Modern Interactions between Algebra, Geometry and Physics

Workshop

Date
April 18, 2016 - April 22, 2016

Venue
TOKYO ELECTRON House of Creativity 3F, Lecture Theater, Katahira Campus, Tohoku University

Invited Speakers
Sergei Gukov (Caltech and MPI, Bonn)
Kazuki Hiroe (Josai University)
Mikhail Kapranov (IPMU, Tokyo)
Maxim Kontsevich (IHES)
Takuro Mochizuki (RIMS, Kyoto)
Ryszard Nest (Copenhagen)
Kyoji Saito (IPMU, Tokyo)
Pierre Schapira (Paris)
Yan Soibelman (Kansas State)
Yoshitsugu Takei (RIMS, Kyoto)
Boris Tsygan (Northwestern)
Daisuke Yamakawa (Tokyo Institute of Technology)

Time schedule

Date	Time	Speaker
18(Mon)	13:30 - 15:00	Pierre Schapira (Paris)
	15:00 - 15:30	Coffee break
	15:30 - 17:00	Kyoji Saito (IPMU, Tokyo)
19(Tue)	10:00 - 11:30	Maxim Kontsevich (IHES)
	11:30 - 13:30	Lunch
	13:30 - 15:00	Sergei Gukov (Caltech and MPI, Bonn)
	15:00 - 15:30	Coffee break
	15:30 - 17:00	Yoshitsugu Takei (RIMS, Kyoto)
20(Wed)	10:00 - 11:30	Daisuke Yamakawa (Tokyo Institute of Technology)
	11:30 - 13:30	Lunch
	13:30 - 15:00	Takuro Mochizuki (RIMS, Kyoto)
	15:00 - 15:30	Coffee break
	15:30 - 17:00	Yan Soibelman (Kansas State)
21(Thu)	10:00 - 11:30	Boris Tsygan (Northwestern)
	11:30 - 13:30	Lunch
	13:30 - 15:00	Kazuki Hiroe (Josai University)
	15:00 - 15:30	Coffee break
	15:30 - 17:00	Ryszard Nest (Copenhagen)
22(Fri)	10:00 - 11:30	Mikhail Kapranov (IPMU, Tokyo)

Title and Abstract

- Pierre Schapira (Paris)

Title: Subanalytic topologies and filtrations on the sheaf of holomorphic functions

Abstract: In [KS01], we have constructed the subanalytic topology M_{sa} on a real analytic manifold M and applied it to construct the sheaf of holomorphic functions with temperate growth on a complex manifold.

In [GS16], we introduce the linear subanalytic topology M_{sal} and the morphism of sites $\rho_{\mathrm{sal}}\colon M_{\mathrm{sa}} \to M_{\mathrm{sal}}$. The derived direct image functor $\mathrm{R}\rho_{\mathrm{sal}*}$ admits a right adjoint $\rho_{\mathrm{sal}}^{!}$ which allows us to associate functorially a sheaf (in the derived sense) on M_{sa} to a presheaf on M_{sa} satisfying suitable properties, this sheaf having the same sections as the presheaf on any open set with Lipschitz boundary.

This construction applies to various presheaves on real manifolds, such as the presheaves of functions with temperate growth of a given order at the boundary or the Sobolev presheaves (a work of G. Lebeau). On a complex manifold, we get various sheaves of holomorphic functions with growth. As an application, we can endow the sheaf of holomorphic functions with various filtrations, in the derived sense, and, using the Riemann-Hilbert correspondence, we can endow functorially regular holonomic $\mathscr{D}$-modules with filtrations. (Joint work with Stéphane Guillermou.)

References

[GS16] Stéphane Guillermou and Pierre Schapira, *Construction of sheaves on the subanalytic site*, Astérisque **234** (2016), available at `arXiv:1212.4326`.

[Kas03] Masaki Kashiwara, *D-modules and Microlocal Calculus*, Translations of Mathematical Monographs, vol. 217, American Math. Soc., 2003.

[KS01] Masaki Kashiwara and Pierre Schapira, *Ind-sheaves*, Astérisque, vol. 271, Soc. Math. France, 2001.

- Kyoji Saito (IPMU, Tokyo)

Title: Dual Artin monoids and zero loci of their skew-growth functions

Abstract: Associated with a finite root system of type P, we consider three combinatorial structures: a dual Artin monoid, a lattice of non-crossing partitions and a cluster complex of finite type. For each structure, we associate certain generating functions: the skew growth function for the dual Artin monoid, the generating function of Moebius invariants of the non-crossing partition lattice and the generating function of dimensions of faces of the cluster complex. Owing to several authors, it is known that all of them give the same polynomial. Our interest and goal is the study of the zero loci of the polynomial. Namely, we show that it has exactly rank of P simple real roots on the interval (0,1]. The proof

is based on a strange fact that the polynomial behaves similarly to orthogonal polynomials, and sometimes may be expressed by using Jacobi polynomials. But we don't have conceptual understanding of this phenomenon (it resembles mirror symmetry). A similar result on the zeros of polynomials for Artin monoids is also observed and conjectured, but it is still open.

- Maxim Kontsevich (IHES)

Title: Resurgence and wall-crossing via complexified path integral

Abstract: I will explain how various instances of resurgence can be explained through analysis of semi-infinite Lefschetz thimbles in complexified path integral in quantum mechanics. The case of zero action give rise to Gaiotto-Moore-Neitzke calculus of spectral networks, while for the case of action quadratic in momenta one obtains fine resurgence properties of heat kernels.

- Sergei Gukov (Caltech and MPI, Bonn)

Title: Mock modularity and categorification of 3-manifold quantum group invariants

Abstract: A long-standing problem of categorifying RTW invariants of 3-manifolds faces an immediate challenge that, unlike invariants of knots, these invariants exhibit no obvious integrality and are not even valued in C[[q]]. Using insights from physics, we first turn RTW invariants of 3-manifolds into q-series with integer coefficients, providing a natural home to previous mysterious role of Eichler integral in Chern-Simons theory. This builds on work of Lawrence-Zagier, Hikami, Ohtsuki, and others. Then, the same physical setup that in the past led to a realization of knot Floer homology and Khovanov-Rozansky homology gives us the desired categorification of Z[[q]]-valued transforms of RTW invariants. Performing explicit computations is not a problem and will be demonstrated in many concrete examples.

- Yoshitsugu Takei (RIMS, Kyoto)

Title: Exact WKB analysis for continuous and discrete Painlevé equations — Stokes geometry, connection formula and wall-crossing formula

Abstract: Generalizing the exact WKB analysis for one-dimensional Schrödinger equations established by Voros, Pham, Delabere and others, Aoki, Kawai and I developed the exact WKB analysis for continuous Painlevé equations and clarified, in particular, their Stokes geometry and connection formula. Later Iwaki discussed the wall-crossing formula for Painlevé equations as well. In this talk, after reviewing these previous works, I would like to talk about my recent research jointly done in part with N. Joshi (Sydney) on the exact WKB analysis for discrete Painlevé equations that are obtained from continuous Painlevé equations through the Bäcklund transformation. In the analysis of such discrete Painlevé equations both connection formula and wall-crossing formula for continuous Painlevé equations play the same role and appear as different kinds of connection formula.

- Daisuke Yamakawa (Tokyo Institute of Technology)

Title: Twisted wild character varieties

Abstract: This is joint work with Philip Boalch. The Riemann-Hilbert-Birkhoff correspondence gives a category equivalence between meromorphic connections on a compact Riemann surface with prescribed irregular classes and the so-called Stokes/monodromy data. The moduli spaces of Stokes/monodromy data are called the wild character varieties, and known to have canonical Poisson structures when the irregular classes are untwisted (shown by Boalch). In this talk we will extend the construction to the twisted case using a slight extension of quasi-Hamiltonian geometry.

- Takuro Mochizuki (RIMS, Kyoto)

Title: Asymptotic behaviour of certain families of harmonic bundles on Riemann surfaces

Abstract: Let $(E, \overline{\partial}_E, \theta)$ be a stable Higgs bundle of degree 0 on a compact connected Riemann surface. Once we fix a flat metric $h_{\det(E)}$ on the determinant of E, we have the harmonic metrics h_t $(t > 0)$ for the stable Higgs bundles $(E, \overline{\partial}_E, t\theta)$ such that $\det(h_t) = h_{\det(E)}$. In this talk, we will discuss two results on the behaviour of h_t when t goes to ∞. First, we show that the Hitchin equation is asymptotically decoupled under some asumption for the Higgs field. We apply it to the study of the so called Hitchin WKB-problem. Second, we discuss the convergence of the sequence $(E, \overline{\partial}_E, \theta, h_t)$ in the case where the rank of E is 2. We explain a rule to determine the parabolic weights of a "limiting configuration", and we show the convergence of the sequence to the limiting configuration in an appropriate sense.

- Yan Soibelman (Kansas State)

Title: Riemann-Hilbert correspondence for difference equations in higher dimensions.

Abstract: I am going to discuss a conjectural Riemann-Hilbert correspondence for holonomic modules over quantum tori. The approach is based on the ideas of Floer theory and tropical geometry. It is a part of the project "Holomorphic Floer theory", joint with Maxim Kontsevich.

- Boris Tsygan (Northwestern)

Title: A microlocal category associated to a symplectic manifold

Abstract: For a symplectic manifold with some topological conditions, we define an infinity category enriched in infinity local systems of modules over the Novikov ring. The construction is based on the category of modules over an extension of the Fedosov deformation quantization, and should be viewed as the De Rham side of the microlocal category defined recently by Tamarkin.

- Kazuki Hiroe (Josai University)

Title: On additive Deligne-Simpson problem

Abstract: The "multiplicative" Deligne-Simpson problem was proposed by P. Deligne and C. Simpson, which asks the existence of irreducible linear monodromy representations of the Riemann sphere with holes after fixing isomorphic classes of local monodromies. V. Kostov considered the additive analogue of this problem, so-called "additive" Deligne-Simpson problem which asks the existence of irreducible Fuchsian systems on the Riemann sphere with fixed local isomorphic classes. This additive analogue can be rephrased as the nonemptiness of the moduli space of stable meromorphic connections with logarithmic singularities on the trivial bundle. For the geometry of this moduli space, W. Crawley-Boevey's work giving a realization of the moduli space as a quiver variety was remarkable and gave a complete answer to the additive Deligne-Simpson problem. In this talk, we will discuss a generalization of the additive DS problem for non-Fuchsian systems with unramified irregular singularities. After the works of Crawley-Boevey, P. Boalch, D. Yamakawa, and so on, an embedding of the moduli space of these non-Fuchsian systems into a quiver variety will be explained. Thanks to this embedding, the connectedness of the moduli space can be shown and also the nonemptiness of the moduli space, which is equivalent to our generalized additive DS problem, is determined.

- Ryszard Nest (Copenhagen)

Title: On analytic construction of the group three-cocycles

Abstract: One of the most important group cocycles is the two-cocycle on the restricted general linear group of a polarised Hilbert space (H, H_+). It has a wide range of applications, like the central extensions of the loop group, theory of Toeplitz operators, gauge theory or invariants of the K_2^{alg}. This two-cocycle can be seen as a two-cocycle associated to the action of the group $GL_{res}(H, H_+)$ on the category of subspaces $K \subset H$ such that the product of orthogonal projections

$$P_{H_+} P_K \colon K \to H_+$$

is in $\mathcal{L}^2(H)$. Morphisms in this category are given by lines $Det(P_{K_1} P_{K_2})$. Similarly, given an action of a group G on an n-category satisfying certain conditions, one can construct a $(n+1)$-cocycle on G. A well known example is the n-Tate space, essentially an algebra of the form $K = k((s1))((s2)) \ldots ((sn))$, where the group is the group of invertibles in K and the n-category structure comes from the natural filtration of K.

The corresponding cocycles, when evaluated on $K_{n+1}^{alg}(K)$, reproduce the Tate tame symbol. However, the constructions are purely algebraic and do not seem to extend to the analytic context, as in the case of $n = 1$.

In this talk we will sketch a construction of a (family of) two-category associated to a pair of commuting idempotents P and Q on a Hilbert space and construct the associated three cocycle on the associated groups. For example, in the case of a two-Tate space, this produces an extension of the Tate symbol from $\mathbb{C}((z1))((z2))$ to, say, $C^\infty(\mathbb{T}^2)$, but also a corresponding invariant of K_3^{alg} of the non-commutative torus $C^\infty(\mathbb{T}^2_\theta)$.

The construction is based on the properties of the determinant of Fredholm operators, in particular on the existence of the canonical perturbation isomorphism $Det(T) \simeq Det(S)$ whenever T and S are two Fredholm operators satisfying $T - S \in \mathcal{L}^1(H)$.

This is a joint work with Jens Kaad and Jesse Wolfson.

- Mikhail Kapranov (IPMU, Tokyo)

Title: Some remarks on D-modules with a large parameter and their Stokes geometry.

Abstract: The classical WKB analysis of linear PDE with a large parameter t amounts, algebraically, to studying modules over the ring $D_X[t]$ and equipping this ring with the non-standard filtration in which $\deg(t) = 1$. This filtration re-defines principal symbols of operators and characteristic varieties of modules which become (after setting $t = 1$), non-homogeneous coisotropic subvarieties $L \subset T^*X$ ("spectral curves", if $\dim(X) = 1$).

For holonomic $D_X[t]$-modules these characteristic varieties are non-homogeneous Lagrangian and dictate the shape of WKB solutions. Higher-dimensional examples include Frobenius manifolds (pencils of flat connections), A-hypergeometric systems and systems for characters and spherical functions in representation theory. (The latter two types of systems involve free parameters which can be made to depend on t.)

I will discuss the possible geometry of Stokes "walls" for holonomic $D_X[t]$-modules, in particular, the higher-dimensional analogs of "new turning points" and their relations to syzygies among elementary matrices generating the group $SL_n(\mathbf{Z})$.