Homological algebra of knots and BPS states

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Abstact: Homological algebra plays an important role in many areas of mathematics and mathematical physics, even such areas as "mirror symmetry" which are intrinsically geometric and at a first glance seem unrelated. In these lectures, I will explain two more applications of homological algebra, one in math and one in physicswhich in the end turn out to be actually the same. The physics problem involves study of the so-called BPS states, that is Q-cohomology of a nilpotent operator Q. surprisingly, even the basic questions, such as the change of the BPS spectrum under various deformations, have not been addressed until recently. In particular, even in the simplest of applications we shall find spectral sequences that relate the elliptic genus to chirals rings of A-model and B-model in two dimensions. These spectral sequences play an important role in physical realizations of knot homologies, where they relate (provide "bridges" between) seemingly different theories. A famous example of such connection is the relation between Khovanov homology and knot Floer homology, which is best understood by embedding both into a larger framework, motivated from physics. The goal of these lectures is to give a friendly introduction into this subject and teach some of the "tricks" which allow to do computations of rather sophisticated objects on the back of the envelope. For example, I will explain how to use these physics-based techniques to compute Khovanov homology of any knot with up to 10 crossings starting just from the Jones polynomial (and with no prior knowledge of Khovanov homology)! If time permits, I will discuss vortex moduli spaces (spaces of Hecke modifications) and how they lead to colored HOMFLY homology. The main references for these lectures will be:

http://arxiv.org/pdf/1112.0030.pdf
http://arxiv.org/pdf/1211.6075.pdf
http://arxiv.org/pdf/1512.07883.pdf