Floer fundamental group

of Lagrangian submanifolds

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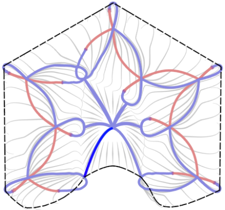
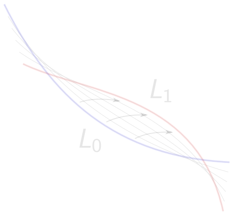


Settings

- (M, ω) : closed monotone symplectic manifold
- $L \subset M$: Lagrangian submanifold.
  - $\omega_{|\pi_2(M,L)} = 0$ (or $N_L > \dim L + 1$).
  - [will discuss the monotone case later].
- $\star \in L$: base point
- Auxiliary data :
  - $H : M \times [0,1] \to \mathbb{R}$,
  - J : (time dependent) ω-compatible almost complex structure,
  - cutoff functions...



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$$L_t = \phi_H^t(L)$$

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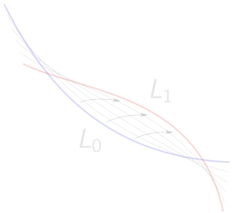

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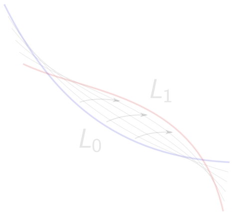

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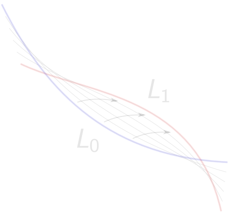

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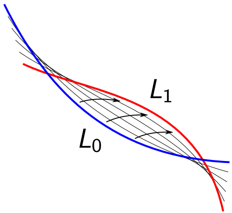

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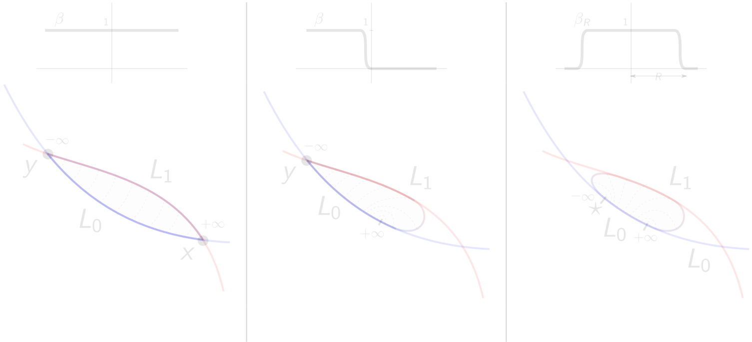
Moduli spaces

Recall the Floer equation for $u : \mathbb{R} \times [0,1] \to M$:

$$\frac{\partial u}{\partial s} + J_t(u)\frac{\partial u}{\partial t} = 0 \qquad \text{and} \qquad \begin{cases} u(s,0) \in L_0 \\ u(s,1) \in L_1 \end{cases}$$



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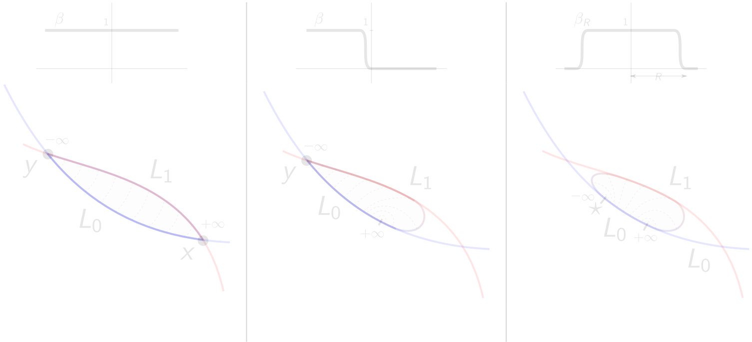

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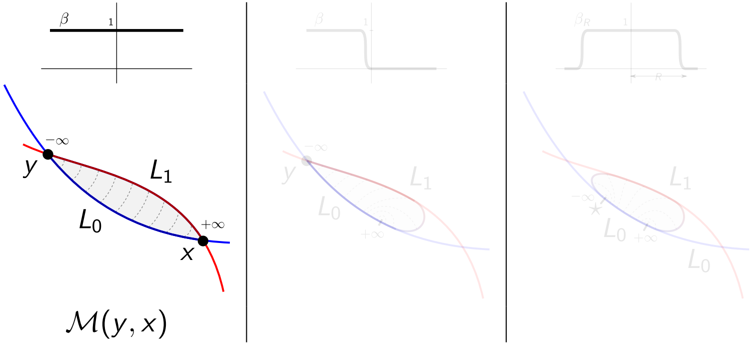

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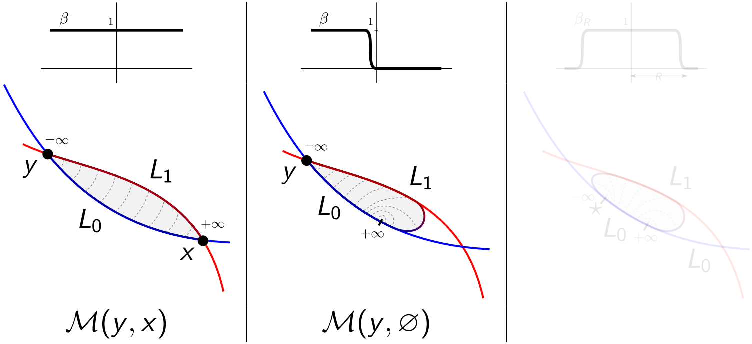

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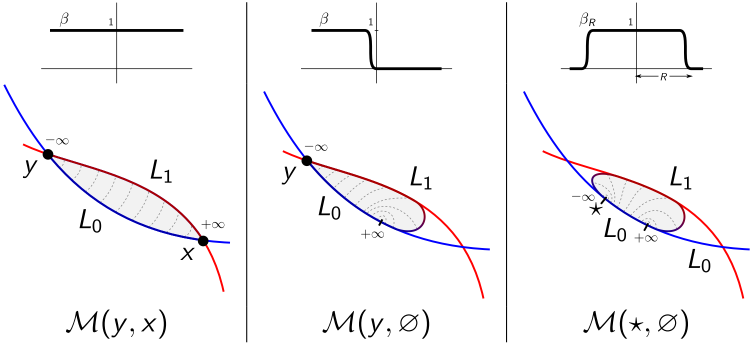

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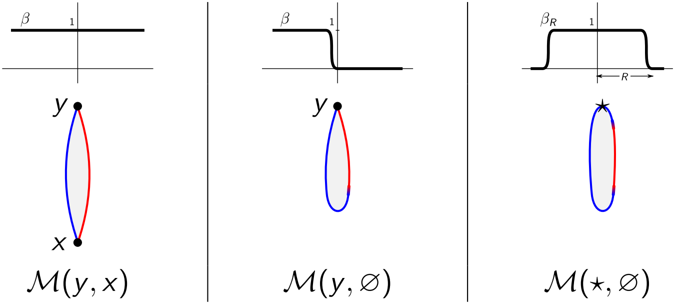

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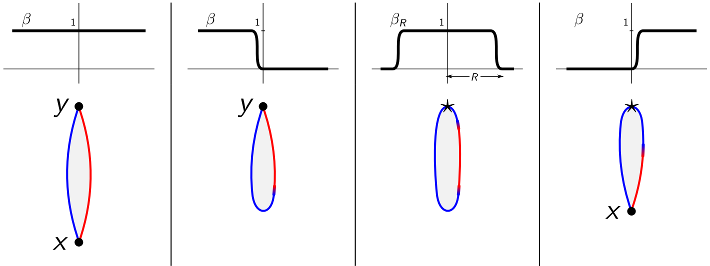

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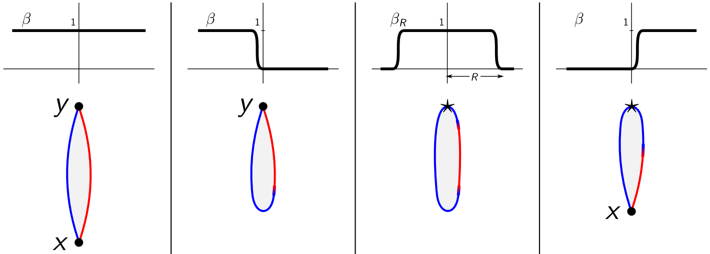

$\mathcal{M}(y,x)$	$\mathcal{M}(y,\varnothing)$	$\mathcal{M}(\star,\varnothing)$	$\mathcal{M}(\star,x)$
$\dim = \lvert y\rvert - \lvert x\rvert - 1$	$\dim = \lvert y\rvert$	$\dim = 1$	$\dim = -\lvert x\rvert$

Floer steps

Definition

A Floer step is an (oriented) component with non empty boundary of one of the moduli spaces $\mathcal{M}(y, \varnothing)$ for $|y| = 1$ or $\mathcal{M}(\star, \varnothing)$.

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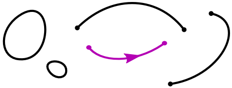

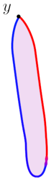

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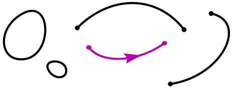
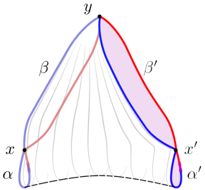
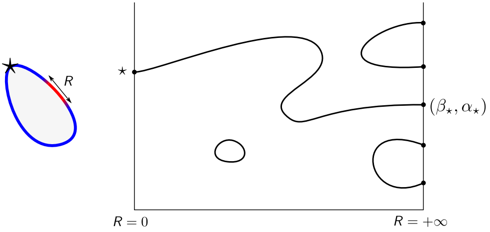




Base point

What about $\mathcal{M}(\star, \varnothing)$?



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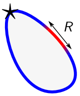

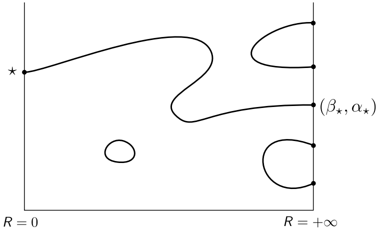

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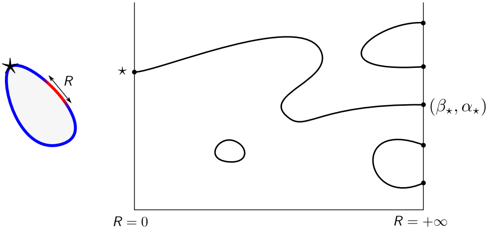

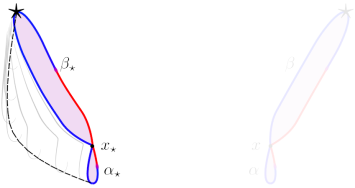

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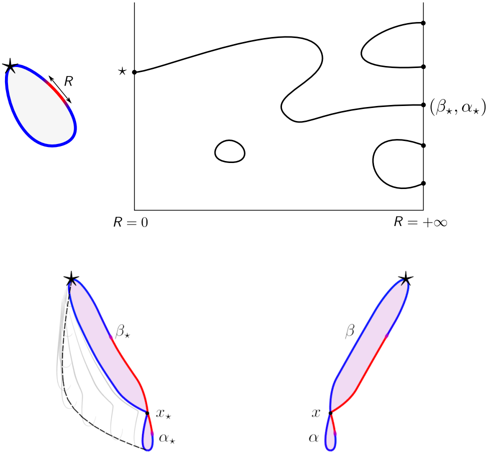

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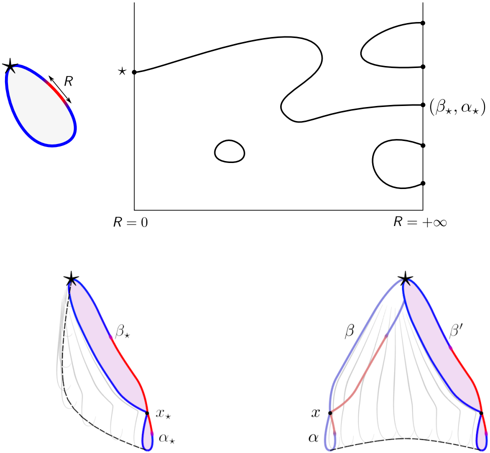

Floer Loops

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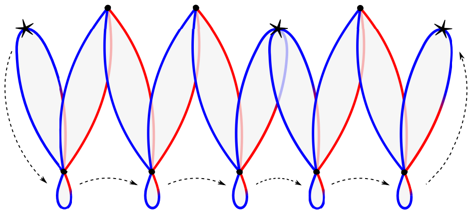
A Floer based loop is a sequence of consecutive Floer steps, that start and end at $\star$.



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$$\mathcal{M}(y,\emptyset) \xrightarrow{\text{ev}} L$$

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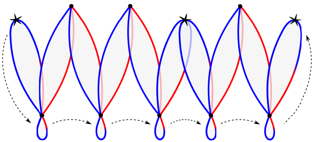

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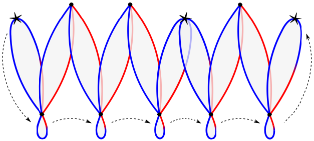

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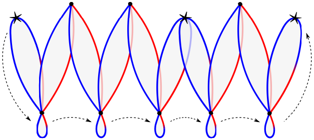

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$$\begin{array}{ccc} \mathcal{L}_H^{(F)}(L,\star) & \xrightarrow{\text{ev}} & \pi_1(L,\star) \\ (u_\tau) & \longmapsto & (u_\tau(+\infty)) \end{array}$$

Main statement

Theorem

The group morphism $\mathcal{L}_H^{(F)}(L,\star) \xrightarrow{ev} \pi_1(L,\star)$ *is onto.*

Postpone

- the proof
- the description of the relations

to first revisit the Morse case...

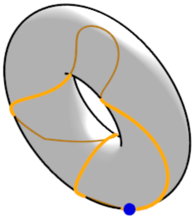
The Morse case

Let

- $f : L \to \mathbb{R}$ be Morse function (with a single minimum),
- g be a metric on L such that (f, g) is Morse-Smale,

Definition

$\mathcal{L}_f^{(M)}(L)$ (Morse loops) is the group of words in the letters $\{y, y^{-1}\}_{y \in \mathrm{Crit}_1(f)}$. $\quad \mathcal{L}_f^{(M)}(L) \xrightarrow{\mathrm{ev}} \pi_1(L, \star)$.



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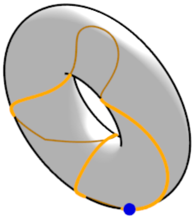

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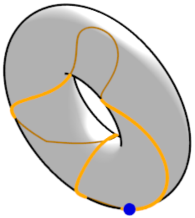

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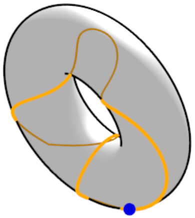

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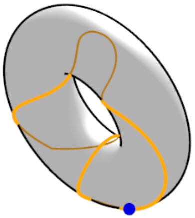

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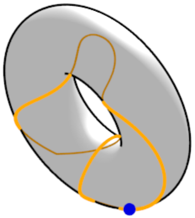

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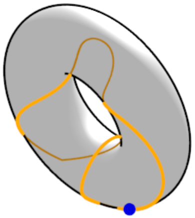

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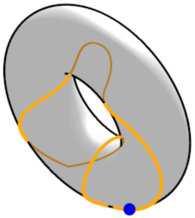

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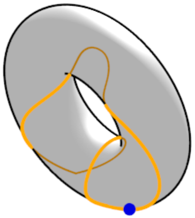

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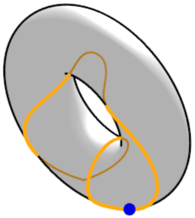

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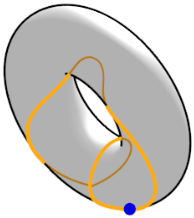

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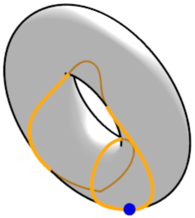

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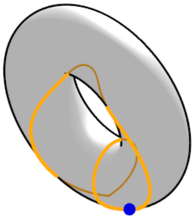

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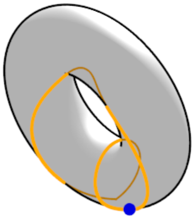

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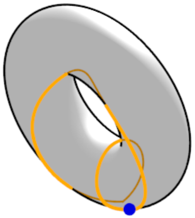

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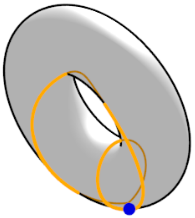

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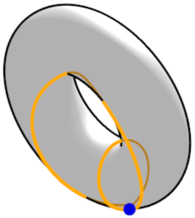

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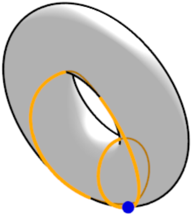

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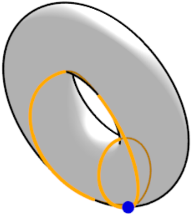

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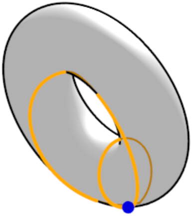

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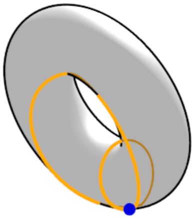

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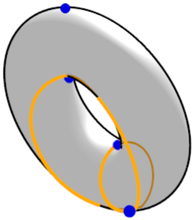

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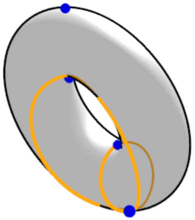

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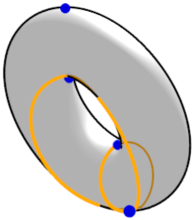

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We need to understand $W^u(y)$ for $y \in \mathrm{Crit}_1(f)$: :

Recall the Morse equation :

$$\frac{d\gamma}{dt} = -X_\gamma.$$

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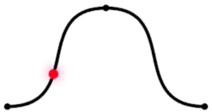

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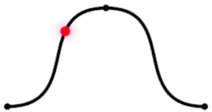

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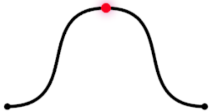

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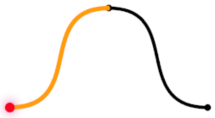

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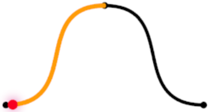

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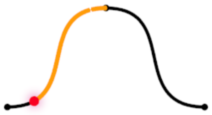

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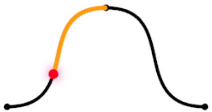

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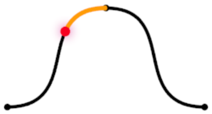

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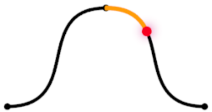

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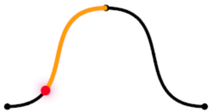

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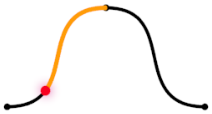

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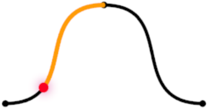

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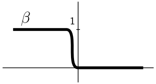

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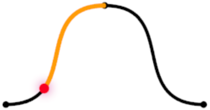

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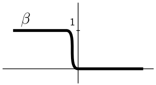

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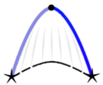
From Morse to Floer loops

Pick f such that it has a single minimum at $\star$.

$$\mathcal{L}_H^{(F)}(L,\star) \xrightarrow{\phi} \mathcal{L}_f^{(M)}(L,\star) : \text{ push down using } f \text{ gradient flow.}$$

$$\mathcal{L}_f^{(M)}(L,\star) \xrightarrow{\psi} \mathcal{L}_H^{(F)}(L,\star) : ?$$



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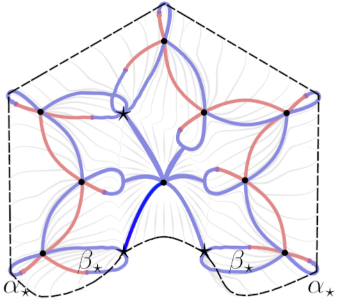

From Morse to Floer loops

$$\begin{array}{ccc}
\mathcal{L}_f^{(M)}(L,\star) & \twoheadrightarrow & \pi_1(L,\star) \\
\downarrow \psi & & \downarrow id \\
\mathcal{L}_f^{(F)}(L,\star) & \longrightarrow & \pi_1(L,\star) \\
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Relations

$$\ker\left(\ \mathcal{L}_H^{(F)}(L,\star) \xrightarrow{\text{ev}} \pi_1(L,\star)\ \right) = \ ?$$

It only depends on L, $\star$, H, J, but we will resort to a Morse function to pick explicit generators.
Let

$$\mathcal{R}_f = \{\partial W^u(z), z \in \mathrm{Crit}_2(f)\},$$

and

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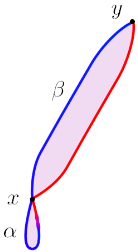

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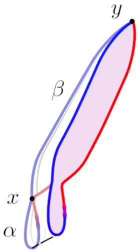

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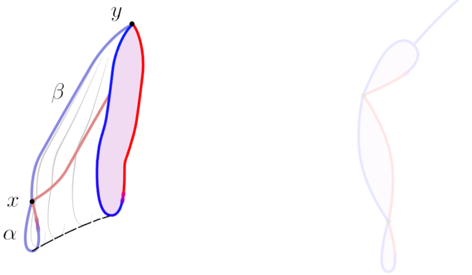

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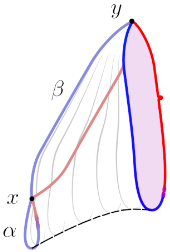

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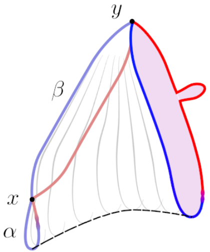

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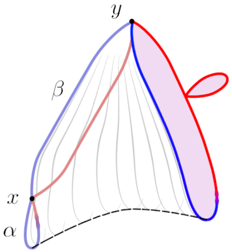

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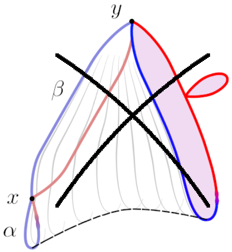

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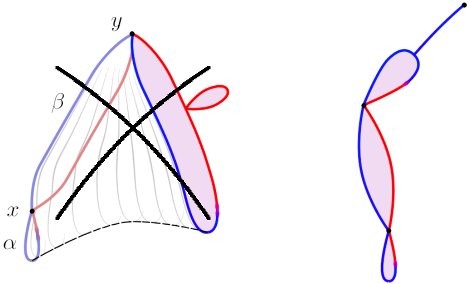

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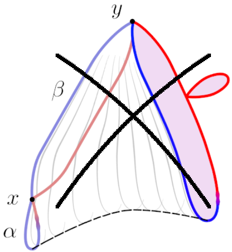

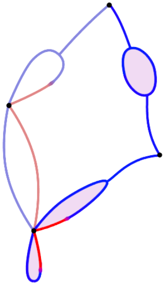

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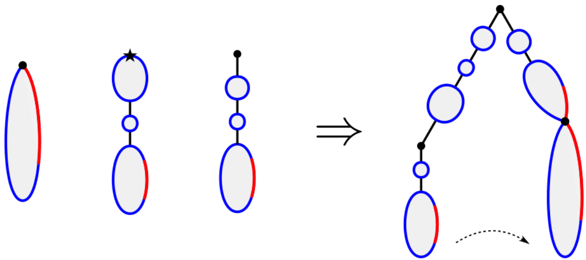
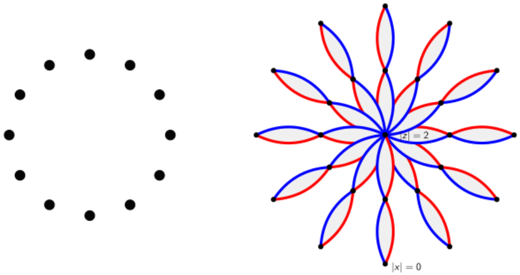
Suppose now L is monotone with $N_L \geq 3$.
Add a Morse function f and a metric to the auxiliary data and "pearlify" all the previous definitions (following Biran, Cornea, Lalonde) :



With this new definition, $\mathcal{L}^{(F)}_{H,f}(L,\star) \to \pi_1(L,\star)$ is still surjective.

Relation to Floer Homology

The homology makes use of 1-dimensional moduli spaces, but only to prove $\partial^2 = 0$:



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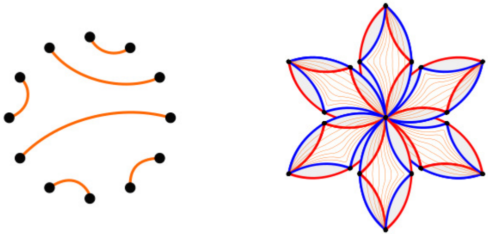

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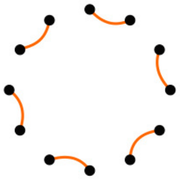

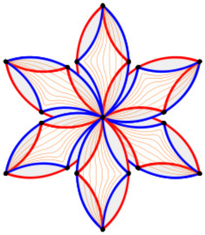

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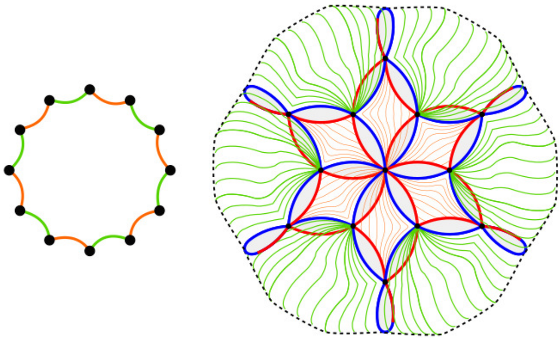

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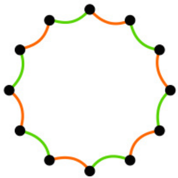

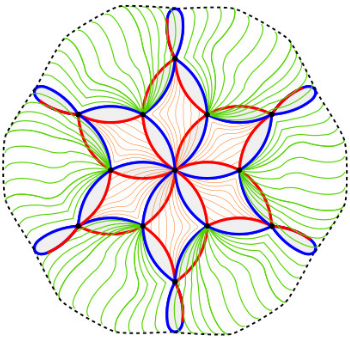

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Variation on the Arnold conjecture.

Suppose L is (weakly) exact. Define (for $|y| = 1$) :

$$\nu(y) = \sharp\{\text{ steps through } y\}$$
$$\nu(\star) = \sharp\{\text{non canonical steps through } \star\}$$

Theorem

Let $\delta(\pi_1)$ be the minimal number of generators of $\pi_1(L)$. Then :

$$\nu(\star) + \sum_{|y|=1} \nu(y) \geq \delta(\pi_1(L))$$

Remark : in the stable Morse setting, exemples are known where $\sharp\text{Crit}(f) < \delta(\pi_1)$.

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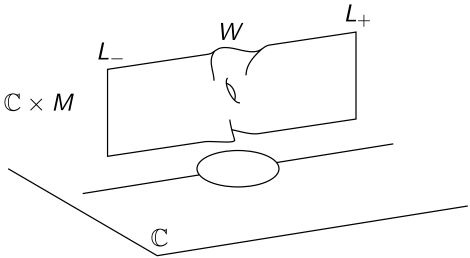
Lagrangian cobordisms

Let L_- and L_+ be two Lagrangian submfds in M, and $W \hookrightarrow \mathbb{C} \times M$ a Lagrangian cobordism from L_- to L_+.



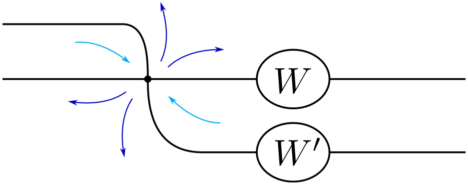
Biran,Cornea : W monotone $\Rightarrow HF_*(L_-) = HF_*(L_+)$.

Proposition (with L. Simone Suarez)

If W is (weakly) exact, then the inclusions $L_\pm \overset{i}{\hookrightarrow} W$ induce surjective maps $\pi_1(L_\pm) \xrightarrow{i_*} \pi_1(W)$.

Lagrangian cobordisms

Sketch of proof: Deform W,



pick $\star$ above the intersection point (and suitable J) : all the strips are confined to this fiber.

Remark : this can also be derived from $HF_\star$ with local coefficients [B. Chantraîne - L. Simone Suarez].

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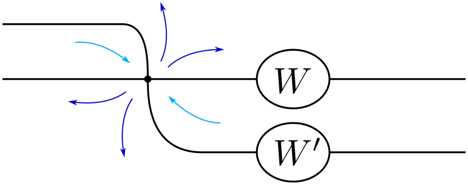

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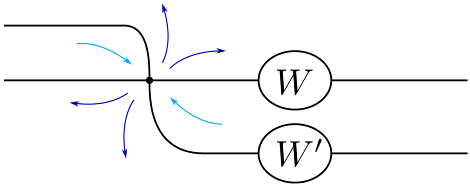

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$$H \overset{(H_\lambda)}{\rightsquigarrow} H' \dashrightarrow HF_*(L_0, L_1) \xrightarrow{\Phi} HF_*(L_0, L_1')$$

However, this construction

- makes sens for small perturbations of generic situations,
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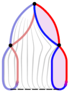

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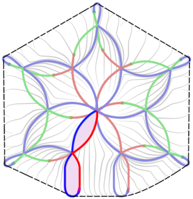

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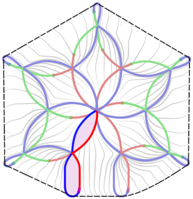

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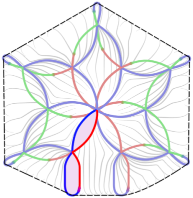

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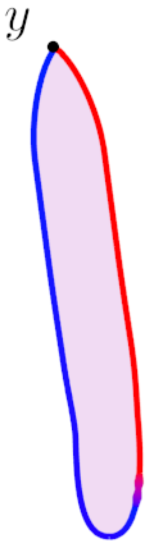
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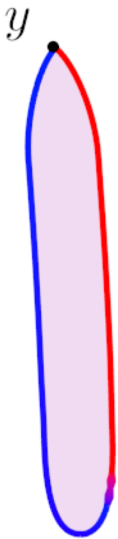
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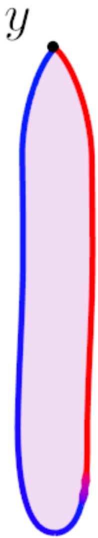
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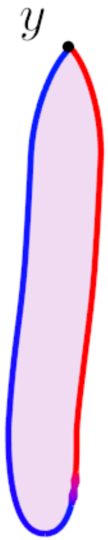
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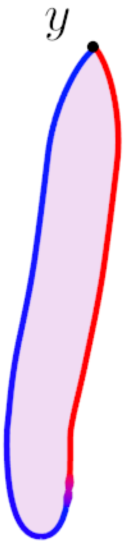
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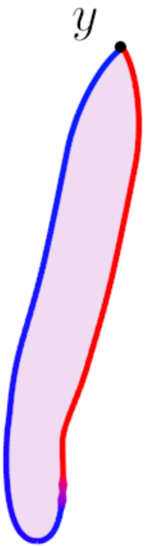
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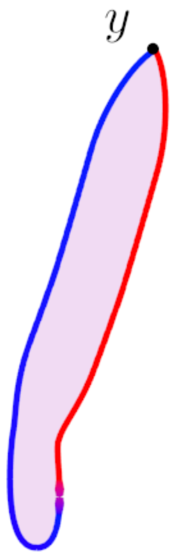
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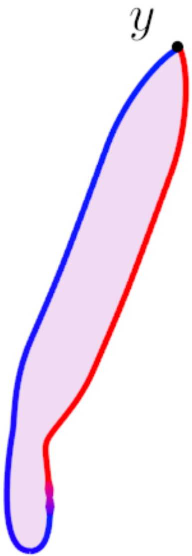
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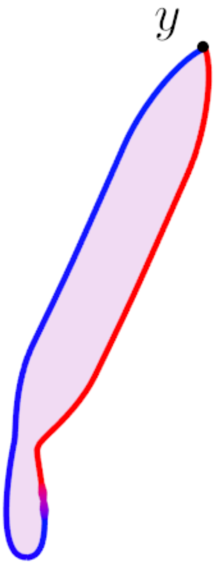
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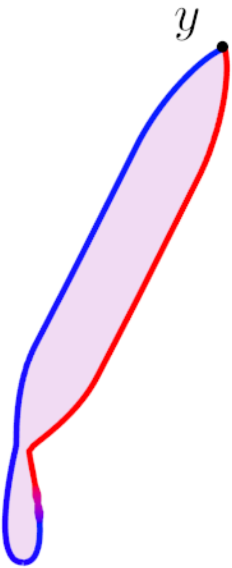
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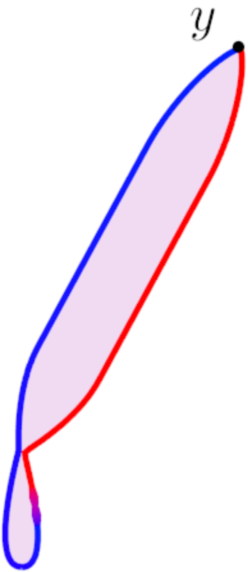
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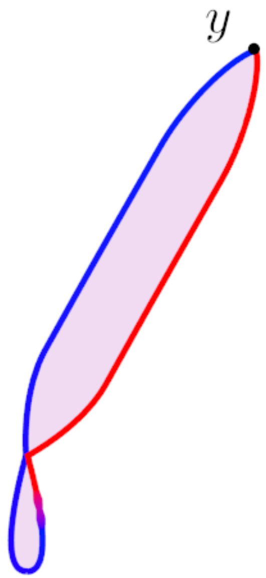
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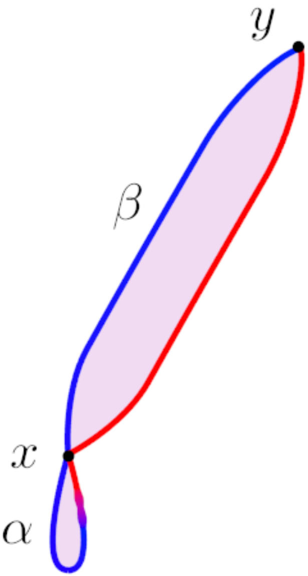
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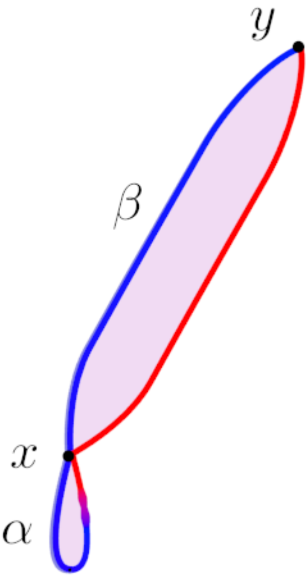
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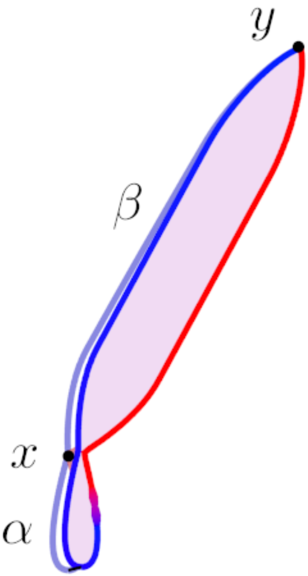
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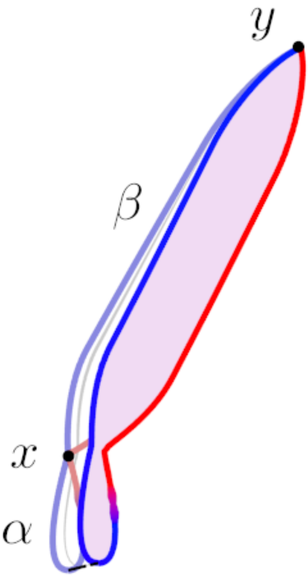
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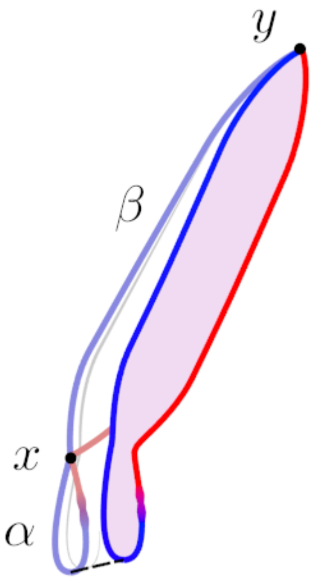
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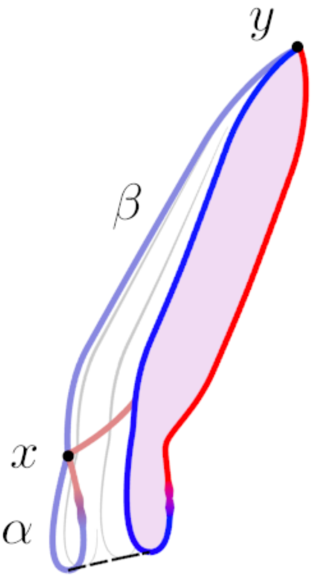
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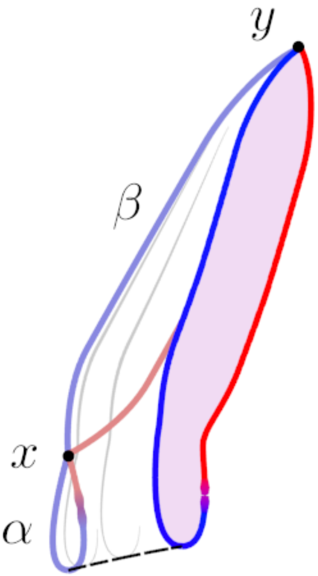
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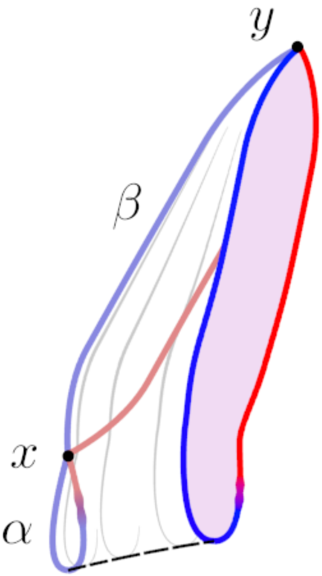
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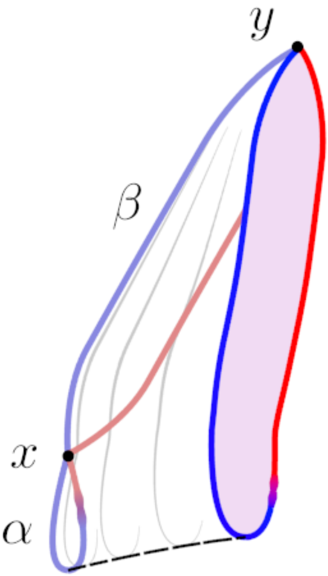
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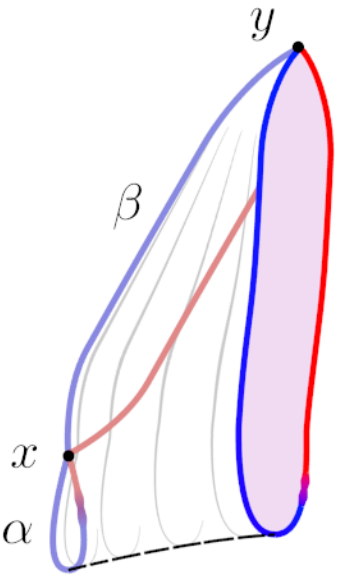
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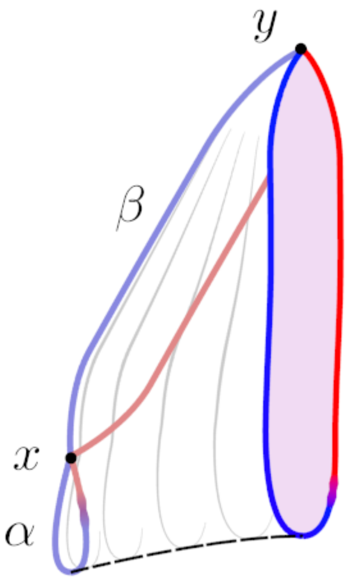
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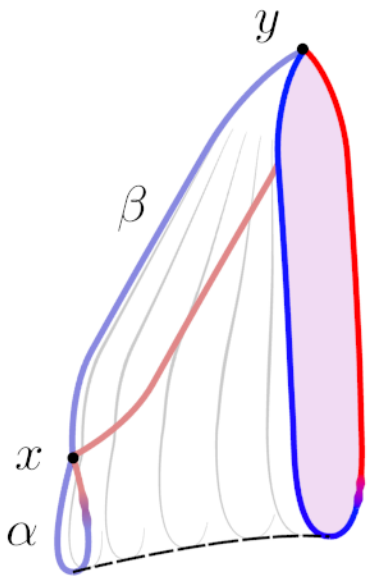
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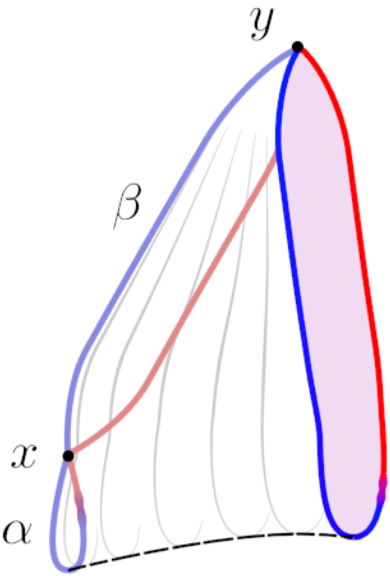

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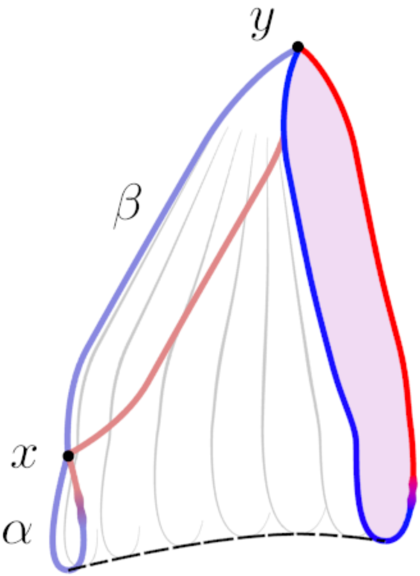
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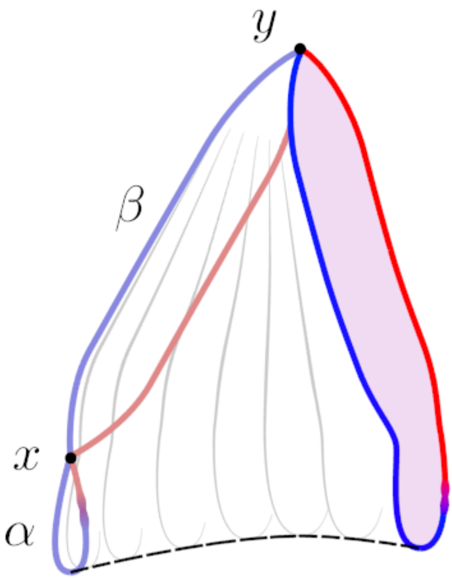
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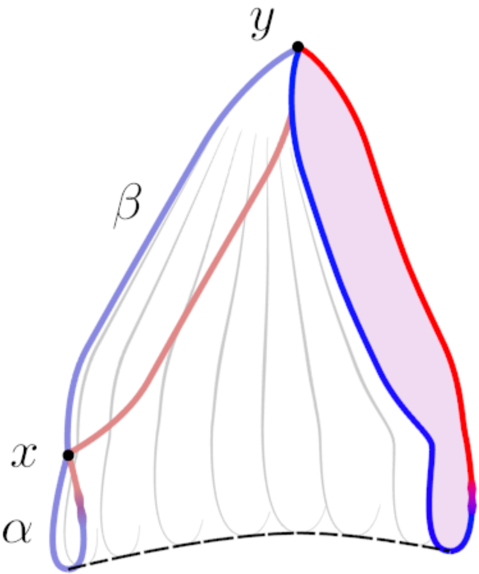
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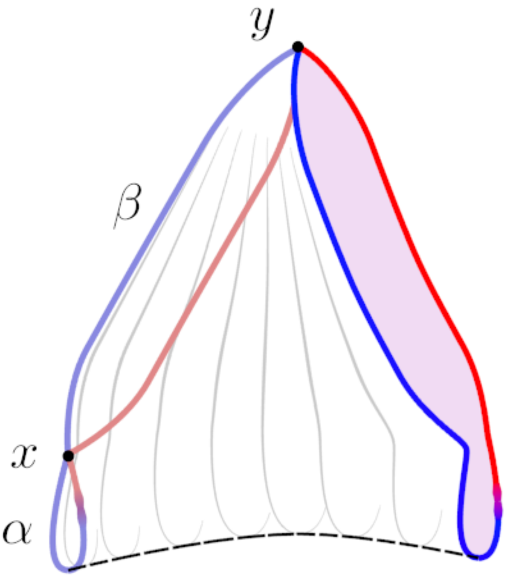
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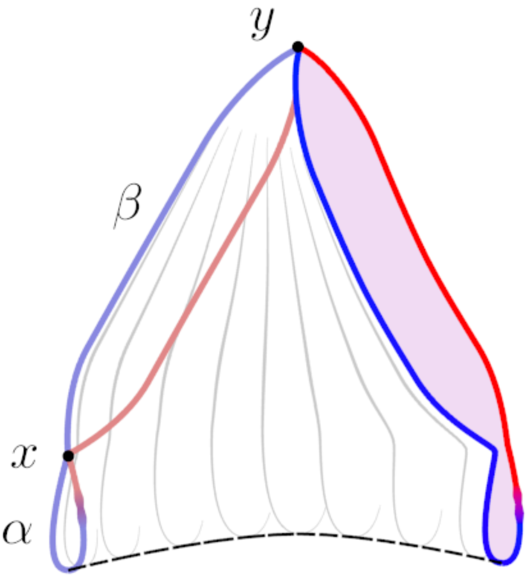
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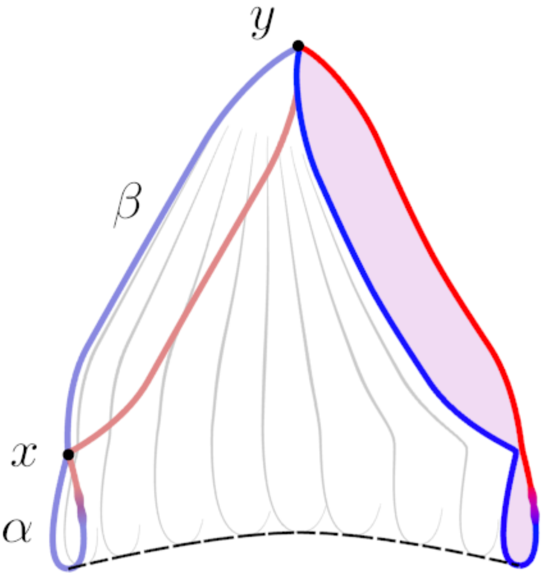
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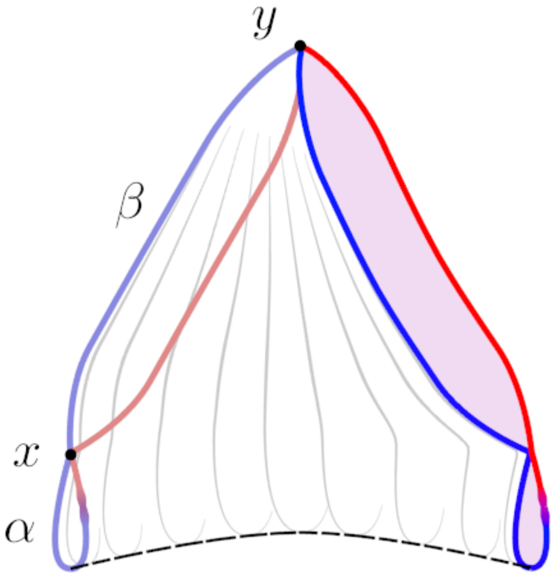
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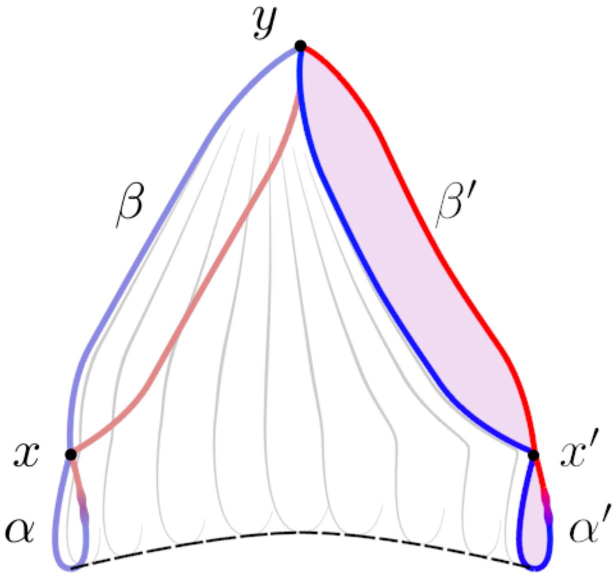
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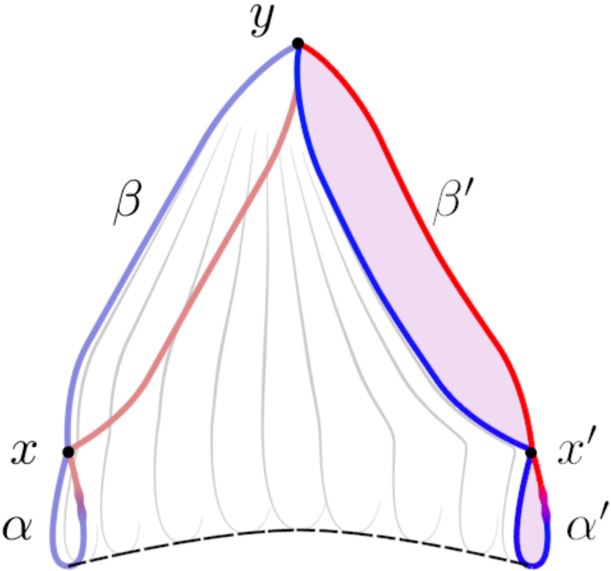
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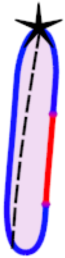
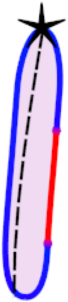
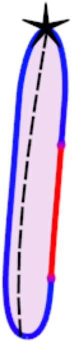
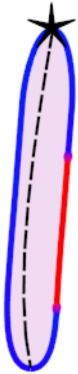
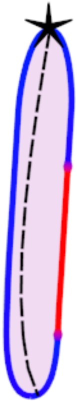
★

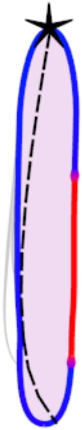


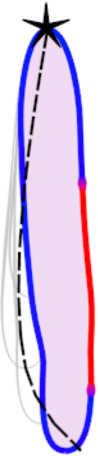


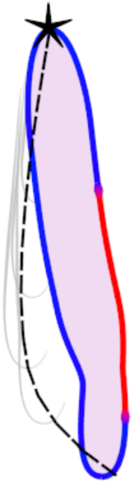


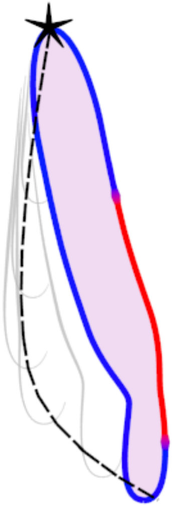


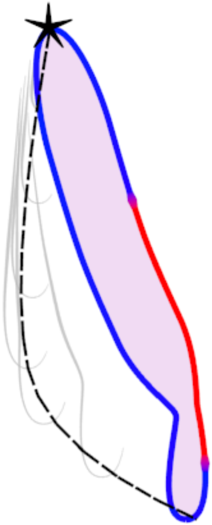


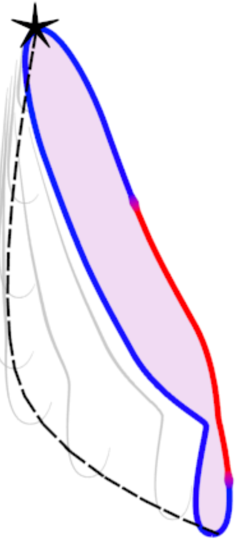


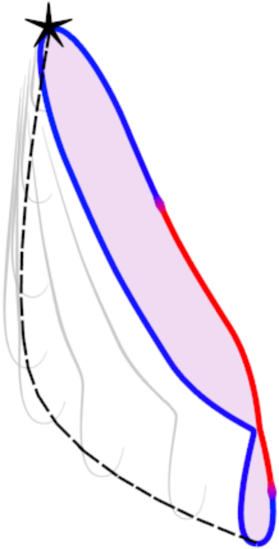


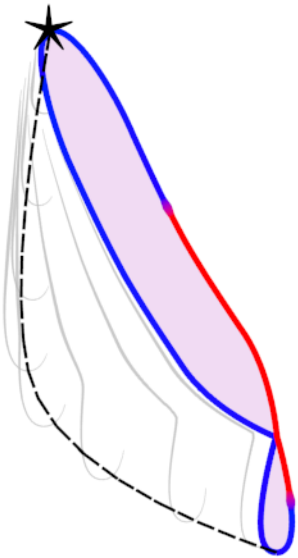


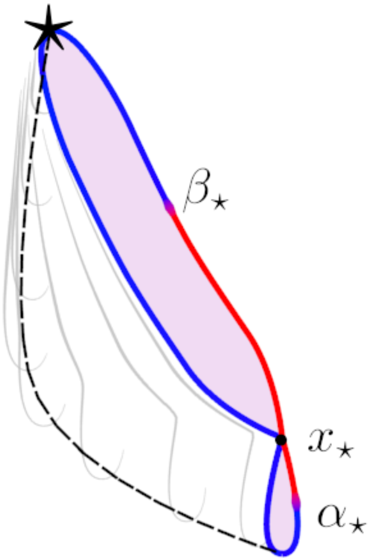








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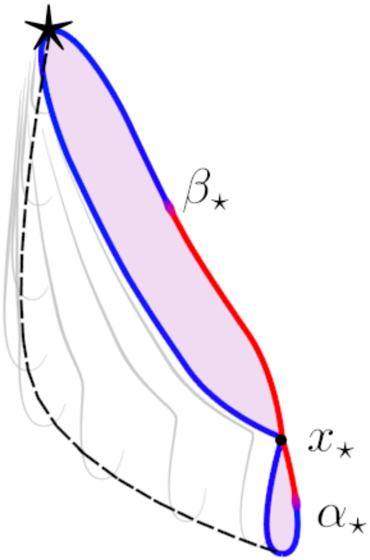
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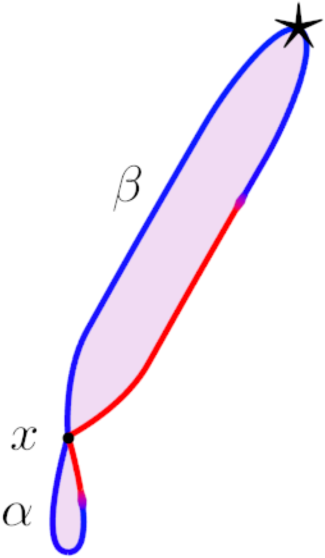
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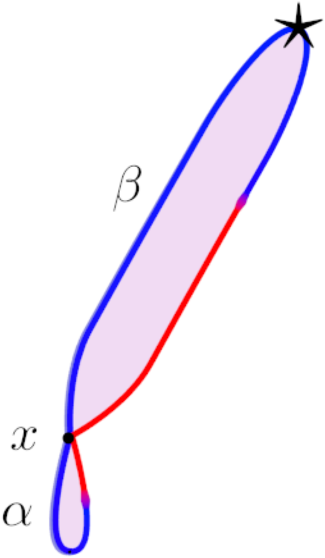
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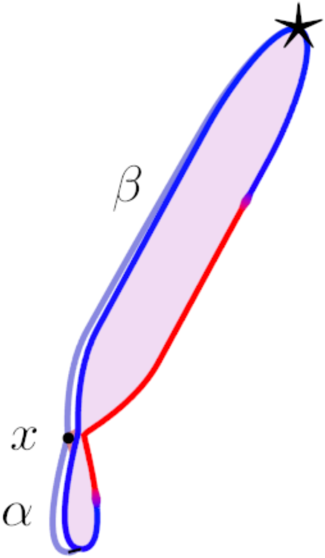
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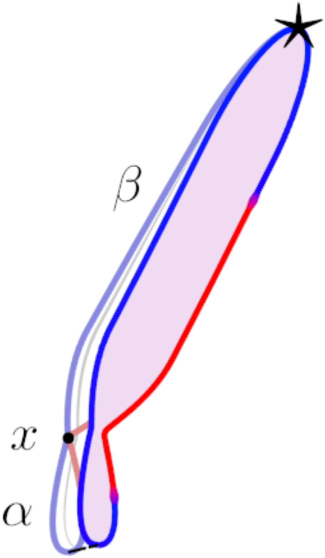
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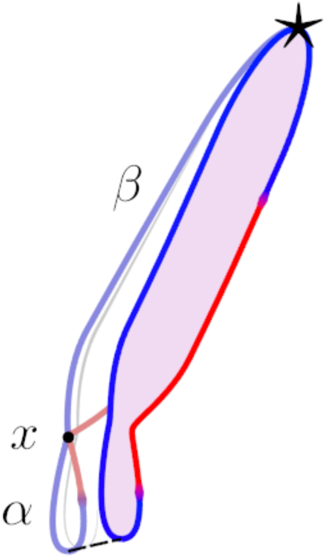
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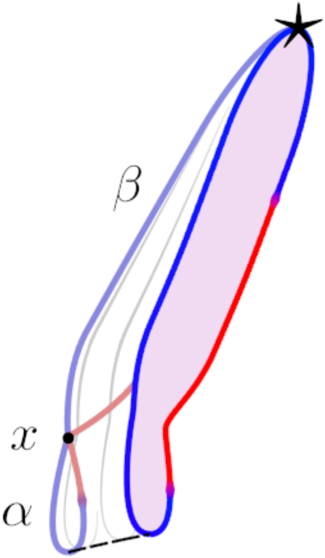
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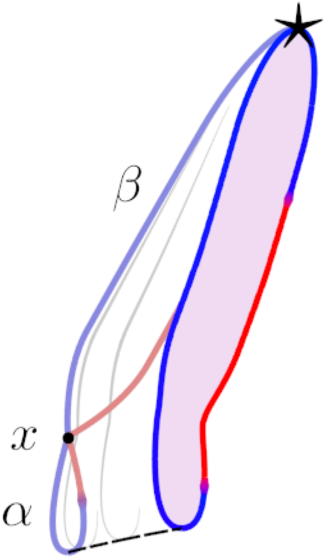
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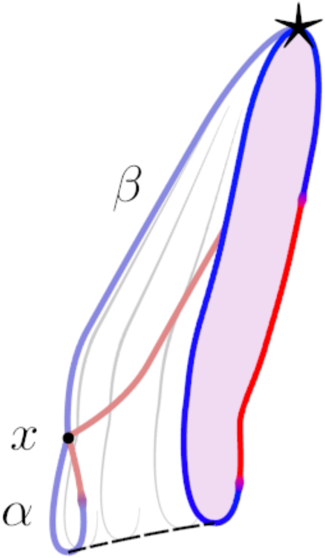
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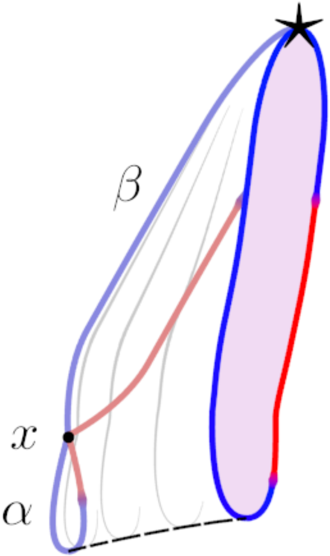
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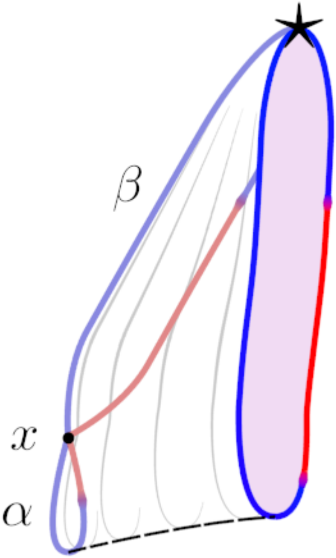
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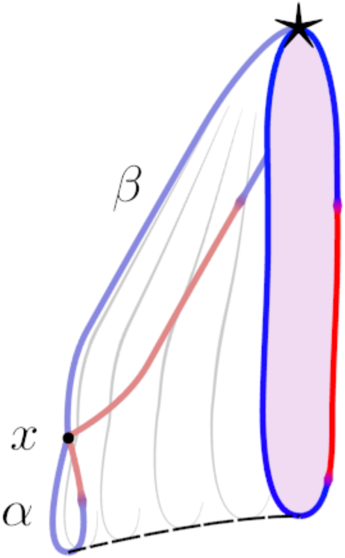
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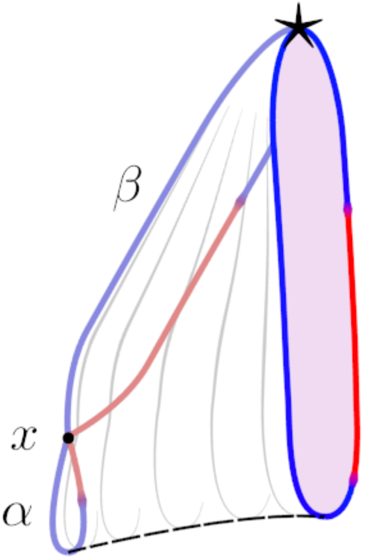
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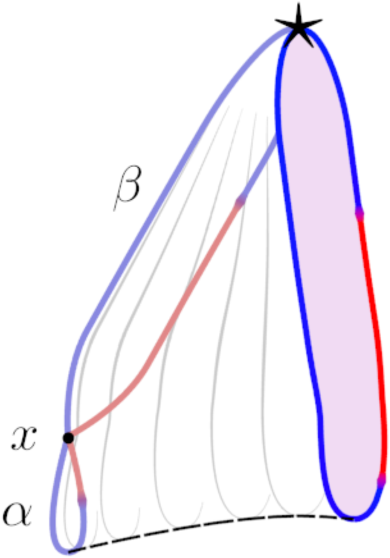
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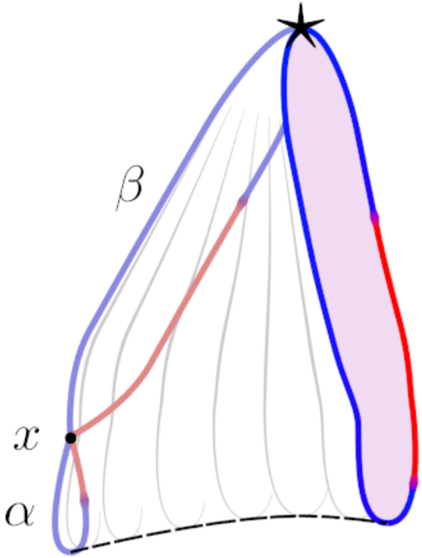
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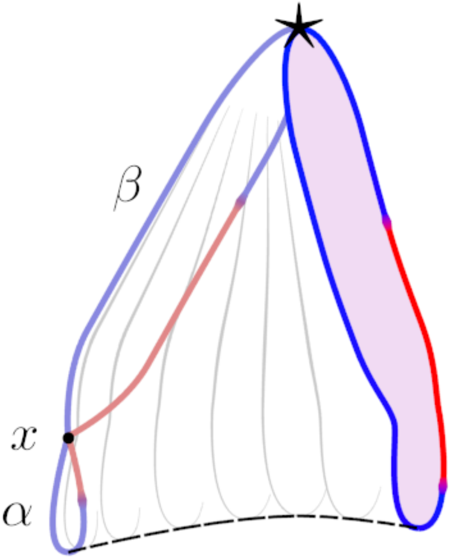
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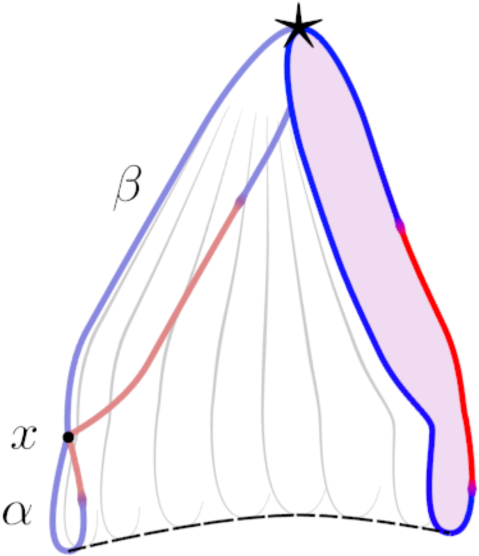
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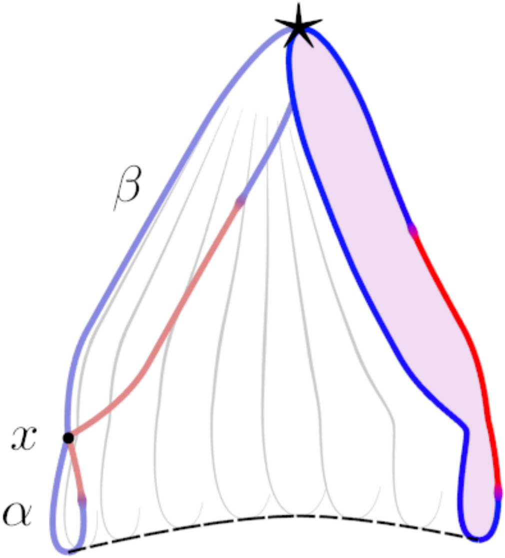

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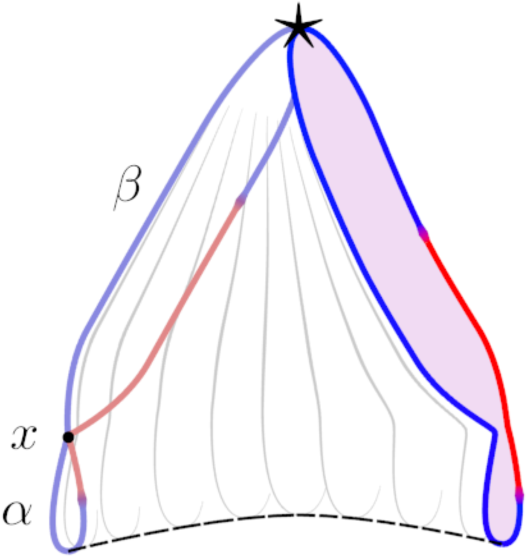
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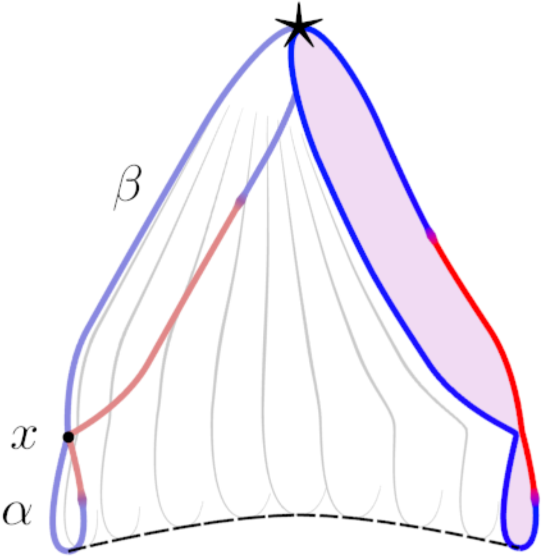
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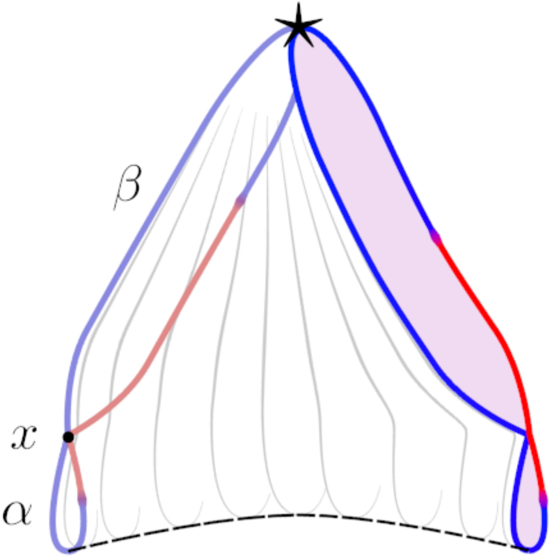
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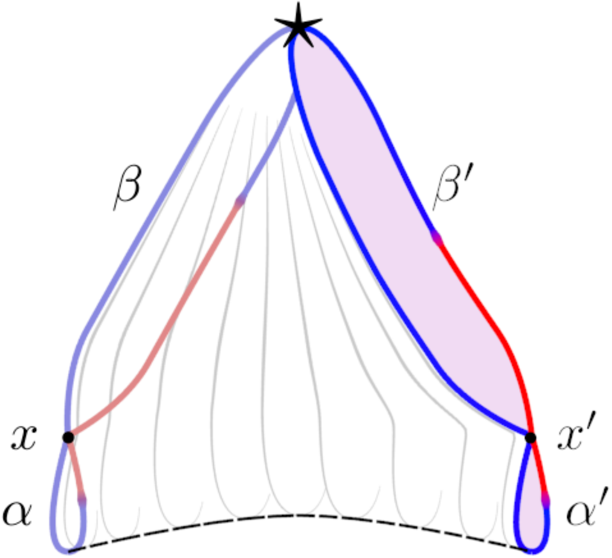
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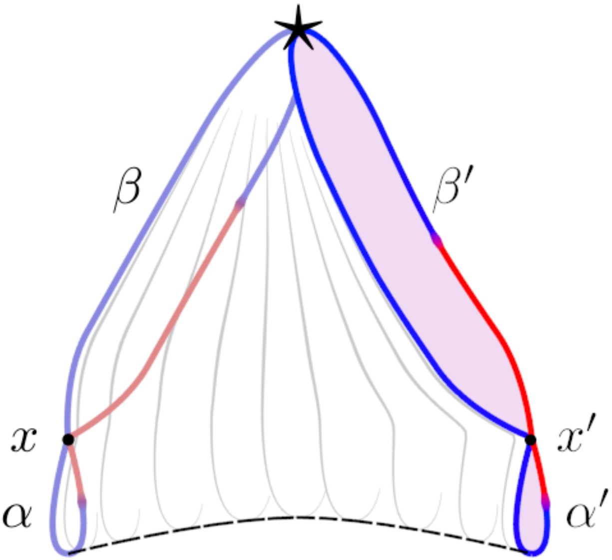
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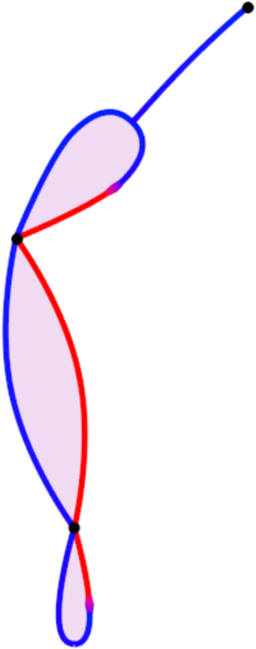

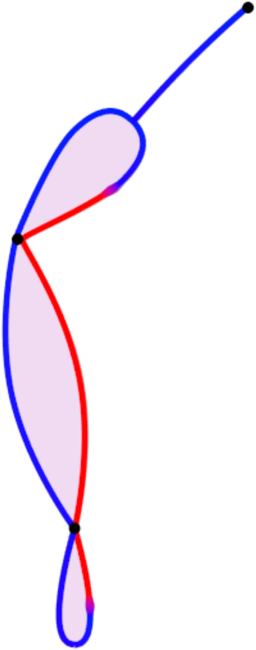

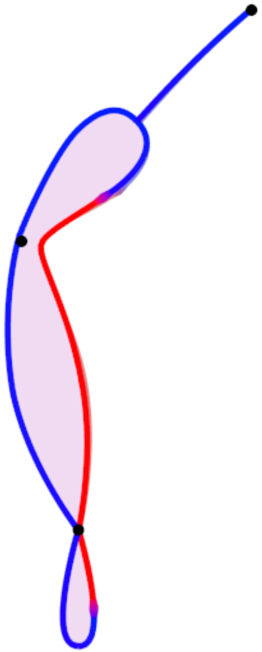

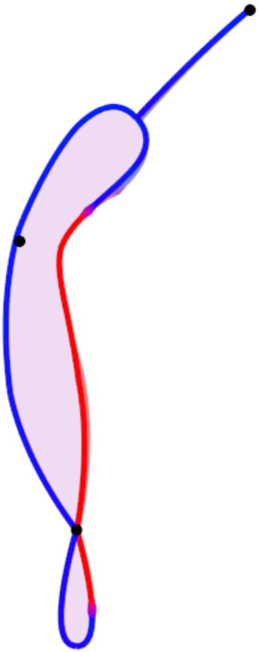

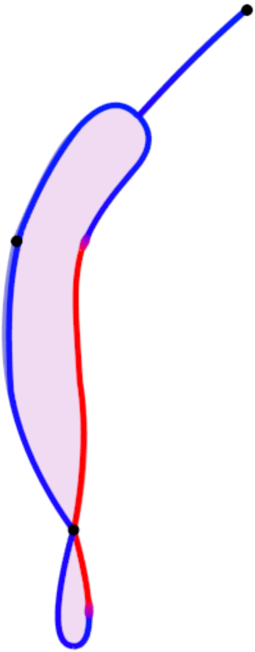

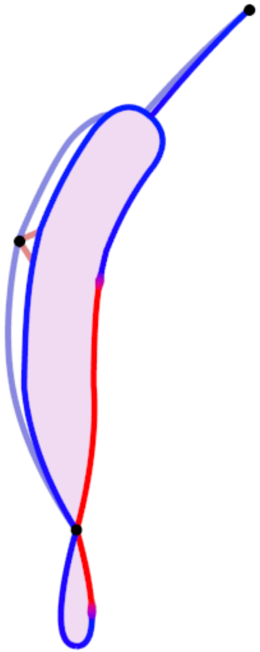

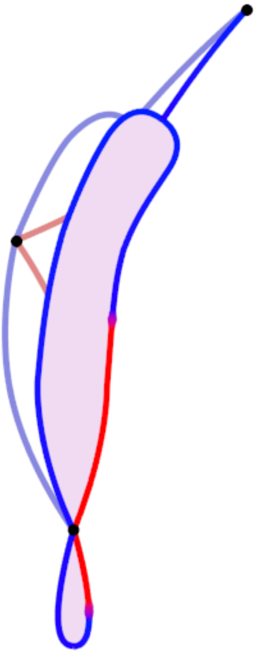

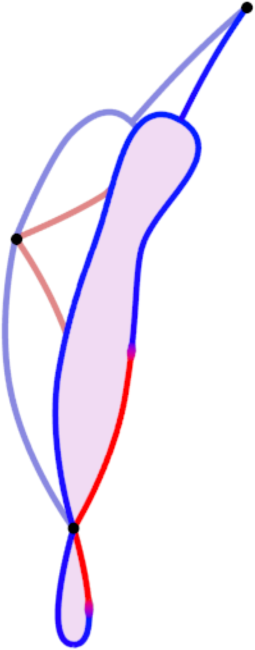

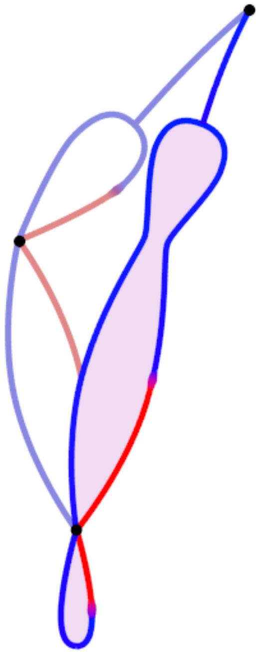

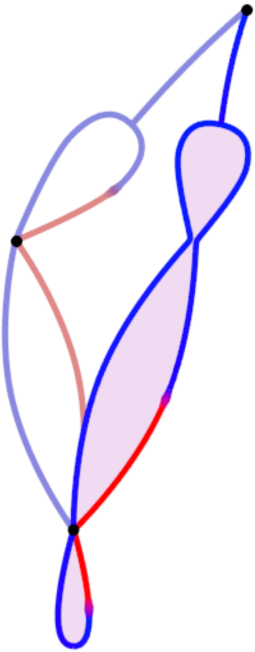

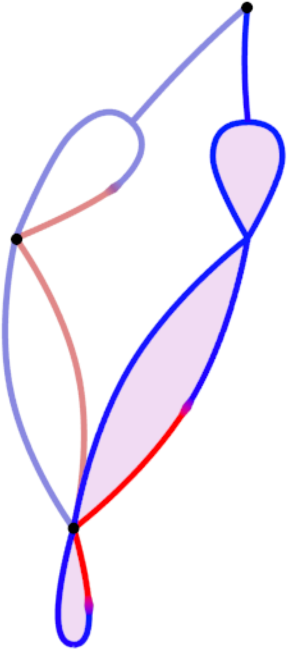

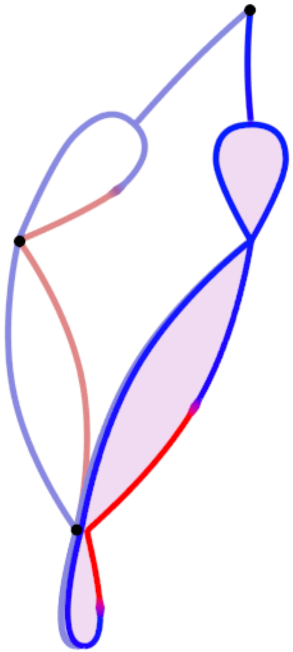

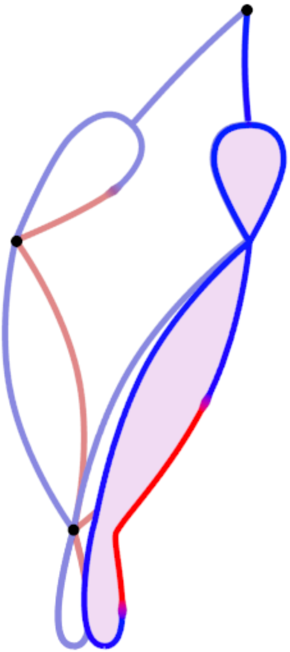

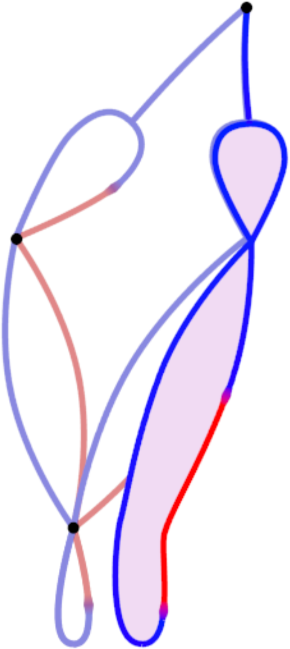

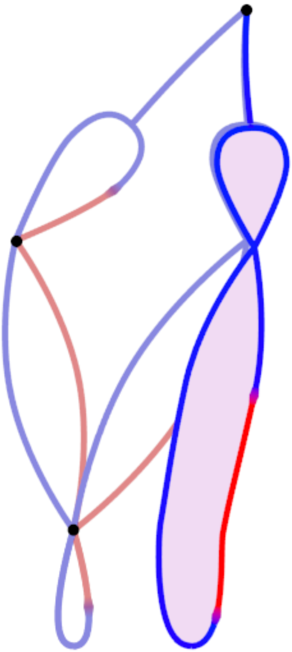

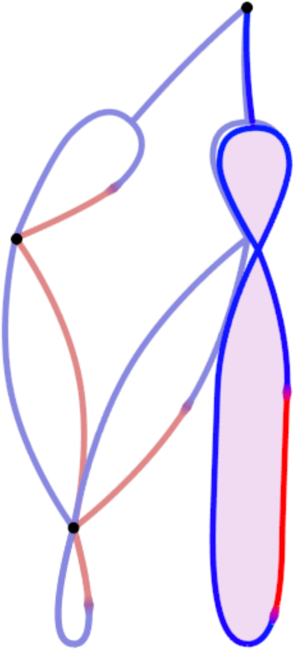

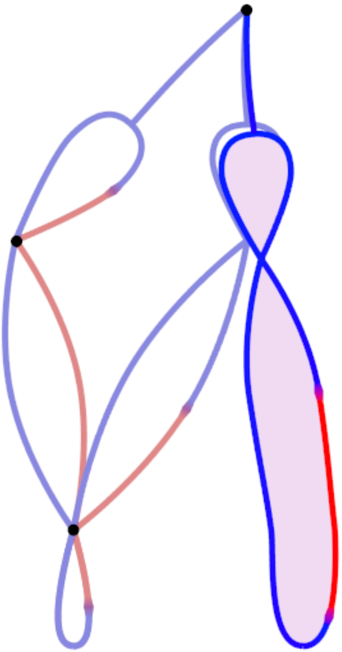

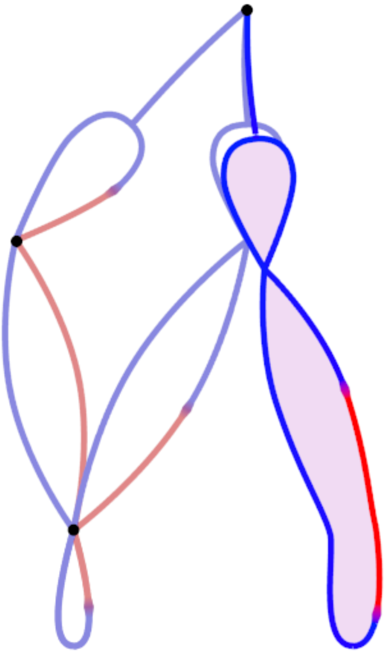

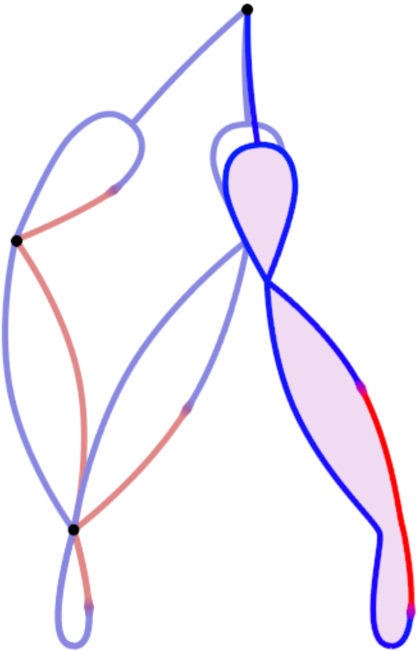

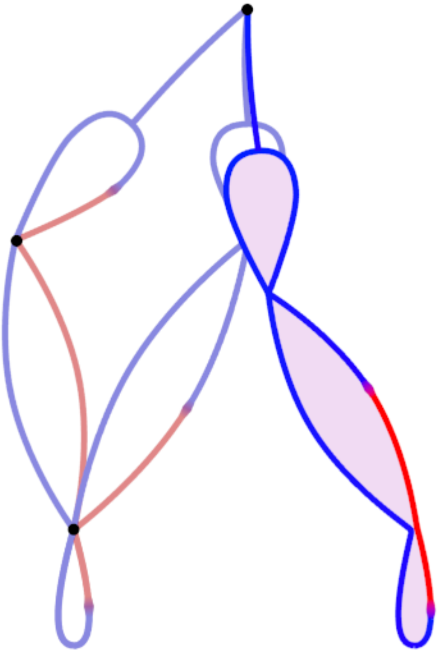

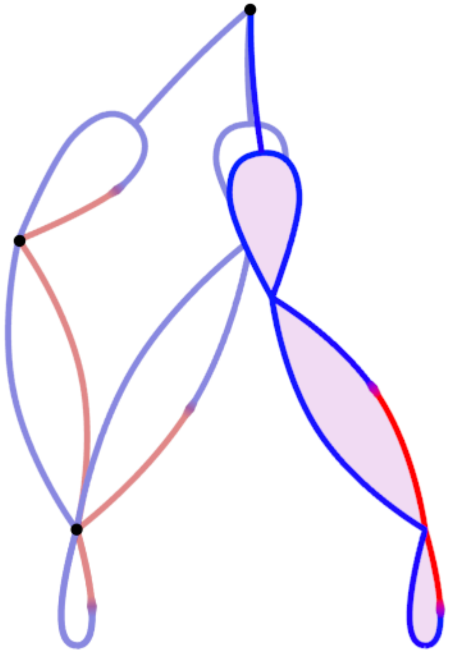

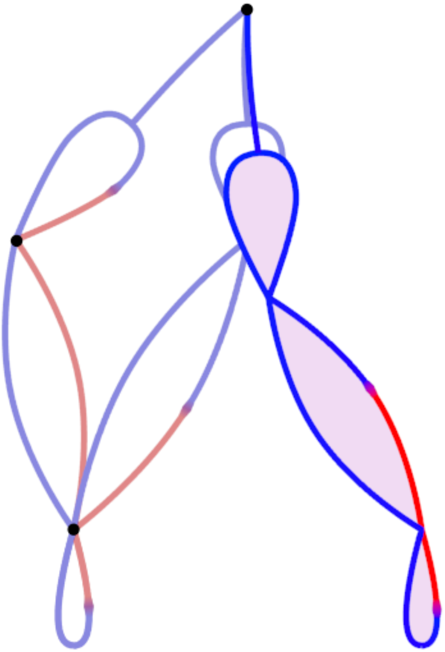

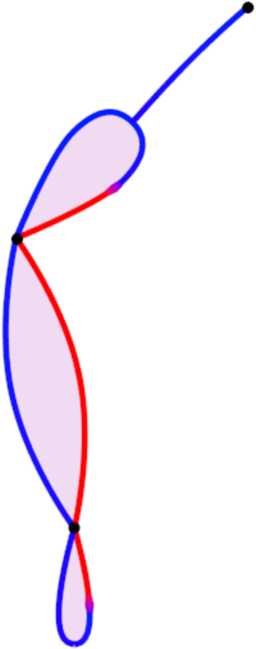

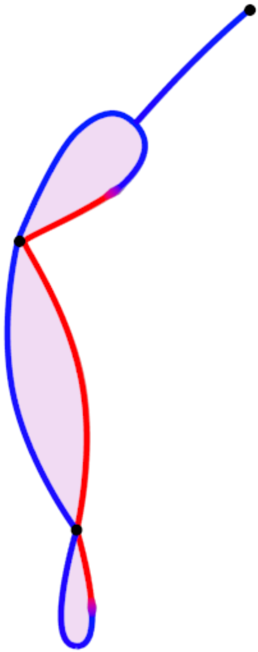

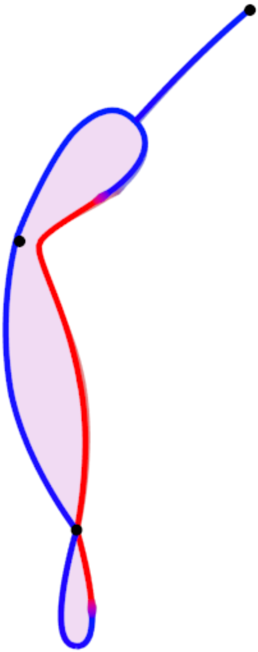

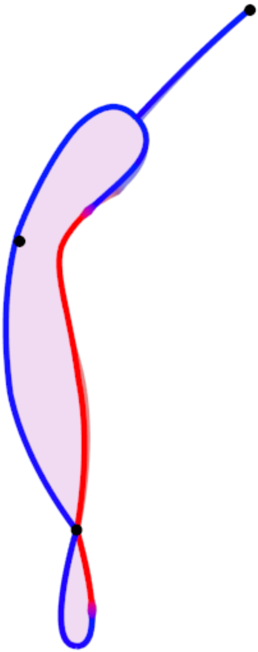

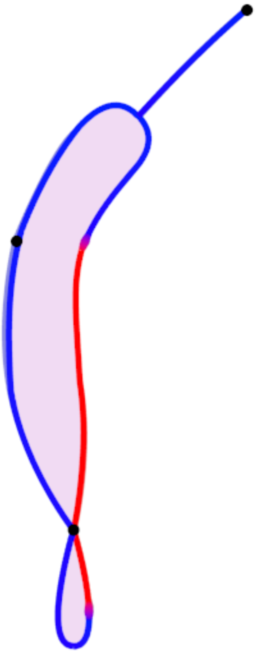

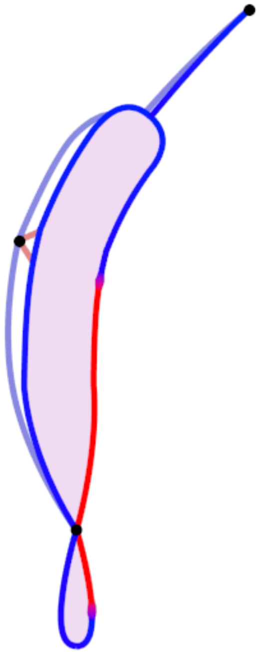

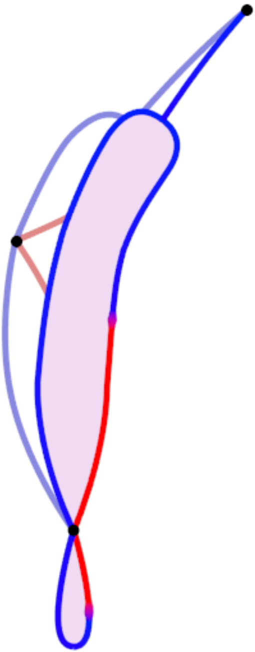

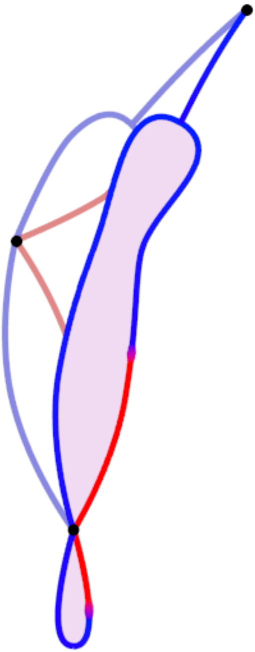

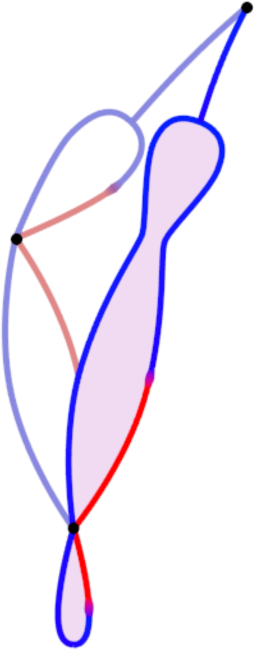

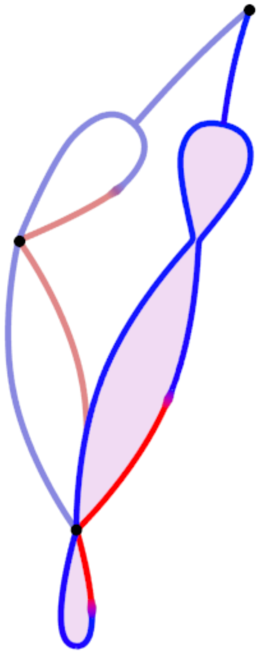

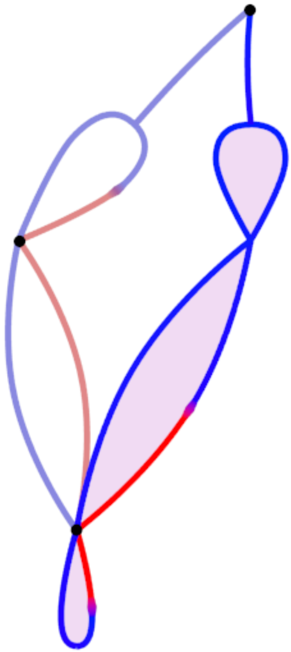

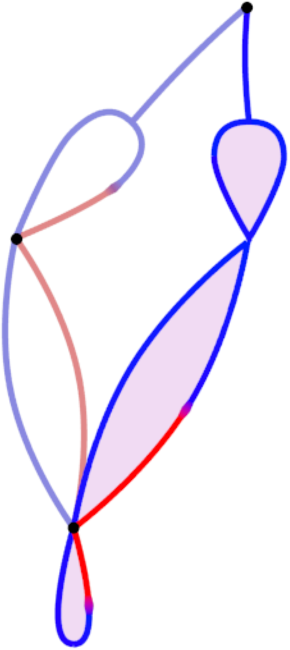

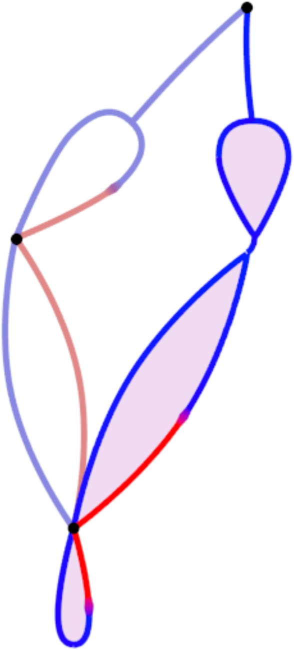

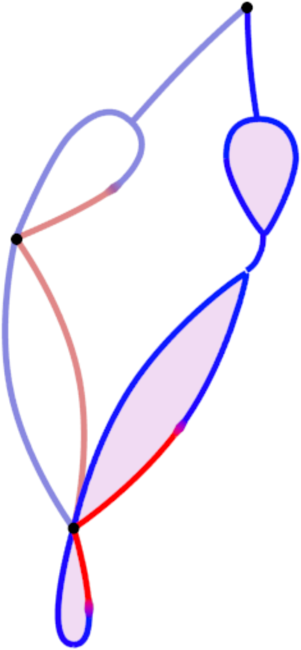

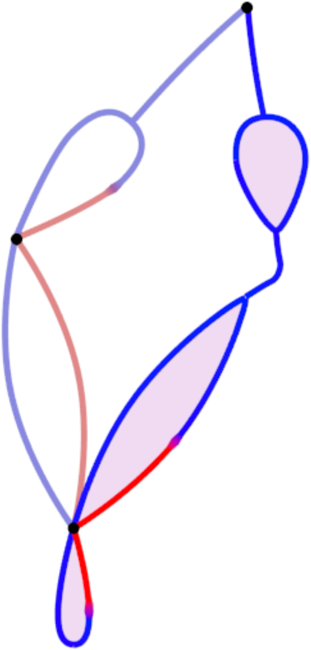

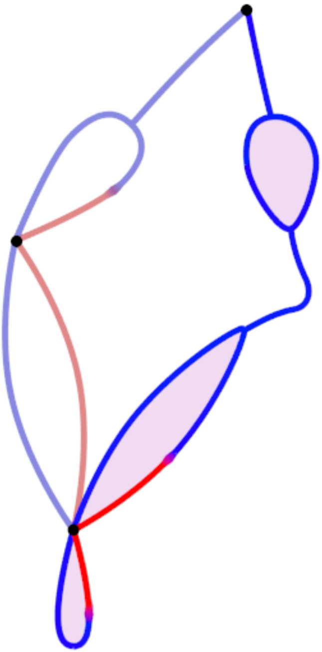

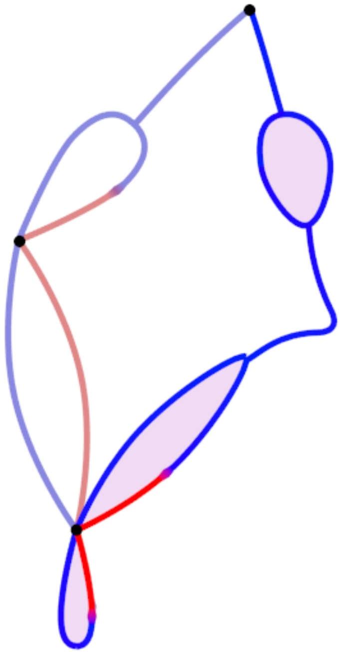

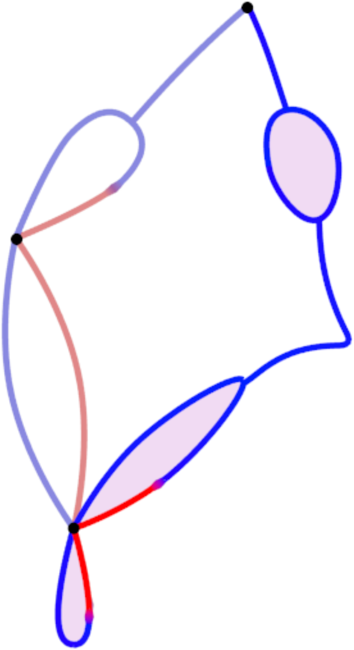

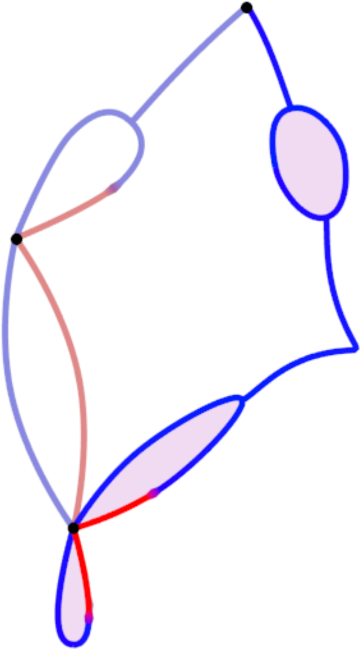

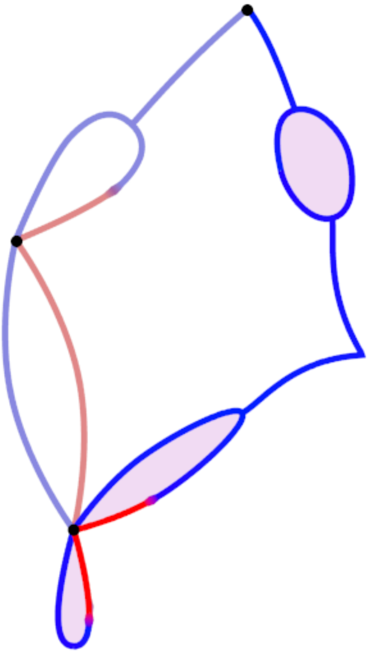

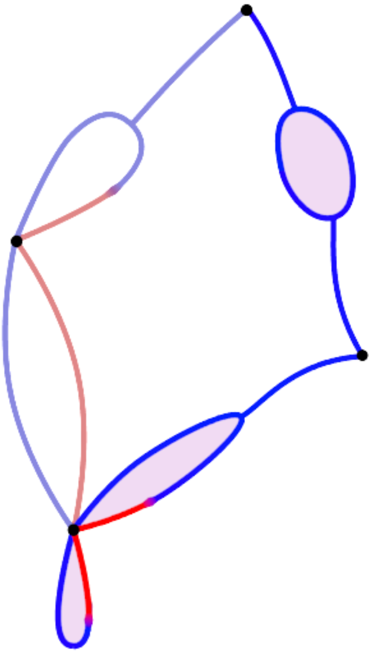

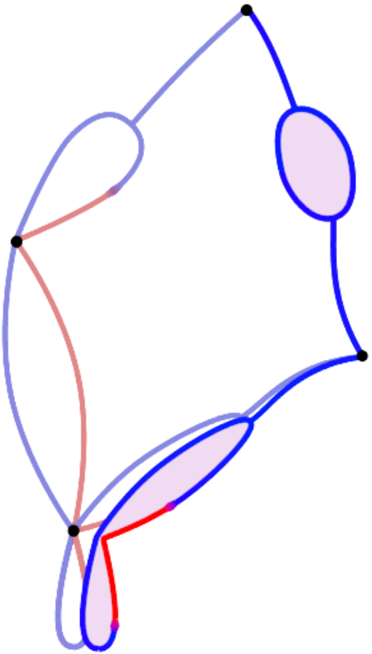

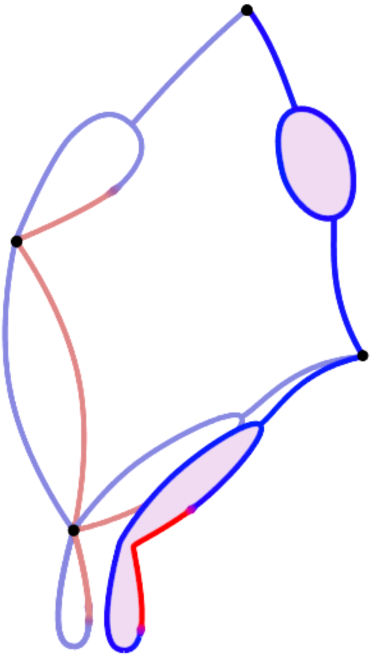

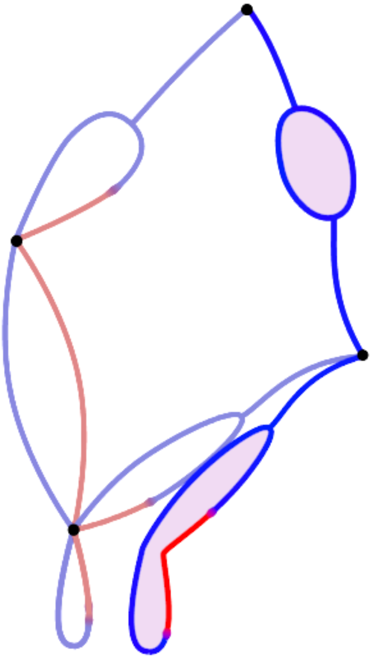

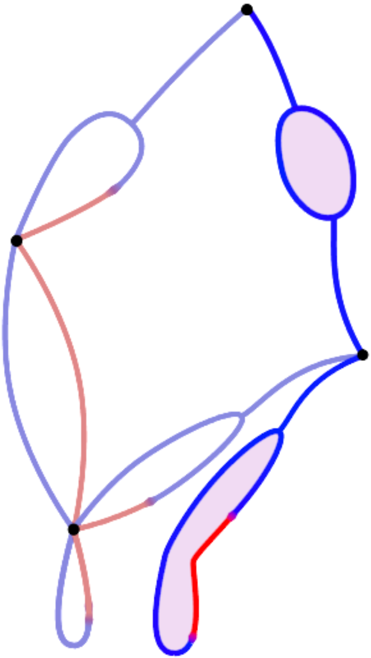

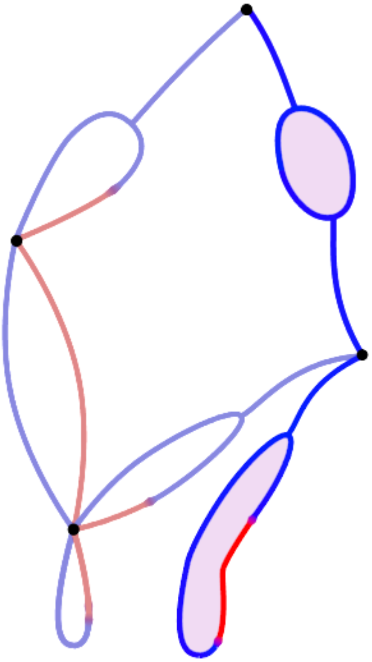

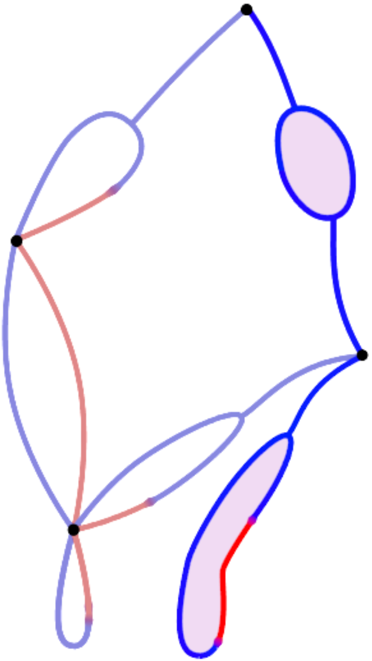

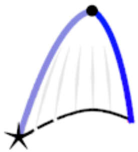

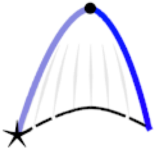

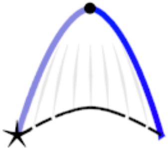

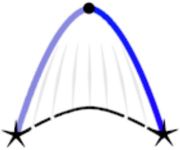

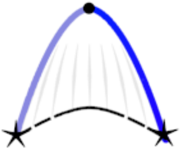

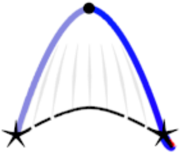

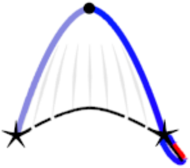

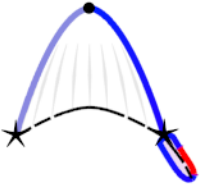

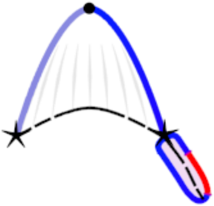

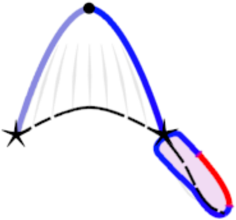

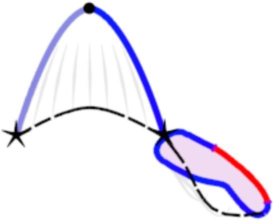

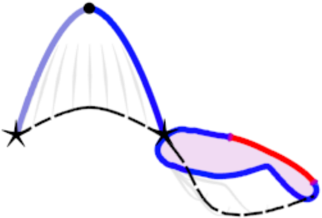

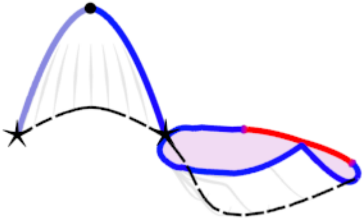

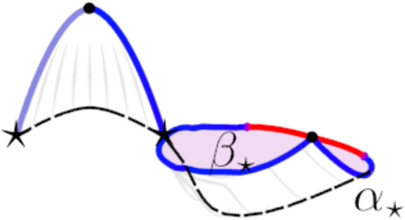
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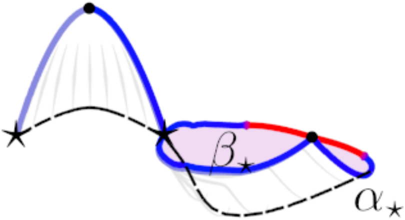
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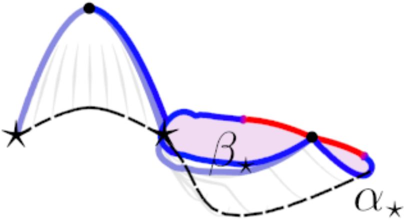
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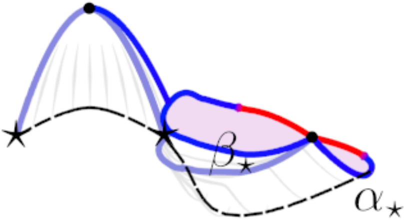
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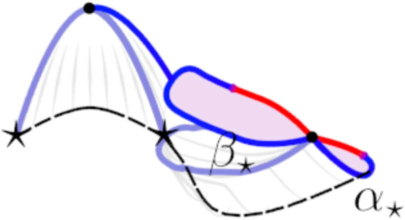
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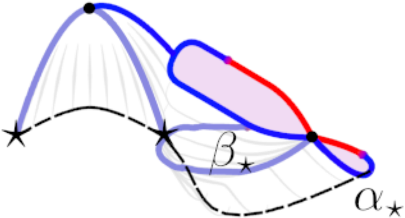
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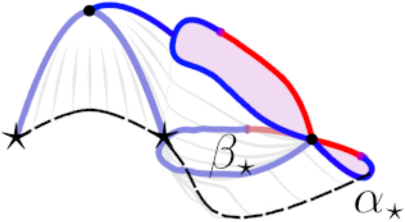
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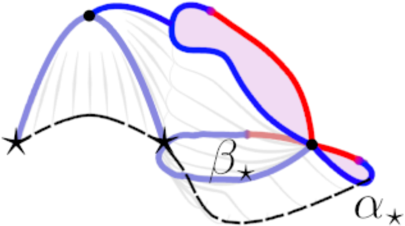
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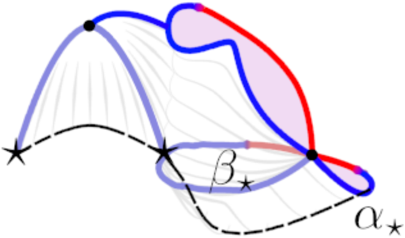
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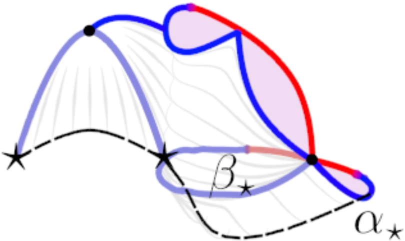
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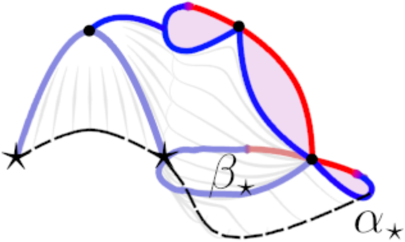
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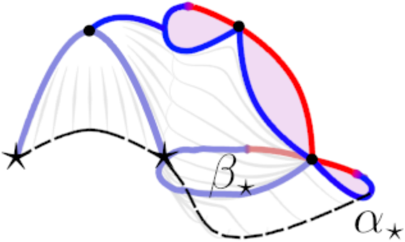
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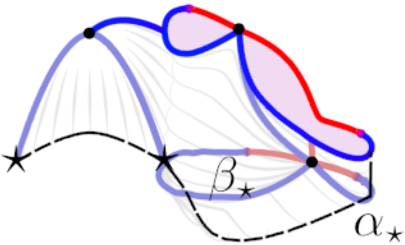
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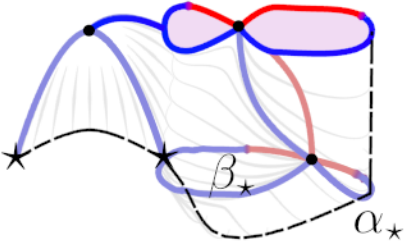
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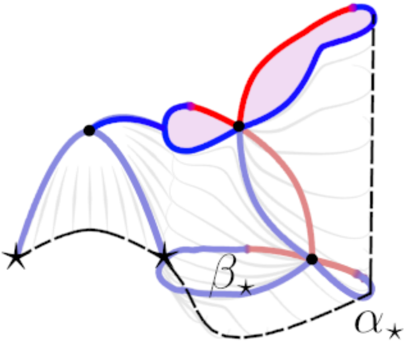
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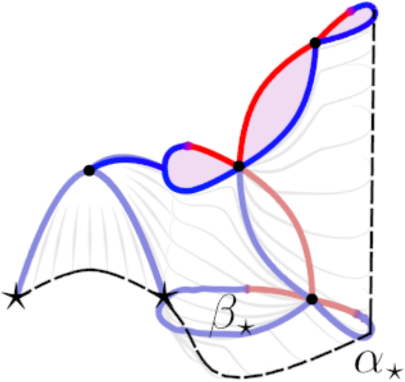
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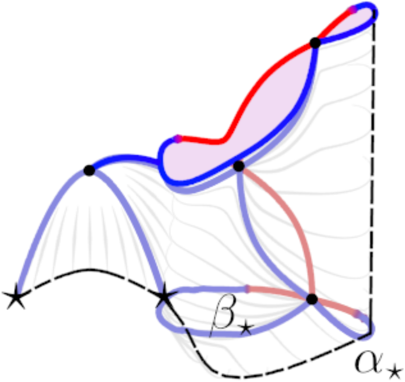
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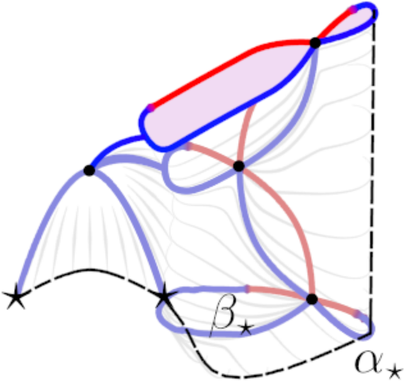
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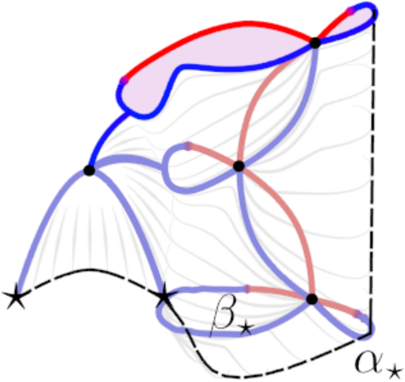
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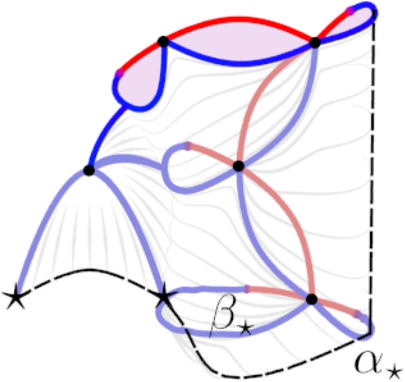
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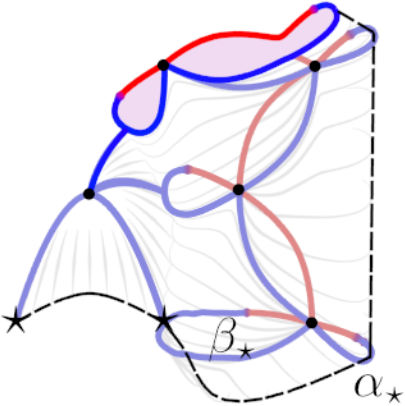
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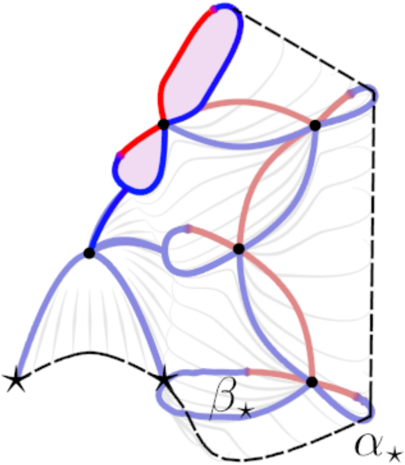
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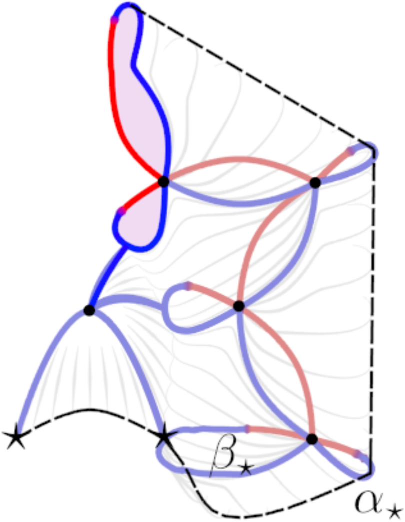
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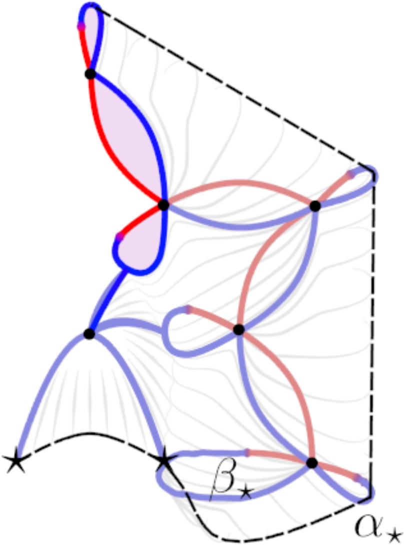
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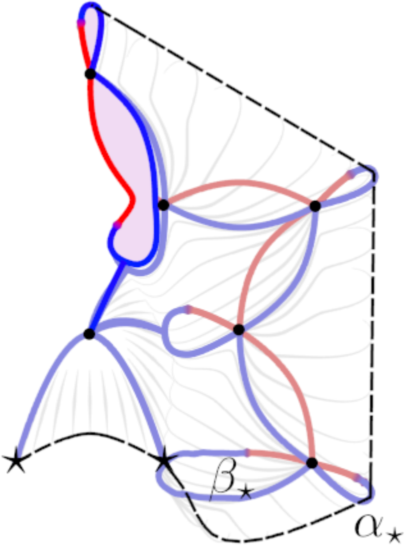
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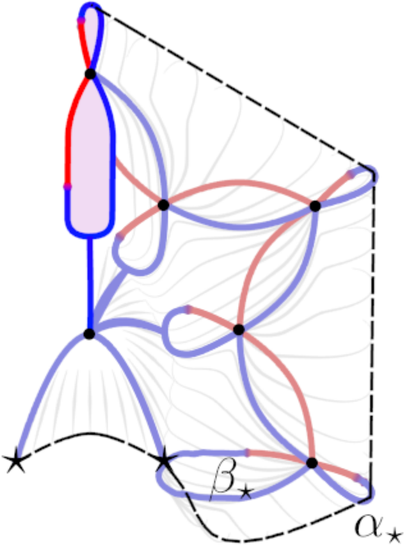
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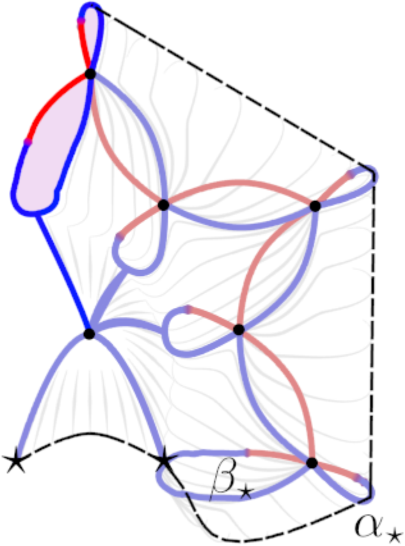
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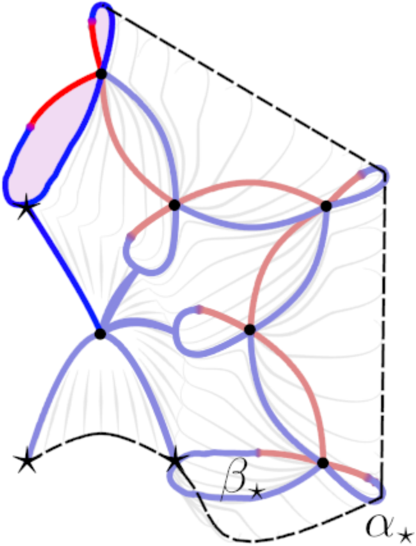
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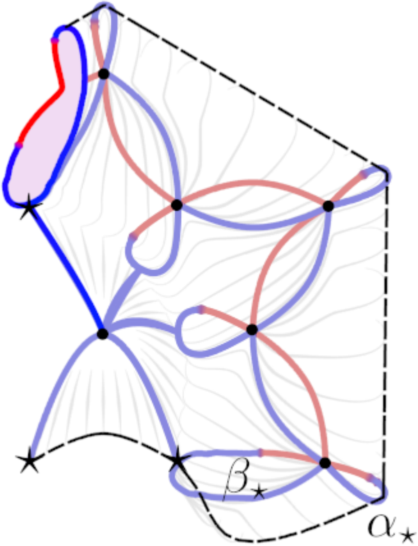
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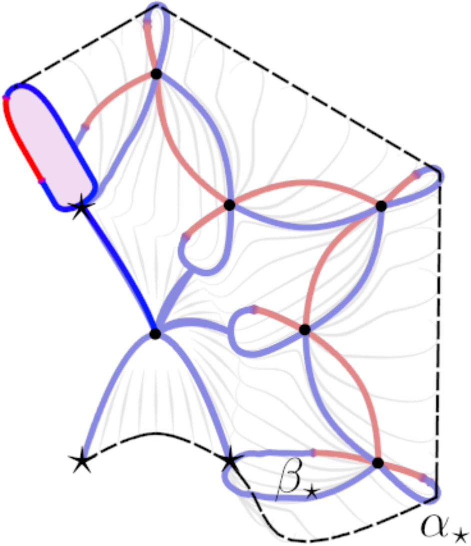
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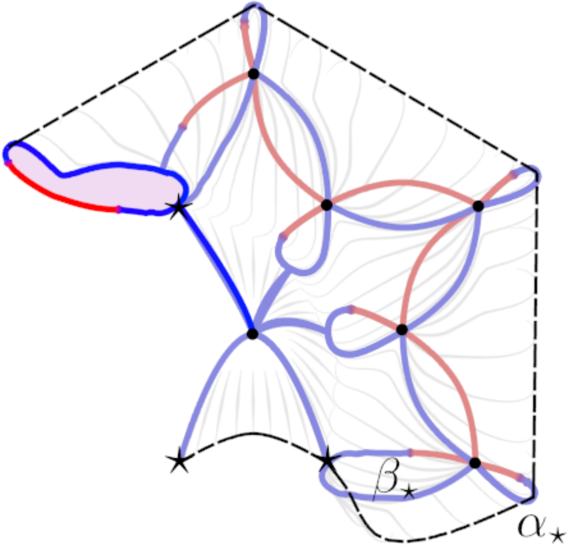
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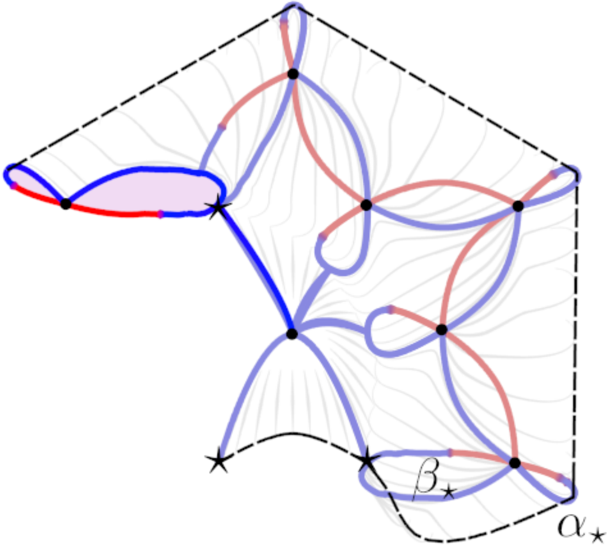
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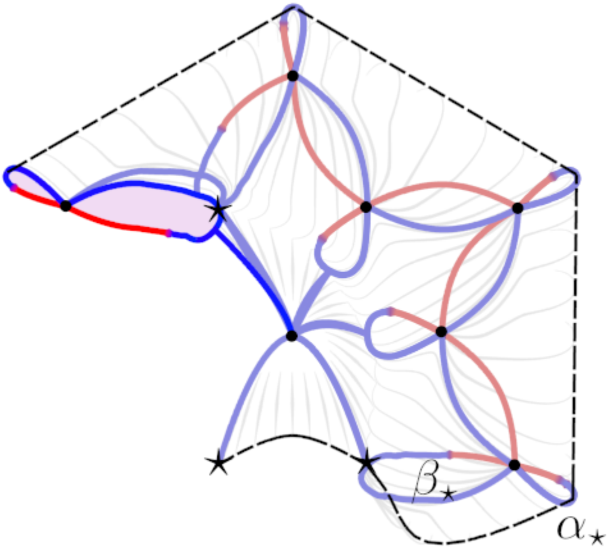
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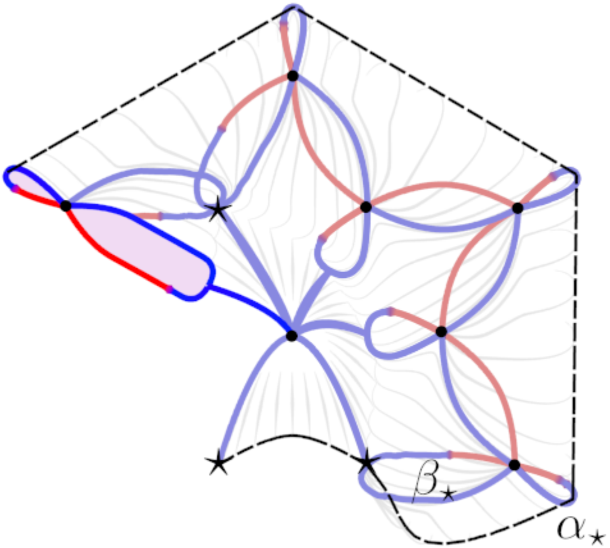
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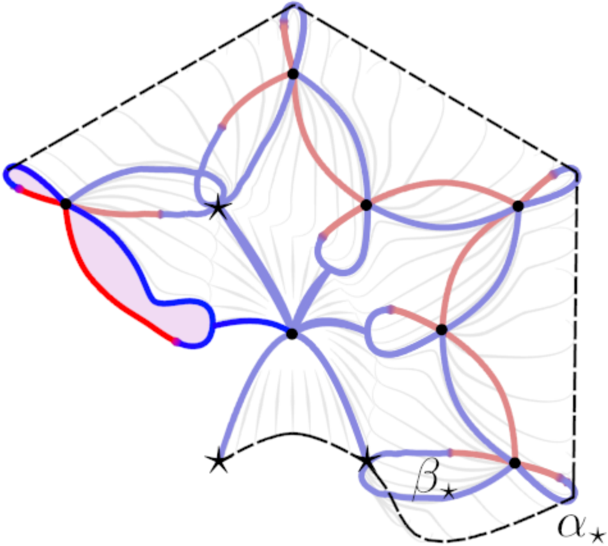
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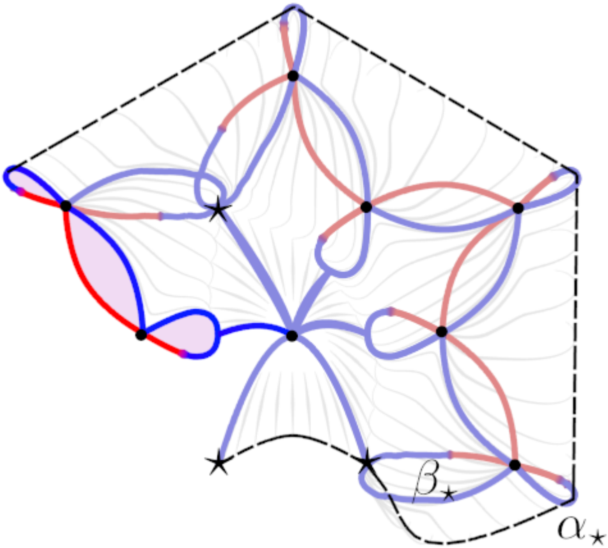
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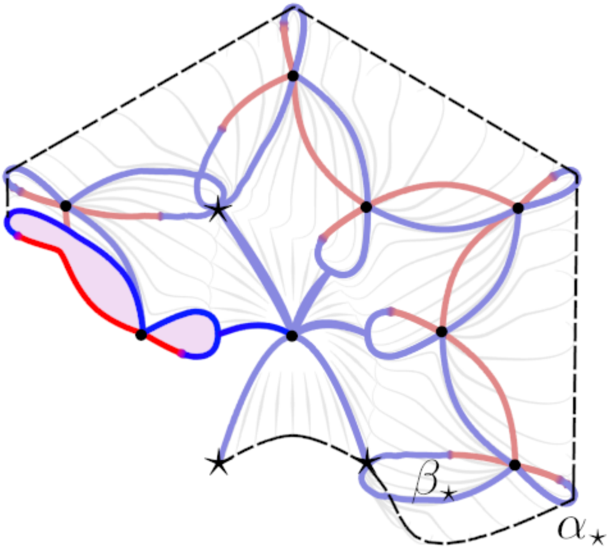
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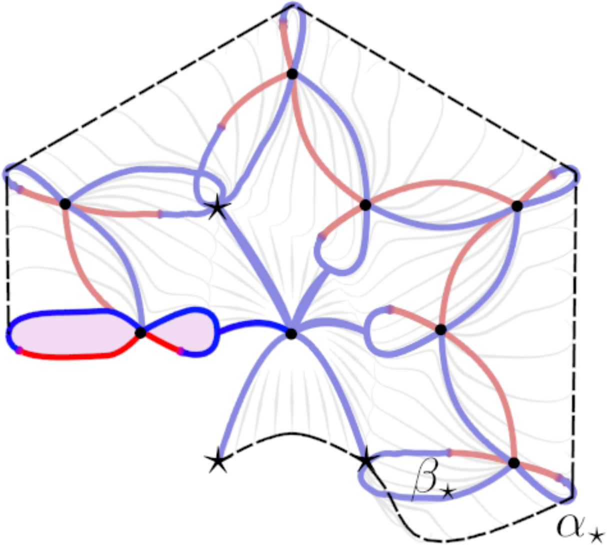
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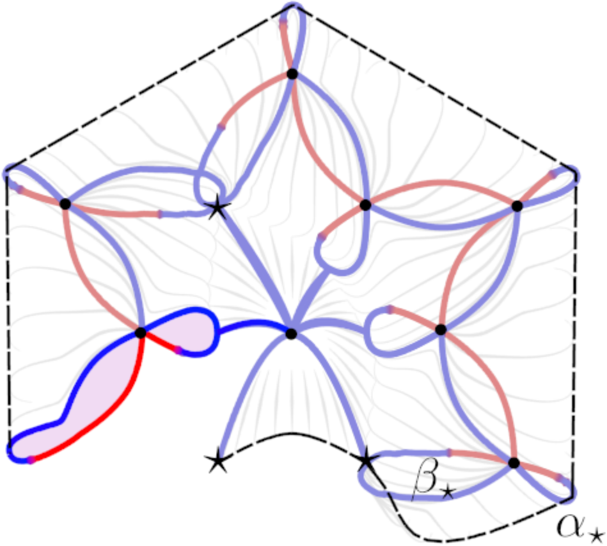
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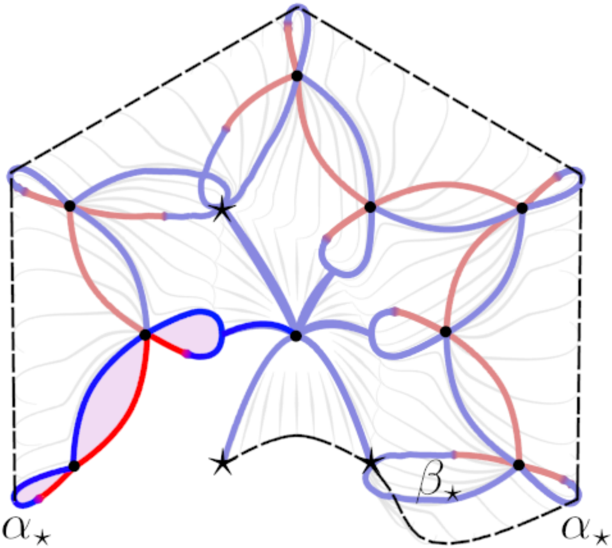
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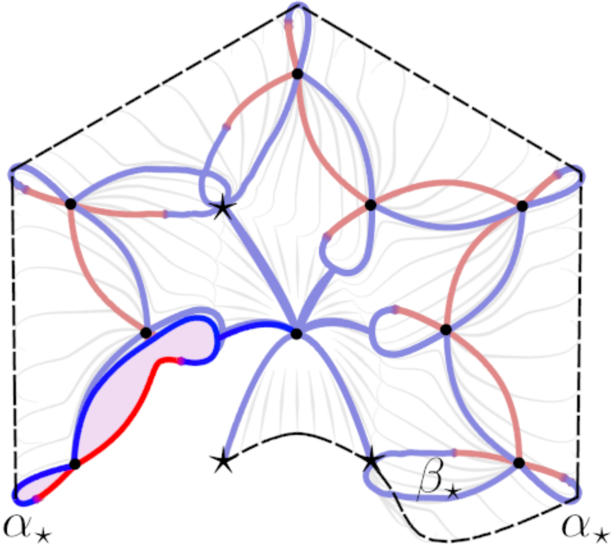
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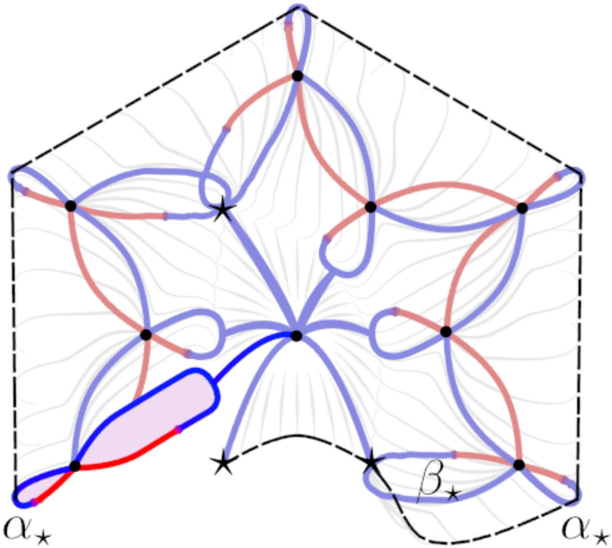
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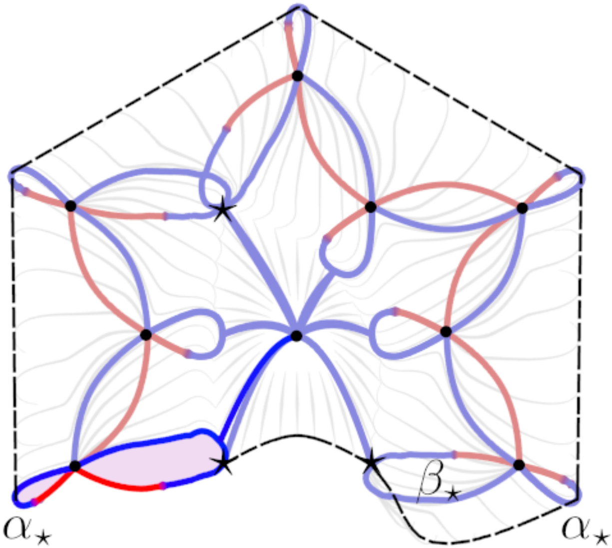
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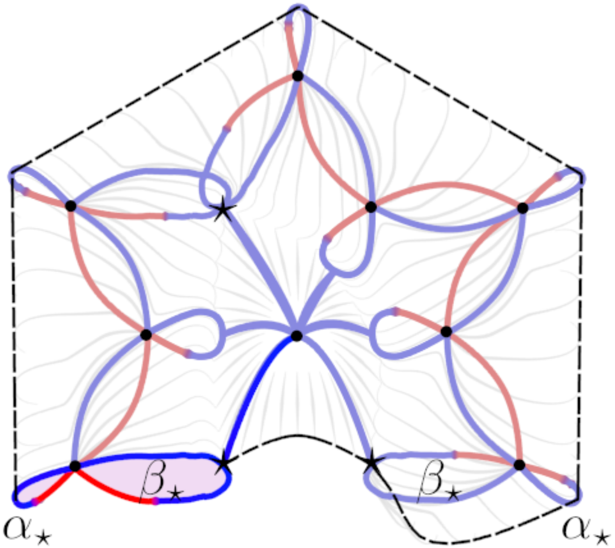
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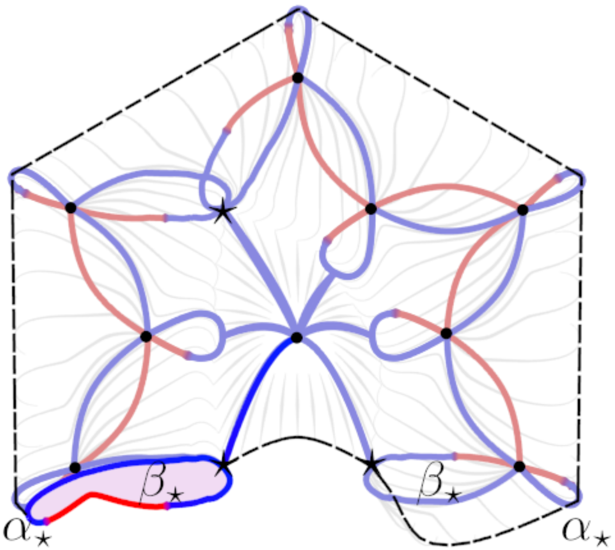
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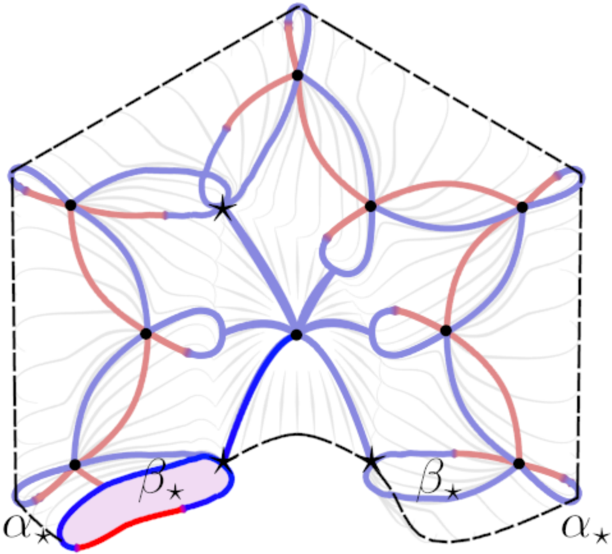
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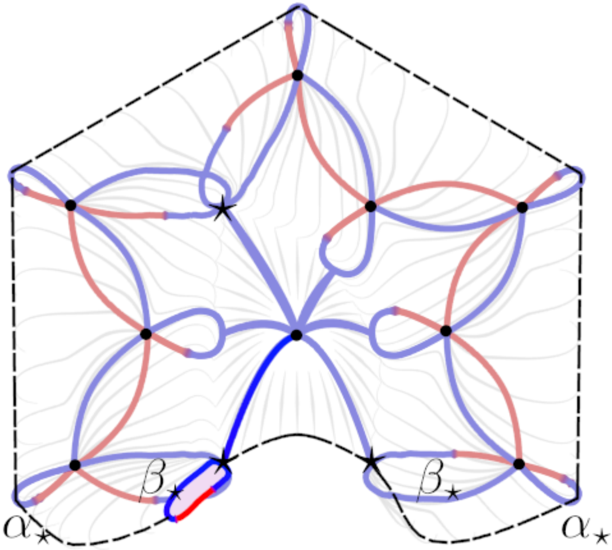
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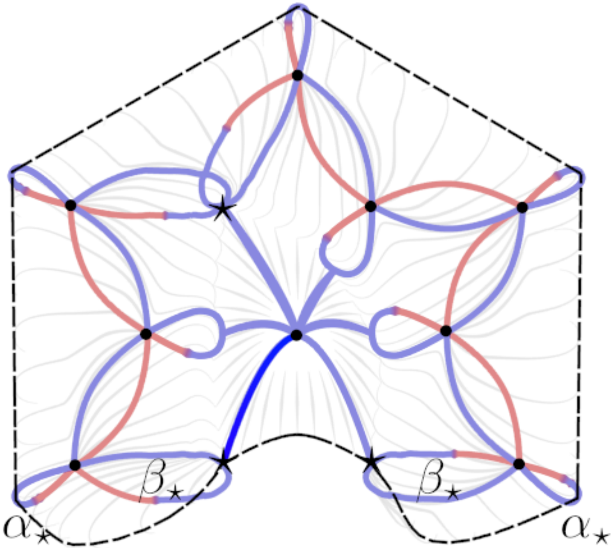
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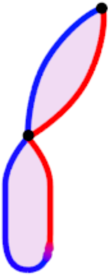

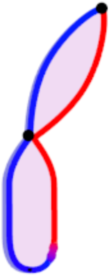

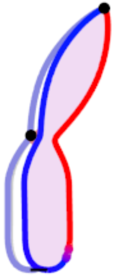

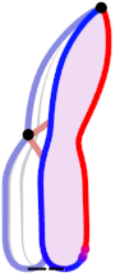

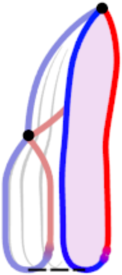

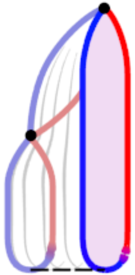

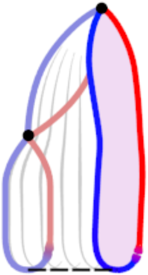

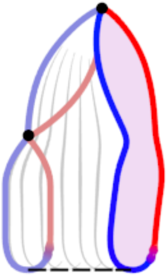

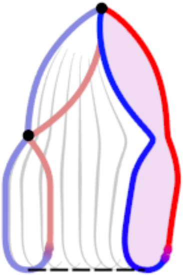

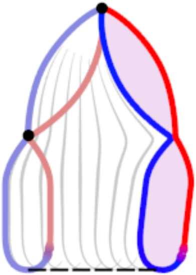

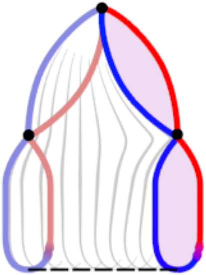

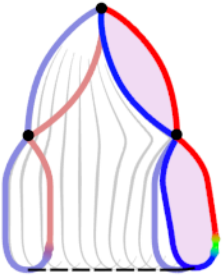

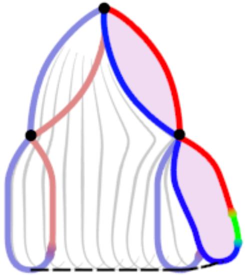

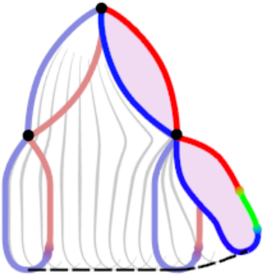

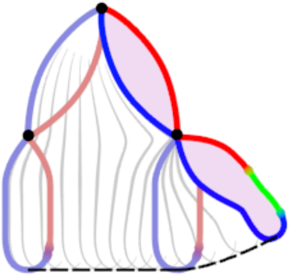

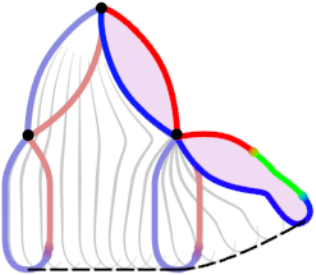

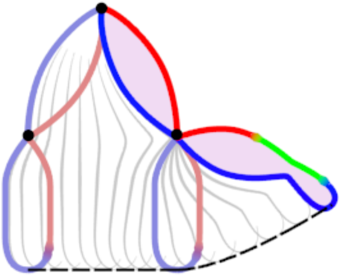

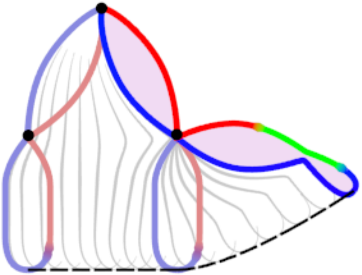

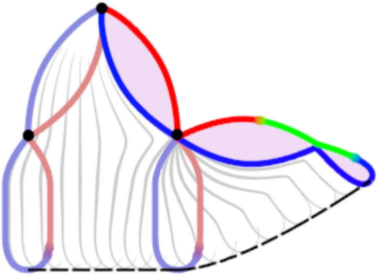

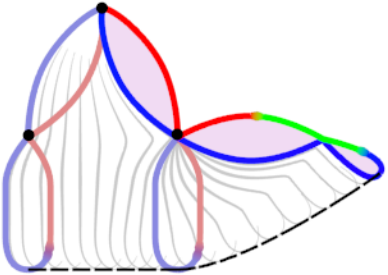

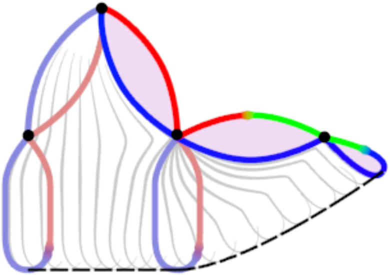

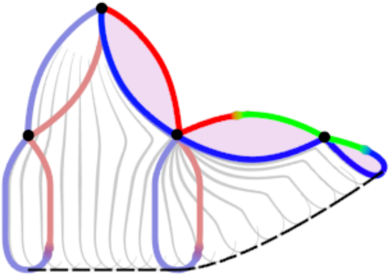

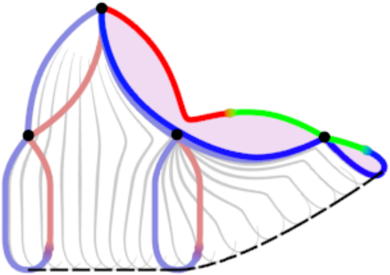

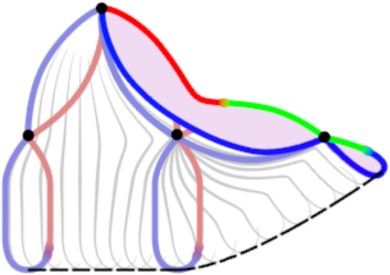

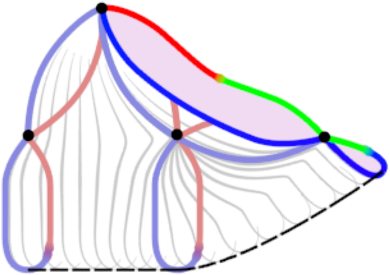

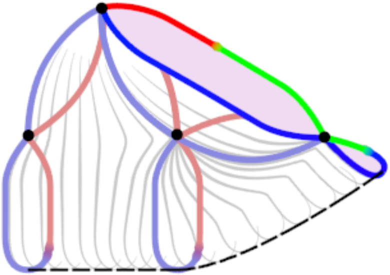

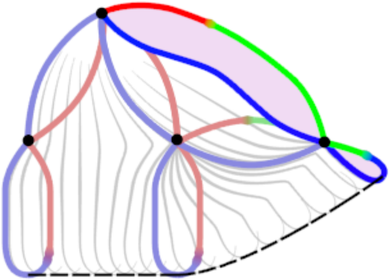

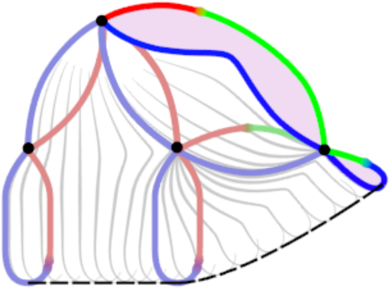

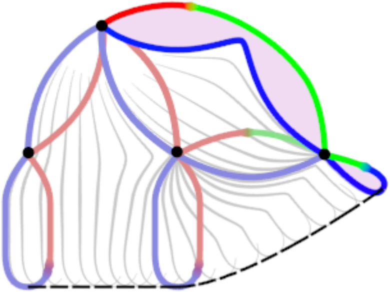

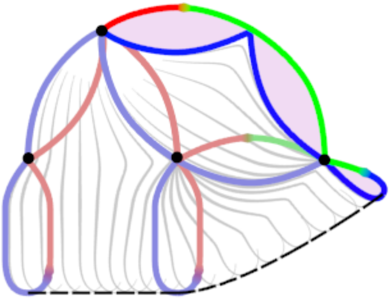

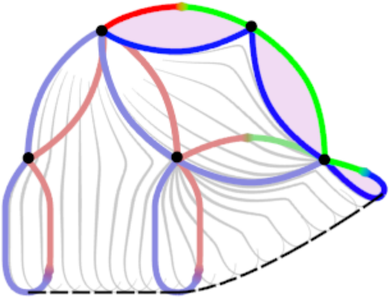

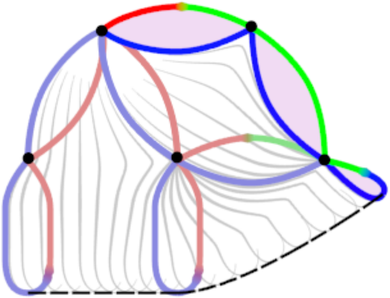

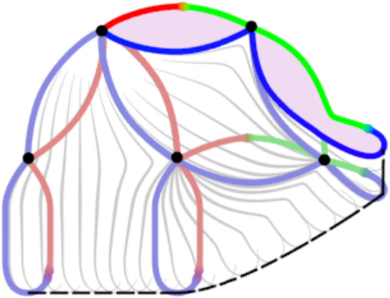

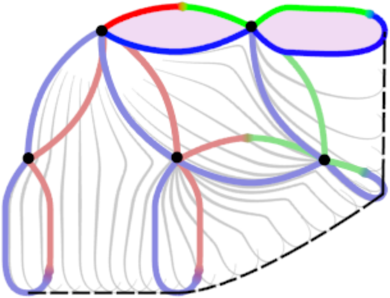

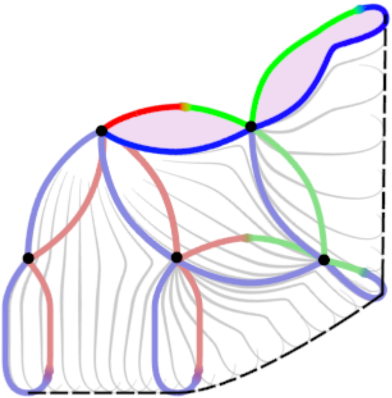

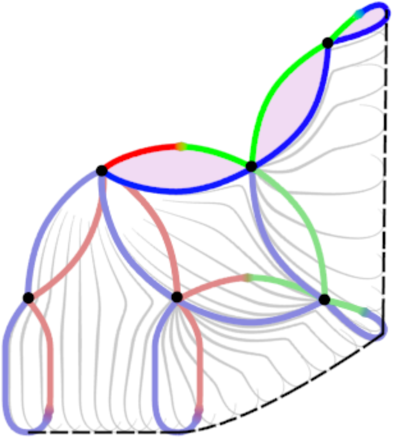

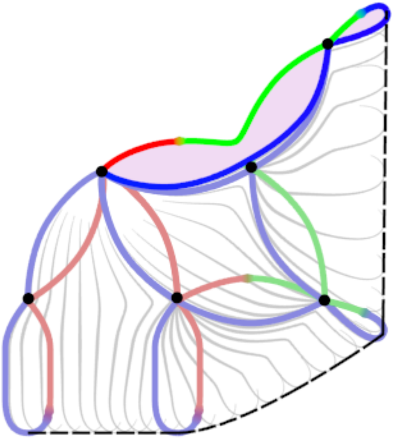

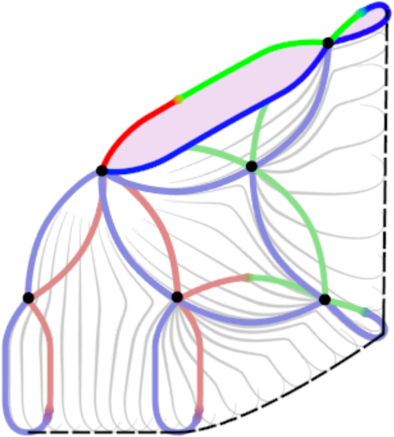

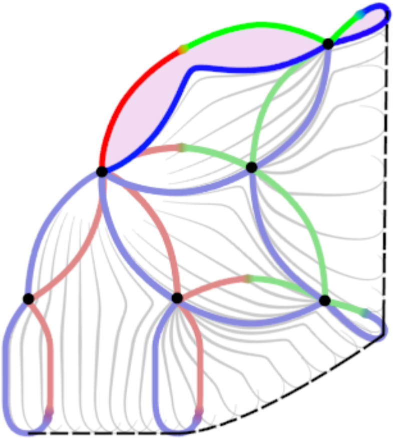

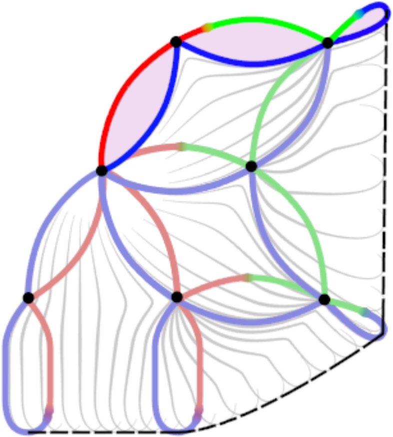

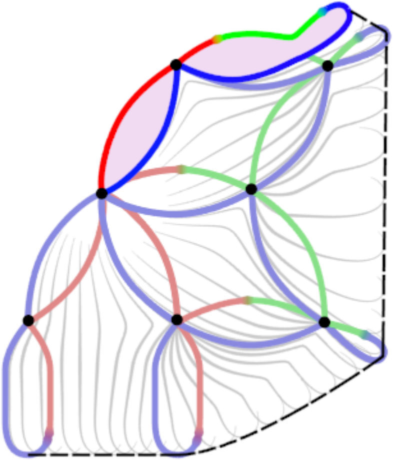

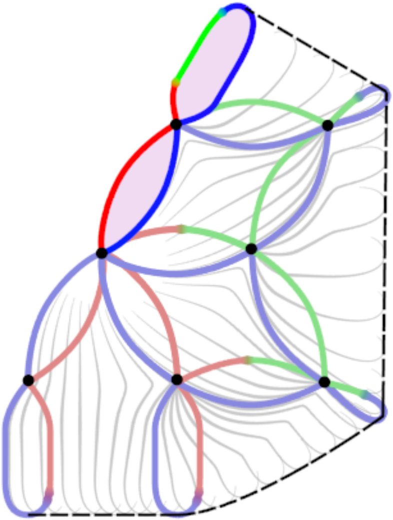

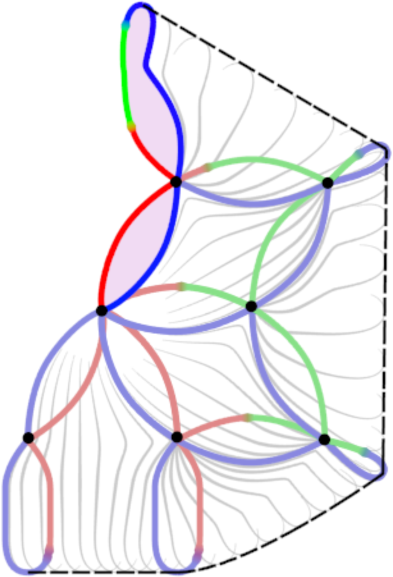

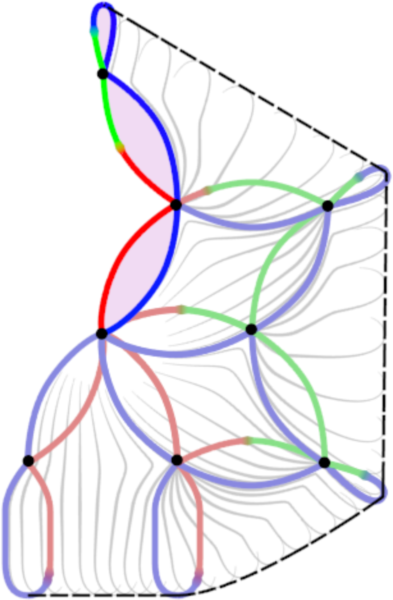

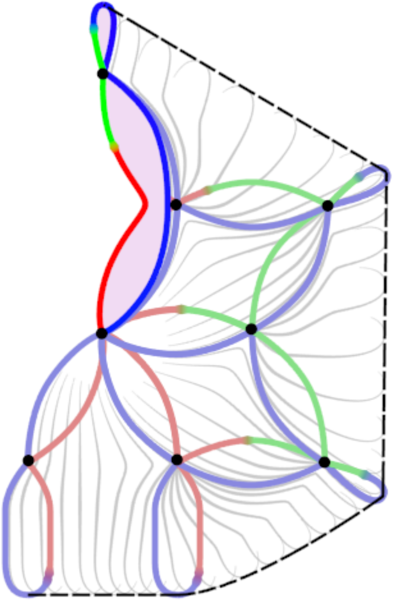

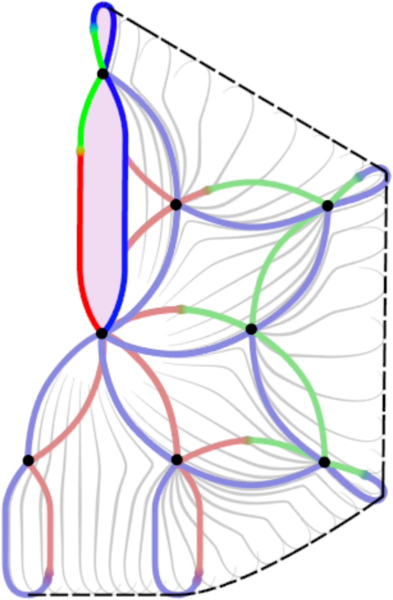

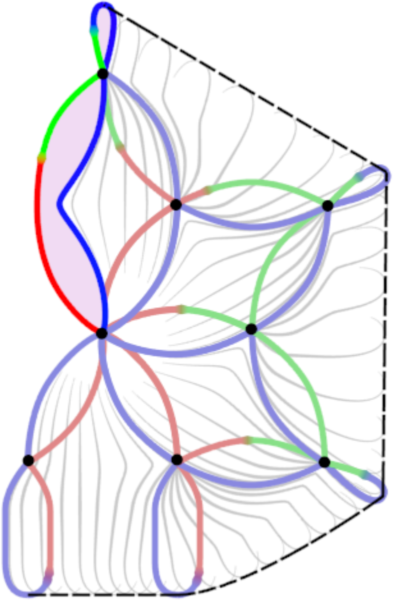

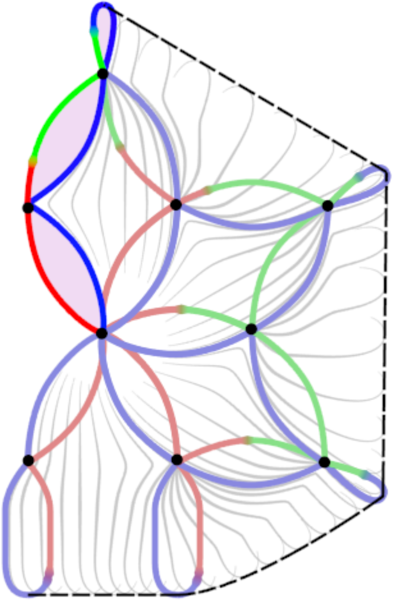

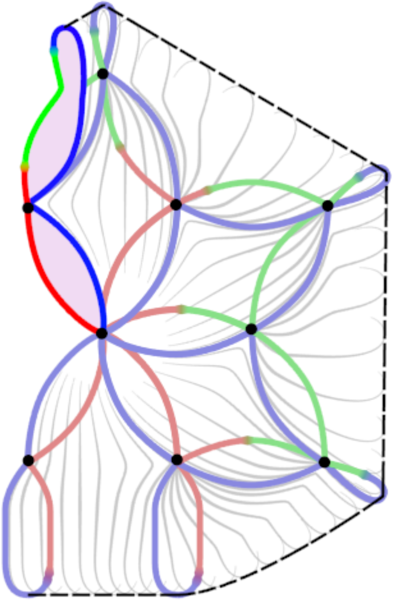

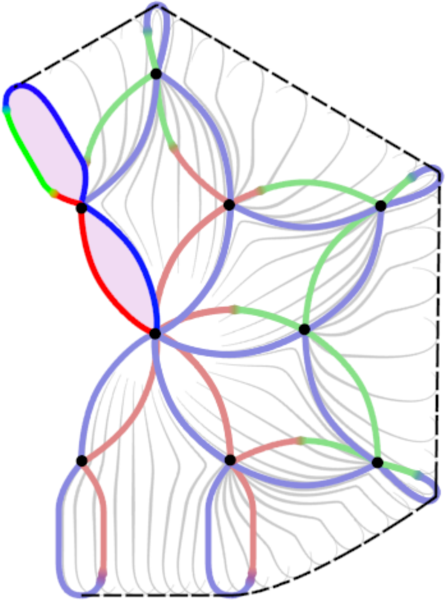

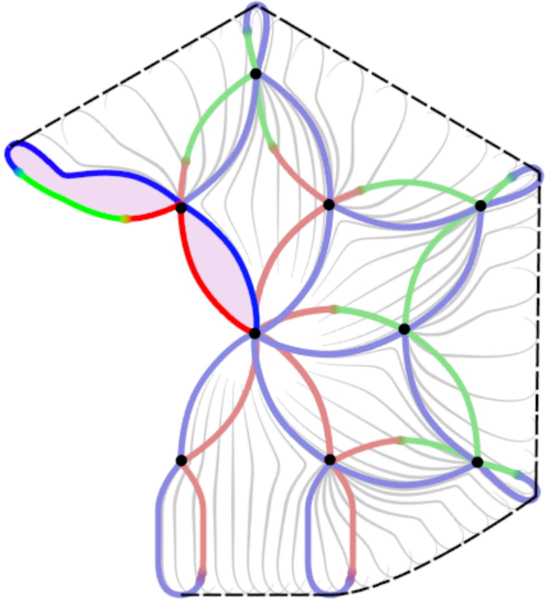

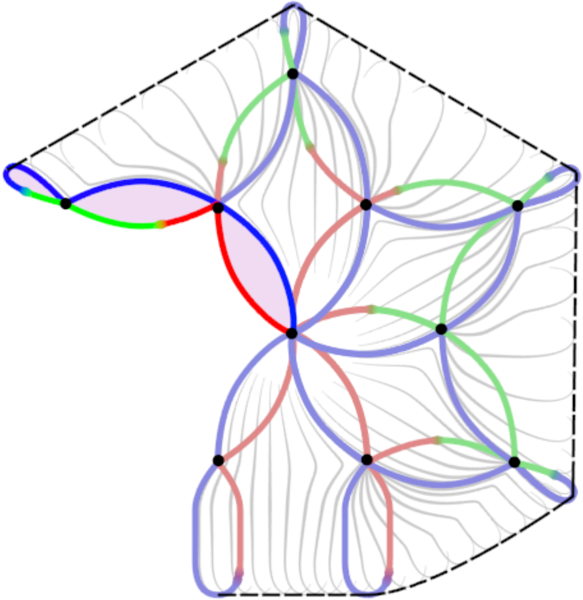

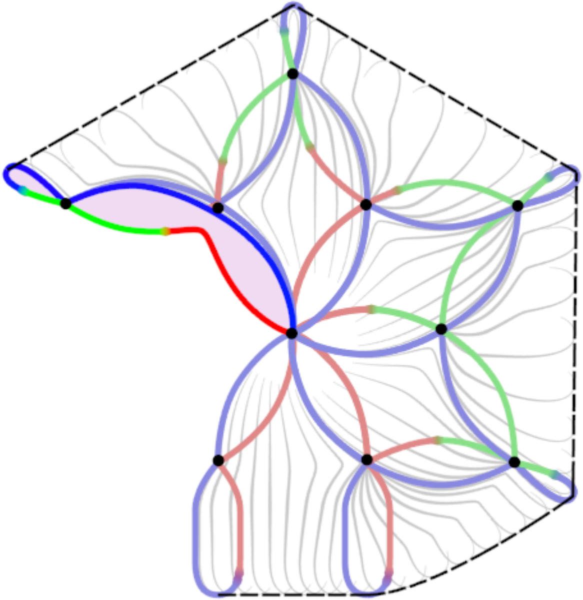

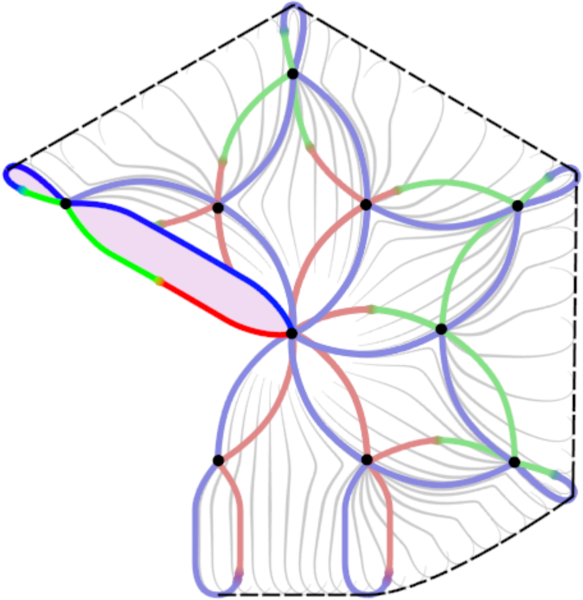

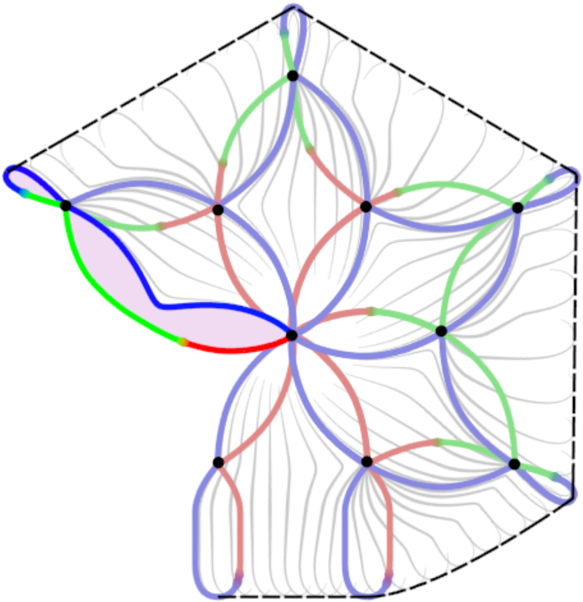

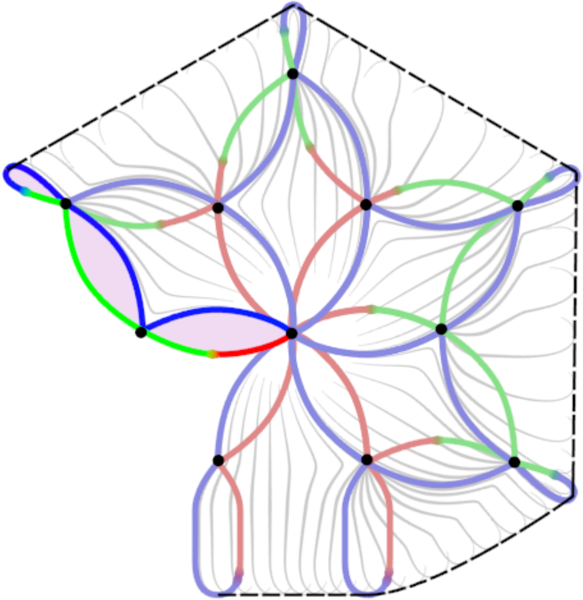

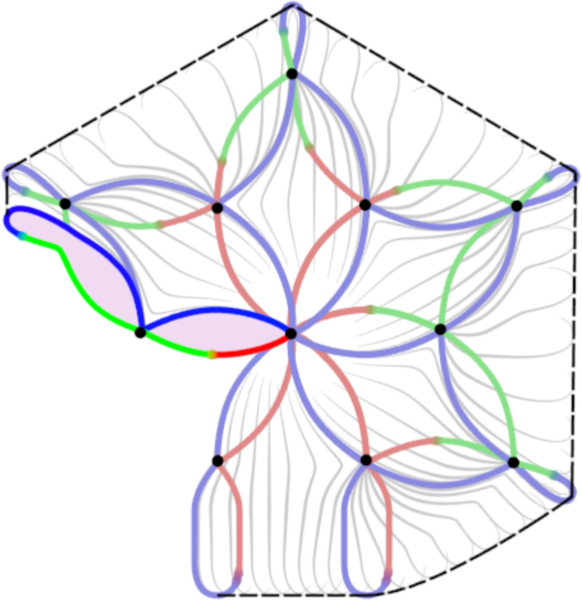

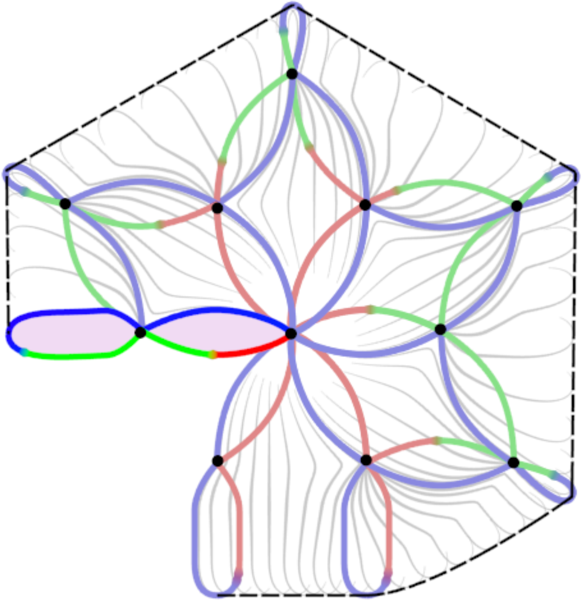

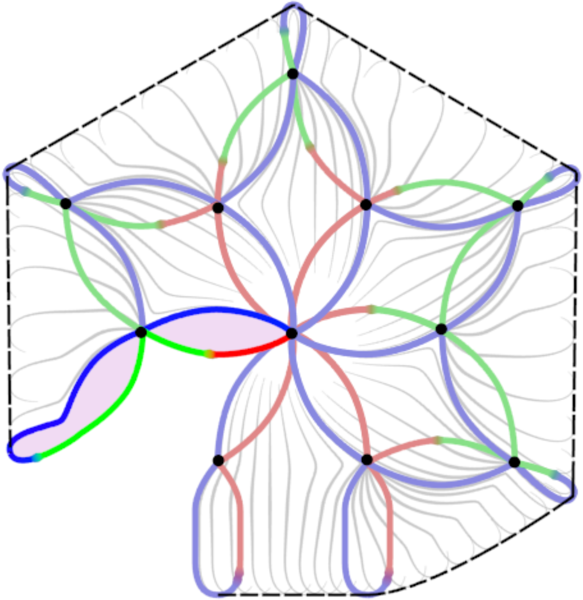

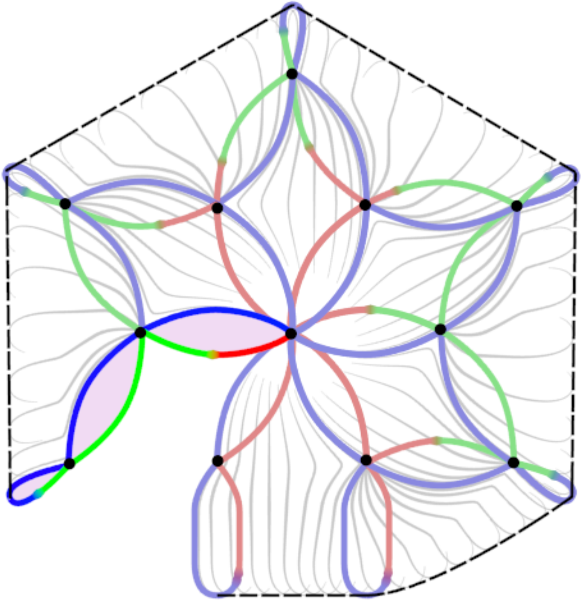

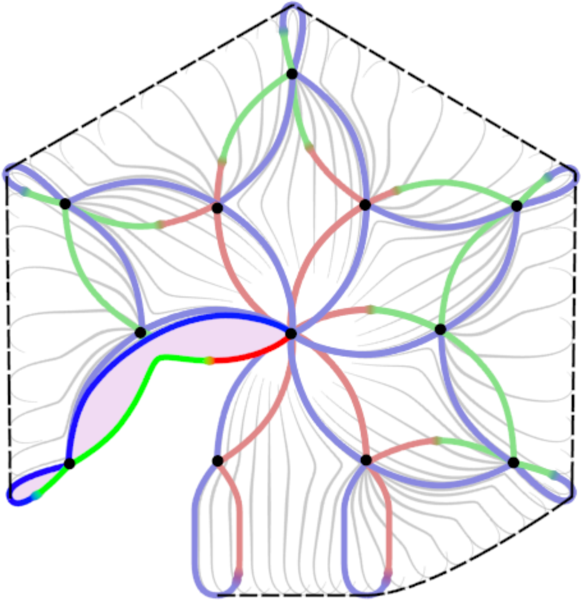

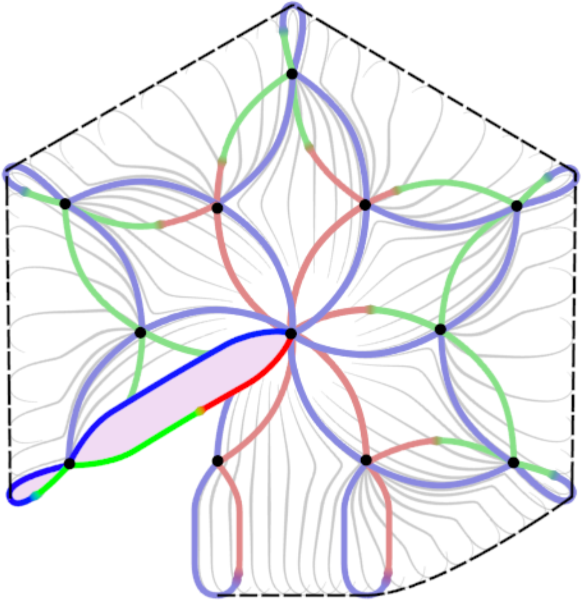

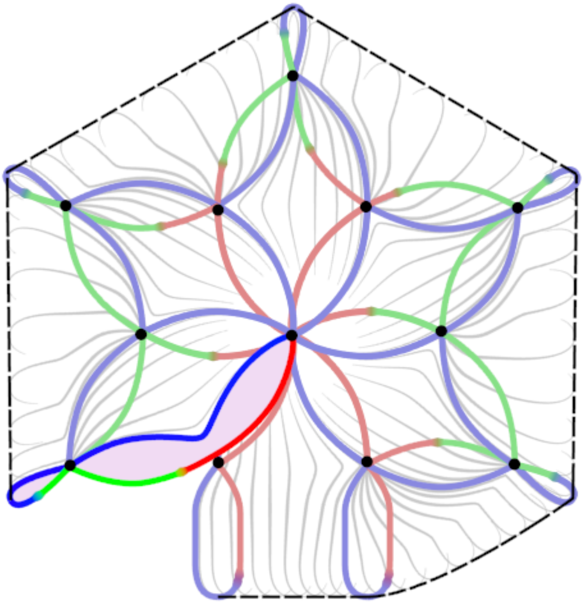

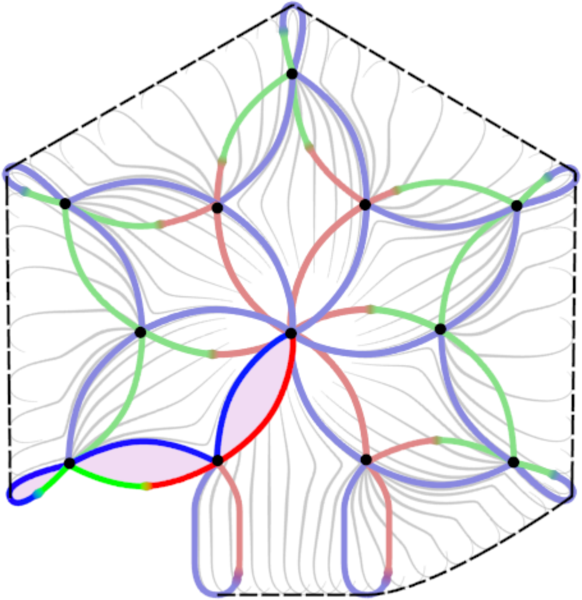

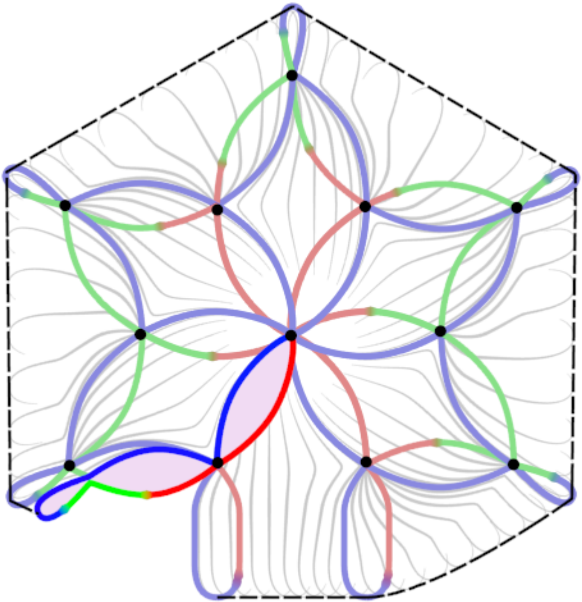

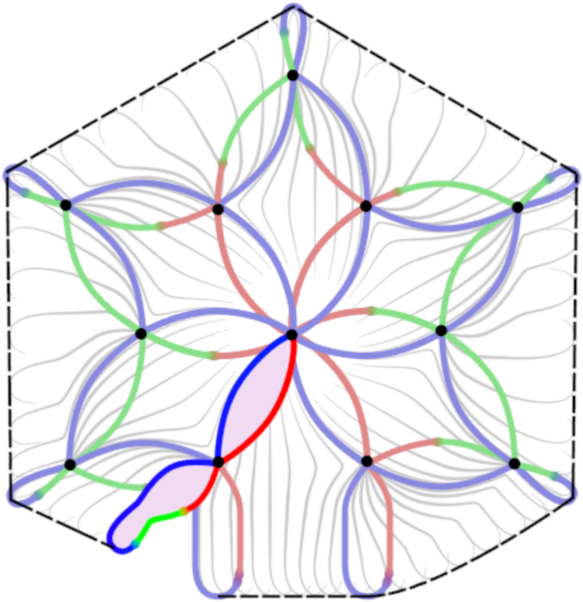

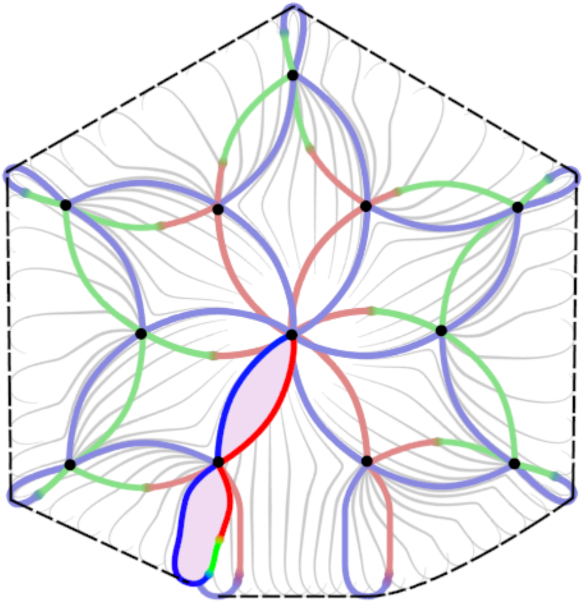

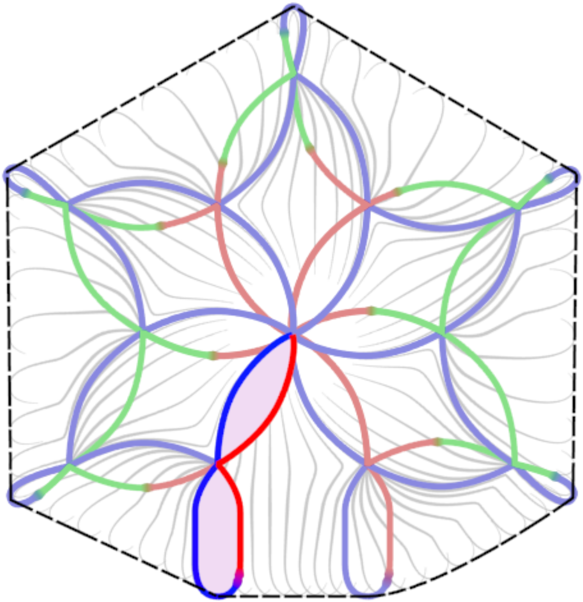

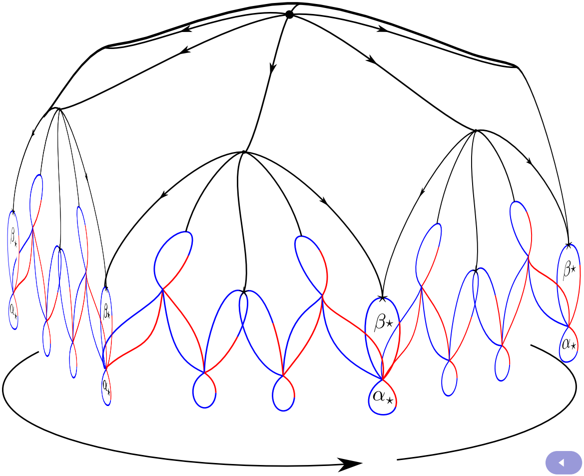
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Thank you !

ありがとう.