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Global Constraints on Matter Representations in F-theory

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Outline:

I. F-theory Compactification: brief overview of
non-Abelian gauge symmetries, matter, couplings;
global particle physics models

II. Global constraints & Abelain symmetries in F-theory
associated w/ additional sections of elliptic fibration
& Mordell-Weil group

a) global constraints on gauge symmetry & matter
implications for F-theory `swampland'

b) novel non-Abelian enhancement & matter
via Mordell-Weil torsion

III. Higher index matter in F-theory

``Exotic" bi-fundamental matter

IV. Concluding Remarks

II.a) Mordell-Weil and global constrains on gauge symmetry

M.C. and Ling Lin,
"The Global Gauge Group Structure of F-theory Compactification with U(1)s,"
JHEP, arXiv:1706.08521 [hep-th]

c.f., Ling Lin's gong show & poster

II.b) Mordell-Weil torsion and novel gauge symmetry enhancement

Florent Baume, M.C., Craig Lawrie and Ling Lin,
"When Rational Sections Become Cyclic: Gauge Enhancement in F-theory via Mordell-Weil Torsion,"
JHEP, arXiv:1709.07453 [hep-th]

III. Higher index matter representations

M.C., Jonathan Heckman and Ling Lin,
"Exotic Bi-Fundamental Matter in F-theory,"
to appear, arXiv:1806.....

F-THEORY BASIC INGREDIENTS

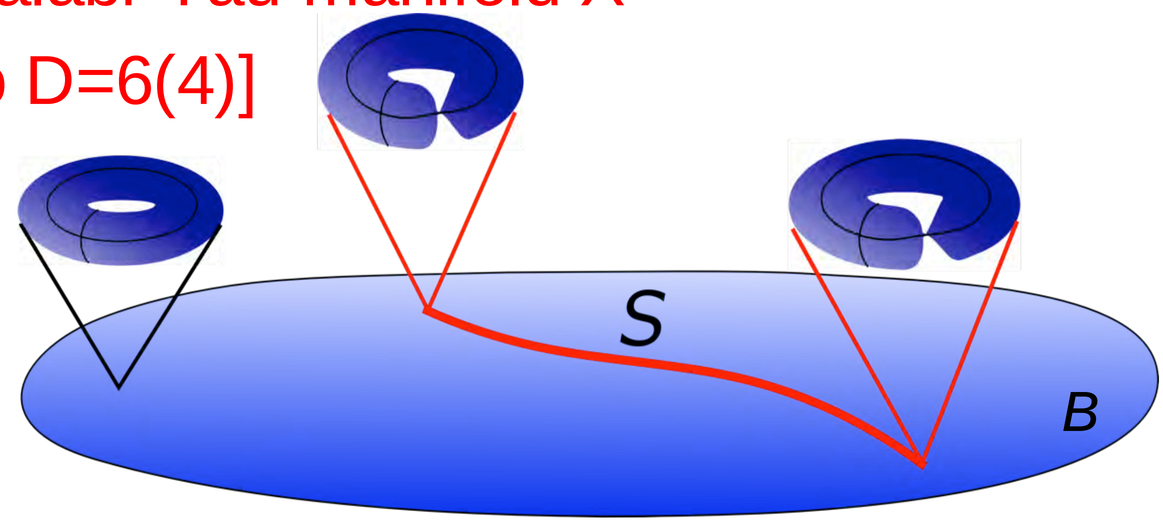
(Type IIB perspective)

F-theory compactification

[Vafa'96], [Morrison,Vafa'96],...

Singular elliptically fibered Calabi-Yau manifold X

[three-(four-)fold for comp. to D=6(4)]



Modular parameter of two-torus

(elliptic curve)

$\tau \equiv C_0 + i g_s^{-1}$

(SL(2,Z) of Type IIB)

Weierstrass normal form for elliptic fibration of *X*

$$y^2 = x^3 + fxz^4 + gz^6$$

[z:x:y] - homogeneous coordinates on $\mathbf{P}^2(1,2,3)$

f, g - sections of anti-canonical bundle of degree 4 and 6

F-theory compactification

[Vafa'96], [Morrison,Vafa'96],...

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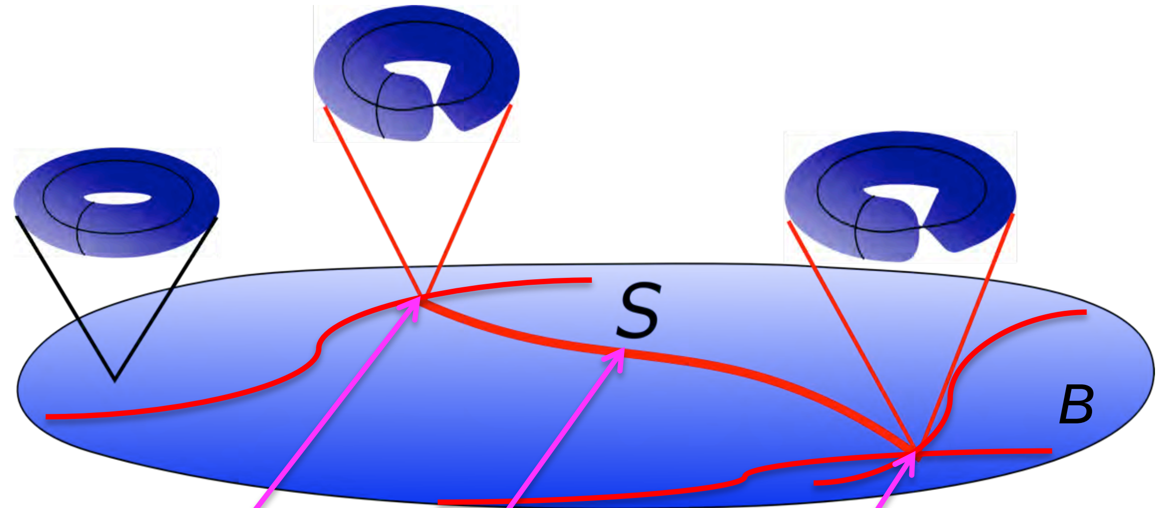


Weierstrass normal form for elliptic fibration of *X*

$$y^2 = x^3 + f x z^4 + g z^6$$

Matter
(co-dim 2; chirality- G_4-flux)

Yukawa couplings
(co-dim 3)

singular elliptic-fibration, $g_s \rightarrow \infty$
location of (p,q) 7-branes

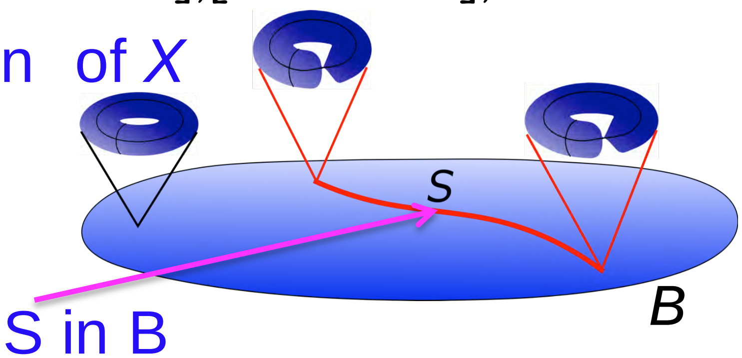
non-Abelian gauge symmetry
(co-dim 1)

Non-Abelian Gauge Symmetry

[Kodaira],[Tate], [Vafa], [Morrison,Vafa],...[Esole,Yau], [Hayashi,Lawrie,Schäfer-Nameki],[Morrison], ...

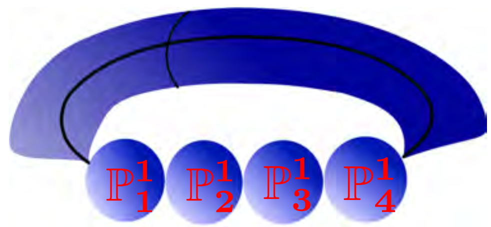
- Weierstrass normal form for elliptic fibration of *X*

$$y^2 = x^3 + fxz^4 + gz^6$$



- Severity of (ADE) singularity along divisor S in B specified by [$ord_S(f)$,$ord_S(g)$,$ord_S(\Delta)$]

- Resolution: structure of a tree of $\mathbb{P}^1$'s over *S* (exceptional divisors)



Resolved I_n-singularity ←→ SU(n) Dynkin diagram

n=5

Cartan gauge bosons: supported by (1,1) form $\omega_i \leftrightarrow \mathbb{P}^1_i$ on resolved X

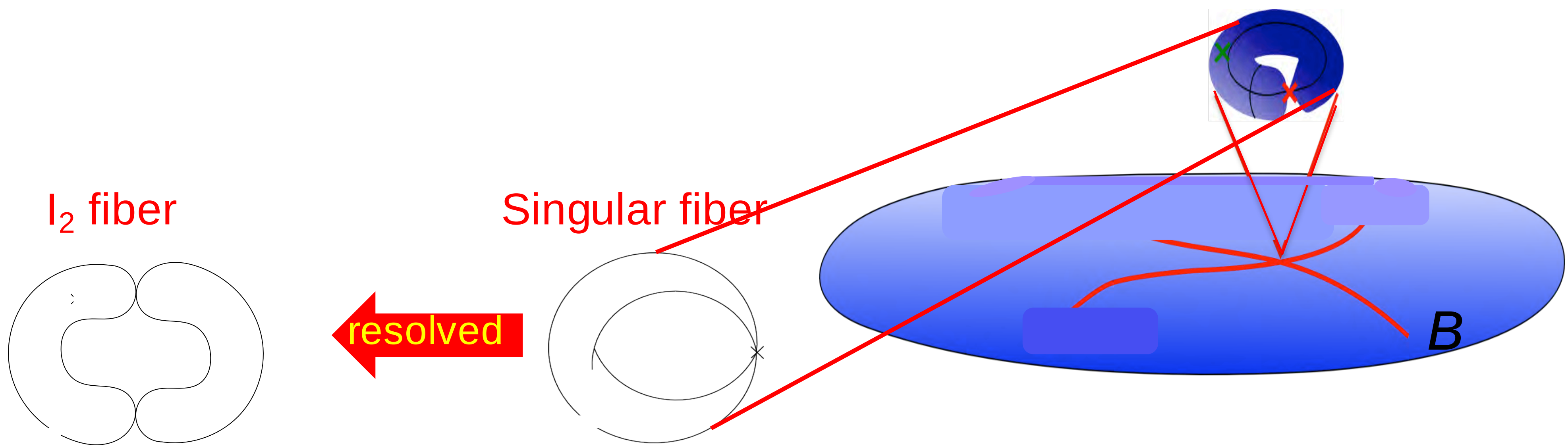
(via M-theory Kaluza-Klein reduction of C_3 potential $C_3 \supset A^i \omega_i$)

Non-Abelian gauge bosons: light M2-brane excitations on $\mathbb{P}^1$'s [Witten]

Deformation: [Grassi, Halverson, Shaneson'14-'15]

Matter

Singularity at co-dimension two in *B*:



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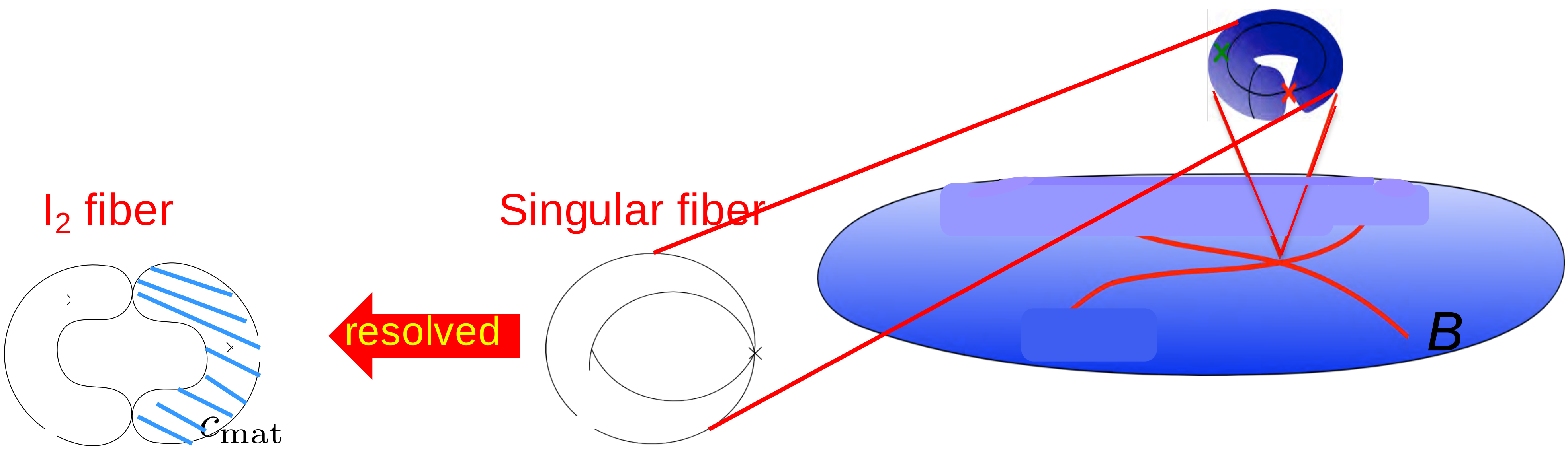


w/isolated (M2-matter) curve wrapping $\mathbb{P}^1$ → charged matter
(determine the representation via intersection theory)

Initial focus: F-theory with SU(5) Grand Unification

[10 10 5 coupling,...] [Donagi,Wijnholt'08][Beasley,Heckman,Vafa'08]...

Model Constructions:

local [Donagi,Wijnholt'09-10]...[Marsano,Schäfer-Nameki,Saulina'09-11]...

Review: [Heckman]

global

[Blumehagen,Grimm,Jurke,Weigand'09][M.C., Garcia-Etxebarria,Halverson'10]...
[Marsano,Schäfer-Nameki'11-12]...[Clemens,Marsano,Pantev,Raby,Tseng '12]...

Other Particle Physics Models:

Standard Model building blocks (via toric techniques)

[Lin,Weigand'14] SM x U(1) [1604.04292]

First Global 3-family Standard, Pati-Salam, Trinification Models
[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]

Global 3-family Standard Model with R-parity!
[M.C., Lin, Muyang Liu, Oehlmann, to appear]

No time; String-Math conference

II. U(1)-Symmetries in F-Theory

Abelian Gauge Symmetries

Different: (1,1) forms ω_m, supporting U(1) gauge bosons, isolated & associated with I_1-fibers, only

[Morrison,Vafa'96]

$\boxed{\text{(1,1) - form } \omega_m \Leftrightarrow \text{rational section of elliptic fibration}}$

Abelian Gauge Symmetry & Mordell-Weil Group

rational sections of elliptic fibr. $\Leftrightarrow$ rational points of elliptic curve

Abelian Gauge Symmetry & Mordell-Weil Group

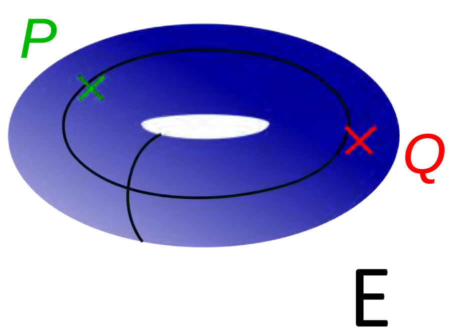
rational sections of elliptic fibr. $\Leftrightarrow$ rational points of elliptic curve

Rational point Q on elliptic curve E with zero point P

- is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

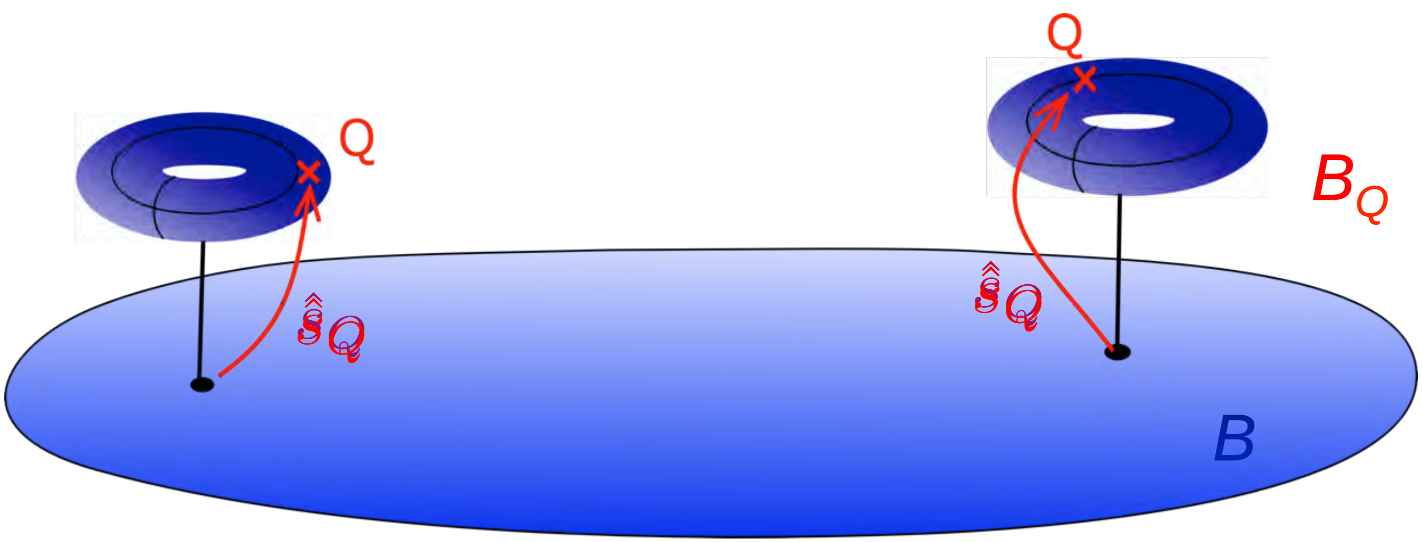
- Rational points form group (addition) on E



$\Rightarrow$ Mordell-Weil group of rational points

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point Q induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of B in X:

new divisor B_Q in X

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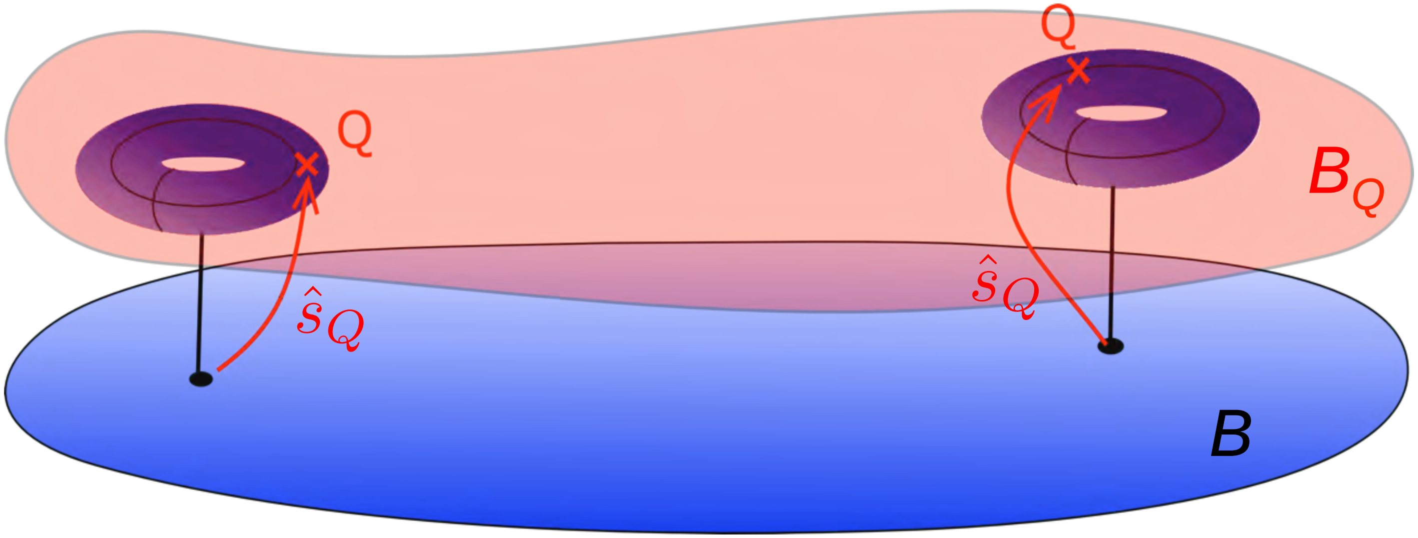

➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

➡ (1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⟺ rational section

Return to it later

Explicit Examples: (n+1)-rational sections – $U(1)^n$

[Deligne]

[via line bundle constr. on elliptic curve E- *CY in (blow-up) of* $W\mathbb{P}^m$]

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park 1208.2695]...

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n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park 1208.2695]...

n=2: with *P, Q, R* - specific example: generic CY in *dP*$_2$

[Borchmann,Mayerhofer,Palti,Weigand 1303.54054,1307.2902]

[M.C.,Klevers,Piragua 1303.6970,1307.6425]

[M.C.,Grassi,Klevers,Piragua 1306.0236]

generalization to nongeneric cubic in $\mathbb{P}^2[u:v:w]$

[M.C.,Klevers,Piragua,Taylor 1507.05954]

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generalization to nongeneric cubic in $\mathbb{P}^2[u:v:w]$

[M.C.,Klevers,Piragua,Taylor 1507.05954]

n=3: with *P, Q, R, S* - CICY in $\mathrm{Bl}_3\mathbb{P}^3$

[M.C.,Klevers,Piragua,Song 1310.0463]

n=4: determinantal variety in $\mathbb{P}^4$

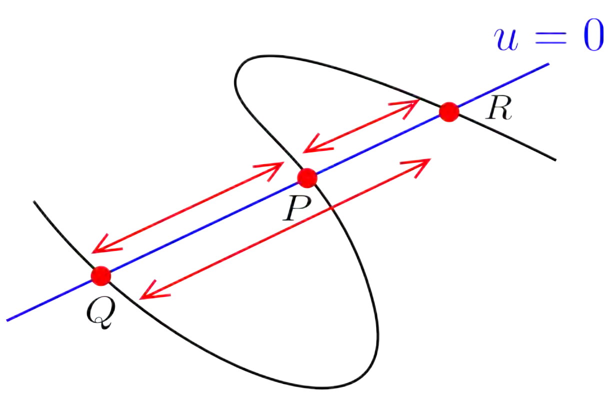
...

higher *n,* not clear…

U(1)xU(1): Further Developments

[M.C., Klevers, Piragua, Taylor 1507.05954]

General U(1)xU(1) construction:



non-generic cubic curve in $\mathbb{P}^2[u:v:w]$:

$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

$f_2(u,v,w)$ degree two polynomial in $\mathbb{P}^2[u:v:w]$

Study of non-Abelian enhancement (unHiggsing) by merging rational points P, Q, R [first symmetric representation of SU(3)]

higher index representations [Klevers, Taylor 1604.01030]

return to it later

[Morrison, Park 1606.0744]

non-local horizontal divisors (Abelian) turn into local vertical ones (non-Abelian) → both in geometry (w/ global resolutions) & field theory (Higgsing matter)

II.a) Global gauge symmetry constraints from Mordell-Weil

[M.C. and Ling Lin 1706.08521]

c.f., Ling Lin's gong show & poster

Shioda map & Non-Abelian Gauge symmetry

Shioda map of section $\hat{s}_Q$ more involved than $\mathcal{B}_Q$:

a map onto divisor complementary to $\mathcal{B}_\mathcal{P}$ divisor of zero section $\hat{s}_P$

& $\mathcal{E}_i$ – resolution (Cartan) divisors of non-Abelian gauge symmetry

$$\sigma\left(\hat{s}_Q\right) = B_Q - B_P - \sum_i l_i\, E_i + \cdots$$

Ensures proper F-theory interpretation of U(1)

(via M-theory/F-theory duality)

$l_i = C_{ij}^{-1}(B_Q - B_P)\,.\,\mathbb{P}_j^1$ - fractional # always an integer κ s.t. $\forall i$: $\kappa\, l_i \in \mathbb{Z}$

Cartan matrix Fiber of divisor E_j

Construct non-trivial central element of U(1) × G:

c.f., Ling Lin's poster

Employing (a) $q_{\mathfrak{u}(1)} = \frac{n}{\kappa}\,, n \in \mathbb{Z}$ & (b) $l_i \mathbf{w}_i = l_i \mathbf{v}_i \mod \mathbb{Z}$:= $L(\mathcal{R}_{\mathfrak{g}}^{(i)})$

$C(\mathbf{w}) := [e^{2\pi i q(\mathbf{w})} \otimes (e^{-2\pi i\, l_i\, \mathbf{w}_i} \times \mathbb{1})]\,\mathbf{w} \overset{(b)}{=} [e^{2\pi i q(\mathbf{w})} \otimes (e^{-2\pi i\, L(\mathcal{R}_{\mathfrak{g}})} \times \mathbb{1})]\,\mathbf{w}$
defines element in centre of $U(1) \times G$; (a) $\Rightarrow C^{\kappa} = 1$.

& $C(\mathbf{w}) = \exp(2\pi i\, \xi(\mathbf{w}))\,\mathbf{w} = \mathbf{w}$.

$\xi(w) = (B_Q - B_P)\,.\,\mathbb{P}^1 \in \mathbb{Z}$

$\Downarrow$

$$G_{\text{global}} = \frac{U(1) \times G}{\langle C \rangle} \cong \frac{U(1) \times G}{\mathbb{Z}_{\kappa}}$$

Global Constraint on Gauge Symmetry:

$$G_{\text{global}} = \frac{U(1) \times G}{\langle C \rangle} \cong \frac{U(1) \times G}{\mathbb{Z}_\kappa}$$

Exemplify for SU(5) GUT's and Standard Model constructions

Including for globally consistent three family SM

[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]

Toric construction with gauge algebra $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$

$\varphi(\sigma) = S - S_0 + \frac{1}{2} E_1^{\mathfrak{su}(2)} + \frac{1}{3}(2\, E_1^{\mathfrak{su}(3)} + E_2^{\mathfrak{su}(3)}) \Rightarrow C^6 = 1,$
so $G_{\text{global}} = [SU(3) \times SU(2) \times U(1)]/\langle C \rangle \cong [SU(3) \times SU(2) \times U(1)]/\mathbb{Z}_6.$

Indeed, geometrically realized (chiral) matter representations:

$(\mathbf{3},\mathbf{2})_{1/6}\,, \quad (\mathbf{1},\mathbf{2})_{-1/2}\,, \quad (\mathbf{3},\mathbf{1})_{2/3}\,, \quad (\mathbf{3},\mathbf{1})_{-1/3}\,, \quad (\mathbf{1},\mathbf{1})_{1}$

Implication for F-theory `Swampland' Criterion

With the choice of Shioda map scaling →
$\exists$ singlet field under $\mathcal{G}$, with U(1) charge Q_{min}=1
`Measure stick'

A necessary condition for a field theory to be in F-theory requires U(1) charge constraint on non-Abelian matter:

(1) If $\mathcal{R}^{(1)} = (q^{(1)}, \mathcal{R}_{\mathfrak{g}})$ and $\mathcal{R}^{(2)} = (q^{(2)}, \mathcal{R}_{\mathfrak{g}})$, then $q^{(1)} - q^{(2)} \in \mathbb{Z}$.
(2) If $\bigotimes_{i=1}^{n} \mathcal{R}_{\mathfrak{g}}^{(i)} = \mathbf{1}_{\mathfrak{g}} \oplus \ldots$, then $\sum_{i=1}^{n} q^{(i)} \in \mathbb{Z}$.

Caveat: Non-Higgsable U(1)'s? [Morrison,Taylor'16], [Wang'17]
In the presence of non-Abelian matter, expect to have singlet representation(s) → probably O.K.

Further comments: studied unHiggsing;
some models with non-minimal codim. 2 loci
→ strongly coupled CFT's [further studies]

II.b) Mordell-Weil torsion &
Novel gauge enhancement

Mordell-Weil torsion & Gauge enhancement

[Baume, M.C., Lawrie, Lin 1709.07453]

Mordell-Weil: $\mathrm{MW}(Y) = \mathbb{Z}^m \oplus \bigoplus_k \mathbb{Z}_{n_k}$

rational sections   torsional sections

[Aspinwall, Morrison '98], [Mayrhofer, Morrison, Till, Weigand '14]

Shioda-map for torsion: $\sigma(\hat{s}_Q) = B_Q - B_P - \sum_i l_i E_i + \cdots$ = 0 - no U(1)

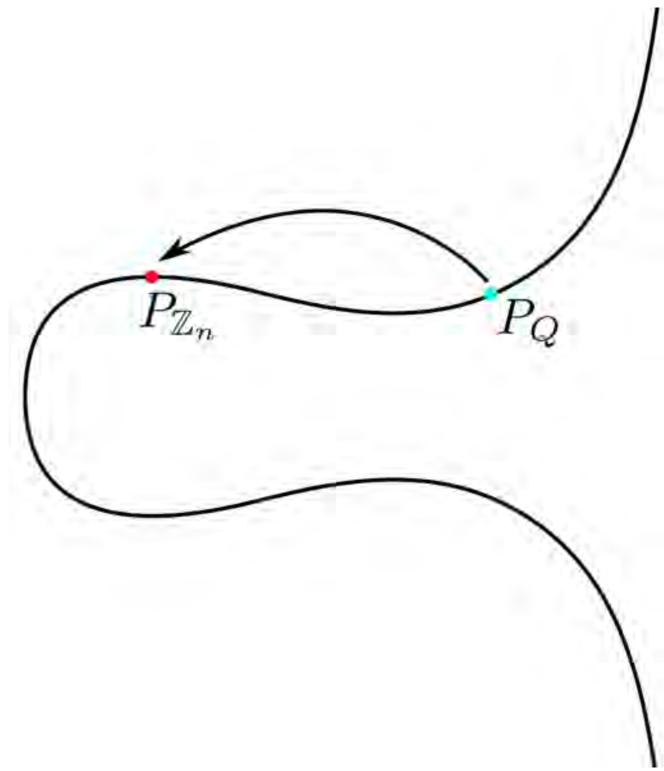
$$l_i \in \frac{1}{n_k}\mathbb{Z}\,.$$

As with U(1): integer condition on Cartan charges: $\sum_i l_i \mathbf{w}_i \in \mathbb{Z}\,.$

Results in the global gauge group: $G \supset \dfrac{G_k}{\mathbb{Z}_{n_k}}$

Gauge enhancement via Mordell-Weil torsion

Gauge enhancement when a section becomes torsional:



Tuning a free section to a torsional one of order n → expect to enhance U(1) to $\frac{G_{..}}{\mathbb{Z}_n} \times \ldots$

Gauge enhancement via Mordell-Weil torsion

Expect U(1) to unHiggs to non-Abelian $\mathcal{G}$ with $\pi_1(\mathcal{G}) = Z_n$

- Similar to unHiggsing through colliding free sections:

U(1)xU(1) w/ **(2,2)** charge matter →SU(3) w/ symm. **6** rep.

[M.C., Klevers, Piragua, Taylor '15]

U(1)-model w/ charge **3** matter→ SU(2) w/ three index symm. **4** rep.

[Klevers, Taylor '16]

- Torsional unHiggsing (to Z_2 torsion-prototype):

U(1) w/ charge **1** matter→SU(2)/Z_2 w/adj. **3** rep.(\`Cartan ch.'**2**)

[Mayrhofer, Morrison, Till, Weigand '14]

U(1) w/ charge **2** matter → Enhanced gauge symmmetry?
Matter representation?

Spoiler alert: NOT **5**-rep. ('Cartan charge' **4**)

→ possible ties to (other) \`swampland' conjectures

[Klevers, Morrison, Raghuram, Taylor '17]

Gauge enhancement via Mordell-Weil torsion

Resulting in Gauge group: $\frac{SU(2)\times SU(4)}{\mathbb{Z}_2}\times SU(2)$

Novel features: explicit global model with

- gauge factor [SU(2)] not affected by torsional section
- resolution of singular co-dim 2 fiber:

new matter rep.: $(\mathbf{3},\mathbf{1},\mathbf{2})$ [no **(5,1,1)**]

III. Higher index matter representations in F-theory

[M.C., Heckman, Lin '18]

Exotic bi-fundamental Matter in F-theory

Motivation:

- F-theory geometric techniques for higher index matter representations limited: via Kodaira classification at most three-index symmetric matter representation of SU(2)...
  [Klevers, Morrison, Raghuram, Taylor '17]
- And yet, Heterotic String Theory on toroidal orbifolds possess ``exotic" matter: e.g., T^4/Z_2 orbifold with $E_7 \times SU(2)$ & bi-fundamental matter (**56,2**)
  [After Higgsing to $SU(2)_{diag}$ leads to four-index symmetric rep.]
- Indeed, heterotic orbifold geometry is singular, but should be open minded about singular F-theory configurations.
  [$E_7 \times SU(2)$ and bi-fundamental **(56,2)** matter compatible with F-theory over Hirzebruch F_{12} base]

[M.C., Heckman, Lin '18]

Global F-theory approach:

Prototype: $\mathbf{E_7}$ **x SU(2)** with **(56,2)** matter

Earlier related work : Lüdeling, Ruehle' 14

Employ Tate's algorithm to construct E_7 and SU(2) devisors on curves with self-intersection +12 (anomaly cancelation)

Result:

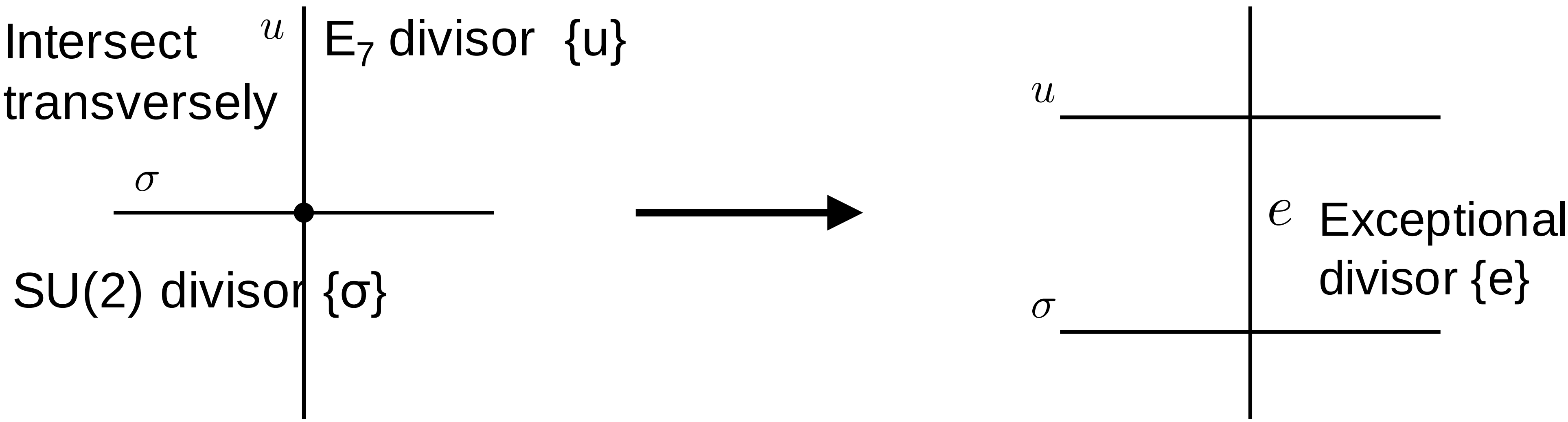
Non-minimal singularity points (beyond Kodaira classification) strongly coupled (superconformal field theory) regime

$\Downarrow$

blow-up in the base (tensor branch)

Result:

non-minimal points (beyond Kodaira classification);
blow-up in the base:

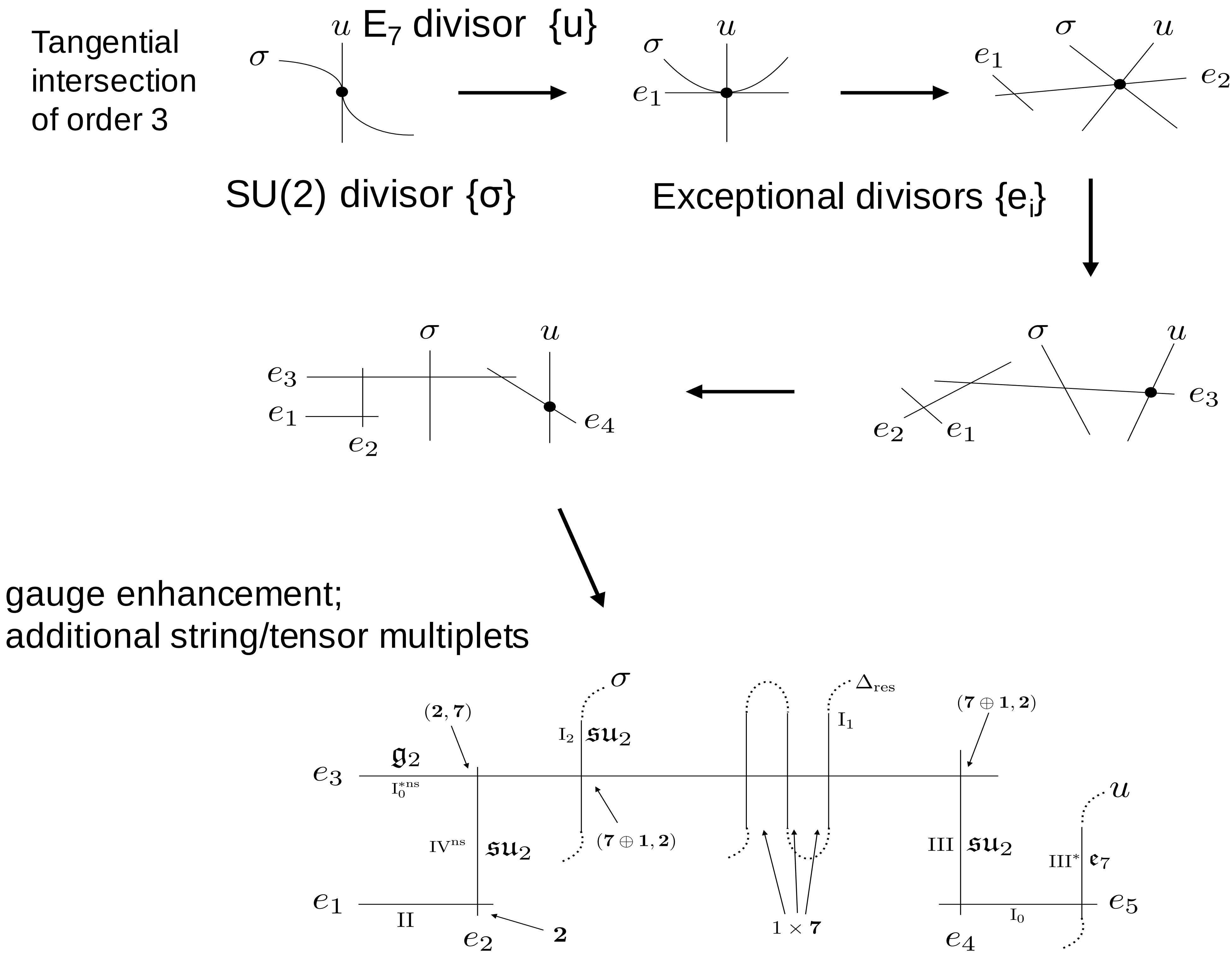


Gives no additional gauge groups, but
incompatible with dual heterotic spectrum:

a) additional strings/tensor multiplets
b) too many singlet fields (complex structure moduli)

Solution:

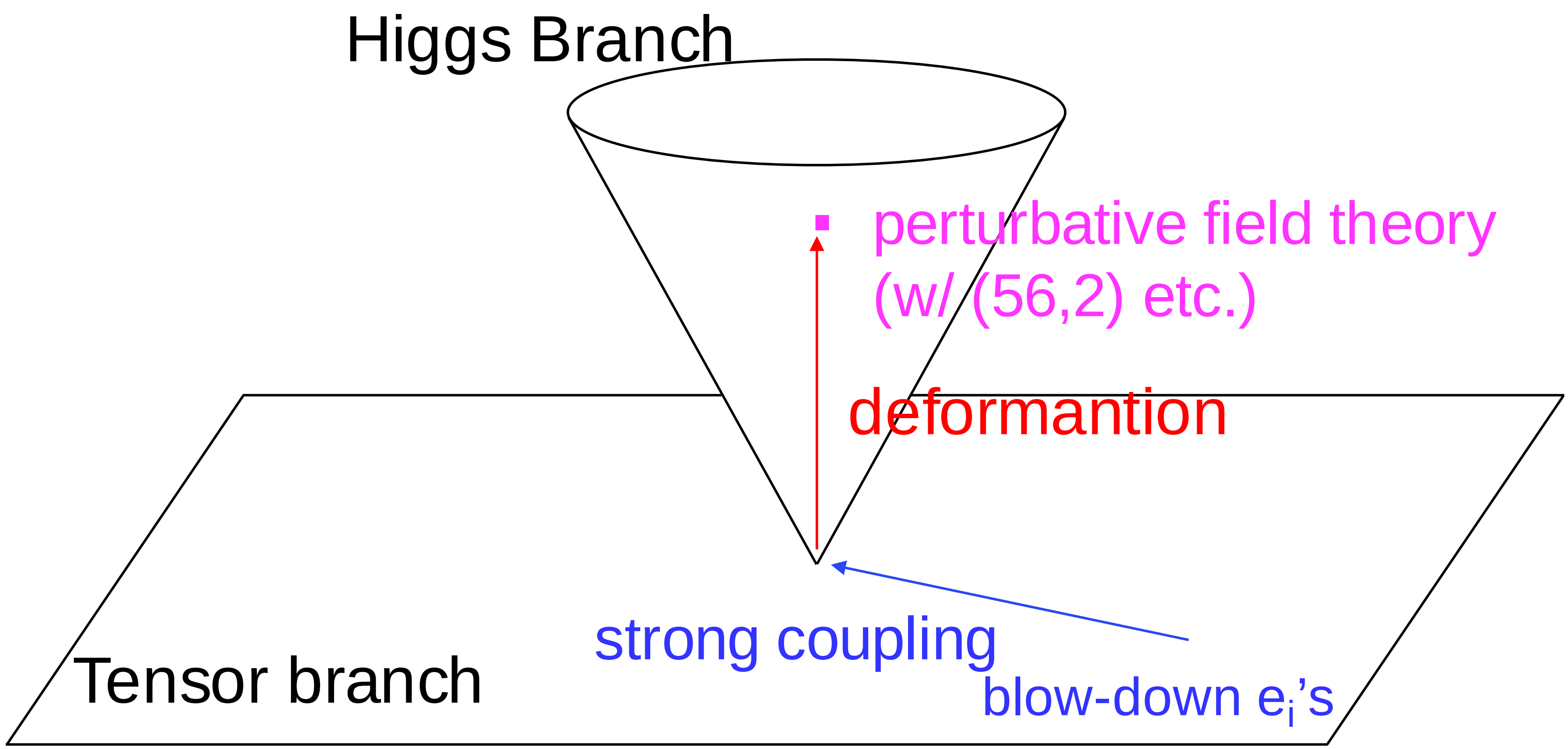
a) further tuning of complex structure – enlarges blow-up sector


Tangential intersection of order 3
u E_7 divisor {u}
σ
SU(2) divisor {σ}
σ u e_1
Exceptional divisors $\{e_i\}$
σ u e_1 e_2
σ u e_3 e_2 e_1
e_3 e_1 e_2 σ u e_4
gauge enhancement;
additional string/tensor multiplets
σ
Δ_{res}
$(\mathbf{2},\mathbf{7})$
I_2 $\mathfrak{su}_2$
I_1
$(\mathbf{7}\oplus\mathbf{1},\mathbf{2})$
$\mathfrak{g}_2$
e_3
$\mathrm{I}_0^{*\text{ns}}$
u
$(\mathbf{7}\oplus\mathbf{1},\mathbf{2})$
IV^{ns} $\mathfrak{su}_2$
III $\mathfrak{su}_2$
III^* $\mathfrak{e}_7$
e_1
II
e_2
$\mathbf{2}$
$1\times\mathbf{7}$
I_0
e_4
e_5

Outcome:

a) further tuning of complex structure – enlarges blow-up sector

b) at strong coupling (origin of tensor branch) activate Higgs branch deformations to flow to 'perturbative' field theory



Consequence:

a) due to complex structure tuning →
lose (complex structure) singlets

b) tensor branch Higgsing →
lose strings/tensor multiplets

Employ the fact that there is the same anomaly contribution in the tensor branch as in the Higgs branch in order to deduce the unique Higgs branch pertubative field theory spectrum with (56,2) etc. of E_7 x SU(2).

[The spectrum compatible with the dual heterotic orbifold one.]

Comments:

On engineering of correct tuning:

a) Tangential intersection: **(56,2)** matter appears only at tangential intersection with order 3 of $\mathbf{E_7}$ divisor with **SU(2)** divisor; in the Weierstrass fibration tune the complex structure to obtain order 3 of tangency
b) In the global construction collapse all co-dimension two enhancement points together; the strong coupling sector enhanced

On Higgs branch deformation:

At strong coupling, cannot analyze the Higgs branch explicitly, and cannot write explicitly the deformation, which is not complex structure, but more like T-brane data (captured in the intermediate Jacobian of elliptic fibration)

On further consistency check: above tuning on F-theory side produces correct dual heterotic $\mathbf{T^4/Z_2}$ orbifold geometry

Summary

II. Global F-theory constraints of Mordell-Weil Group

Encountered subtle issues:

a) Free part: presence of U(1) → global constraints on gauge symmetry & U(1) charges of non-Abelian matter (`swampland' conjecture)

b) Torsion part: novel gauge symmetry enhancements and representations

III. Exotic bi-fundamental matter in F-theory

employ geometric techniques to probe strongly coupled sector & identify **(56,2)** of **E_7xSU(2)**

[also **(27,3)** of **E_6xSU(3)**]

Higgsing to higher index reps.& further to non-Abelian discrete symmetries

Further studies

Thank you!