Puzzles and Microstructures of Black Holes

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September 10, 2015
Int'l Workshop on Strings, Black Holes and Quantum Info
Tohoku Forum for Creativity, Tohoku U

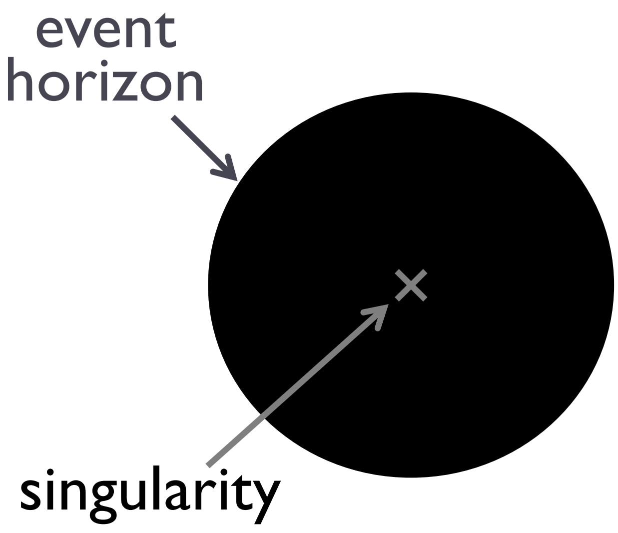
https://en.wikipedia.org/wiki/File:BH_LMC.png

Plan

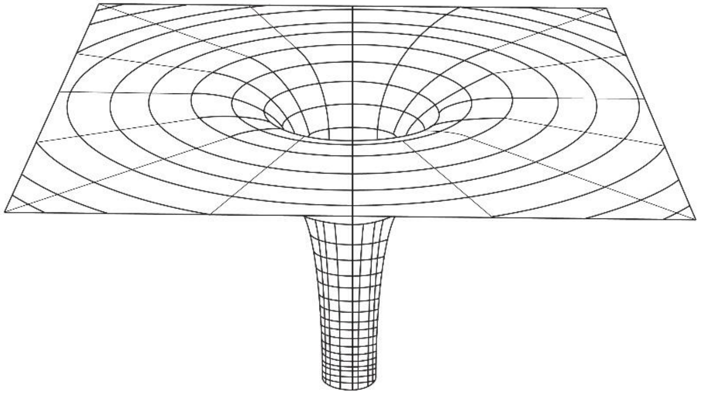
- BH microstates
- Microstate geometries
- Fuzzball conjecture &
  microstate geom program
- Microstate geom in 5D
- Microstate geom in 6D
- Conclusions

Black hole microstates

Black holes



- Solution to Einstein equations
- Boundary of no return: event horizon
- Spacetime breaks down at spacetime singularity



BH entropy puzzle

- BH entropy:

$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{N}}}$$

⇒ Stat mech: $N_{\mathrm{micro}} = e^{S_{\mathrm{BH}}}$

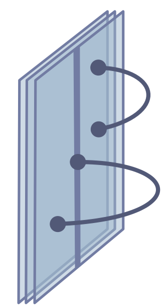
— ***Where are the microstates?***

- Uniqueness theorems
- Need quantum gravity?

AdS/CFT correspondence

string theory / gravity in AdS space ⟷ quantum field theory (CFT)

black hole → thermodyn. ensemble

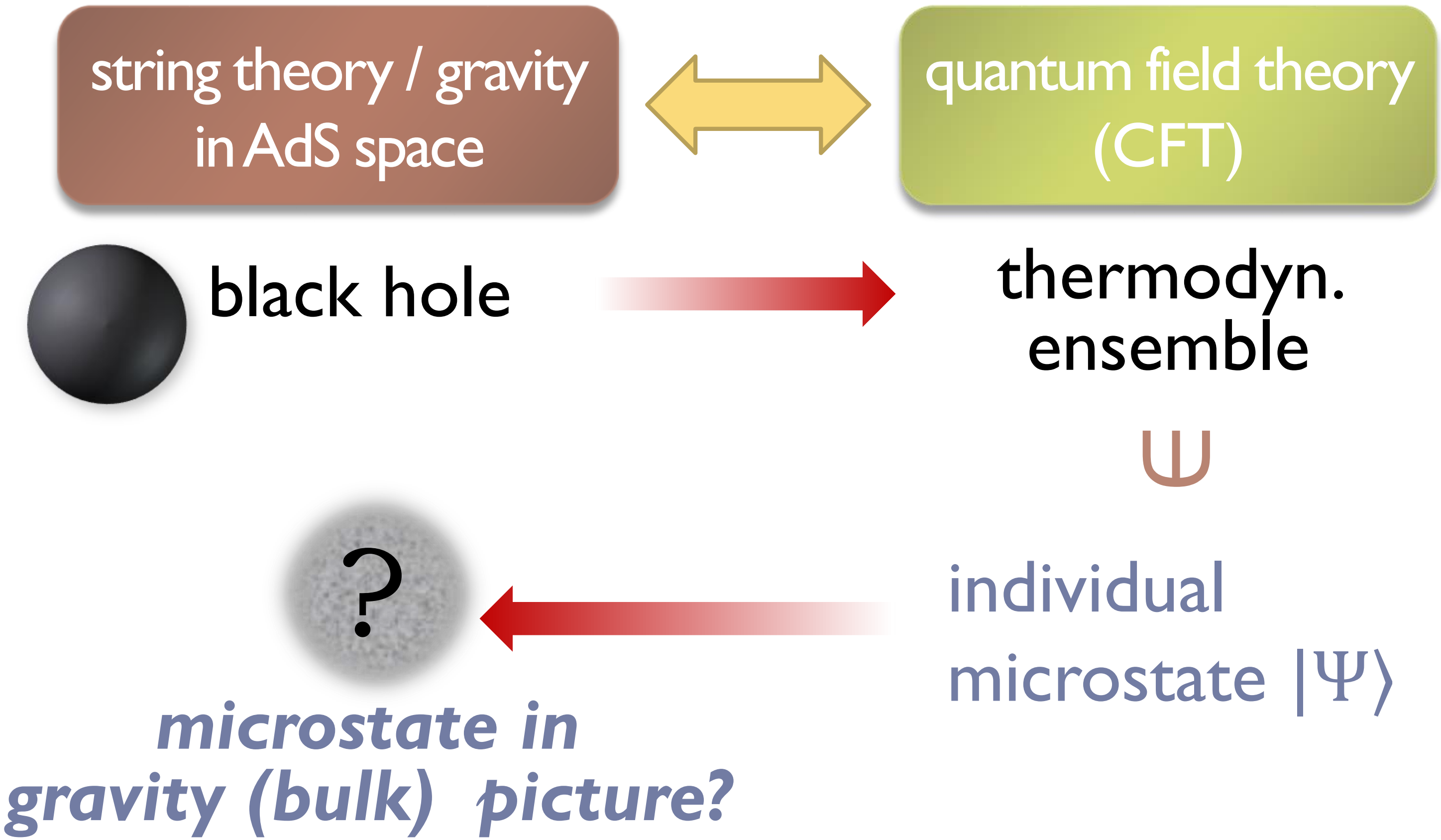


[Strominger-Vafa '96]

$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{N}}} \overset{!}{=} S_{\mathrm{CFT}} = \log N_{\mathrm{micro}}$$

→ Stat mech interpretation of BH put on firm ground

BH microstates



- Must be a state of quantum gravity / string theory in general

<u>Summary:</u>

We want gravity picture of BH microstates!

Microstate geometries

Are examples of gravity microstates known?

– Yes!

We know examples of microstates called *microstate geometries*.

- Solution of *classical* gravity
- Has same mass & charge as the BH
- Smooth & horizonless

Example 1: LLM geometries

[Lin-Lunin-Maldacena 2004]

LLM geometries (1)

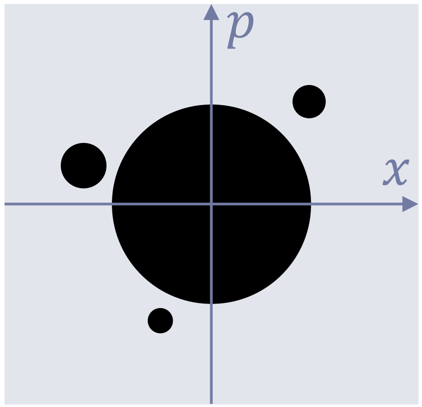
Sugra in AdS_5xS^5 ⟷ D=4, $\mathcal{N}$=4 SYM

½ BPS BH: "superstar"
(singular, $A = 0$) → ½ BPS states

||

fermion gas in
harmonic potential

$H = \frac{1}{2}(p^2 + x^2)$



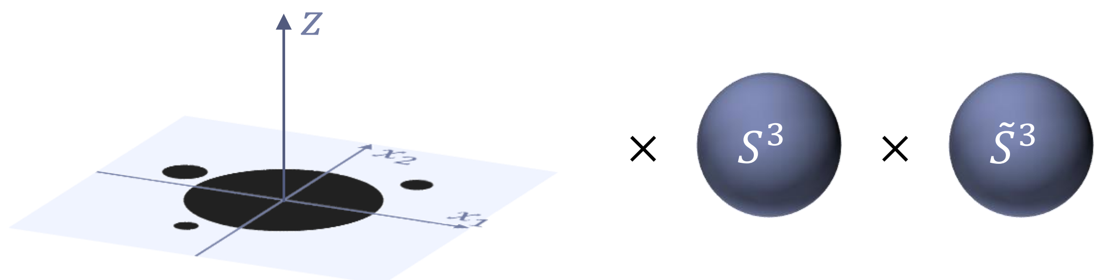
LLM (bubbling) geometries ←

LLM geometries (2)

$$ds^2 = -h^{-2}(dt+V)^2 + h^2\left(dy^2 + dx_1^2 + dx_2^2\right) + ye^{G}d\Omega_3^2 + ye^{-G}d\tilde{\Omega}_3^2$$

$$h^{-2} = 2y\cosh G \qquad e^{2G} = \frac{1/2+z}{1/2-z}$$

$$[\partial_1^2 + \partial_2^2 + y\partial_y(y^{-1}\partial_y)]z(x_1,x_2,y) = 0$$



- LLM diagram encodes how S^3's shrink
- Smooth horizonless geometries
- Non-trivial topology supported by flux
- 1-to-1 correspondence with coherent states in CFT

no uniqueness thm in 10D

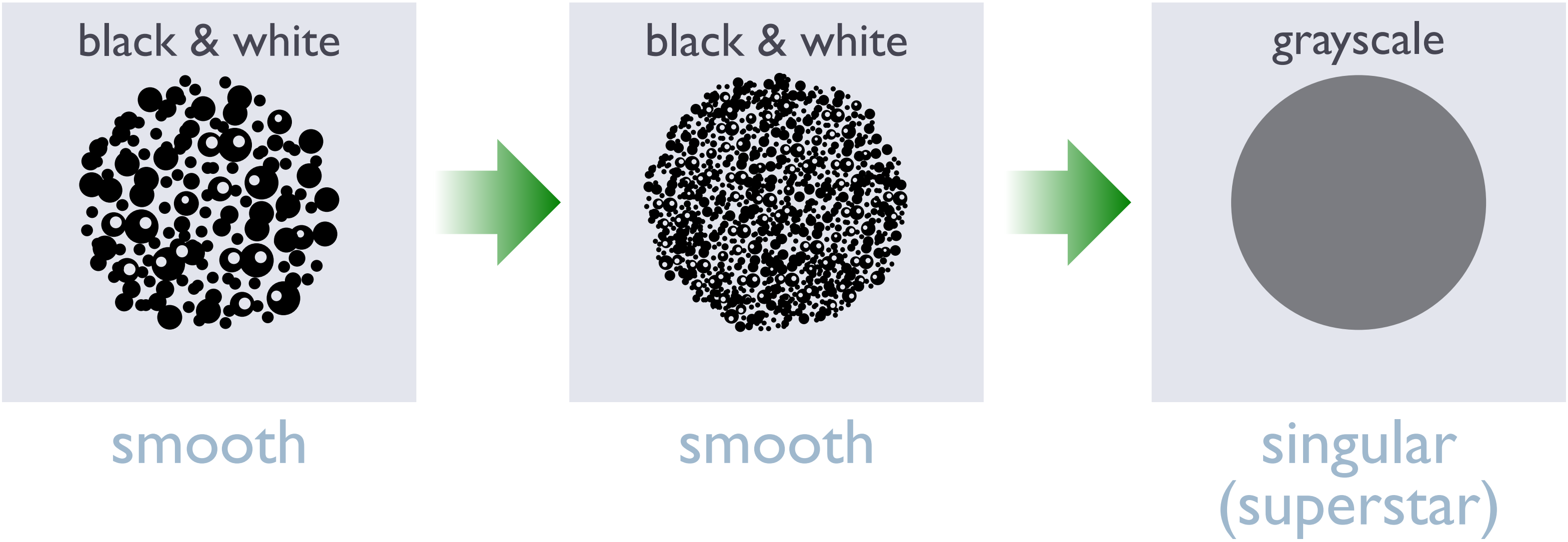
Classical limit

How is naive singular geometry (superstar) recovered?

- Bubble area quantized

$$(\text{area}) = 4\pi^2 l_p^4 N, \qquad h = 4\pi^2 l_p^4$$

- Classical limit: $l_p \to 0, \ N \to \infty$



Example 2:
LM geometries

[Lunin-Mathur 2001]

[Lunin-Maldacena-Maoz 2002]

LM geometries (1)

Sugra in $AdS_3 x S^3$ ⟺ D=2, $\mathcal{N}$=(4,4) CFT

2-charge BH
(singular, $A = 0$)

N_1 D1-branes
N_2 D5-branes

→

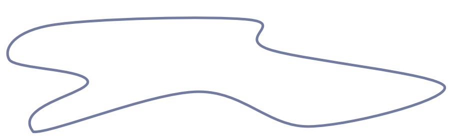
½ BPS states
||
free bosons in 2D

Parametrized by integers
$n_1, n_2, n_3, \ldots$

$$\sum_k k n_k = N_1 N_2$$

←

"LM geometries"



LM geometries (2)

$$ds^2 = -\frac{2}{\sqrt{Z_1 Z_2}}(dv+\beta)(du+\omega) + \sqrt{Z_1 Z_2}\,dx_{1234}^2 + \sqrt{Z_1/Z_2}\,dx_{6789}^2$$

$$Z_1(\vec{x}) = 1 + \frac{Q_2}{L}\int_0^L \frac{|\dot{\vec{F}}|^2 d\lambda}{|\vec{x}-\vec{F}(\lambda)|^2}, \qquad Z_2(\vec{x}) = 1 + \frac{Q_2}{L}\int_0^L \frac{d\lambda}{|\vec{x}-\vec{F}(\lambda)|^2} \qquad \ldots$$

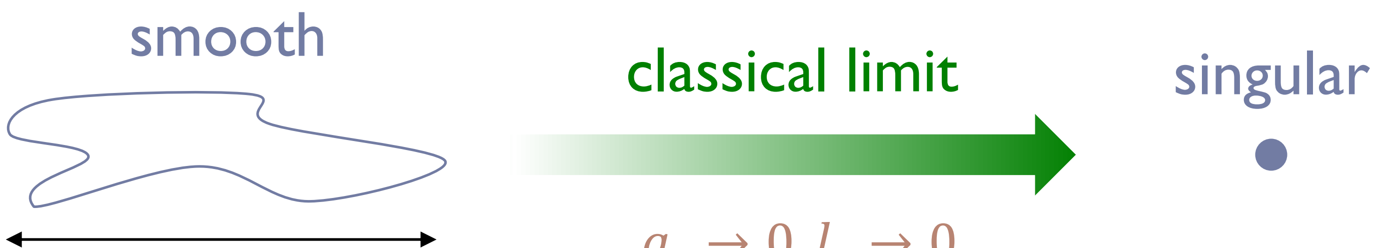
arbitrary curve $\vec{x} = \vec{F}(\lambda) \in \mathbb{R}^4_{1234}$

Fourier coeffs of $\vec{F}(\lambda) \Leftrightarrow \{n_k\}$

- LM curve encodes how S^1 shrinks
- Smooth horizonless geometries supported by flux
- 1-to-1 correspondence with CFT states: $\vec{F}(\lambda) \leftrightarrow \{n_k\}$
- Entropy reproduced geometrically: $S \sim \sqrt{N_1 N_2}$

Classical limit

How is naive singular geometry recovered?



$$\mathcal{R} \sim g_s^{1/3} l_s^{1/3} (Q_1 Q_2)^{1/6}$$
$$\sim g_s^{2/3} l_s (N_1 N_2)^{1/6}$$

$g_s \to 0, l_s \to 0,$

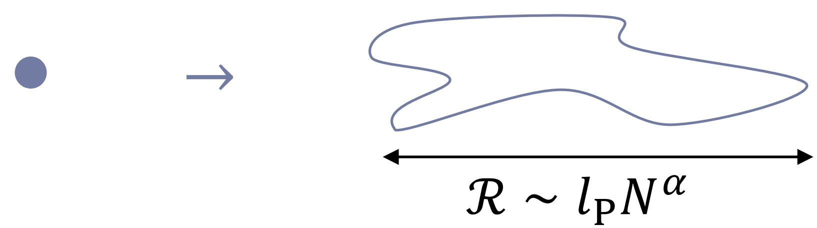
$N_{1,2} \to \infty$

Fix $Q_{1,2} \sim g_s l_s^2 N_{1,2}$

Summary:

Some BH microstates *are* represented by *microstate geometries*.

— Naive BH solutions are replaced by bubbling geometries with *finite spread*.

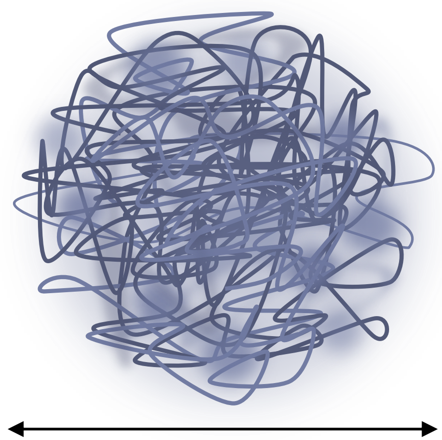


(but recall $A = 0$ so far)

Fuzzball conjecture & microstate geometry program

Maybe the same is true for genuine black holes?

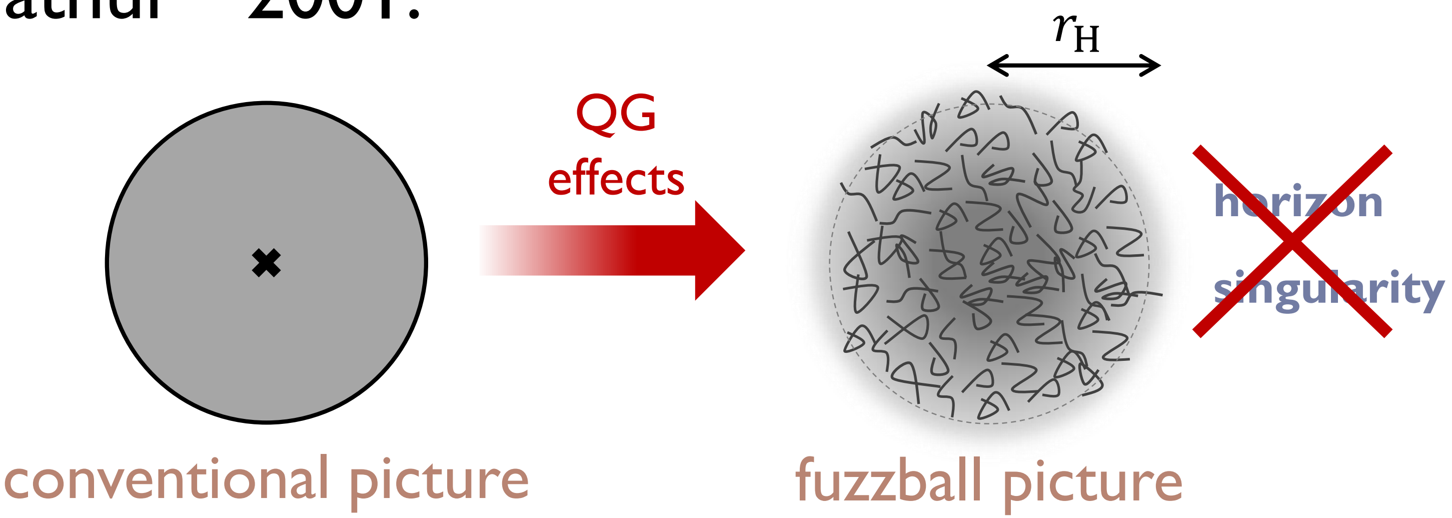
— BH microstates are some stringy configurations *spreading over a wide distance*?



$\mathcal{R} \sim l_{\mathrm{P}} N^{\alpha} \sim r_H$??

Fuzzball conjecture

- Mathur ~2001:



- BH microstates = QG/stringy "fuzzballs"
- No horizon, no singularity
- Spread over horizon scale

Sugra fuzzballs (1)

Are fuzzballs describable in sugra?

- Unlikely in general
  - General fuzzballs must involve all string modes
  - Massive string modes are not in sugra

- Hope for *supersymmetric* states
  - Massive strings break susy
    - → Only massless (sugra) modes allowed?
  - "Example": MSW (wiggling M5)

    [Maldacena+Strominger+Witten 1997]

Sugra fuzzballs (2)

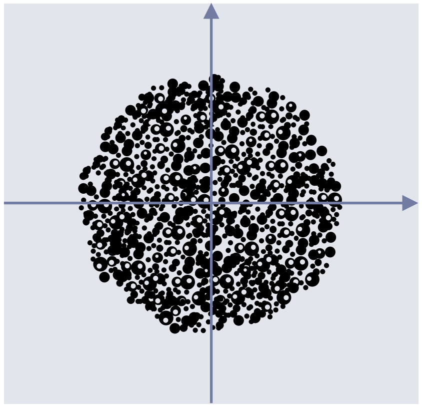
Are supersymmetric states any good?

- More tractable
  - First order PDEs
- Can tell us about *mechanism*
  - Mechanism for horizon-sized structure
- String theory objects are locally susy

Sugra fuzzballs (3)

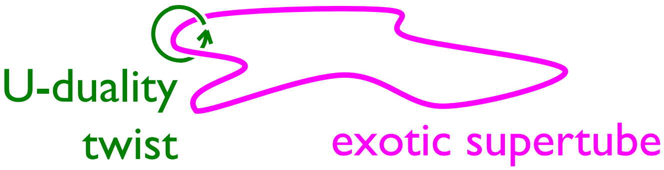
Caveats:

- Generic states have large curvature
  - Higher derivative corrections nonnegligible
  - But qualitative picture must be robust; DoF must be the same (cf. LLM)



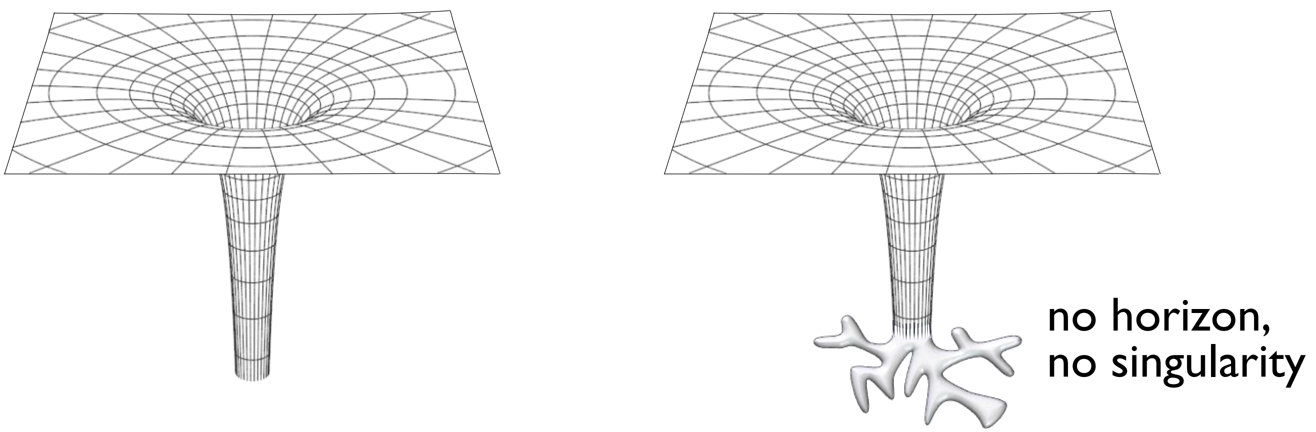
smooth, but curvature large

- Non-geometries
  - Non-geometric microstates possible [Park+MS 2015]
  - Need to extend framework (DFT, EFT)



Microstate geometry program:

What portion of the BH entropy
of (supersymmetric) BHs is accounted for
by smooth, horizonless solutions of classical sugra?



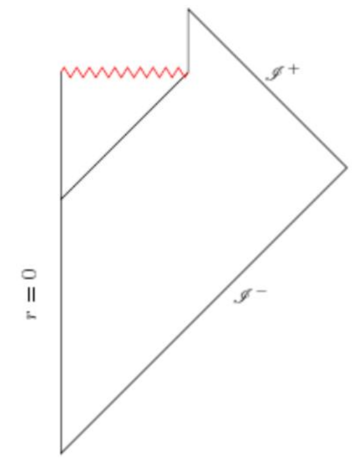
Comment: bottom-up vs. top-down

[Mathur '09] $O(1)$ deviation from flat space is needed for Hawking radiation to carry information

- Based on Q info (strong subadditivity)

[AMPS '12] "Firewall"

- Same result, same Q info (monogamy etc.)



These arguments are "bottom-up"

→ Mechanism to support finite size not explained

Microstate geometry program is "top-down"

→ Finite size supported by topology with fluxes

Microstate geometries in 5D

Setup

- $D = 5, \mathcal{N} = 1$ sugra with 2 vector multiplets

gauge fields: $A^I_\mu\,,\ I = 1,2,3. \quad F^I \equiv dA^I.$

scalars: $X^I,\ \ X^1X^2X^3 = 1$

- Action

$$S_{\text{bos}} = \int \Big(*_5 R - Q_{IJ}\, dX^I \wedge *_5\, dX^I - Q_{IJ} F^I \wedge *_5\, F^J - \tfrac{1}{6} C_{IJK} \underbrace{F^I \wedge F^J \wedge A^K}_{\text{Chern-Simons interaction}} \Big)$$

$$C_{IJK} = |\epsilon_{IJK}|\,, \qquad Q_{IJ} = \tfrac{1}{2}\text{diag}(1/X^1, 1/X^2, 1/X^3)$$

11D interpretation

- M-theory on $T^6_{56789\mathrm{A}}$

$\mathrm{A}=10$

$$ds_{11}^2 = ds_5^2 + X^1\left(dx_5^2 + dx_6^2\right) + X^2\left(dx_7^2 + dx_8^2\right) + X^3\left(dx_9^2 + dx_\mathrm{A}^2\right)$$

$$\mathcal{A}_3 = \underbrace{A^1}_{\text{M2(56)}} dx_5 \wedge dx_6 + \underbrace{A^2}_{\text{M2(78)}} dx_7 \wedge dx_8 + \underbrace{A^3}_{\text{M2(9A)}} dx_9 \wedge dx_\mathrm{A}$$

M2(56) ↕ M5(λ789A)

M2(78) ↕ M5(λ569A)

M2(9A) ↕ M5(λ5678)

BPS solutions [Gutowski-Reall '04] [Bena-Warner '04]

- Require susy

4D base $\mathcal{B}^4$ (hyperkähler)

$$ds_5^2 = -Z^{-2}(dt+k)^2 + Z\,\underbrace{ds_4^2}$$

$$A^I = \underbrace{-Z_I^{-1}(dt+k)}_{\text{elec}} + \underbrace{B^I}_{\text{mag}}, \qquad dB^I = \Theta^I$$

$$Z = (Z_1 Z_2 Z_3)^{1/3}; \quad X^1 = \left(\frac{Z_2 Z_3}{Z_1^2}\right)^{1/3} \text{ and cyclic}$$

All depends only on B_4 coordinates

- Linear system

$$\Theta^I = *_4\,\Theta^I,$$

$$\nabla^2 Z_I = C_{IJK}\; *_4\left(\Theta^J \wedge \Theta^K\right)$$

$$(1+*_4)dk = Z_I\Theta^I$$

Sol'ns with U(1) sym [Gutowski-Gauntlett '04]

Solving eqs in general is difficult.
Assume $U(1)$ symmetry in $\mathcal{B}^4$

$$ds_4^2 = V^{-1}(d\psi + A)^2 + V\underbrace{\left(dy_1^2 + dy_2^2 + dy_3^2\right)}_{\text{flat } \mathbb{R}^3},$$

(Gibbons-Hawking space)

V is harmonic in $\mathbb{R}^3$:

$$V = v_0 + \sum_p \frac{v_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

Complete solution

All eqs solved in terms of harmonic functions in $\mathbb{R}^3$:

$$H = \left(V, K^I, L_I, M\right), \qquad H = h + \sum_p \frac{Q_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

$$\Theta^I = d\left(\frac{K^I}{V}\right) \wedge (d\psi + A) - V *_3 d\left(\frac{K^I}{V}\right)$$

$$Z_I = L_I + \frac{1}{2V} C_{IJK} K^J K^K$$

$$k = \mu(d\psi + A) + \omega$$

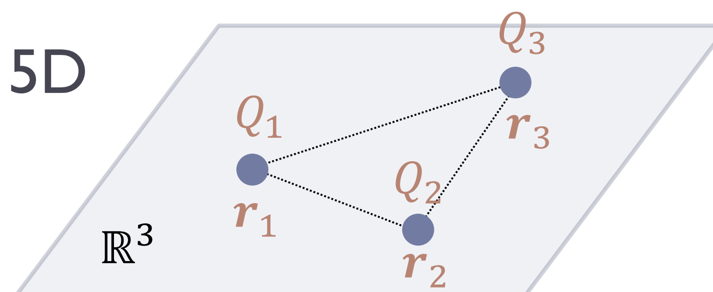
$$\mu = M + \frac{1}{2V} K^I L_I + \frac{1}{6V^2} C_{IJK} K^I K^J K^K$$

$$*_3\, d\omega = V dM - M dV + \frac{1}{2}\left(K^I dL_I - L_I dK^I\right)$$

Multi-center solution

$$H = (V, K^I, L_I, M), \qquad H = h + \sum_p \frac{Q_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

- KK monopole → V
- mag (M5) → K^I
- elec (M2) → L_I
- KK momentum along ψ → M



- Multi-center config of BHs & BRs in 5D
- Positions $\boldsymbol{r}_p$ satisfy "bubbling eq"
  (force balance)
- Reducing on ψ gives 4D BHs
  (same as Bates-Denef 2003)

Microstate geometries (1)

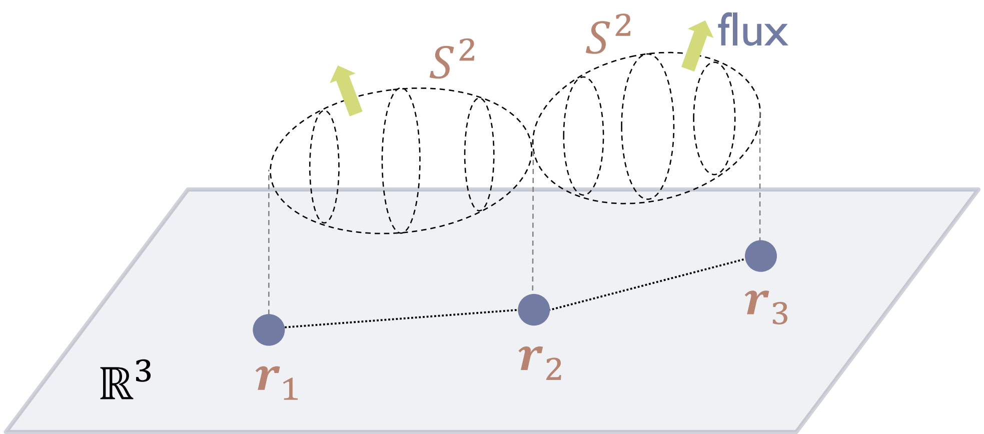
Tune charges:

$$l_p^I = -\frac{C_{IJK}}{2}\frac{k_p^J k_p^K}{v_p}$$

$$m_p = \frac{C_{IJK}}{12}\frac{k_p^I k_p^J k_p^K}{v_p^2}$$

Smooth horizonless solutions

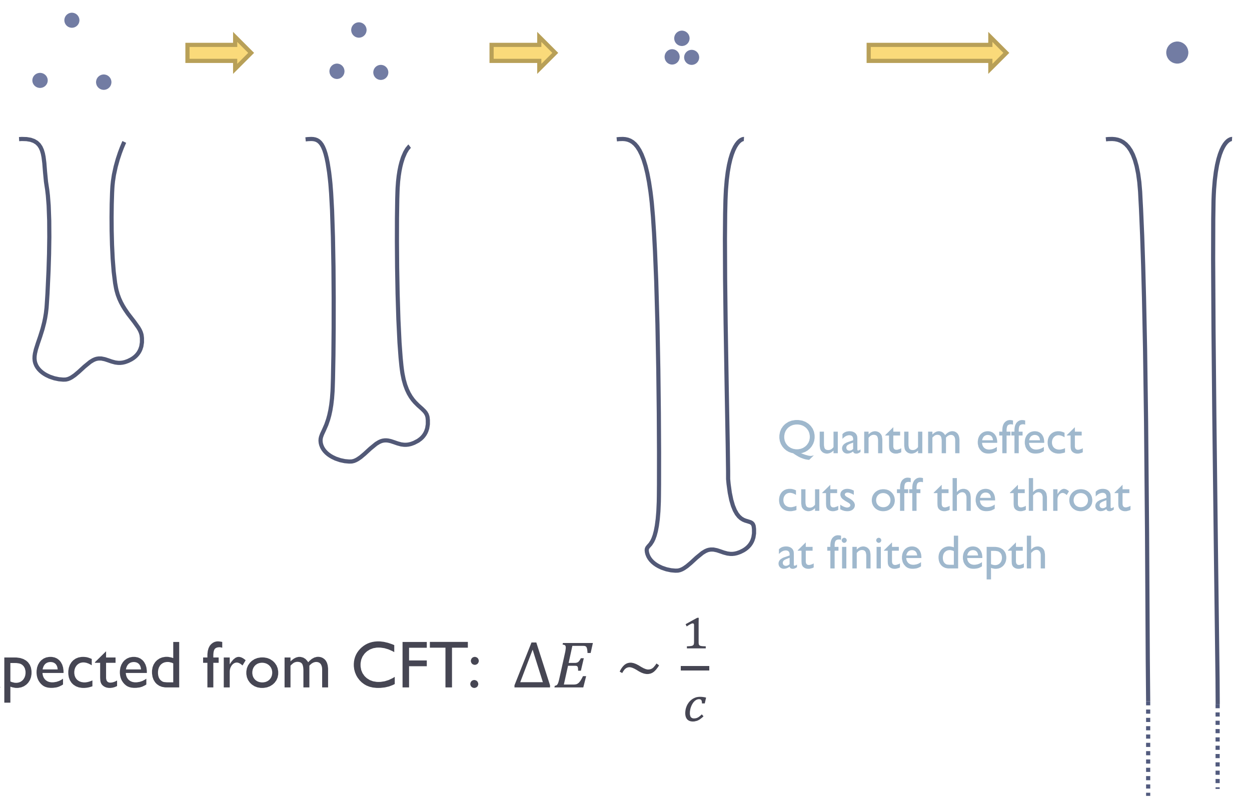
[Bena-Warner 2006] [Berglund-Gimon-Levi 2006]



- Microstate geometries for 5D (and 4D) BHs ☺
  - Same asymptotic charges as BHs
- Topology & fluxes support the soliton
- *Mechanism* to support horizon-sized structure!

Microstate geometries (2)

- Various nice properties ☺
  - Scaling solutions [BW et al., 2006, 2007]



  - Gap expected from CFT: $\Delta E \sim \frac{1}{c}$

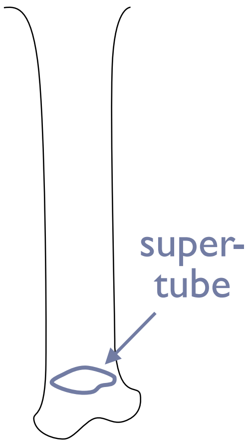
The real question:

Are there enough?

- 3-chage sys (+ fluctuating supertube)
  - Entropy enhancement mechanism [BW et al., 2008]

    → Much more entropy?
  - An estimate [BW et al., 2010]

    $$S \sim Q^{\frac{5}{4}} \ll Q^{\frac{3}{2}}$$ *Parametrically smaller* ☹



- 4-chage sys [de Boer et al., 2008-09]
  - Quantization of $D6$-$\overline{D6}$-$D0$ config → *much less entropy* ☹

<u>Summary:</u>

We found microstate geometries
for genuine BHs,
but they are *too few*.

Possibilities:

A) Sugra is not enough

B) Need more general ansatz ⇐ this talk

Microstate geometries in 6D

New hope

- 5D microstate geometries are not enough
- String theory and AdS/CFT suggest:
  - There are solutions fluctuating along 6th direction
  - They are parametrized by functions of ≥ 2 variables

"superstratum" [Bena, de Boer, Warner, MS 2010–14]

⇩

Look for superstrata in 6D sugra!

Can use $\mathrm{AdS}_3/\mathrm{CFT}_2$ as guide:

IIB on $\mathrm{AdS}_3 \times S^3 \times T^4$ ⟺ 2D CFT (D1-D5 CFT)

6D sugra

- 6D $\mathcal{N} = 2$ sugra with a vector multiplet
- Bosonic fields
  - Metric $g_{\mu\nu}$
  - Dilaton ϕ
  - 2-form B_2, field strength $G_3 = dB_2$
- IIB on T^4_{6789}:

  $\mathrm{D1(5)}$ → 1-brane coupled to B_2

  $\mathrm{D5(56789)}$ → 1-brane coupled to $\tilde{B}_2$

Susy sol'n (1): Base [Bena-Giutso-MS-Warner '11]

6D spacetime: (u, v, x^m)

u: isometry, $v \sim x^5$
x^m: 4D base

- 4D base $\mathcal{B}^4(v)$: almost hyper-Kähler

$$ds_4^2 = h_{mn}(x, v) dx^m dx^n, \quad m, n = 1,2,3,4$$

$\beta(x, v)$: 1-form (↔ KKM)

$J^{(A)}(x, v)$, $A = 1,2,3$: almost HK 2-forms

$$J^{(A)m}{}_n J^{(B)n}{}_p = \epsilon^{ABC} J^{(C)m}{}_p - \delta^{AB} \delta^m_p$$

$$d_4 J^{(A)} = \partial_v \left(\beta \wedge J^{(A)}\right), \qquad D \equiv d_4 - \beta \wedge \partial_v$$

Susy sol'n (2): Fields

- Fields on $\mathcal{B}^4$

Z_1: scalar ↔ D1(v) Z_2: scalar ↔ D5(v6789)
Θ_1: 2-form ↔ D1(λ) Θ_2: 2-form ↔ D5(λ6789)
ω: 1-form ↔ J $\mathcal{F}$: scalar ↔ P(v)

- 6D fields

$$ds_6^2 = \frac{2}{\sqrt{Z_1 Z_2}}(dv+\beta)\left(du+\omega+\frac{1}{2}\mathcal{F}(dv+\beta)\right) - \sqrt{Z_1 Z_2}\, ds_4^2$$

$$G_3 = d[-\frac{1}{2}Z_1^{-1}(du+\omega)\wedge(dv+\beta)] + \frac{1}{2} *_4 \left(DZ_2 + \dot{\beta} Z_2\right) + (dv+\beta)\wedge\Theta_1$$

$$e^{\sqrt{2}\phi} = \sqrt{Z_1/Z_2}$$

Susy sol'n (3): Linear structure

- First layer (Z, Θ)

$$D *_4 \left(DZ_I + \dot{\beta} Z_I\right) + 2D\beta \wedge \Theta_J = 0 \qquad \{I, J\} = \{1,2\}$$

$$D\Theta_J - \dot{\beta} \wedge \Theta_J - \partial_v \left[\frac{1}{2} *_4 \left(DZ_I + \dot{\beta} Z_I\right)\right] = 0 \qquad \dot{} \equiv \partial_v$$

- Second layer $(\mathcal{F}, \omega)$

$$\begin{aligned} *_4 D *_4 L = {} & \dot{Z}_1 \dot{Z}_2 + \ddot{Z}_1 Z_2 + Z_1 \ddot{Z}_2 + \tfrac{1}{2} \partial_v (Z_1 Z_2) h^{mn} \dot{h}_{mn} \\ & + \tfrac{1}{2} Z_1 Z_2 \left(h^{mn} \ddot{h}_{mn} - \tfrac{1}{2} h^{mn} \dot{h}_{np} h^{pq} \dot{h}_{qm}\right) - 2\, \dot{\beta}_m L^m - 2 *_4 (\Theta_1 \wedge \Theta_2 - \hat{\psi} \wedge D\omega) \end{aligned}$$

$$(1 + *_4) D\omega = 2(Z_1 \Theta_1 + Z_2 \Theta_2) - \mathcal{F} D\beta - 4 Z_1 Z_2 \hat{\psi}$$

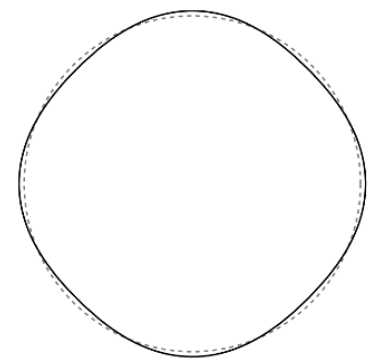
$$L \equiv \dot{\omega} + \frac{1}{2} \mathcal{F} \dot{\beta} - \frac{1}{2} D\mathcal{F} \qquad \hat{\psi} = \frac{1}{16} \epsilon^{ABC} J^{(A)mn} \dot{J}^{(B)}_{mn} J^{(C)}$$

— Linear if solved in the right order

Superstratum around $\mathrm{AdS}_3 \times S^3$ (1)

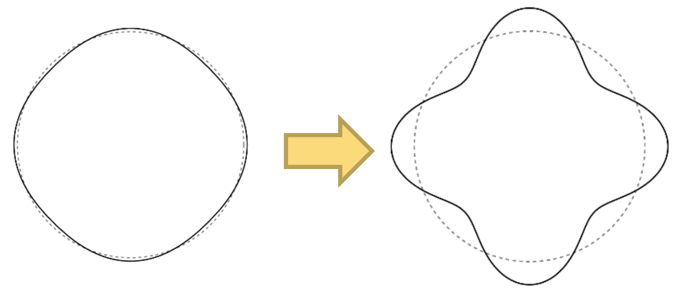
[Bena-Giusto-Russo-MS-Warner '15]

- Easiest to start from simplest background: $AdS_3 \times S^3$
- AdS/CFT dictionary for linear fluctuation known [Deger et al. '98]
  - Correspond to descendants of chiral primaries in CFT
  - Labeled by 3 quantum numbers (k, m, n)
  - "Supergraviton gas"



Superstratum around $AdS_3 \times S^3$ (2)

- Can use linear structure of 6D eqs to nonlinearly complete it



- Superposing multiple modes

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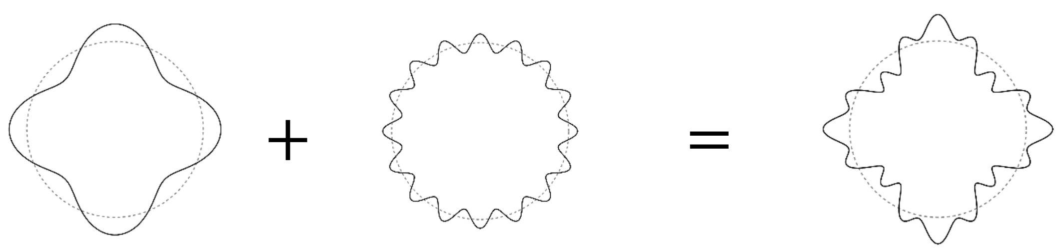
Sol's parametrized by funcs of 3 variables



- Correspond to *non-chiral* primaries in CFT

most general microstate geom with CFT dual known!

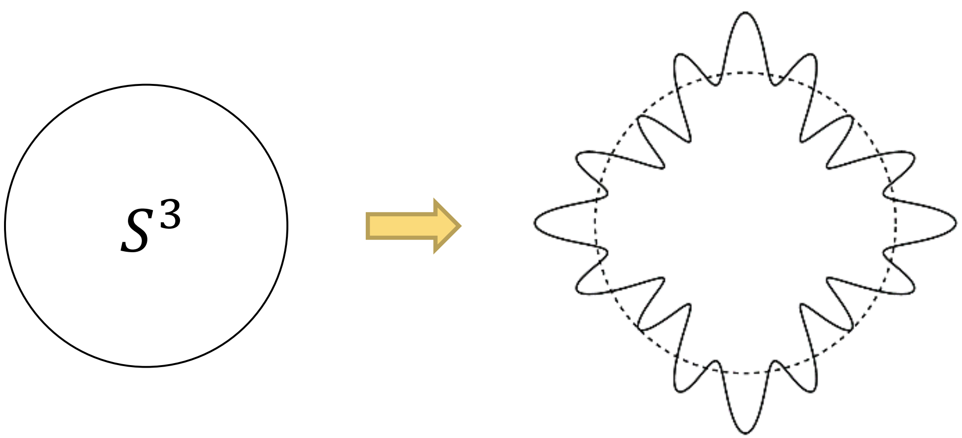
What's missing?

- Does this class of superstrata reproduce S_{BH}?

  → Not yet ☹

  These correspond to
  supergraviton gas = fluct around S^3.

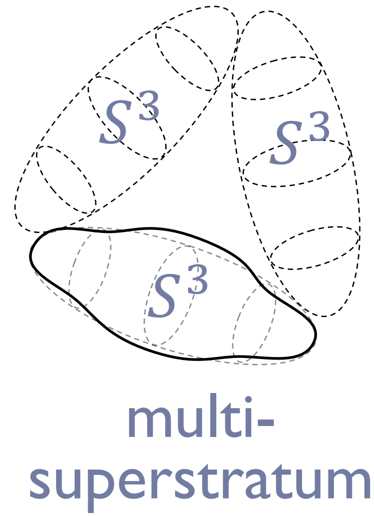
  Entropy parametrically smaller. [de Boer '98]

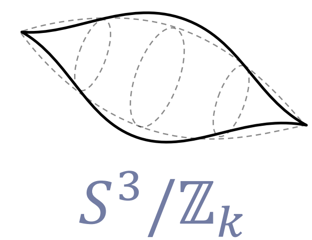


More general superstrata

Next steps:

- Other backgrounds
  - → multiple S^3's, $\mathbb{Z}_k$ orbifolds





- CFT side:
  - → Need higher and fractional modes of $SL(2,\mathbb{R})_L \times SU(2)_L$

$$(J^+_{-1})^m \,|\psi\rangle \quad \rightarrow \quad J^+_{-2}|\psi\rangle \qquad J^+_{-\frac{1}{k}} J^+_{-\frac{2}{k}}|\psi\rangle$$

Conclusions

Conclusions

- Microstate geometry program
  - Interesting enterprise elucidating micro nature of BHs, whether answer turns out to be yes or no
- Microstate geom in 5D
  - Have properties expected from CFT, but too few
- 6D: superstrata
  - A new class of microstate geometries
  - CFT duals precisely understood
  - More general superstrata are crucial to reproduce S_{BH}

Future directions

- Superstratum
  - More general solution, multi-strata
  - Count states, reproduce entropy (or not)
  - Non-geometric microstates (exotic branes, DFT/EFT)
- More
  - Non-extremal BHs
  - Information paradox
  - Observational consequences?
  - Early universe
  - …