Geometric Classification of 5d $\mathcal{N} = 1$ SCFTs

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Introduction

The subject of this talk is the classification of 5d $\mathcal{N} = 1$ SCFTs on the Coulomb branch using CY 3-folds

Based on 1801.04036 (PJ, Sheldon Katz, Hee-Cheol Kim, Cumrun Vafa)

Classification of 5d $\mathcal{N} = 1$ SCFTs is an open problem:

- Gauge theory classification in terms of $(\mathfrak{g}, \mathbf{R}, k)$ misses some theories predicted by string theory
- Geometric classification previously only done for rank one theories, which correspond to shrinking del Pezzo surfaces $\mathrm{dP}_{n\leq 8}$

New catalog of candidate gauge theories proposed in 1705.05836 captures all field theories previously missed; however, field theory methods cannot detect non-perturbative physics due to BPS instantons

Stringy/geometric constructions needed to verify existence of UV fixed points, along with non-Lagrangian theories (e.g. rank one dP_0)

M-theory compactifications on local CY 3-fold

It is possible to "construct" 5d $\mathcal{N} = 1$ field theories by compactifying M-theory on a local CY 3-fold Y

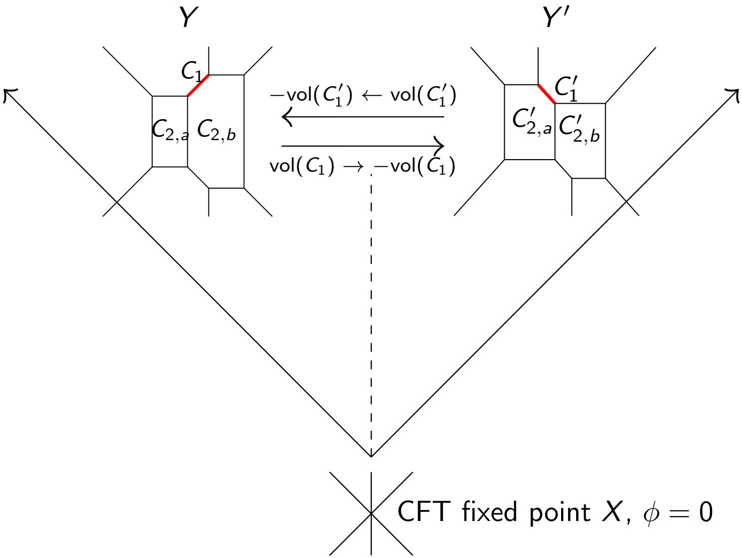
Mn branes wrapping complex p-cycles C_p correspond to BPS particles/monopole strings whose mass/tension is controlled by the volume $\text{vol}(C_p) = \int_{C_p} J^p$ in the 3-fold Y (J is a Kähler class)

The extended Kähler cone, the closure of the set $\{\phi\}$ of all Kähler classes $J = \phi^i [C_{2,i}]$ such that $\int_{C_p} J^p > 0$, is identified with the Coulomb branch of the 5d theory

CFT fixed point $\phi = 0$ where all BPS states become massless/tensionless is thus a singularity $Y \to X$ where all cycles collapse to zero volume

Hence the existence of a 5d fixed point with Coulomb branch deformations implies the existence of a 3-fold Y with positive Kähler class J and a singular limit $Y \to X$

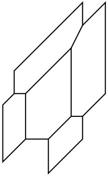
M-theory compactifications on local CY 3-fold



Classification program

We assume all 3-folds engineering 5d fixed points can be realized as a neighborhood of a Kähler surface $C_2 = \cup_{i=1}^{\text{rank}} C_{2,i}$ with $C_{2,i}$ intersecting transversely in a curve, which maps to a canonical singularity

Problem reduces to classification of all graphs corresponding to 5d theories:



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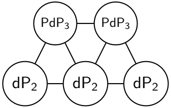

Classification results!

rank	graphs
1	dP_n or $\mathbb{F}_0$
2	$\text{Bl}_p\mathbb{F}_m \overset{\mathbb{P}^1}{—} \text{dP}_n$ or $\text{Bl}_p\mathbb{F}_m \overset{\mathbb{P}^1}{—} \mathbb{F}_0$

Interesting conclusions and future prospects

Geometric classification confirms almost all rank two gauge theories predicted by field theory analysis, with two exceptions:

1. $\mathfrak{su}(3)_{k=\pm 8}$, excluded
2. $\mathfrak{su}(3)_{k=\frac{1}{2}} + 1\mathrm{Sym}$, related to $\mathrm{O7}^+$/frozen singularity

All 5d SCFTs up to rank two descend from 6d $(1,0)$ SCFTs compactified on S^1

- Rank one has one parent, dP_9
- Rank two has five "parents":
  1. $\mathrm{Bl}_5\mathbb{F}_1 \cup \mathrm{dP}_6$
  2. $\mathrm{Bl}_9\mathbb{F}_4 \cup \mathbb{F}_0$
  3. $\mathbb{F}_2 \cup \mathrm{dP}_7$
  4. $\mathbb{F}_6 \cup \mathrm{dP}_4$
  5. $\mathbb{F}_{10} \cup \mathbb{F}_0$

Can we classify higher rank theories?

Flavor symmetry enhancements for rank two and higher?

Thank you!