Higher qq-characters and S-duality of Wilson loops/surfaces

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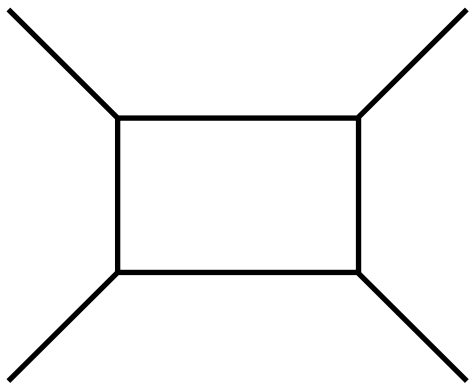
KIAS

arXiv:1804.09932 (with J. Kim, S. Kim, P. Agarwal) + arXiv:1806.XXXXX (with B. Assel)

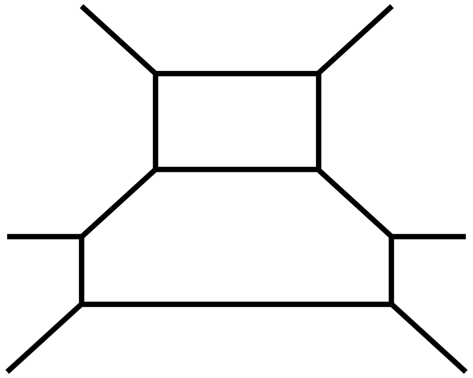
String-Math 2018 - Gong Show

Goal: understand Wilson loops in 5d $\mathcal{N} = 1$ Lagrangian theories (on $\mathbb{R}^4_{\epsilon_{1,2}} \times S^1$)

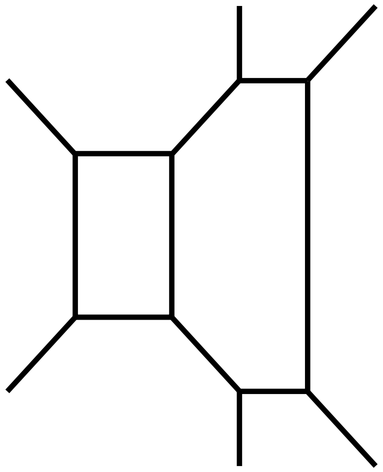
Setting: webs of (p,q) 5-branes in type IIB string theory



$SU(2)$



$SU(3)\ \ N_F = 2$



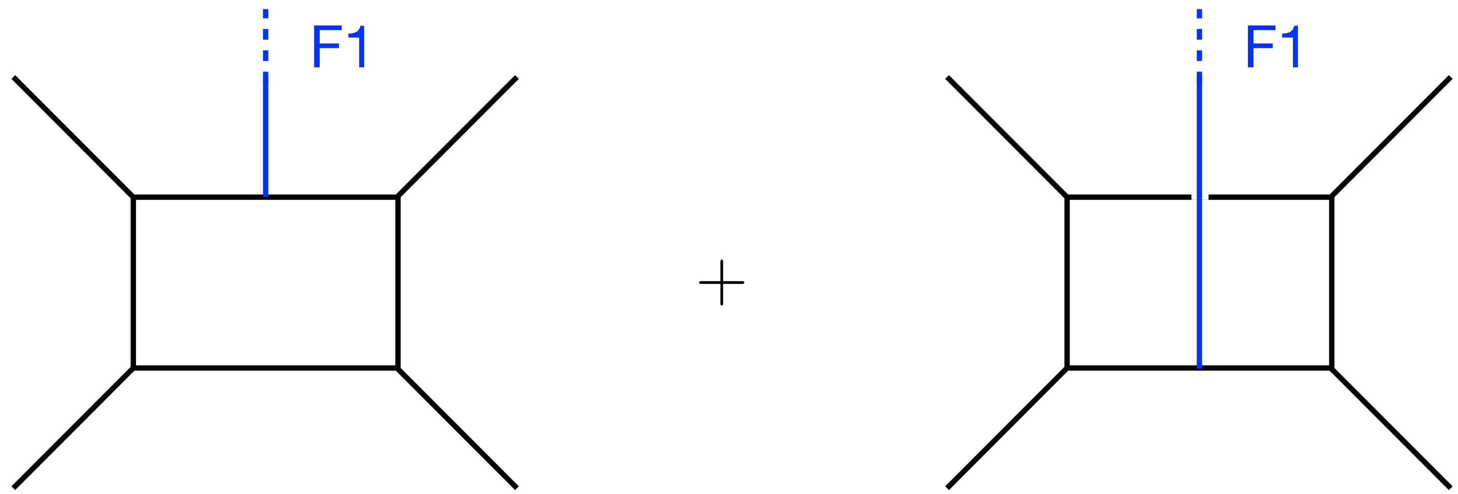
$SU(2) \times SU(2)$

A few possible questions:

- How to realize Wilson loops in the brane picture?
- How to compute VEV of Wilson loops?
- What are their properties?

In brane picture, Wilson loops usually realized via semi-infinite F1 ending on D5:

$\langle \mathcal{W}_{\mathbf{2}}^{SU(2)} \rangle$:



However, technical problems in localization computation of Wilson loop VEVs

(regularization ADHM moduli space and bundles over it, poles at infinity in ADHM QM, …)

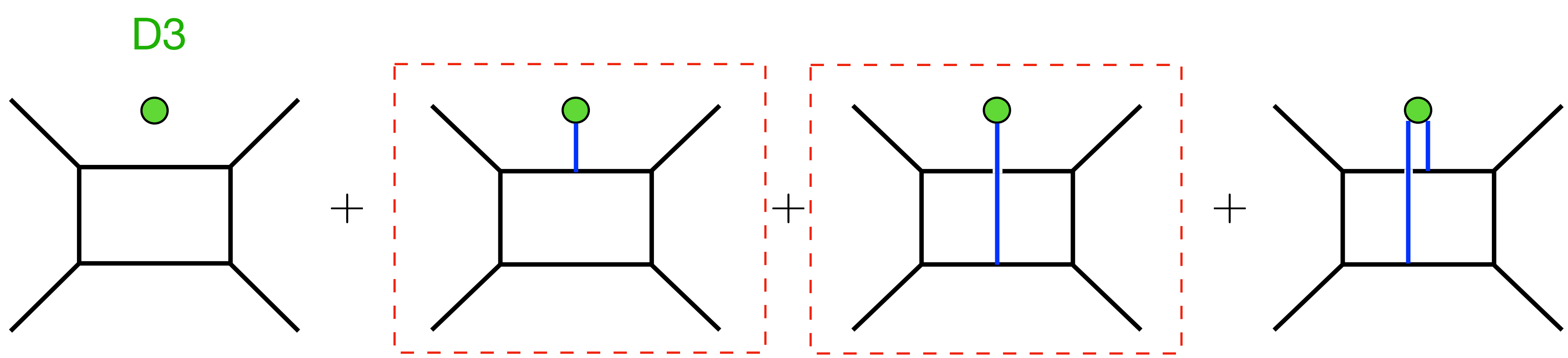
How to proceed? Take finite-length F1 stretched between D5 and N' external D3;

D3 intersect D5 along $S^1 \implies$ loop observable $\langle \mathcal{L}_{\text{SQM}}^{N'} \rangle$ for 5d $\mathcal{N} = 1$ theory

$\langle \mathcal{L}^{N'}_{\rm SQM} \rangle$ observable (qq-character): 1d $\mathcal{N}=(0,4)$ SQM coupled to 5d theory

Claim: contains Wilson loops in tensor product of antisymmetric representations*

$\langle \mathcal{L}^{N'=1}_{\rm SQM} \rangle$:



Advantage: no technical problems in localization computation (modified ADHM)

Disadvantage: how to extract Wilson loops from $\langle \mathcal{L}^{N'}_{\rm SQM} \rangle$?

*Also loop observable for 4d $\mathcal{N}=2^*$ $U(N')$ theory on D3; information on 4d Wilson, 't Hooft loops

Case $N' = 1$: $\langle \mathcal{L}^{N'=1}_{\rm SQM} \rangle$ Laurent polynomial in $x = e^M$ (mass 1d lowest mode),

whose coefficients are Wilson loops in rank-l antisymmetric representations

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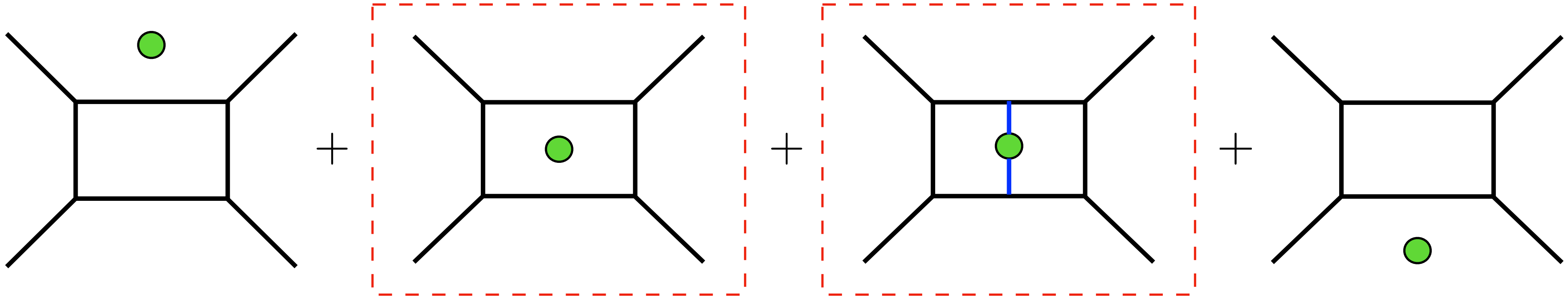

$$\langle \mathcal{L}^{N'=1}_{\rm SQM} \rangle = x - \langle \mathcal{W}_{\mathbf{2}} \rangle + x^{-1}$$

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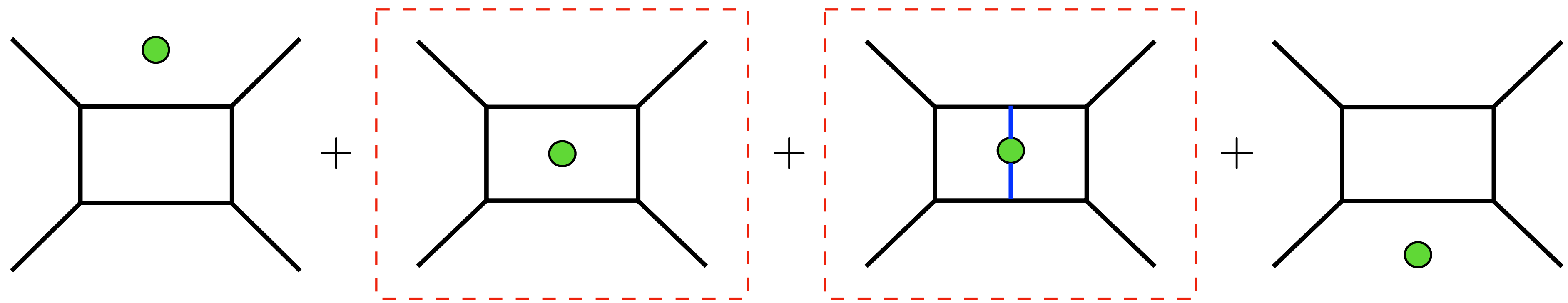
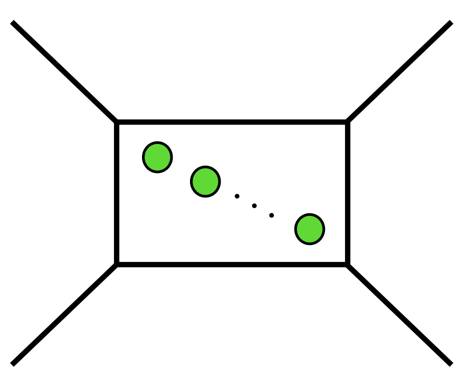


$$\langle \mathcal{L}^{N'=1}_{\mathrm{SQM}} \rangle = x - \langle \mathcal{W}_{\mathbf{2}} \rangle + x^{-1}$$

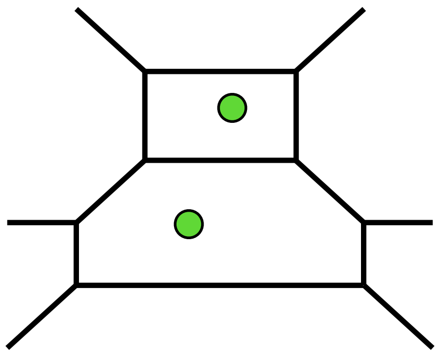
Case $N' > 1$: $\langle \mathcal{L}^{N'}_{\mathrm{SQM}} \rangle$ rational function in $x_1, \ldots, x_{N'}$; what is a Wilson loop?

Proposal: subsector of particular 4d $U(1)^{N'} \subset U(N')$ charge (tensor product reps.)



$$\langle \mathcal{W}^{SU(2)}_{\mathbf{2}^{\otimes N'}} \rangle = (-1)^{N'} \oint \prod_{j=1}^{N'} \frac{dx_j}{x_j} \langle \mathcal{L}^{N'}_{\mathrm{SQM}} \rangle$$

Test: under S-duality different Wilson loops mapped between different theories



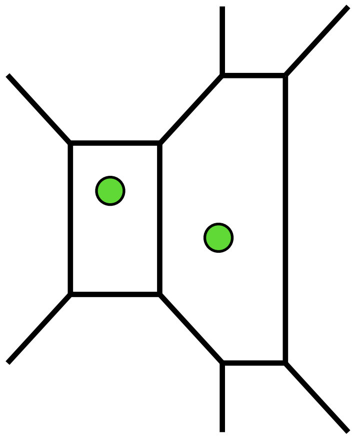
$SU(3)\ \ N_F = 2$

$\langle \mathcal{W}_{\mathbf{3}\otimes\overline{\mathbf{3}}} \rangle$

$\Longleftrightarrow$

$SU(2) \times SU(2)$

$\langle \mathcal{W}_{\mathbf{2},\mathbf{2}} \rangle$



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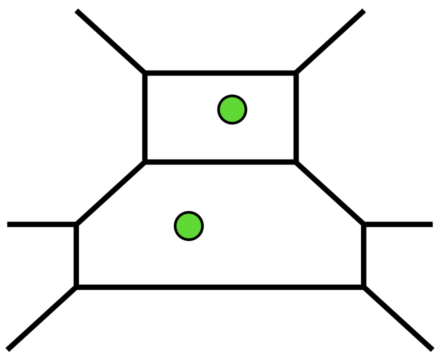

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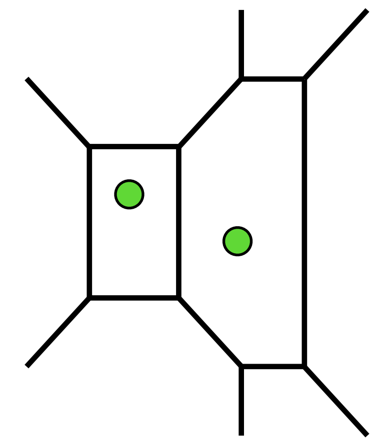
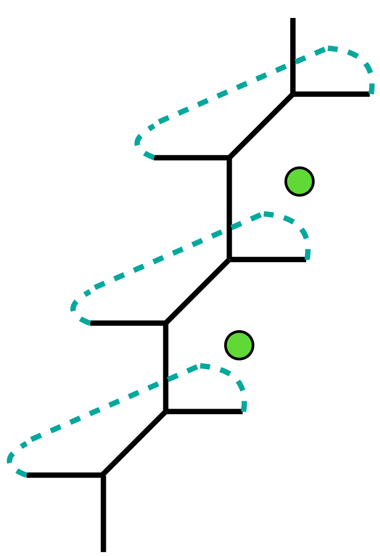


For 5d $\mathcal{N} = 1^*\ \ SU(N)$, map to Wilson surfaces in 6d $\mathcal{N} = (2,0)\ \ A_{N-1}$ theory



$\mathcal{N} = 1^*\ \ SU(3)$

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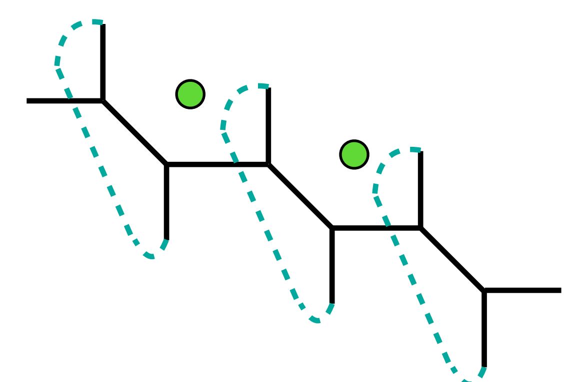

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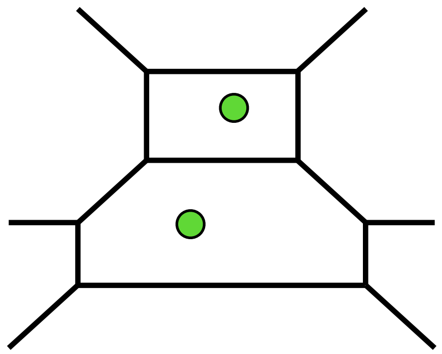

$SU(3)\ N_F=2$ $\Longleftrightarrow$ $SU(2)\times SU(2)$

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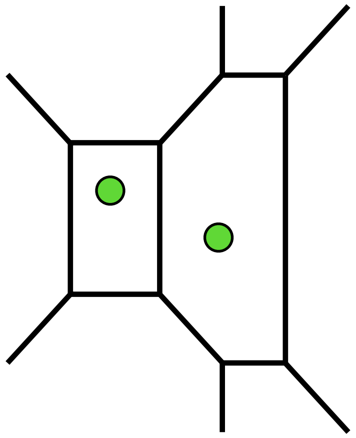
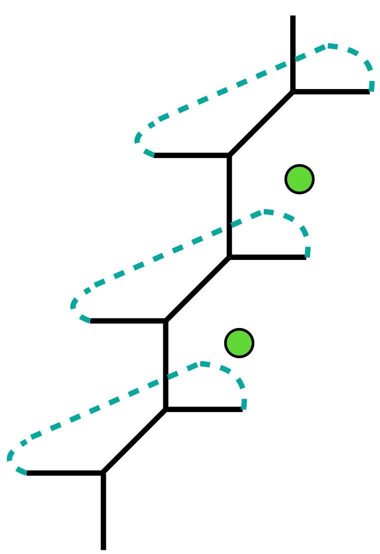
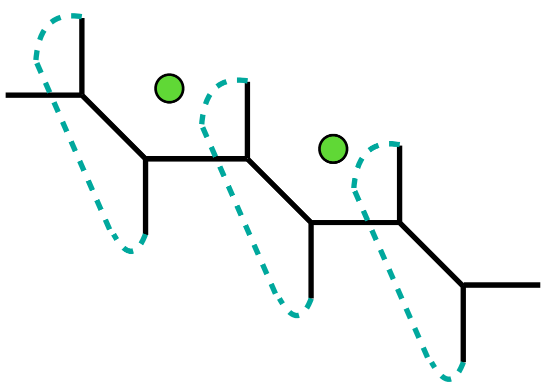


For 5d $\mathcal{N}=1^*\ SU(N)$, map to Wilson surfaces in 6d $\mathcal{N}=(2,0)\ A_{N-1}$ theory



$\mathcal{N}=1^*\ SU(3)$ $\Longleftrightarrow$ $\mathcal{N}=(2,0)\ A_2$

$\langle \mathcal{W}_{\mathbf{3}\otimes\overline{\mathbf{3}}}\rangle$ $\langle \mathcal{S}_{\mathbf{2},\mathbf{2}}\rangle$



Also: Wilson loops exhibit enhanced flavor symmetry (E_{N_F+1} for $SU(2)\ N_F$)

For more details:

look for me at the poster session

(or anytime!)