Nonlinear PDE for Future Applications

— Evolution Eq. and Mathematical Fluid Dynamics —

Date	10 July, 2017 – 14 July, 2017
Place	TOKYO ELECTRON House of Creativity 3F, Lecture Theater, Katahira Campus, Tohoku University
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Program

10 July (Mon)

18:30 – 20:30	Reception Venue: The restaurant HAGI

11 July (Tue)

9:50 – 10:00	Opening
10:00 – 11:30	Robert Denk (University of Konstanz) Maximal regularity for parabolic evolution equations Lecture 1: Lp-Sobolev spaces and maximal regularity
13:30 – 15:00	Michel Ruzicka (University of Freiburg) Theoretical and numerical analysis of generalized Newtonian fluids I
15:30 – 16:20	David Wegmann (TU Darmstadt) Existence of strong solutions and decay of turbulent solutions of Navier-Stokes flow with nonzero Dirichlet boundary data

12 July (Wed)

10:00 – 11:30	Robert Denk (University of Konstanz) Maximal regularity for parabolic evolution equations Lecture 2: The concept of $\mathcal{R}$-boundedness and the theorem of Mikhlin
Special Session of CFD and Plasma Physics	
13:00 – 13:40	Masaru Furukawa (Tottori University) A new method for 3D MHD equilibrium calculation via Hamiltonian field theory
13:50 – 14:15	Makoto Hirota (Tohoku University) Magnetohydrodynamic relaxation process sustained by AC magnetic helicity
14:15 – 14:40	Yasuhide Fukumoto (Kyushu University) Gyroscopic analogy of a rotating stratified flow confined in a spheroid and its implication to stability

15:00 − 15:40	Dmitry Kolomenskiy (JAMSTEC) Spectral method with volume penalization for numerical simulation of flapping flight of insects
15:50 − 16:15	Keiji Onishi (RIKEN) Immersed Boundary Method considering the handling of 'dirty' CAD data
16:15 − 16:40	Hiroshi Suito (Tohoku University) Application of immersed boundary method to environmental and biomedical problems
16:40 − 17:05	Yuji Hattori (Tohoku University) Corrected volume penalization method for direct numerical simulation of compressible flow and aeroacoustic sound
18:00 − 20:00	Banquet Venue: The restaurant HAGI

13 July (Thu)

10:00 − 11:30	Robert Denk (University of Konstanz) Maximal regularity for parabolic evolution equations Lecture 3: Maximal regularity for linear parabolic boundary value problems
13:30 − 15:00	Michel Ruzicka (University of Freiburg) Theoretical and numerical analysis of generalized Newtonian fluids II
15:30 − 16:20	Ken Abe (Osaka City University) Global well-posedness of the two-dimensional exterior Navier-Stokes equations for non-decaying data

14 July (Fri)

9:30 − 11:00	Robert Denk (University of Konstanz) Maximal regularity for parabolic evolution equations Lecture 4: Quasilinear parabolic evolution equations
11:20 − 12:50	Michel Ruzicka (University of Freiburg) Theoretical and numerical analysis of generalized Newtonian fluids III
12:50 − 13:00	Closing

Maximal regularity for parabolic evolution equations

Robert Denk

University of Konstanz

Lecture 1: L^p-Sobolev spaces and maximal regularity

Maximal regularity is one of the standard approaches to investigate semilinear and quasilinear parabolic evolution equations. The basic idea of maximal regularity is to find appropriate spaces for the right-hand side and for the solution, where the operator associated to the linearized equation induces an isomorphism. A closed operator $A\colon X \supset D(A) \to X$ acting in a Banach space has maximal L^p-regularity if the Cauchy problem

$$\partial_t u - Au = f \quad (t \in (0,T)), \quad u(0) = u_0$$

has a unique solution $u \in H^1_p((0,T);X) \cap L^p((0,T);D(A))$ depending continuously on the data f and u_0.

In our lectures, we will consider maximal regularity in L^p-Sobolev spaces in time t and space x with $p \in (1,\infty)$. For the initial value $u|_{t=0}$ or for boundary values, one has to describe the trace spaces of L^p-Sobolev spaces. It turns out that these are Besov spaces of non-integer order. If one considers L^p-spaces in time and L^q-spaces in space, so-called Triebel-Lizorkin spaces appear as trace spaces.

Lecture 2: The concept of $\mathcal{R}$-boundedness and the theorem of Mikhlin

Taking Fourier (or Laplace) transform $\mathscr{F}_t$ in time, one can see that maximal regularity is equivalent to the condition that

$$\mathscr{F}_t^{-1} i\tau(i\tau - A)^{-1}\mathscr{F}_t$$

defines a continuous operator in $L^p(\mathbb{R};X)$. In this case, the (operator-valued) symbol $\tau \mapsto i\tau(i\tau - A)^{-1}$ is said to be an L^p-multiplier.

The theorem of Mikhlin gives a sufficient condition for a symbol to be an L^p-multiplier. In the vector-valued case, one assumes $\mathcal{R}$-boundedness of the symbol ("$\mathcal{R}$" standing for Rademacher or randomized). Several results (e.g., due to Lutz Weis) can be used to prove $\mathcal{R}$-boundedness and, consequently, maximal regularity for operator valued symbols. Here, the Banach space X has to be a UMD space and, in particular, X has to be reflexive.

Lecture 3: Maximal regularity for linear parabolic boundary value problems

An application of the vector-valued Mikhlin theorem gives maximal regularity for linear parabolic differential operators in the whole space $\mathbb{R}^n$. For boundary value problems in domains, however, one has to study model problems in the half space $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n : x_n > 0\}$ of the form

$$\lambda u - Au = f \quad \text{in } \mathbb{R}^n_+,$$
$$B_j u = g_j \ (j = 1, \dots, m) \quad \text{on } \mathbb{R}^{n-1},$$

where A is a linear partial differential operator of order $2m$ and $B_1, \dots, B_m$ are boundary operators. We assume the boundary value problem to be parabolic which means, in particular, that the Shapiro-Lopatinskii condition has to be satisfied.

For a parabolic boundary value problem, one can explicitly describe the solution operator to the above boundary value problem in the half space. This makes it possible to prove $\mathcal{R}$-boundedness and to apply Mikhlin's theorem.

Lecture 4: Quasilinear parabolic evolution equations

The above results on maximal regularity can be applied to quasilinear parabolic problems of the form

$$\partial_t u + A(u)u = F(u) \quad (t > 0), \quad u(0) = u_0.$$

By an application of the contraction mapping principle, one can show that this problem is locally well-posed if the linearized operator $A(u_0)$ has maximal regularity. As an example, one can show well-posedness for the graphical mean curvature flow.

By construction, the maximal regularity approach yields a short-time solution (or a global solution for small data) belonging to some L^p-Sobolev space. However, additional regularity of A and F also implies additional regularity for the solution, as can be shown by the so-called "parameter trick". Concerning the stability of equilibria of the equations, we mention some results on the generalized principle of linearized stability due to Jan Prüss.

References

[1] R. Denk, M. Hieber, and J. Prüss. $\mathcal{R}$-boundedness, Fourier multipliers and problems of elliptic and parabolic type. *Mem. Amer. Math. Soc.*, 166(788):viii+114, 2003.

[2] P. C. Kunstmann and L. Weis. Maximal *Lp*-regularity for parabolic equations, Fourier multiplier theorems and H1-functional calculus. In *Functional analytic methods for evolution equations*, volume 1855 of *Lecture Notes in Math.*, pages 65–311. Springer, Berlin, 2004.

[3] J. Prüss and G. Simonett. *Moving interfaces and quasilinear parabolic evolution equations*, volume 105 of *Monographs in Mathematics*. Birkhäuser/Springer, [Cham], 2016.

Theoretical and numerical analysis of generalized Newtonian fluids

Michel Ruzicka

University of Freiburg

The motion of generalized Newtonian fluids has attracted a huge research activity in the last 20 years. In the lectures we will present in detail the state of the art concerning the existence of weak solutions for the system describing steady motions of generalized Newtonian fluids. Moreover, we will present the proof for convergence rates for a Finite Element approximation of the steady p-Stokes problem. Both approaches make strong use of so called shifted N-functions.

Existence of strong solutions and decay of turbulent solutions of Navier-Stokes flow with nonzero Dirichlet boundary data

David Wegmann

TU Darmstadt

We consider the Navier-Stokes equations in a domain with compact boundary and nonzero Dirichlet boundary data β. A solution is constructed as a sum of a very weak solution b to the instationary Stokes equations with nonzero boundary data and a weak solution v to a system of Navier-Stokes type with zero boundary data.

Assuming that $\beta(t) \to 0$ as $t \to \infty$, we proved in [2] that there exists a solution v which fulfills $\|v(t)\|_2 \to 0$ as $t \to \infty$. Furthermore, in a bounded domain the solution v tends exponentially to 0 if the corresponding data is exponentially decreasing.
As a last result, we calculated a lower polynomial bound for the decay rate if Ω is unbounded. Therefore, we used a suitable spectral decomposition of the Stokes operator as introduced in [1] and Duhamel's formula.

Recently, we proved the same decay result for an arbitrary turbulent solution. The main tool for the proof is to show the existence of a strong solution after some time $T > 0$ and to identify the turbulent solution with the strong solution in the time interval $[T, \infty)$.

This is a result of a collaboration with Prof. Dr. Reinhard Farwig and Prof. Dr. Hideo Kozono.

References

[1] W. Borchers and T. Miyakawa. Algebraic L^2 decay for Navier-Stokes flows in exterior domains. II. *Hiroshima Math. J.*, **3**, 621–640,1991.

[2] R. Farwig, H. Kozono, and D. Wegmann. Decay of non-stationary Navier-Stokes flow with nonzero Dirichlet boundary data. *Indiana Univ. Math. J.*, 2016, in press.

[3] R. Farwig, H. Kozono, and D. Wegmann. Existence of Strong Solutions and Decay of Turbulent Solutions of Navier-Stokes Flow with Nonzero Dirichlet Boundary Data. *J. Math. Anal. Appl.*, 453(1): 271-286, 2017.

A new method for 3D MHD equilibrium calculation via Hamiltonian field theory

Masaru Furukawa

Department of Applied Mathematics and Physics, Tottori University

A new method for calculating three-dimensional (3D) magnetohydrodynamics (MHD) equilibrium has been developed. The method is based on the theory of Hamiltonian mechanics. Dissipationless fluid dynamics and MHD are described by Hamiltonian. The dynamical equation described by the Hamiltonian conserves energy of the system. In fluid systems, other conserved quantities, called Casimir invariants, can exist. For example, enstropy is a Casimir invariant in two-dimensional neutral fluid dynamics. Among a variety of states with the same values of the Casimir invariants, an energy extremum gives an equilibrium of the system. To reach the equilibrium, we solve an artificial dynamics derived from the original one, especially the artificial dynamics that changes the energy monotonically while preserves the Casimir invariants. Thus the system reaches an equilibrium. This method is called simulated annealing (SA) [1]. The SA assumes nothing that is assumed in existing numerical codes for 3D MHD. Moreover, the equilibrium obtained by the SA is classified systematically by the values of the Casimir invariants, which is good for theoretical studies. Our previous studies demonstrated that the SA succeeds to give various low-beta reduced MHD equilibria in two-dimensional rectangular domain [2,3]. Recently it has been extended to the 3D equilibrium of ideal, low-beta reduced MHD [4]. In this talk, the theoretical basis is explained, and then an equilibrium with 3D structure, or magnetic islands, is shown that is obtained as a lower energy state than a 2D state. Several issues, i.e. non-uniqueness of the construction of the artificial dynamics and dependence on the initial condition, are discussed.

References

[1] G. R. Flierl, P. J. Morrison, Physica D 240, 212 (2011).

[2] Y. Chikasue and M. Furukawa, Phys. Plasmas 22, 022511 (2015).

[3] Y. Chikasue and M. Furukawa J. Fluid Mech. 774, 443 (2015).

[4] M. Furukawa and P. J. Morrison, Plasma Phys. Control. Fusion 59, 054001 (2017).

Magnetohydrodynamic relaxation process sustained by AC magnetic helicity injection

Makoto Hirota

Institute of Fluid Science, Tohoku University

Driving a steady current in plasma as long as possible is an important issue in the study of magnetic confinement fusion reactor, which is typically a torus-shaped vessel called Tokamak. Since the plasma current is usually generated inductively by means of a transformer (applying a DC voltage), the finite flux swing of the transformer leads to a finite discharge time and, hence, it is problematic that the fusion reactor is limited to pulsed operation. As an attractive way of non-inductive current drive, oscillating toroidal and poloidal voltages are known to generate a steady current via magnetohydrodynamic (MHD) relaxation process. In contrast to the DC voltage, the two AC voltages generates oscillating magnetic field in the edge region, which amounts to the periodic injection of magnetic helicity. Then, a nonlinear turbulent transport causes self-organization of a DC current in the core region, following Taylor's conjecture that magnetically confined plasmas tend to relax to minimum energy states while conserving total helicity. In this talk, a direct numerical simulation of this 3D MHD relaxation process will be introduced to verify Taylor's conjecture. The efficiency of current drive and the optimal parameters for helicity injection will be discussed based on numerical results.

Gyroscopic analogy of a rotating stratified flow confined in a spheroid and its implication to stability

Yasuhide Fukumoto, Yuki Miyachi

Institute of Mathematics for Industry and Graduate School of Mathematics, Kyushu University

In the investigation of the Rayleigh-Taylor instability (RTI) in a rotating frame, Chandrasekhar (1961) claimed that rotation does not affect the instability or stability of a stratified flow. However, the dispersion relation of the internal inertia-gravity wave reveals the stabilizing action of the Coriolis force. We address the suppression of the gravitational instability of rotating stratified flows in a confined geometry in two ways, continuous and discontinuous stratification. A rotating flow of a stratified fluid confined in an ellipsoid, subject to gravity force, whose velocity and density fields are linear in coordinates, bears an analogy with a mechanical system of finite degrees of freedom, that is, a heavy rigid body. An insight is gained into the mechanism of system rotation for the ability of a lighter fluid of sustaining, on top of it, a heavier fluid when the angular velocity is greater than a critical value. The sleeping top corresponds to such a state. First we show that a rotating stratified flow confined in a tilted spheroid is equivalent to a heavy symmetrical top with the center of gravity off the symmetric axis. This off-axis effect of the gravity center on the linear stability of the sleeping top and its bifurcation is investigated in some detail. Second, we explore the incompressible two-layer RTI of a discontinuously stratified fluid confined in the lower-half of a spheroid rotating about the axis of symmetry oriented parallel to the vertical direction. The gyroscopic analogy accounts for decrease of the critical rotation rate with oblateness.

Spectral method with volume penalization for numerical simulation of flapping flight of insects

Dmitry Kolomenskiy

JAMSTEC

Numerical simulation of flapping flight presents multiple challenges such as relative flapping motion of the wings, their complex geometrical shape and deformation. In the talk, I will present a dedicated open source software for numerical modeling of the flapping flight of insects, FluSI. The computational framework is optimized to high performance computers with distributed memory. The discretization of the three-dimensional incompressible Navier-Stokes equations is based on a Fourier pseudospectral method with adaptive time stepping. The complex time varying geometry of flapping wings and moving body is handled using the volume penalization method. I will present multiple examples of application of this software to biological problems, including flight in turbulent environment and maneuvering.

Immersed Boundary Method considering the handling of ‘dirty’ CAD data

Keiji Onishi

RIKEN

When conducting simulations based on Computational Fluid Dynamics (CFD) in the industrial field, we still have a major problem on calculation grid generation with good quality and short work load. The geometry preparation process including shape reproduction to make geometries ‘clean’ and ‘water-tight’ is still required even if we used an immersed boundary method. The human work load of this process costs a lot, even we used recently designed commercial grid generation software. This is mainly caused by gaps / overlaps which were generated at the data conversion process from CAD software, or many surfaces which have zero-thickness, or unnecessary surfaces inside the parts and tiny parts which have a size less than grid resolution. This kind of data is called as ‘dirty’ dataset. We are worried that this user’s heavy work load will increase further in near future, and will be a major reason for hindering the broadening of applications in the industrial CFD field, as such it is an important issue to solve.

The authors propose the method to avoid the problem of gap / overlap by discretizing the wall, and the method for zero thickness wall by arranging dummy cells. So that shape data can be contracted on a cell basis and immersed imaginary points can be arbitrarily set. The discretization can be easily obtained by applying binary space partitioning from the geometric relation between the grid line (surface) and the polygon (subdivided into triangles). The dummy cell is handled as shadow data existing only on the computer memory. Furthermore, we introduce a technique to avoid search failure coming from ‘dirtiness’ of geometry by projecting the searching direction of imaginary points in the space into an axial direction and replacing the operation of divergence with the projected vector. This is a technique derived by exploiting the assumption that the divergence around the cell obtained by the original stencil calculation is expanded, then the tensor component of the second order derivative of the 3 dimensional velocity gradient which appears in the complement calculation at the imaginary point can be ignored. This made it possible to calculate immersed boundary with a robust and small number of operations. In this presentation, we introduce details of the numerical scheme applied to the immersed boundary methods and discuss their relevance.

Application of immersed boundary method to environmental and biomedical problems

Hiroshi Suito

Advanced Institute for Materials Research, Tohoku University

Immersed boundary method is a promising numerical method that is applicable to a wide range of real-world problems. This talk presents some examples of numerical simulations using immersed boundary method. The first example is the motions of aquatic plants during interaction with flowing water. In this example, the boundary between a solid (plant) and a fluid (water) deforms along with time. Immersed boundary method can be applied easily to such moving boundary problems. The second example is the blood flow in the aorta, wherein the various shapes of aneurysms are represented using immersed boundary method on a curvilinear coordinate system, where one coordinate axis is taken along the centerline of the aorta. Using these examples, both strong and weak points of immersed boundary method are discussed.

Corrected volume penalization method for direct numerical simulation of compressible flow and aeroacoustic sound

Yuji Hattori

Institute of Fluid Science, Tohoku University

The volume penalization (VP) method for compressible flows is investigated as a tool of direct numerical simulation of aeroacoustic sound in problems where not only acoustic pressure but also hydrodynamic pressure depends on time and position. First, it is shown that the method proposed by Liu and Vasilyev (2007) is not Galilean invariant. It is corrected to satisfy Galilean invariance. Next, numerical accuracy of the corrected VP method is investigated in problems of simple geometry which can be simulated also by a standard method on a body-fitted coordinate system: sound generation in (i) flow past a fixed square/circular cylinder, (ii) flow past an oscillating square/circular cylinder, and (iii) flow past two square cylinders. The results confirm that the corrected VP method gives reasonably accurate results for sound pressure which is much smaller than hydrodynamic pressure within 5% error. Finally, the corrected method is applied to a problem of complex geometry, which cannot be simulated by standard methods using body-fitted coordinate systems without considerable difficulty: sound generation in flow past an oscillating cylinder and a fixed cylinder behind it. The results show that the present method is in principle applicable to aeroacoustic problems in any complex geometry including practical engineering ones.

Global well-posedness of the two-dimensional exterior Navier-Stokes equations for non-decaying data

Ken Abe

Osaka City University

We consider the two-dimensional Navies-Stokes equations in an exterior domain, subject to the Dirichlet boundary condition. Stationary solutions of this problem and their asymptotic behavior have been studied in a large literature, while a few results are known about the non-stationary problem for non-decaying initial data. We report some global well-posedness result for bounded initial data with a finite Dirichlet integral, and unique existence of asymptotically constant solutions for arbitrary large Reynolds numbers.