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{-} = \epsilon_-^2 < \varepsilon^2$$

$${}_{j-1}^{+}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{-} = \epsilon_{+-} < \varepsilon$$

$$\rho_{\Xi_j} = \begin{pmatrix} {}_{j-1}^{+}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{+} & {}_{j-1}^{+}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{-} \\ {}_{j-1}^{-}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{+} & {}_{j-1}^{-}\langle\tilde{\Psi}|\tilde{\Psi}\rangle_{j-1}^{-} \end{pmatrix} = \begin{pmatrix} 1-\epsilon_-^2 & \epsilon_{+-} \\ \epsilon_{+-}^{*} & \epsilon_-^2 \end{pmatrix}$$

$$S(\Xi_j) = \left(\epsilon_{+-}^2 - \epsilon_-^2\right)\log\left(\epsilon_-^2 - \epsilon_{+-}^2\right) < \varepsilon$$

Density Matrix for Inside Partner

$$\rho_{I_j} = \frac{1}{2}\begin{pmatrix} 1+\mathrm{Re}\,\epsilon_{+-} & 0 \\ 0 & 1-\mathrm{Re}\,\epsilon_{+-} \end{pmatrix}$$

$$S(I_j) > \log 2 - 2\epsilon_{+-}^2 > \log 2 - \varepsilon$$

Entanglement Entropy for Created Pair $S\left(\Xi_j\right) < \varepsilon$

Entanglement Entropy for Inside Partner $S\left(I_j\right) > \log 2 - \varepsilon$

Subadditivity

$$S(\rho_{AB}) + S(\rho_{BC}) \geq S(\rho_A) + S(\rho_C) \qquad S(\rho_{AB}) \geq \left| S(\rho_A) - S(\rho_B) \right|$$

$$S(\{\mathrm{O}_{j-1}, \Xi_j\}) \geq \left| S(\{\mathrm{O}_{j-1}\}) - S(\Xi_j)\}) \right| \geq S(\{\mathrm{O}_{j-1}\}) - \epsilon$$

$$S(\{\mathrm{O}_j\})_{ent} + S(\Xi_j)_{ent} = S(\{\mathrm{O}_{j-1}\}, \mathrm{O}_j)_{ent} + S(\mathrm{O}_j, \mathrm{I}_j)_{ent} > S(\{\mathrm{O}_{j-1}\}) + S(\mathrm{I}_j)_{ent}$$

$$S(\{\mathrm{O}_j\})_{ent} > S(\{\mathrm{O}_{j-1}\}) + S(\mathrm{I}_j)_{ent} - S(\Xi_j)_{ent} = S(\{\mathrm{O}_{j-1}\}) + \log 2 - 2\varepsilon$$

Entropy of Outgoing Radiation always increases by at least $\log 2 - 2\varepsilon$

Normal Matter

- Each Quanta of emission entangled many possible ways
- Correlations change with each emission

Black Holes

- Each Quanta of emission entangled same way
- Correlations same with each emission

The Required Final State

Suppose $|\Phi\rangle_{in} = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle$

What we have: $$|\Psi\rangle_n \approx \frac{1}{2^{n/2}}|\Phi\rangle_{In}\prod_{m=1}^{n}\otimes\left(|0_k\rangle_{Im}|0_{-k}\rangle_{Om} + |1_k\rangle_{Im}|1_{-k}\rangle_{Om}\right)$$

?

$$|\Psi\rangle_n \approx \frac{1}{2^{n/2}}\left(|\Phi_0\rangle|1_1\rangle_I + |\Phi_1\rangle|0_1\rangle_I\right)\left(\alpha|0_1\rangle_O + \beta|1_1\rangle_O\right)$$
$$\prod_{m=1}^{n-1}\otimes\left(|0_k\rangle_{Im}|0_{-k}\rangle_{Om} + |1_k\rangle_{Im}|1_{-k}\rangle_{Om}\right)$$

Information-retaining but Mixed

?

$$|\Psi\rangle_n \approx \left(\alpha|\Phi_0\rangle|1_1 1_2 \cdots 1_n\rangle_I + \beta|\Phi_1\rangle|0_1 0_2 \cdots 0_n\rangle_I\right)$$
$$\otimes\left(|0_1 0_2 \cdots 0_n\rangle_O + |1_1 1_2 \cdots 1_n\rangle_O\right)$$

Pure but not Information-retaining

?

$$|\Psi\rangle_n \approx \left(|\Phi_0\rangle|1_1 1_2 \cdots 1_n\rangle_I + |\Phi_1\rangle|0_1 0_2 \cdots 0_n\rangle_I\right)$$
$$\otimes\left(\alpha|0_1 0_2 \cdots 0_n\rangle_O + \beta|1_1 1_2 \cdots 1_n\rangle_O\right)$$

Pure AND Information-retaining

Remedies?

- Niceness Conditions break down?
  - Need new physics inside horizon
- Exotic End-states
  - Fuzzballs: stringy degrees of freedom prevent formation of both horizon and singularity
  - Need a generic mechanism
- Quantum Hair
  - Horizon is distorted according to characteristics of collapsing matter (keeps information out of hole)
  - Must avoid divergent stress-energy, ensure hair transfers information, elude no-hair theorems

Mathur Fort Phys **53** (2005) 793

Hotta PR**D66** (2002) 124001

Complementarity

Susskind/Thorlacius/Uglum
PR**D48** (1993) 3743

- Basic idea: No super-observer exists that can perform experiments both inside and outside of the black hole
- Outside Observer
  - Horizon induces a boundary condition: a brick wall
  - Wall absorbs all infalling matter and unitarily emits it as Hawking radiation, similar to normal matter
- Infalling Observer
  - No wall exists as observer crosses horizon
  - Infalling Observer exponentially unlikely to measure any emitted quanta

Complementarity Postulates

- Unitarity
  - There exists a unitary S-matrix describing evolution from collapsing matter to outgoing Hawking radiation
- Locality
  - Physics is described by local semi-classical field theory anywhere outside of the horizon
  - Hilbert space factorizes: (interior)x(exterior)
- Placidity (no-drama)
  - Gravity is locally indistinguishable from acceleration
  - Freely-falling observers are exponentially unlikely to see any state at the horizon other than the vacuum

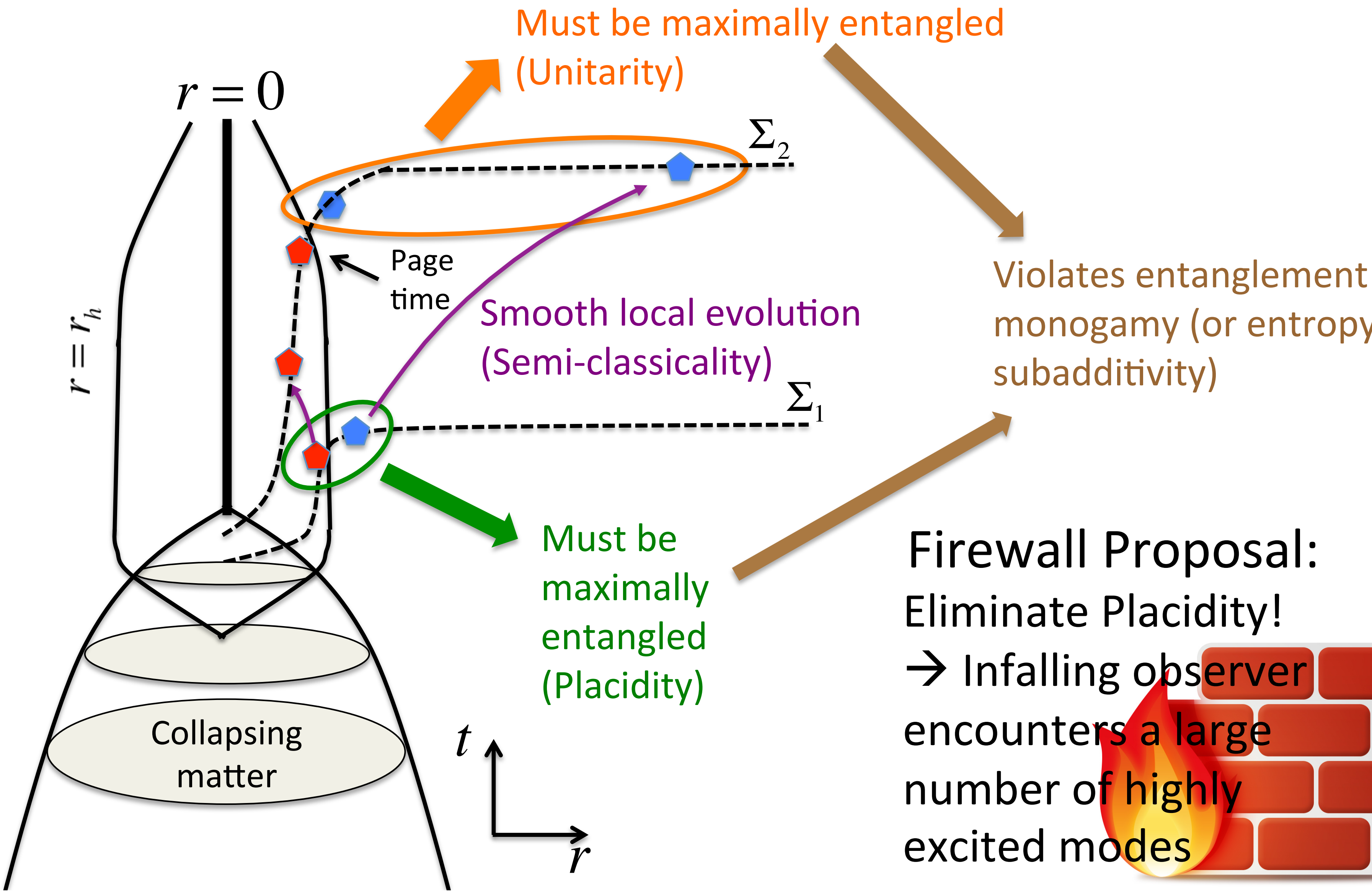
Firewalls

Almheiri/Marolf/Polchinski/Sully JHEP **1302** (2012) 62
Almheiri/Marolf/Polchinski/Stanford/Sully JHEP **1309** (2013) 18

- Complementarity Postulates are not self-consistent
- Locality → Late-time quanta smoothly evolve from early-time quanta via local physics
- Unitarity → After half the black hole mass is radiated away (Page time), entanglement entropy of created pairs must decrease

- Placidity → Regular horizon implies increasing entanglement entropy of created pairs

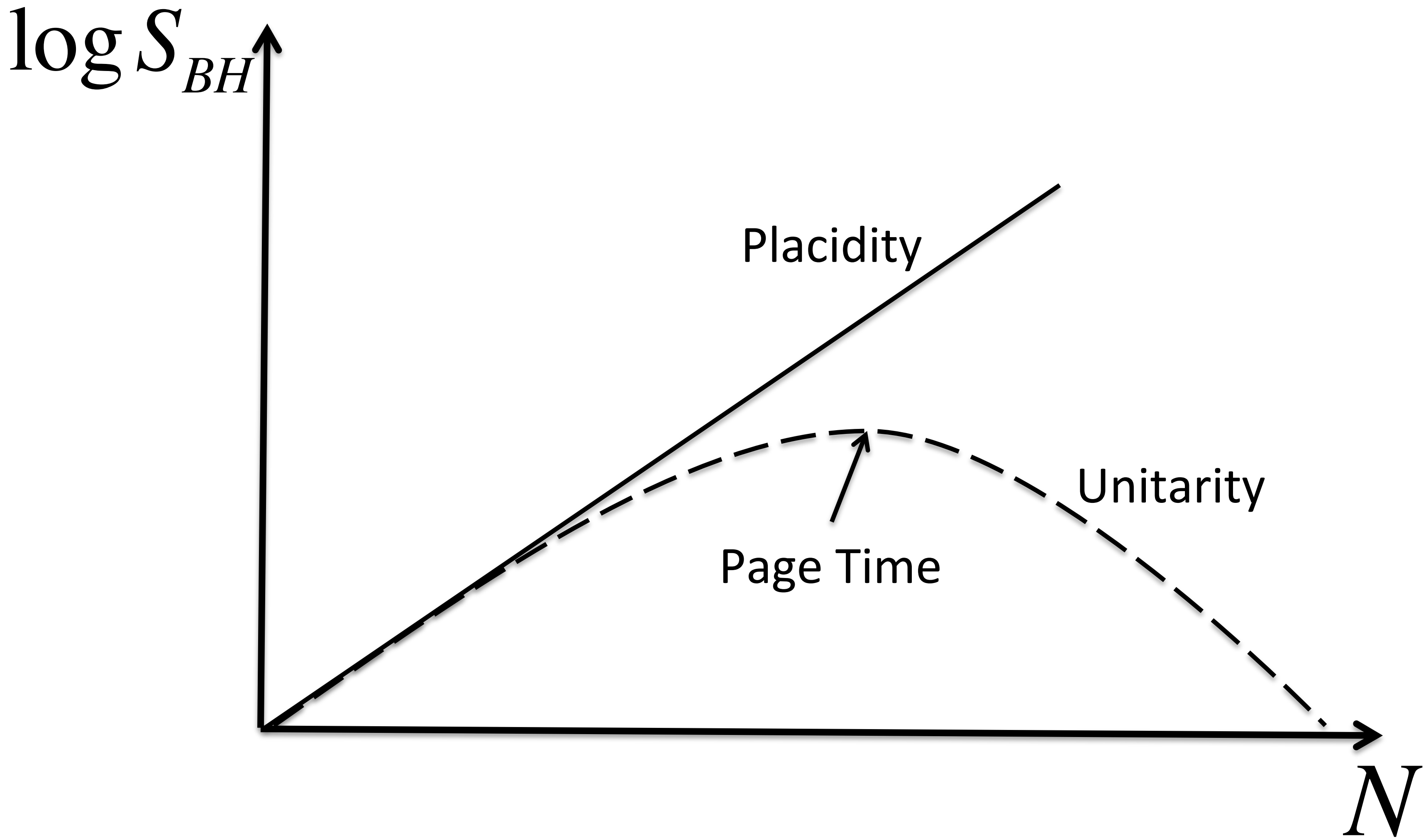
$$|\Psi\rangle_n \approx \frac{1}{2^{n/2}} |\Phi\rangle_{In} \prod_{m=1}^{n} \otimes \left(|0_k\rangle_{Im} |0_{-k}\rangle_{Om} + |1_k\rangle_{Im} |1_{-k}\rangle_{Om} \right) \rightarrow |\tilde{\Phi}\rangle_I |\Xi\rangle_O$$



Firewall Proposal:

Eliminate Placidity!

→ Infalling observer encounters a large number of highly excited modes


$\log S_{BH}$
Placidity
Unitarity
Page Time
N

Firewall Responses

- Correct: Firewall exists
  - Mechanism of formation?
  - Physical Characteristics?
- Correct: Firewall removed by other physics
  - Black holes never form (exotic objects instead)
  - Non-local physics is present (how?)
  - Unitarity violated (what of AdS/CFT?)
  - Modified Quantum Physics is polygamous (how?)
- Wrong: Firewall not there in the first place
  - Number of degrees of freedom not properly accounted for because of quantum gravity effects?
  - Factorization of state into localized degrees of freedom invalid?
  - Many "cures" violate the Born rule

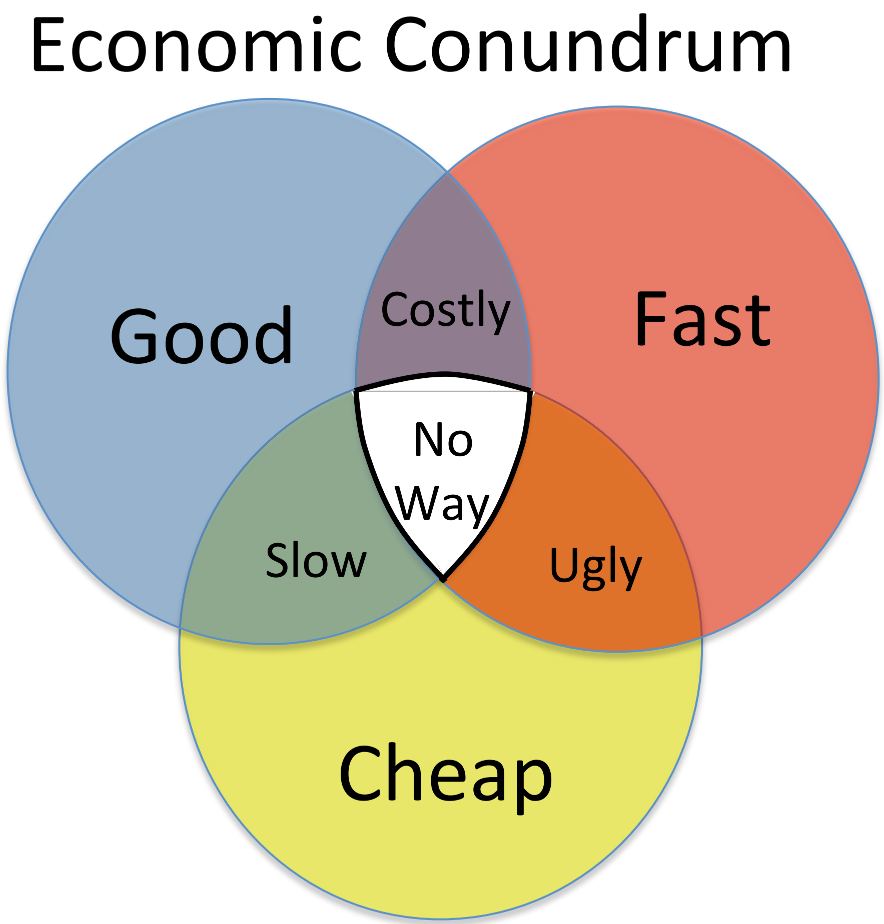
Marolf/Polchinksi
arXiv:1506.01337

A Toy Firewall

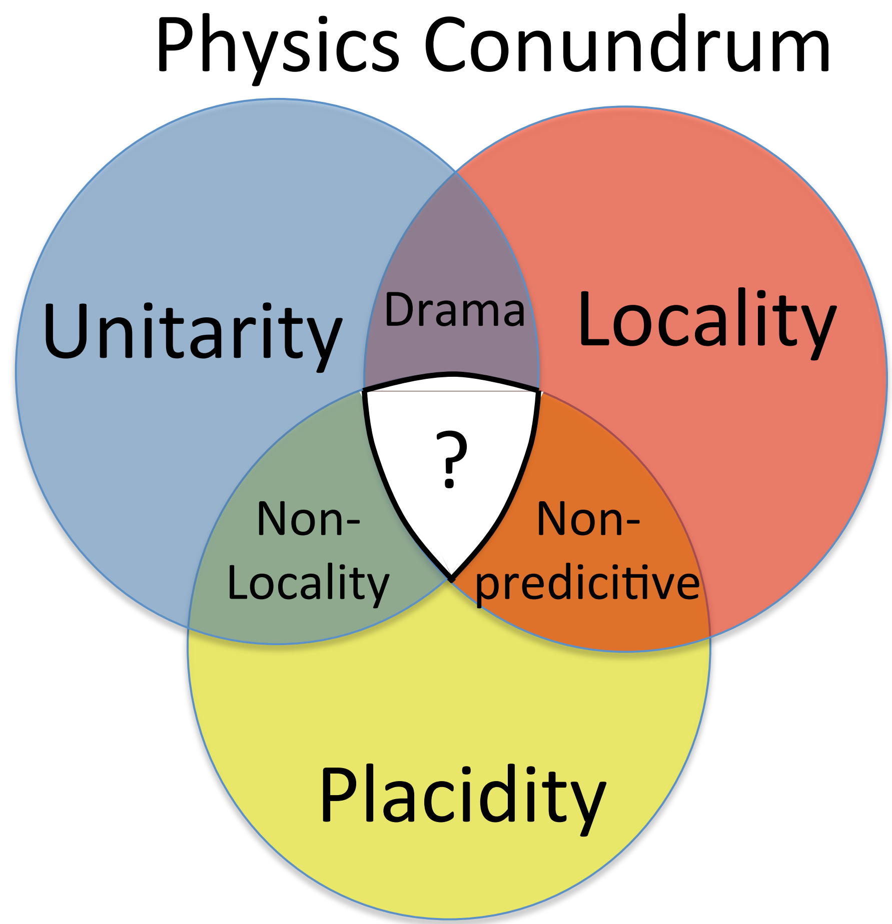
Louko
JHEP **1409** (2014) 142
Louko/Martin-Martinez
PRL **115** (2013) 031301

- Consider (1+1)-Dimensional Rindler Spacetime
- Break Quantum Correlations across acceleration horizons ``by hand"
- What happens to UdW Detectors?
- What happens to Quantum Entanglement?

Conclusion?



Economic Conundrum
Good
Costly
Fast
No
Way
Slow
Ugly
Cheap



Physics Conundrum
Unitarity
Drama
Locality
?
Non-
Locality
Non-
predicitive
Placidity