Surface defects and mock theta functions

George Korpas
Trinity College Dublin

May 15, 2018

Abstract for String Math 2018

We study Donaldson-Witten theory, that is $\mathcal{N} = 2$ topologically twisted pure SYM theory, on a compact and without boundary four-manifold X with an embedded surface $S \in H_2(X)$. This theory is the physical counterpart of the ramified Donaldson theory introduced by Mrowka and Kronheimer that computes the so-called "ramified Donaldson invariants" and it was studied by Tan in the context of the u-plane integral of Moore and Witten. The Coulomb branch integral, or u-plane integral, is the path integral of a specific operator over the Coulomb branch parameter u, or equivalently the generator of the equivariant cohomology of a point. We introduce a $\mathcal{Q}$-exact deformation to this integral that allows us to re-write it in terms of a mock theta function. The result is a contour integral, whose integrand is the mock theta function we mentioned, localized at the cusps of the Coulomb branch $\mathcal{B}$ of the ramified theory. This allows for a straight forward computation of the integral on a specific class of four-manifolds. Such a $\mathcal{Q}$-exact insertion raises the point of wether it is well-defined. A novel regularization for such u-plane integrals is required. Our technique shows that there exists a deep connection between such (Coulomb branch) integrals and mock modular forms and we hope that it will help towards understanding better other topological $\mathcal{N} = 2$ theories (with surface defects as well) since such theories are candidates to provide new four-manifold invariants.