POLYLOGS, MULTIPLE ZETAS, AND RELATED TOPICS

Date: November 11-12, 2017

Venue: TOKYO ELECTRON House of Creativity 3F, Lecture Theater, Katahira Campus, Tohoku University

Organizers: Herbert Gangl (Kyushu University, Durham University)
Masanobu Kaneko (Kyushu University)
Yasuo Ohno (Tohoku University)

Program

November 11, Saturday

10:00 – 10:10 Opening

10:10 – 11:00 Kenji Sakugawa (RIMS)
A p-adic analogue of Zagier's polylogarithm conjecture

11:20 – 12:10 Steven Charlton (Tuebingen Univ.)
Relating multiple polylogarithms in weight ≥ 5

——— Lunch Break ———

14:00 – 14:50 Masataka Ono (Keio Univ.)
New functional equations of finite multiple polylogarithms

15:10 – 16:00 Danylo Radchenko (MPIM Bonn)
Special values and functional equations of the Herglotz function

16:20 – 17:10 Shin-ichiro Seki (Osaka Univ.)
Generalizations of finite multiple zeta values

November 12, Sunday

10:00 – 10:50 Erik Panzer (Oxford Univ.)
Conical zeta values

11:10 – 12:00 Nobuo Sato (National Taiwan Univ.)
On Charlton's conjectural identities of the multiple zeta values

——— Lunch Break ———

14:00 – 14:50 Nils Matthes (Kyushu Univ.)
The elliptic double shuffle Lie algebra

15:10 – 16:00 Minoru Hirose (Kyushu Univ.)
A generalized cyclic sum formula for iterated integrals

Co-hosted by: Multiple Zeta Research Center, Kyushu University
Tohoku Forum for Creativity, Tohoku University

Polylogs, multiple zetas, and related topics

Abstracts

Kenji Sakugawa (RIMS)

"A p-adic analogue of Zagier's polylogarithm conjecture"

Zagier's polylogarithm conjecture states that the Borel regulators can be described by polylogarithms. A weak version of this conjecture (WZPC) was proved by Rob de Jeu. In the early 2000s, Amnon Besser and Rob de Jeu formulated a p-adic analogue of WZPC and obtained partial results. In this talk, we will review the p-adic analogue of WZPC and we will present results obtained by the speaker.

Steven Charlton (Tuebingen Univ.)

"Relating multiple polylogarithms in weight ≥ 5"

Multiple polylogarithms, a multi-variable variant of the classical polylogarithms, are important functions both in number theory, and in theoretical physics. Understanding their identities and functional equations is of considerable interest. Here we investigate some of the symmetries and relations between multiple polylogarithms at weight 5. Using an observation due to Goncharov, on the co-boundary of $I^+_{4,1}(x,y) = \frac{1}{2}(I_{4,1}(x,y) + I_{4,1}(x,1/y))$ where $I_{4,1}$ is a certain more useful change of variables of the multiple polylogarithm $\mathrm{Li}_{4,1}$, we are able to obtain identities reducing certain combinations $I^+_{4,1}(\mathrm{Li}_2 \text{ functional equation}, y)$ or $I^+_{4,1}(x, \mathrm{Li}_3 \text{ functional equation})$ to Li_5's and so obtain new functional equations for Li_5. We can generalise this approach to weight 6 using $I^+_{5,1}(\mathrm{Li}_3 \text{ functional equation}, \mathrm{Li}_3 \text{ functional equation}) = \mathrm{Li}_6$'s to obtain new Li_6 functional equations. We indicate some potential approaches and partial results for higher weight ≥ 7.

Masataka Ono (Keio Univ.)

"New functional equations of finite multiple polylogarithms"

In this talk, we give "$t \leftrightarrow 1-t$" type new functional equations of finite multiple polylogarithms of Ono-Yamamoto type and Sakugawa-Seki type.

Danylo Radchenko (MPIM Bonn)

"Special values and functional equations of the Herglotz function"

The Herglotz function is a special function whose principal occurrence is in the formulation of a Kronecker limit formula for real quadratic fields. While by itself it is not very well-known, this function has strong ties to polylogarithms and to modular forms. I will describe these connections and several other curious properties of this and related functions with emphasis on special values and functional equations. The talk is based on a joint work in progress with Don Zagier.

Shin-ichiro Seki (Osaka Univ.)

"Generalizations of finite multiple zeta values"

Two types of finite multiple zeta values are known; A-finite multiple zeta values (A-FMZVs) and symmetrized multiple zeta values (SMZVs). In this talk, we introduce generalizations of A-FMZVs and SMZVs respectively. The generalizations of A-FMZVs are called $\hat{A}$-FMZVs. We further generalize these values to $\hat{A}$-finite multiple polylogarithms and investigate their functional equations.

Erik Panzer (Oxford Univ.)

"Conical zeta values"

Terasoma studied sums, over the lattice points in a rational cone, of products of powers of inverse linear forms. These sums are a vast generalization of MZV and include, for example, zeta functions of root systems and also sums arising from closed superstring perturbation theory (studied recently by Zerbini). Terasoma proved that all of these sums evaluate to polylogarithms at roots of unity. In this talk, I will discuss an algorithm for the explicit computation of such conical sums, using Francis Brown's integration theory on moduli spaces of marked genus zero curves, and its implementation with HyperInt.

Nobuo Sato (National Taiwan Univ.)

"On Charlton's conjectural identities of the multiple zeta values"

In this talk, we give a proof of a special case of the generalized cyclic insertion conjecture on the MZVs, which was formulated by Steven Charlton in his thesis. The conjecture is stated in terms of the block notation for MZVs introduced by himself. Charlton 's conjecture is a broad generalization of several long unproven families of identities such as Borwein-Bradley-Broadhurst-Lisonek 's cyclic insertion conjecture and certain conjectural identities posed by Hoffman. Our proof is based on certain identities among iterated integrals on a punctured projective line which we found by a search with the aid of computers. This is a joint work with Minoru Hirose at Kyushu University.

Nils Matthes (Kyushu Univ.)

"The elliptic double shuffle Lie algebra"

The classical double shuffle Lie algebra provides an algebraic framework for the study of double shuffle relations between multiple zeta values. In this talk, we describe an elliptic analog, recently introduced by Schneps, and its relation to the study of elliptic multiple zeta values. If time permits, we also indicate the relation between the elliptic double shuffle Lie algebra, the elliptic Grothendieck–Teichmller Lie algebra and the fundamental Lie algebra of the category of universal mixed elliptic motives. This talk is based on joint work with Pierre Lochak and Leila Schneps.

Minoru Hirose (Kyushu Univ.)

"A generalized cyclic sum formula for iterated integrals"

Iterated integrals are generalization of multiple zeta values. In this talk, we give a certain family of linear relations of iterated integrals. This relation gives a generalization of the cyclic sum formula and the derivation relation for the multiple zeta values This is a joint work with Nobuo Sato at National Taiwan University.