Modern Interactions between Algebra, Geometry and Physics

Workshop on Symplectic Geometry and Physics - Floer theory and Hamiltonian dynamics -

Date

May 23, 2016 - May 27, 2016

Venue

TOKYO ELECTRON House of Creativity 3F, Lecture Theater, Katahira Campus, Tohoku University

Invited Speakers

Viktor Ginzburg (UC Santa Cruz)
Basak Gurel (Florida)
Ely Kerman (University of Illinois at Urbana-Champaign)
Andrei Pajitnov (Nantes)
Roman Golovko (Budapest)
Doris Hein (Freiburg)
Youngjin Bae (IBS Center for Geometry and Physics, POSTECH)
Jean Francois Barraud (Universite Paul-Sabatier, Toulouse)
Morimichi Kawasaki (IBS Center for Geometry and Physics, POSTECH)
Yoshihiro Sugimoto (Kyoto)
Manabu Akaho(Tokyo Metropolitan University)
Ryuma Orita (Tokyo)

Organizers

Kaoru Ono (RIMS, Kyoto)
Reiko Miyaoka (Tohoku)

Time schedule

Date	Time	Speaker
23(Mon)	9:30 - 11:00	Viktor Ginzburg I (UC Santa Cruz)
	11:15 - 12:15	Manabu Akaho (Tokyo Metropolitan University)
	12:15 - 13:30	Lunch
	13:30 - 15:00	Basak Gurel I (University of Central Florida)
	15:30 - 16:30	Morimichi Kawasaki (IBS Center for Geometry and Physics)
24(Tue)	9:30 - 11:00	Viktor Ginzburg II (UC Santa Cruz)
	11:15 - 12:15	Roman Golovko I (Budapest)
	12:15 - 13:30	Lunch
	13:30 - 15:00	Ely Kerman I (University of Illinois at Urbana-Champaign)
	15:30 - 16:30	Yoshihiro Sugimoto (Kyoto)
25(Wed)	9:30 - 11:00	Viktor Ginzburg III (UC Santa Cruz)
	11:15 - 12:15	Jean Francois Barraud (Universite Paul-Sabatier, Toulouse)
	12:15 - 13:30	Lunch
	13:30 - 15:00	Basak Gurel II (University of Central Florida)
	15:30 - 16:30	Andrei Pajitnov (University of Nantes)
	16:45 - 17:45	Ryuma Orita (Tokyo)
	18:00 -	Reception (Restaurant Hagi)
26(Thu)	9:30 - 11:00	Viktor Ginzburg IV (UC Santa Cruz)
	11:15 - 12:15	Doris Hein (University of Freiburg)
	12:15 - 13:30	Lunch
	13:30 - 15:00	Ely Kerman II (University of Illinois at Urbana-Champaign)
	15:30 - 16:30	Youngjin Bae I (IBS Center for Geometry and Physics)
27(Fri)	10:00 - 11:00	Roman Golovko II (Budapest)
	11:15 - 12:15	Youngjin Bae II (IBS Center for Geometry and Physics)

A few short talks by young reseachers may be added.

Title and Abstract

- Viktor Ginzburg (UC Santa Cruz)

Title: Periodic Points of Hamiltonian Systems: the Conley Conjecture and Beyond

Abstract: One distinguishing feature of Hamiltonian dynamical systems is that such systems, with very few exceptions, tend to have numerous periodic points. In 1984 Conley conjectured that a Hamiltonian diffeomorphism of a torus has infinitely many periodic points. This conjecture was established by Hingston some twenty years later in 2004, and by now the Conley conjecture has been proved for a broad class of symplectic manifolds by Floer theoretic methods. Yet the conjecture does not hold unconditionally for all symplectic manifolds as the example of an irrational rotation of the 2-sphere shows. In fact, the conjecture also fails for many "nice" symplectic manifolds, e.g., for the projective spaces and the Grassmannians.

In this series of lectures we will outline the proof of the Conley conjecture and the relevant symplectic topological techniques. We will discuss some related results such as the generic existence of infinitely many periodic orbits, the manifolds for which the conjecture fails and the analogs of the conjecture in this case. We will also examine how the methods and results from Hamiltonian dynamics translate to the realm of Reeb flows and connect with questions from differential geometry and "classical calculus of variations".

- Manabu Akaho(Tokyo Metropolitan University)

Title: Symplectic displacement energy for exact Lagrangian immersions

Abstract: Displaceability or non-displaceability is interesting phenomenon in symplectic topology. For example, the Arnold conjecture asks the non-displaceability of Lagrangian submanifolds under Hamiltonian isotopies. In this talk, we discuss the displacement energy of exact Lagrangian immersions. More precisely, we show an lower bound of the displacement energy by the symplectic area of punctured holomorphic disks. Our approach is based on some version of Floer homology of Lagrangian immersions and Chekanov's homotopy technique of continuations.

- Basak Gurel (University of Central Florida)

Title: Non-contractible periodic orbits in Hamiltonian dynamics on closed symplectic manifolds

Abstract: The main theme of these two lectures is the question of the existence of non-contractible periodic orbits for Hamiltonian dynamical systems on closed symplectic manifolds, one of the least explored directions in symplectic topology. We will discuss several recent results and some conjectures concerning the existence of such orbits, relating them to other phenomena such as the Conley conjecture. We will examine the relevant aspects of Floer theory and introduce some new tools as well. We will show that, in a variety of settings, the presence of one non-contractible periodic orbit implies the existence of infinitely many such orbits in a specific collection of free homotopy classes.

- Morimichi Kawasaki (IBS Center for Geometry and Physics)

Title: Non-contractible orbits found by the Floer theory on contractible orbits

Abstract: Biran, Polterovich and Salamon defined a relative symplectic capacity which indicates existence of non-contractible trajectories(orbits) of certain Hamiltonian isotopies(flows). On the other hand, Entov and Polterovich defined heaviness for subsets of symplectic manifolds in terms of the Hamiltonian Floer theory on contractible trajectories. We give an upper bound of the Biran-Polterovich-Salamon capacity of heavy subsets.

- Roman Golovko (Budapest)

Title: First talk:
On Legendrian submanifolds, exact Lagrangian cobordisms and the homological Arnold chord conjecture I

Second talk:
On Legendrian submanifolds, exact Lagrangian cobordisms and the homological Arnold chord conjecture II

Abstract: We will discuss some obstructions to the existence of exact Lagrangian cobordisms in the symplectization of the contactization of a Liouville manifold, and some constructions of exact Lagrangian cobordisms. In addition, we will discuss the role of exact Lagrangian fillings in the linearization of a Chekanov-Eliashberg

DGA and in the proof of the homological Arnold chord conjecture.

Then, extending the work of Ekholm-Etnyre-Sullivan and Ekholm-Etnyre-Sabloff, we will prove the homological Arnold chord conjecture for a chord-generic horizontally displaceable Legendrian submanifold L of the contactization of a Liouville manifold with the property that its Chekanov-Eliashberg DGA admits a finite-dimensional matrix representation. Moreover, if L admits an exact Lagrangian filling, we prove that the number of Reeb chords on L is bounded from below by the stable Morse number of the filling. In general, this bound is stronger than the bound coming from the homological Arnold chord conjecture. We will also discuss some additional bounds whose proof is based on the machinery developed by Ono-Pajitnov. This is joint work with Georgios Dimitroglou Rizell.

- Ely Kerman (University of Illinois at Urbana-Champaign)

Title: Hamiltonian Floer theory and a theorem of Ekeland and Lasry

Abstract: A classic rigidity theorem of Ekeland and Lasry establishes the existenceof multiple closed characteristics on convex hypersurfaces that aresuitably pinched between two spheres. I will describe how Hamiltonian Floer theory can be used to both recover this result and to generalize this rigidity phenomenon to Reeb flows on any closed contact manifold.

- Yoshihiro Sugimoto (Kyoto)

Title: Hofer's metric and wrapped Floer homology

Abstract: In this talk, I will explain a relationship between displaceability and vanishing of wrapped Floer homology or symplectic homology. I also explain an application of this relationship to the Hofer's metric of the space of Lagrangian submanifolds.

- Jean Francois Barraud (Universite Paul-Sabatier, Toulouse)

Title: The Floer fundamental group for monotone Lagrangian submanifolds

Abstract: I will explain how the fundamental group of monotone (with minimal maslov number at least 3) Lagrangian submanifolds can be recovered from Floer theoretic objects and discuss some applications, limitations and perspectives.

While most of the symplectic invariants derived from pseudo holomoprhic curves rest on 0 dimensional moduli spaces and a counting process, there are situations where higher dimensional moduli spaces do contain information that can not be reached through 0 dimensional ones. In particular, I will explain how a closer look at 1 dimensional moduli spaces can enrich the familiar homology relation $\partial^2 = 0$ to keep track of some cyclic order among the generators and give access to the fundamental group beyond the first homology group. This point of view leads to a reformulation of the classical construction of the fundamental group in Morse theory in terms of moduli spaces, which extends to the Floer setting. I will discuss the new light this construction sheds on the Arnold conjecture regarding Lagrangian intersections or fixed points of Hamiltonian flows, as well as a (partial!) application to exact Lagrangian cobordism (this is joint with Lara Suarez). Finally, the construction of natural morphisms among such fundamental groups is both a crucial and a non trivial problem, which I may also evoque if not out of time.

- Andrei Pajitnov (University of Nantes)

Title: Arnold conjecture, Floer chain complexes, and the augmentation ideals of finite groups.

Abstract: Let H be a time-dependent Hamiltonian H on a closed symplectic manifold M. Assume that the periodic orbits of the corresponding vector field are non-degenerate. The Arnold Conjecture says that the number P(H) of the periodic orbits is not less than the Morse number of M. The Floer chain complex, associated to the pair (M,H) allows in many cases to prove that P(H) is not less than the sum of Betti numbers of M.

We construct a refined version of the Floer chain complex associated to (M,H) and any regular covering of M. This chain complex gives rise to a sequence of numerical invariants, which provide new lower bounds for the number of periodic orbits of H.

For the case of finite fudamental group the first of these invariants is computable from the augmentation ideal of the group ring. Using these invariants we prove in particular that if the fundamental group of M is finite and solvable or simple, then P(H) is not less than the minimal number of generators of the fundamental group. This is joint work with Kaoru Ono.

- Ryuma Orita (Tokyo)

Title: Non-contractible periodic orbits in Hamiltonian dynamics on tori

Abstract: We show that the presence of a non-contractible periodic orbit in tori yields the existence of infinitely many periodic orbits. This was an open question by V. Ginzburg and B. Gurel who proved that the same arguments hold for atoroidal and toroidally monotone cases.

- Doris Hein (University of Freiburg)

Title: Morse theory with symmetries and applications

Abstract: Local behavior of contact dynamics can be studied using Morse homology of a discrete action functional of Hamiltonian dynamics. For iterated periodic orbits, time shift presents a symmetry and the interesting part is the invariant Morse homology. In the degenerate case, we construct an invariant local perturbation of the action for which Morse homology can be defined and we can still see critical points as generators. This gives rise to local homological invariants of Reeb orbits where generators of local homology are still periodic orbits. In this talk, I will define the discrete action functional and discuss the Morse theory with symmetries with a focus on invariant perturbations.

- Youngjin Bae (IBS Center for Geometry and Physics)

Title: Rabinowitz Floer homology and symplectic deformations

Abstract: In these lectures, I will introduce Rabinowitz Floer homology(RFH) which is useful in the study of fixed energy hypersurfaces of a given dynamical system. After giving computational results for RFH,

we discuss about symplectic deformations in RFH which can be interpreted as magnetic effects on a given dynamical system. A concept of (magnetic) Leafwise intersections and its growth rate will be also discussed.