Modular Solutions to Minimal 6d SCFTs

and how to blow them up

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Based on:
arXiv:1701.00764: M-x Huang, JG, A-K Kashani-Poor, A. Klemm
arXiv:1712.07017: M-x Huang, JG, A-K Kashani-Poor, A. Klemm, G. Lockhart, M. Del Zotto

Minimal 6d (0,1) SCFTs

- F-theory compactified on CY3 X: elliptic fibration over $O(-n) \to \mathbb{P}^1$

n	1	2	3	4	5	6	7	8	12
G_{min}	–	–	$SU(3)$	$SO(8)$	F_4	E_6	$E_7^{\frac{1}{2}\mathbf{56}}$	E_7	E_8

- Partition function $Z_{\text{SCFT}}(t_b, \tau, \mathbf{m}, \epsilon_\pm)$ of a minimal model on $(\mathbb{R}^4 \times T^2)_{\epsilon_\pm}$
  t_b: tensor modulus, τ: cxpl str. of T^2, $\mathbf{m}$: gauge/flavor fugacity
  - It has instanton contributions from elliptic genera of non-critical strings

$$Z_{\text{SCFT}}(t_b, \tau, \mathbf{m}, \epsilon_\pm) = Z_0(\tau, \mathbf{m}, \epsilon_\pm) \cdot \left(1 + \sum_{k=1}^{\infty} \mathrm{e}^{k \cdot 2\pi \mathrm{i} t_b} \cdot \mathbb{E}_k(\tau, \mathbf{m}, \epsilon_\pm)\right) \tag{1}$$

  - It is the same as refined topological string amplitude $Z_{\text{top}}(X)$ as a result of F-/M-theory duality.

Computation of elliptic genera

- $\mathbb{E}_k$ is a weight zero meromorphic Jacobi form with multiple elliptic parameters. Let $\mathbf{z} = (\epsilon_+, \epsilon_-, \mathbf{m})$

$$\mathbb{E}_k(-\tfrac{1}{\tau}, \tfrac{\mathbf{z}}{\tau}) = e^{2\pi i f_k(\mathbf{z})/\tau}\mathbb{E}_k(\tau, \mathbf{z}) \tag{2}$$

- Ansatz: $\mathbb{E}_k$ is a ratio of weak Jacobi forms
  - denominator fixed from known pole structure.
    GV formula of $F_{\text{top}}(X)$, Z_{SCFT} reduced to Nekrasov partition function of 5d SYM when $\tau \to i\infty$.
  - numerator has finite terms, and is fixed by Castelnuovo bounds.
- Results: $n = 1, 2$ completely solved; $n = 3, 4$ for low numbers of strings.
- Blow them up:
  - Göttsche-Nakajima-Yoshioka blowup equations: recursion formulas for Nekrasov partition functions of 5d SYM.
  - Geometric formulation of GNY blowup equations.
  - Partially valid for E-string ($n = 1$).

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