Special subspaces in symplectic vector spaces
(colloquium and geometry seminar talks)

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In a vector space V carrying a symplectic (i.e. nondegenerate, skew-symmetric) bilinear form ω, each subspace A has a **symplectic orthogonal** space A^ω consisting of those elements v for which $\omega(v, w) = 0$ whenever $w \in A$. Subspaces for which the intersection of A with A^ω is equal to A, A^ω, or $\{0\}$ are especially important; they are called **isotropic**, **coisotropic**, or **symplectic** respectively. When both of the first conditions hold, the subspace is called **lagrangian**.

Linear maps $(V, \omega_V) \leftarrow (W, \omega_W)$ whose graphs in the product $(V \times W, \omega_V \times -\omega_W)$ are isotropic, coisotropic, or lagrangian also have a special importance; they are symplectic embeddings, Poisson submersions, or symplectic isomorphisms respectively. It is also useful to look at subspaces of these types in the product which may not be graphs of mappings. Those which are lagrangian are called **canonical relations** or **lagrangian correspondences** and are of particular importance as the morphisms in symplectic categories.

In a manifold M carrying a closed non-degenerate 2-form, submanifolds whose tangent spaces are of the distinguished types above are the subject of many interesting problems, both solved and unsolved. Even at the linear level, there are still important problems in both finite and infinite-dimensional (for field theory) cases, having to do with classification of (k-tuples of) subspaces, composition of relations, and quantization. All of these problems can be formulated within a general problem of studying representations of quivers (objects consisting of vertices and arrows between them) by symplectic vector spaces and special relations.

In the colloquium talk, I will describe the basic structures and problems, and some of the solutions. The second talk will begin with a quick review of the first one, followed by more details.