Deep Learning and AdS/CFT

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Akinori Tanaka (RIKEN),
Akio Tomiya (CCNU→RIKEN)

Based on arXiv:1802.08313 (PRD 98, 046019)
& 1809.?????

Discrete Approaches to the Dynamics of Fields and Space-Time 2018
Tohoku univ., Sep. 10, 2018

Title was changed by PRD.

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Deep ~~L~~earning

and AdS/CFT

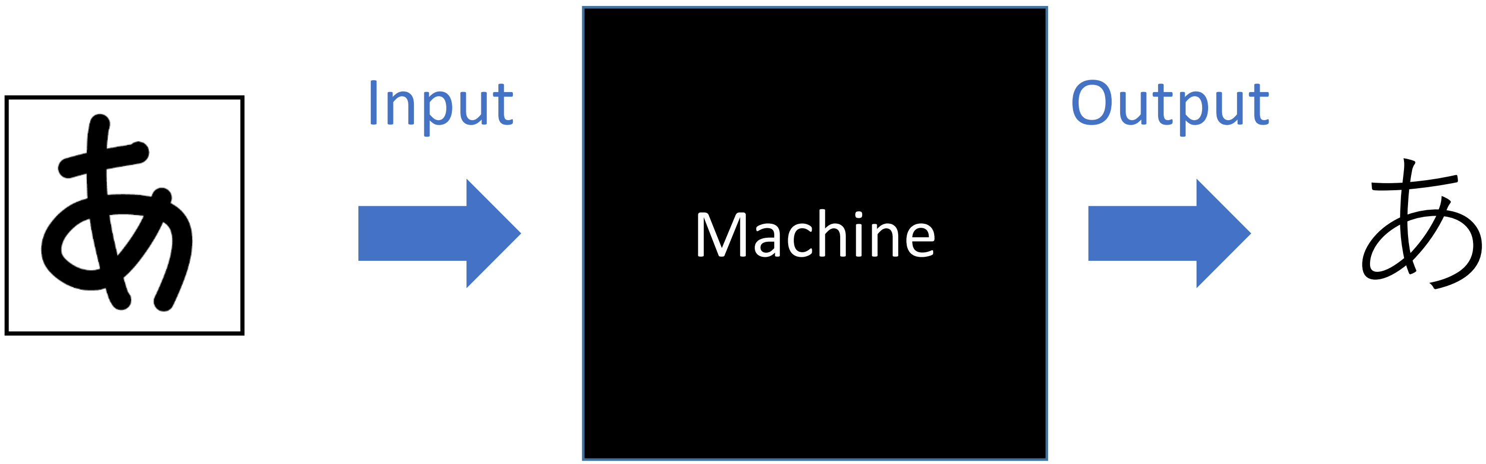
the correspondence

Outline

- Introduction
- Deep learning
- AdS/CFT
- Deep learning and AdS/CFT
- Summary

What is deep learning?

Deep learning is part of machine learning.



What is deep learning?

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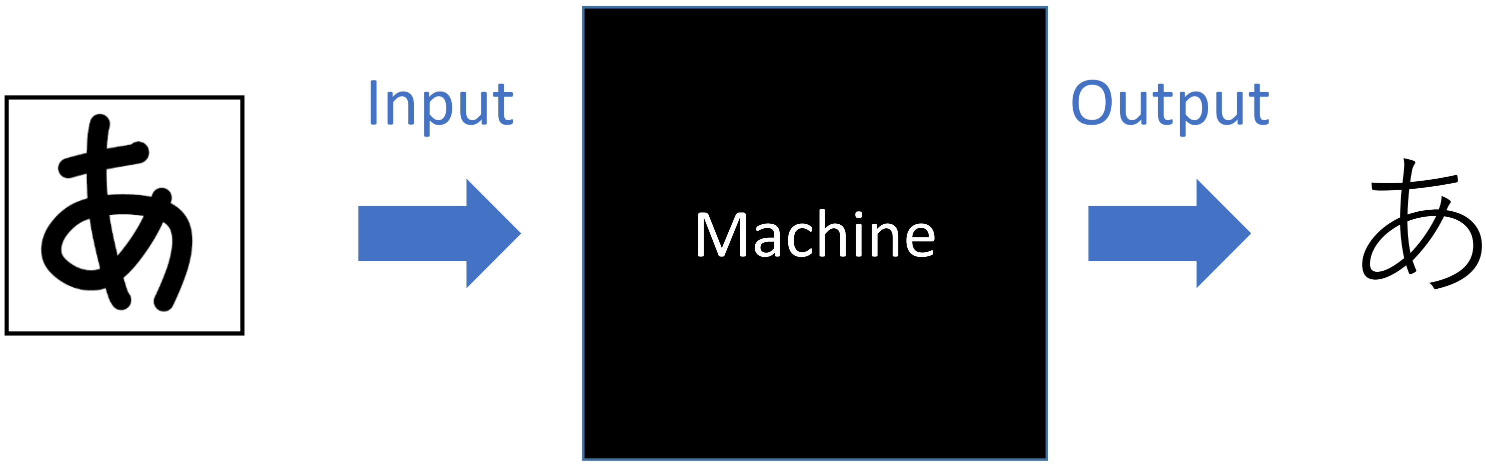

In the machine, $y = f_N \circ f_{N-1} \cdots \circ f_1(x)$

$x \in \mathcal{D}^{\text{input}}$

f_i : non-linear transformation w/ tunable parameters

- "Learning" is tuning of the parameters.
- "Deep" means large N.

Related to coarse-graining?

$$x \longrightarrow y = f_N \circ f_{N-1} \cdots \circ f_1(x)$$

$x^{(1)} =$ あ

$x^{(2)} =$ あ

$x^{(3)} =$ あ　$\longrightarrow$　あ

$x^{(4)} =$ あ

- Output captures some essential characteristics of input.

Related to RG?

It reminds us of the renormalizaton group.

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Or theoretical physicists are tuned so.

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Or theoretical physicists are tuned so.

There are papers discussing the relation between machine learning and RG.

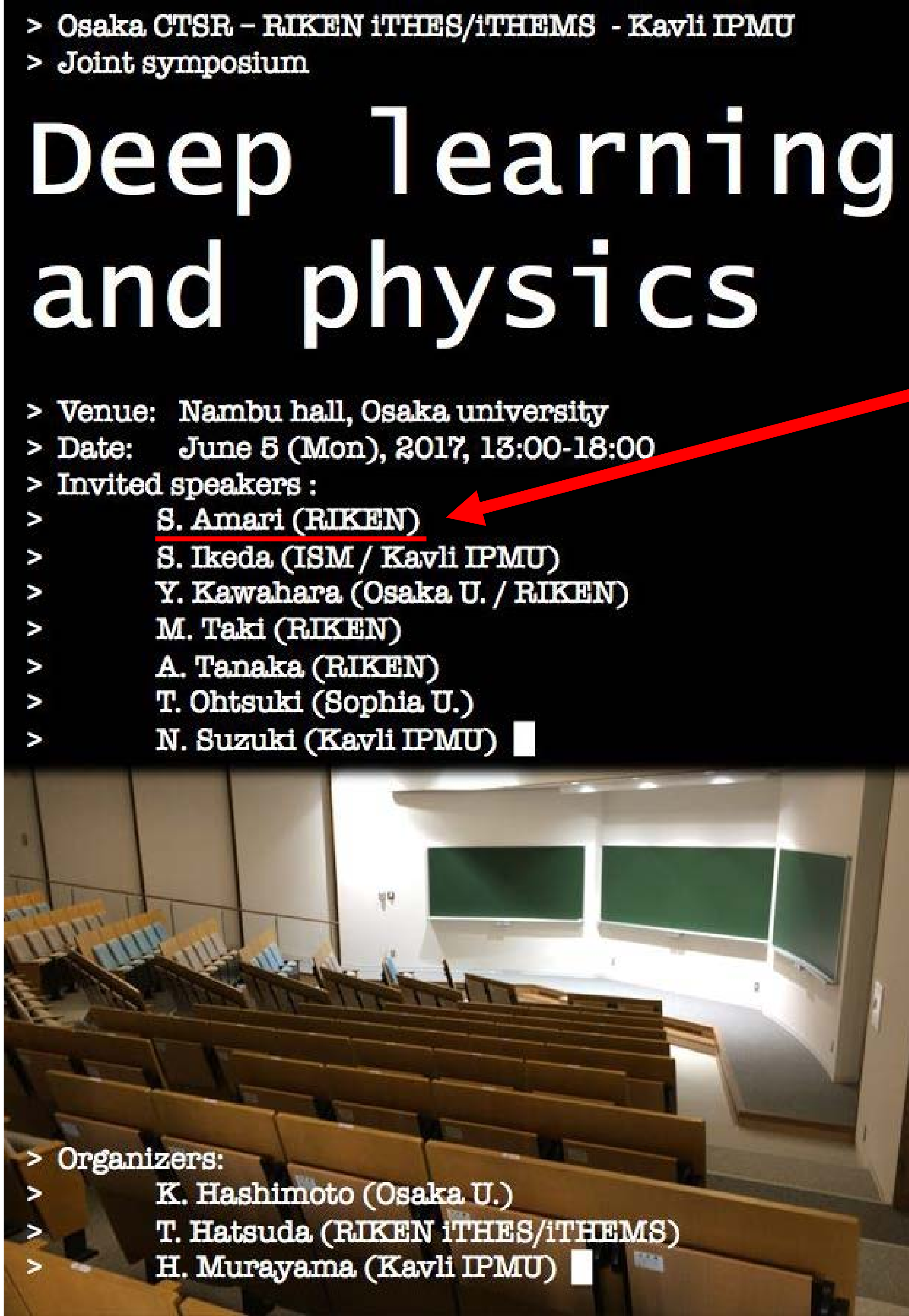
[Beny (2013), Mehta & Schwab (2014),
Koch-Janusz & Ringel (2017), Iso, Shiba & Yokoo (2018), …]

> Osaka CTSR – RIKEN iTHES/iTHEMS - Kavli IPMU
> Joint symposium

Deep learning and physics

> Venue: Nambu hall, Osaka university
> Date: June 5 (Mon), 2017, 13:00-18:00
> Invited speakers :
> S. Amari (RIKEN)
> S. Ikeda (ISM / Kavli IPMU)
> Y. Kawahara (Osaka U. / RIKEN)
> M. Taki (RIKEN)
> A. Tanaka (RIKEN)
> T. Ohtsuki (Sophia U.)
> N. Suzuki (Kavli IPMU)

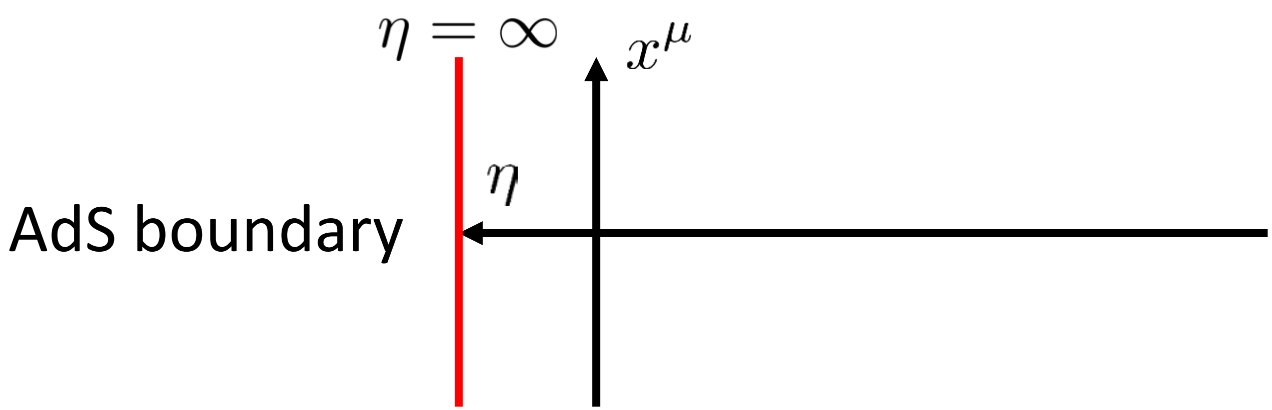
> Organizers:
> K. Hashimoto (Osaka U.)
> T. Hatsuda (RIKEN iTHES/iTHEMS)
> H. Murayama (Kavli IPMU)



Inspired by
Dr. Amari's talk

AdS/CFT and RG

AdS space $$ds^2 = d\eta^2 + e^{2\eta}(dx_\mu dx^\mu)$$

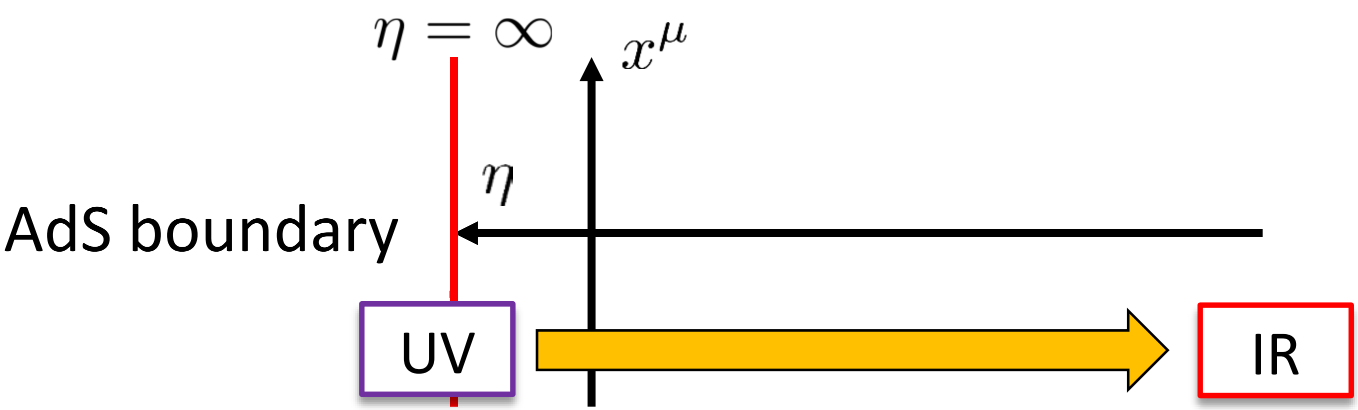


scale transformation = η-translation

$$x^\mu \to e^\lambda x^\mu$$

AdS/CFT and RG

AdS space $$ds^2 = d\eta^2 + e^{2\eta}(dx_\mu dx^\mu)$$



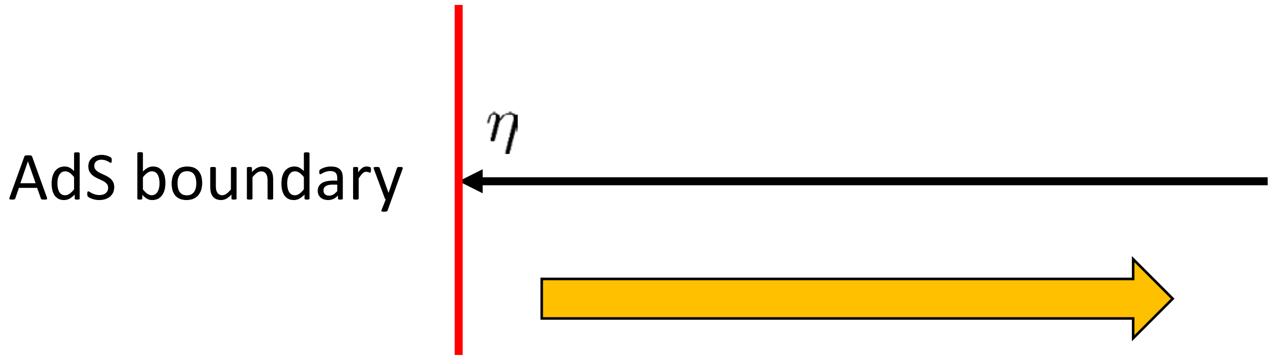
scale transformation = η -translation

$$x^\mu \to e^\lambda x^\mu$$

holographic RG

AdS space as neural network

Can we regard the evolution in the bulk direction as the propagation in a deep neural network?



Outline

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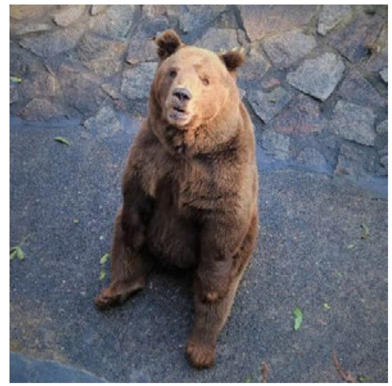
Machine Learning

- classification

Hey, is this a bear?



The bear machine

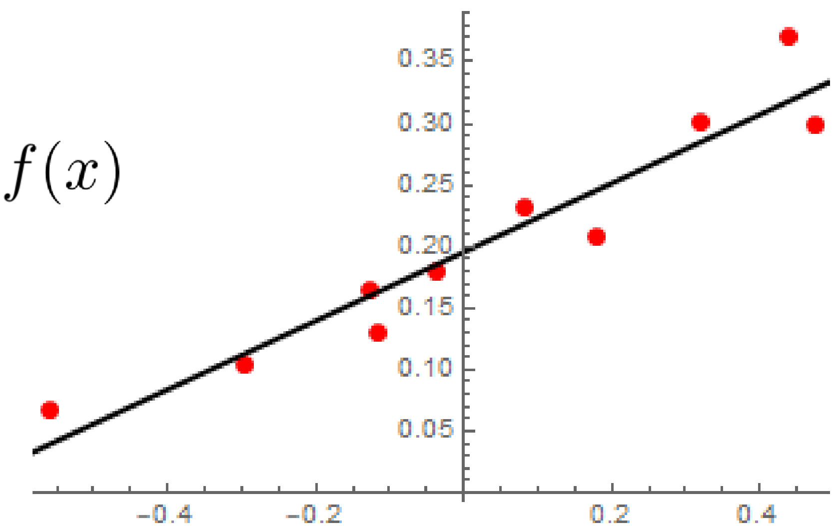
Yes.

- regression

data $\{(x^{(i)}, y^{(i)})\}$ $y = f(x)$

$$f(x) = ax + b$$

Learn $(a\,,\ b)$.



Neural network

input

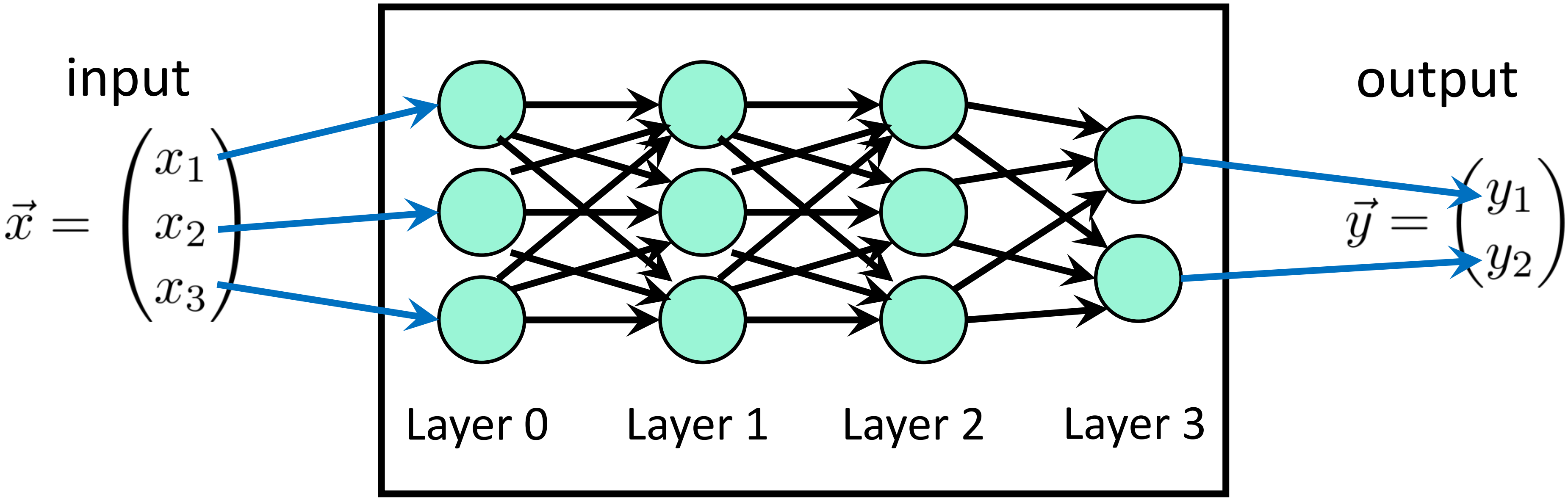
$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Machine

output

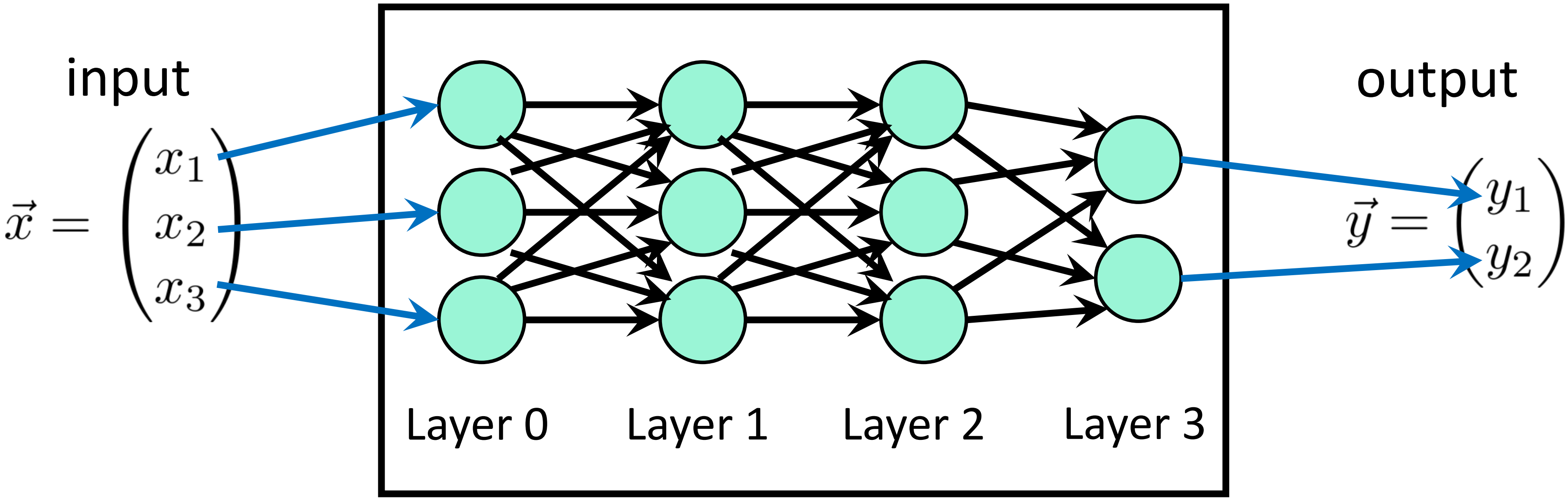
$$\vec{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Neural network



$$\vec{x} = \vec{x}^{(0)} \Rightarrow \vec{x}^{(1)} \Rightarrow \vec{x}^{(2)} \Rightarrow \vec{x}^{(3)} = \vec{y}$$

Neural network



$$\vec{x} = \vec{x}^{(0)} \rightarrow \vec{x}^{(1)} \rightarrow \vec{x}^{(2)} \rightarrow \vec{x}^{(3)} = \vec{y}$$

#(Layers) is large $\Rightarrow$ Deep Learning

Neural network

$$\vec{x} = \vec{x}^{(0)} \longrightarrow \vec{x}^{(1)} \longrightarrow \vec{x}^{(2)} \longrightarrow \vec{x}^{(3)} = \vec{y}$$

- Proceed to next layer by linear trsf and non-linear trsf.

$$\vec{x}^{(a)} \longrightarrow \vec{x}^{(a+1)} = \underline{\varphi^{(a)}}(\underline{W^{(a)}\vec{x}^{(a)} + \vec{b}^{(a)}})$$

Neural network

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- parameters of linear transformation is tunable.

$W^{(a)}$: weight, $\vec{b}^{(a)}$: bias ← updated in the learning process

Neural network

$$\vec{x} = \vec{x}^{(0)} \longrightarrow \vec{x}^{(1)} \longrightarrow \vec{x}^{(2)} \longrightarrow \vec{x}^{(3)} = \vec{y}$$

- Proceed to next layer by <u>linear trsf</u> and <u>non-linear trsf</u>.

$$\vec{x}^{(a)} \longrightarrow \vec{x}^{(a+1)} = \underline{\varphi^{(a)}}(\underline{W^{(a)}\vec{x}^{(a)} + \vec{b}^{(a)}})$$

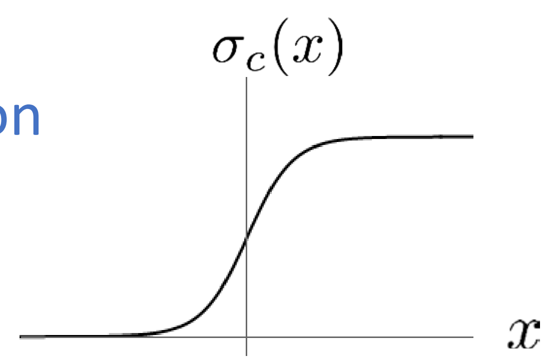
- parameters of linear transformation is tunable.

  <u>$W^{(a)}$: weight, $\vec{b}^{(a)}$: bias</u> $\longleftarrow$ updated in the learning process

- non-linear transformation $\underline{\varphi^{(a)}(\vec{x})}$ is not changed.

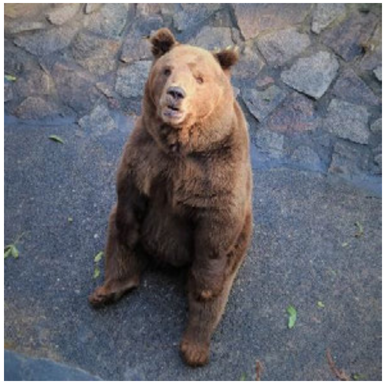
  activation function

  e.g., sigmoid function $\sigma_c(x) = \dfrac{1}{1 + e^{-cx}}$



Supervised learning

- Teach answers to the machine.



This is a bear.

The bear machine

This is not a bear.

The bear machine

Supervised learning

- Prepare a data set $\{(\vec{x}^{(i)}, \vec{y}^{(i)})\}$

$\left(\vec{x}^{(1)} = \right.$  $\left. , \vec{y}^{(1)} = \text{yes} \right),$ …

… $\left(\vec{x}^{(10)} = \right.$  $\left. , \vec{y}^{(10)} = \text{no} \right),$ …

- Feed input data $\vec{x}^{(i)}$ to the machine.

$\vec{x}^{(i)}$ → The bear machine → $\vec{y}(\vec{x}^{(i)}, \omega)$

ω

Supervised learning

- Prepare a data set $\{(\vec{x}^{(i)}, \vec{y}^{(i)})\}$

$\left(\vec{x}^{(1)} = \right.$  $\left. , \vec{y}^{(1)} = \text{yes} \right),$ …

… $\left(\vec{x}^{(10)} = \right.$  $\left. , \vec{y}^{(10)} = \text{no} \right),$ …

- Feed input data $\vec{x}^{(i)}$ to the machine.

$\vec{x}^{(i)}$ → **The bear machine** → $\vec{y}(\vec{x}^{(i)}, \omega)$ ↔ $\vec{y}^{(i)}$

ω

Compare and
adjust parameters ω

Supervised learning

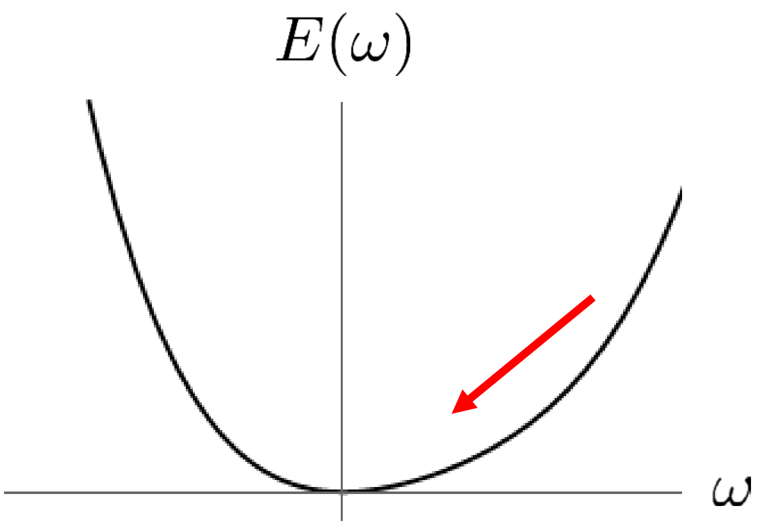
- Adjust parameters $\{W^{(a)}, \vec{b}^{(a)}\}$ so that a loss function (error function, cost function) decreases.

$$E(\omega) = \underline{d(\vec{y}(\vec{x}^{(i)}, \omega), \vec{y}^{(i)})} \longleftarrow$$ optimize it.

- mean squared error
- L1 norm
- cross entropy
- …

- gradient descent

$$\omega \to \omega - \epsilon \partial_\omega E(\omega)$$



Regularization

$$E(\omega) \to E(\omega) + E_{\mathrm{reg}}$$

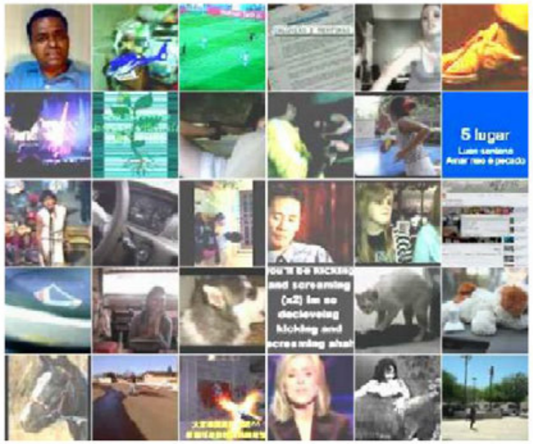
e.g.

- to avoid the overfitting
- to get the expected properties

Unsupervised learning

For example, [Google Brain (2012)]

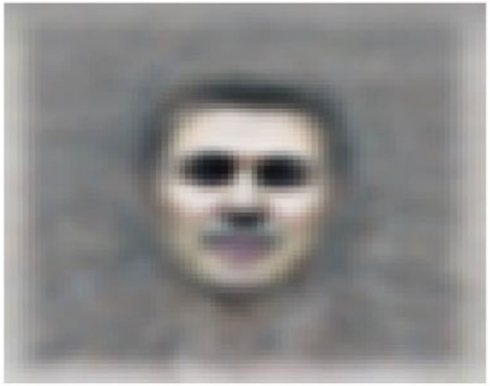
10 million images



Google's AI

The concept of cats

The concept of faces



Machine learning for physics

- Quantum many body system

[Carleo & Troyer (2016), Fujita, Nakagawa, Sugiura & Oshikawa (2017),…]

- Detection of phase transition in statistical systems

[Wang (2016), Tanaka & Tomiya (2016), Ohtsuki^2 (2016),…]

- String landscape

[He (2017), Ruehle (2017), Carifio, Halverson, Krioukov & Nelson (2017),…]

- LHC, Monte Calro, cosmology, neutron star, …, **AdS/CFT**

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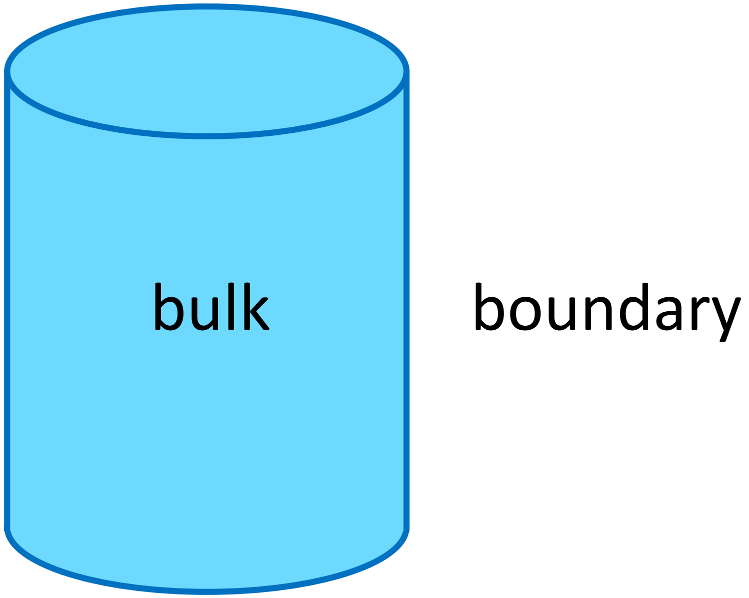
AdS/CFT correspondence

[Maldacena (1997)]

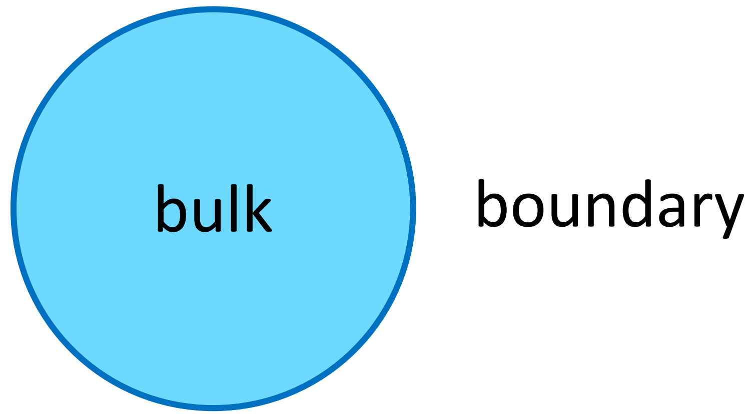
Quantum Gravity on AdS = CFT on boundary of AdS



Many works about this conjecture.

Emergence of geometry

- AdS/CFT states that the bulk direction emerges from the dynamics of boundary QFT.



- Holography

Gravity can be defined in terms of low-dimension theories.

A definition of quantum gravity

AdS ← CFT

Holographic approach

AdS → CFT

In some parameter regions, a QFT can be described by a classical Einstein gravity with matter fields.

⇒ We can analyze the QFT with strong couplings by using classical gravity.

hard

easy

- holographic QCD,…

Note

- CFTs which have gravity dual descriptions are very specific.

  e.g. 4d $\mathcal{N} = 4$ SYM $\longleftrightarrow$ string theory on $AdS_5 \times S^5$

In order to have a "good gravity description", CFT should satisfy some criteria: [Heemskerk, Penedones, Polchinski, Sully (2009), El-Showk & Papadodimas (2011)]
e.g.

- large DOF (large N, large c,...)
- gapped spectrum (strong coupling)

We assume QFTS we will consider have semiclassical gravity descriptions.

GKP-Witten

[Gubser, Klebanov, Polyakov (1998), Witten (1998)]

$$\boxed{\langle e^{\int d^d x J(x)\mathcal{O}(x)}\rangle_{\mathrm{CFT}} \simeq e^{-S^{grav}_{cl}[J(x)]}}$$

- Correlation functions in CFT can be computed from the <u>classical solution of the dual field</u> in gravitational theory.

boundary condition is fixed by $J(x)$

Dictionary in AdS/CFT

scalar operator $\mathcal{O}$ in CFT$_d$ with dim Δ

(2-pt function in the g.s. $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle \propto |x|^{-2\Delta}$)

$\Leftrightarrow$ scalar field $\phi(x)$ with mass m $[m^2 = \Delta(\Delta - d)]$
on $(d+1)$-AdS

Free scalar on AdS

$$S = \int d^{d+1}x\sqrt{-g}\,\frac{1}{2}(-g^{MN}\partial_M\phi\partial_N\phi - m^2\phi^2)$$

$$[m^2 = \Delta(\Delta - d)]$$

$$ds^2 = d\eta^2 + e^{2\eta}(dx_\mu dx^\mu)$$

AdS boundary $\eta \sim \infty$

- Asymptotic behavior of the classical solution near $\eta \sim \infty$

$$\boxed{\phi_{\rm cl}(\eta, x) \sim e^{-(d-\Delta)\eta}(J(x) + \cdots) + e^{-\Delta\eta}(B(x) + \cdots)}$$

$$B(x) = \frac{\langle \mathcal{O}(x)\rangle_J}{2\Delta - d}$$

1-pt func w/ source in CFT

[Klebanov & Witten (1999)]

$$\Rightarrow \quad e^{-S[\phi_{\rm cl}]} \sim e^{-\int d^dx J\langle\mathcal{O}\rangle}$$

Asymptotically AdS

- Background does not need to be the exact AdS space.

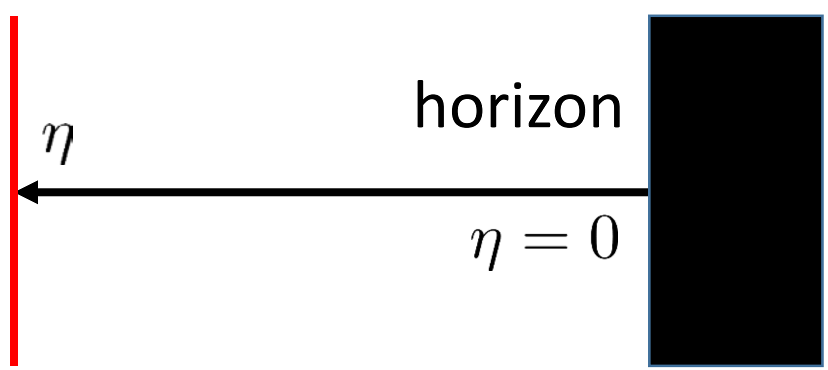
$$ds^2 = d\eta^2 + e^{2\eta}(dx_\mu dx^\mu)$$

Asympto AdS is OK. $g_{MN} \sim g_{MN}^{\mathrm{AdS}}$ around $\eta \sim \infty$

It corresponds to an excited state in CFT.

- AdS Schwarzschild $\Longleftrightarrow$ finite temperature

$$ds^2 = d\eta^2 + C\cosh^{\frac{4}{d}}\left(\frac{d\eta}{2}\right)\left[-\frac{\sinh^2\left(\frac{d\eta}{2}\right)}{\cosh\left(\frac{d\eta}{2}\right)}dt^2 + dx^i dx^i\right]$$



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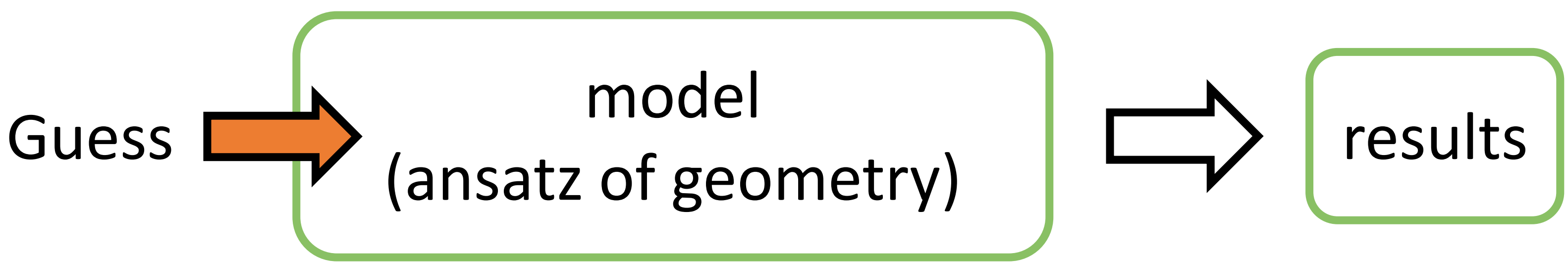
Holographic modeling

- Two approaches
  - Top-down — embed into string theory
  - Bottom-up — prepare a phenomenological model

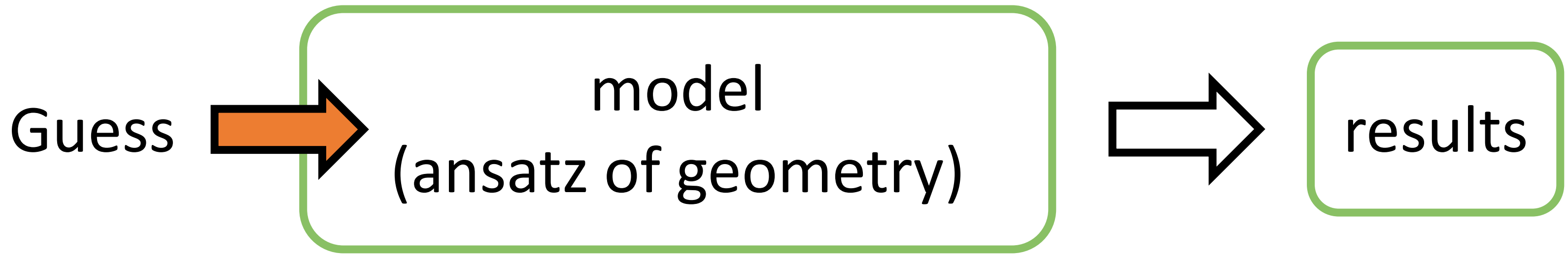
- conventional holographic QCD models in the bottom up approach
  - Hard wall, Soft wall,...

  [Erlich, Katz, Son & Stephanov (2005), Da Rold & Pomarol (2005),...]



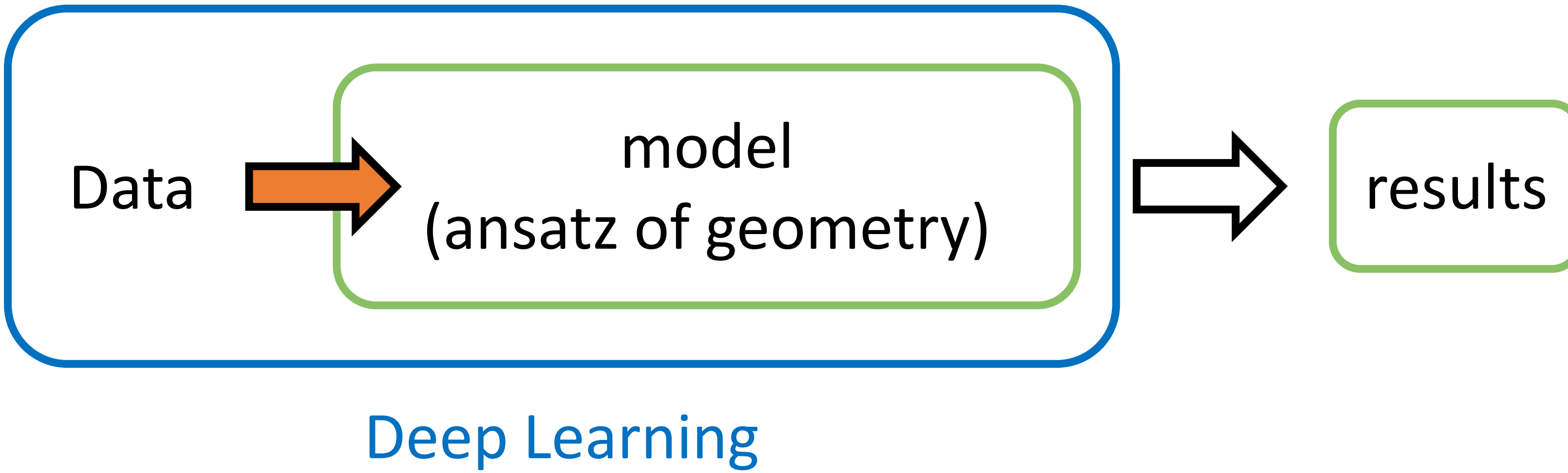
DL holographic modeling

- Conventional approach



DL holographic modeling

- Our approach
  [Hashimoto, SS, Tanaka, Tomiya (1802.08313)]



Setup

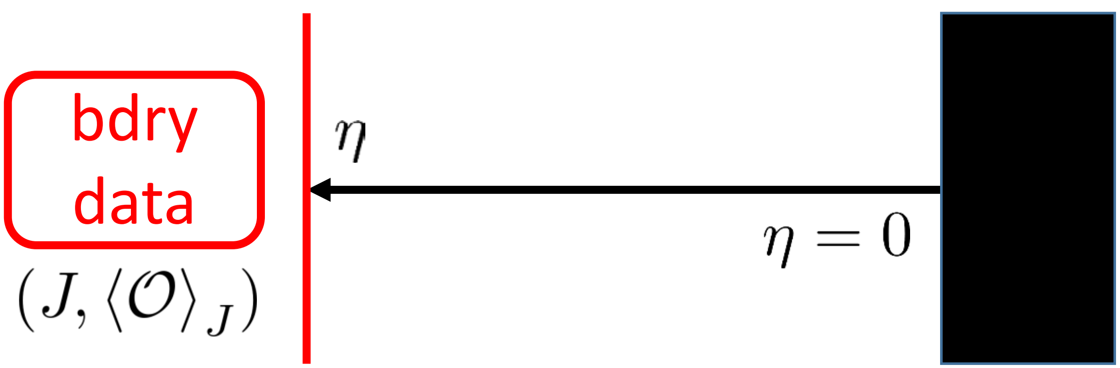
We consider general backgrounds (*not* sol. of Einstein eq.).

$$ds^2 = d\eta^2 - f(\eta)dt^2 + g(\eta)dx^i dx^i$$

- bulk scalar dual to boundary op. $\mathcal{O}$

$$S = \int d^{d+1}x \sqrt{f g^{d-1}} \, \frac{1}{2}\left(-g^{MN}\partial_M\phi\partial_N\phi - m^2\phi^2 - \frac{\lambda}{2}\phi^4\right)$$

general metric with horizon



Learn the metric

AdS bdry

horizon

bdry data

$(J, \langle \mathcal{O} \rangle_J)$

$\phi(\eta \sim \infty)$

$\phi(\eta \sim 0)$

smooth b.c.

η

$$\phi(\eta \sim \infty) \sim e^{-(d-\Delta)\eta} J + e^{-\Delta\eta} \frac{\langle \mathcal{O} \rangle_J}{2\Delta - d}$$

Learn the metric

AdS bdry

horizon

bdry data

$(J, \langle \mathcal{O} \rangle_J)$ $\Longleftrightarrow$ $\phi(\eta \sim \infty)$

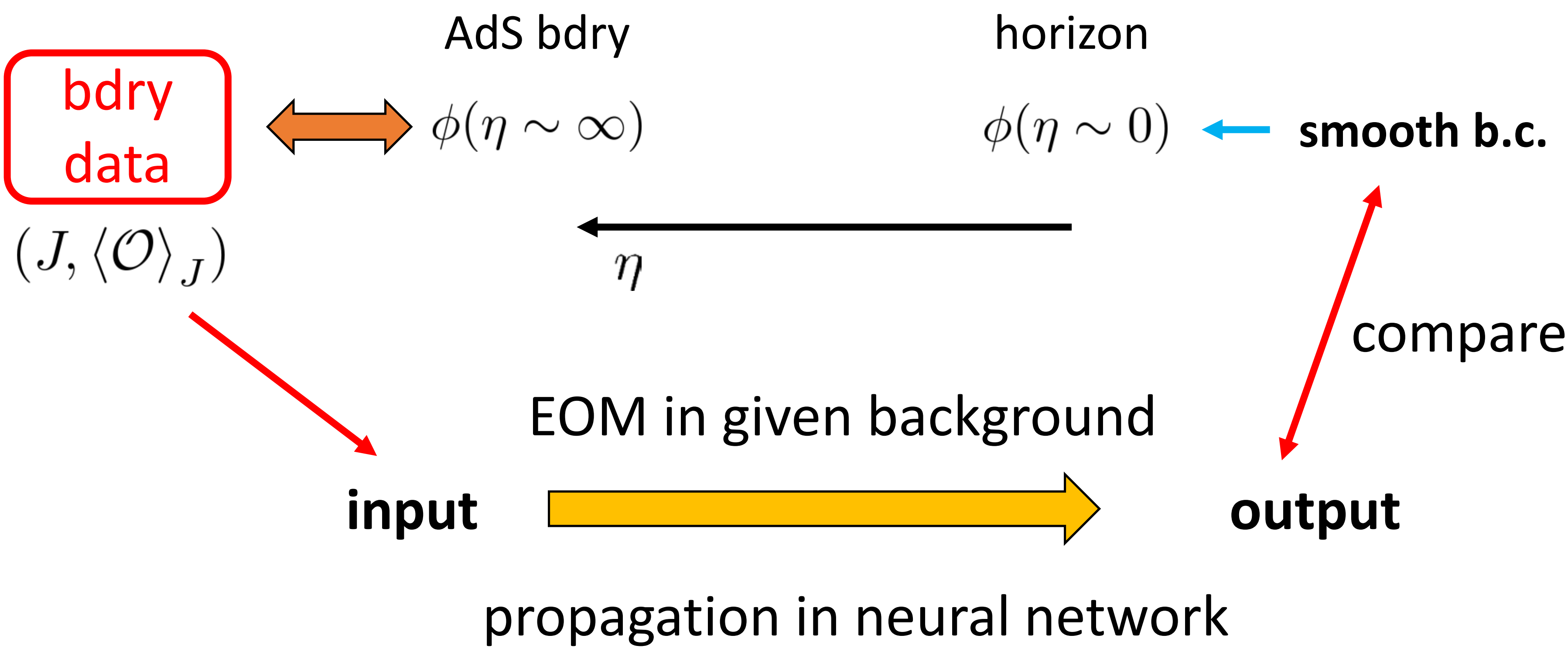
$\phi(\eta \sim 0)$ $\leftarrow$ **smooth b.c.**

$\xleftarrow{\quad\eta\quad}$

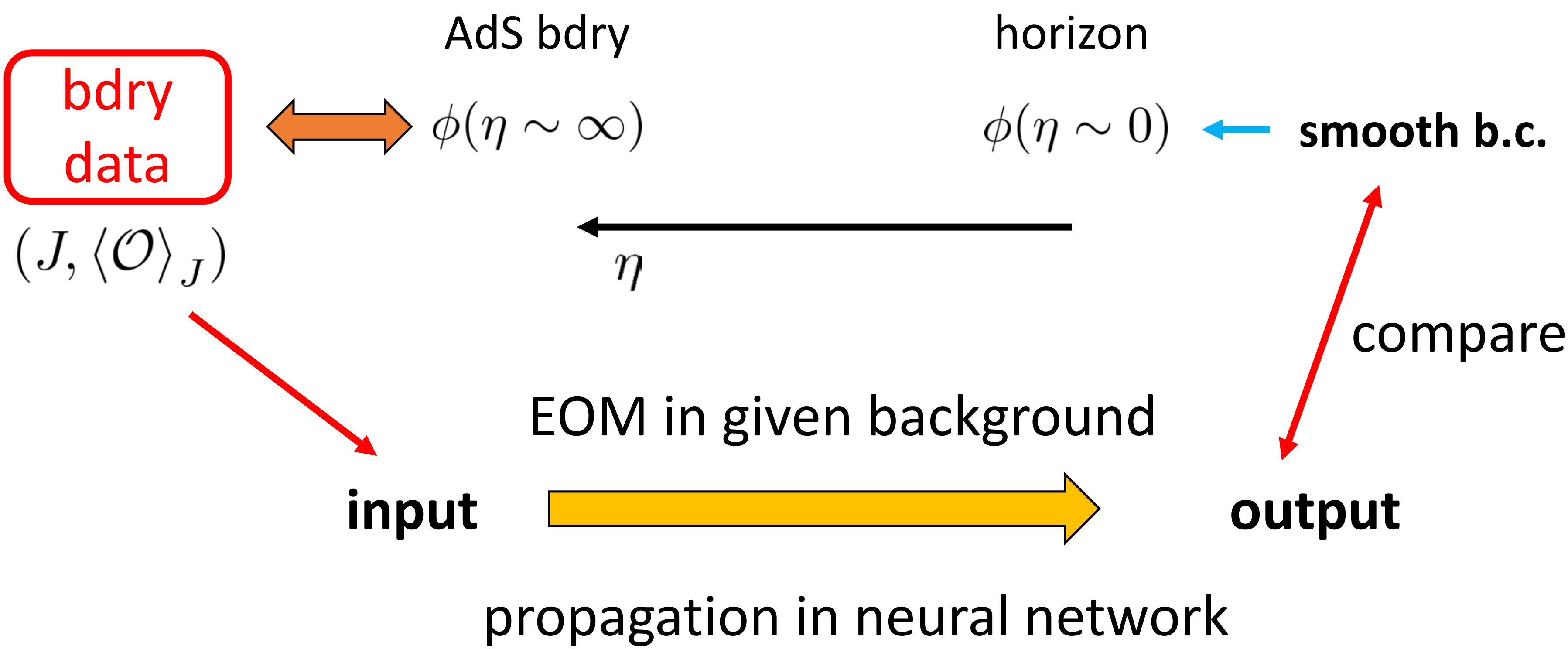
EOM in given background

input $\Longrightarrow$ **output**

Learn the metric



Learn the metric



- In the appropriate metric, output should satisfy the correct boundary condition.

background metric = parameters to be updated

Neural network rep. of EOM

- Suppose the field is homogeneous $\phi(\eta, \cancel{t, x^i})$

$$\boxed{\partial_\eta \pi + h(\eta)\pi - m^2\phi - \lambda\phi^3 = 0\,, \quad \pi = \partial_\eta \phi}$$

$$h(\eta) = \partial_\eta \log\sqrt{f(\eta)g(\eta)^{d-1}}$$ ← to be learned

Neural network rep. of EOM

- Suppose the field is homogeneous $\phi(\eta, \cancel{t, x^i})$

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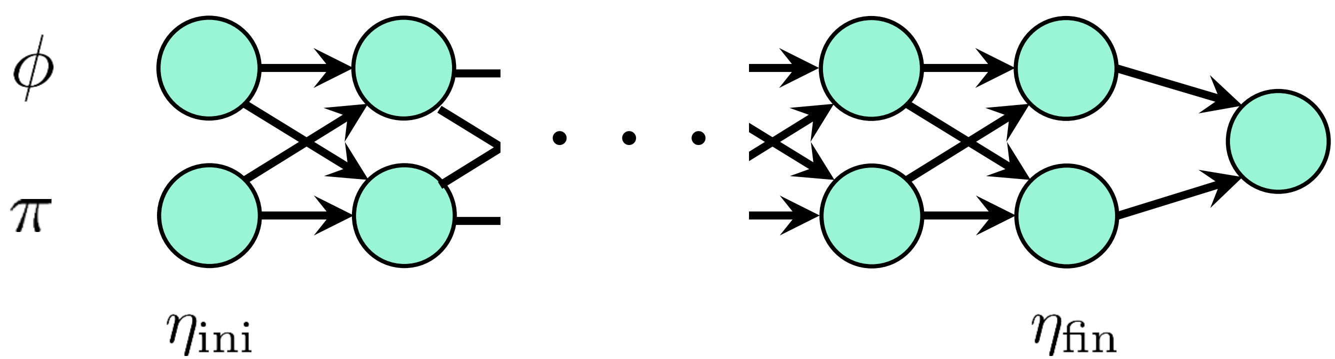
- Discretize η $(\eta_{\rm ini} > \eta > \eta_{\rm fin})$

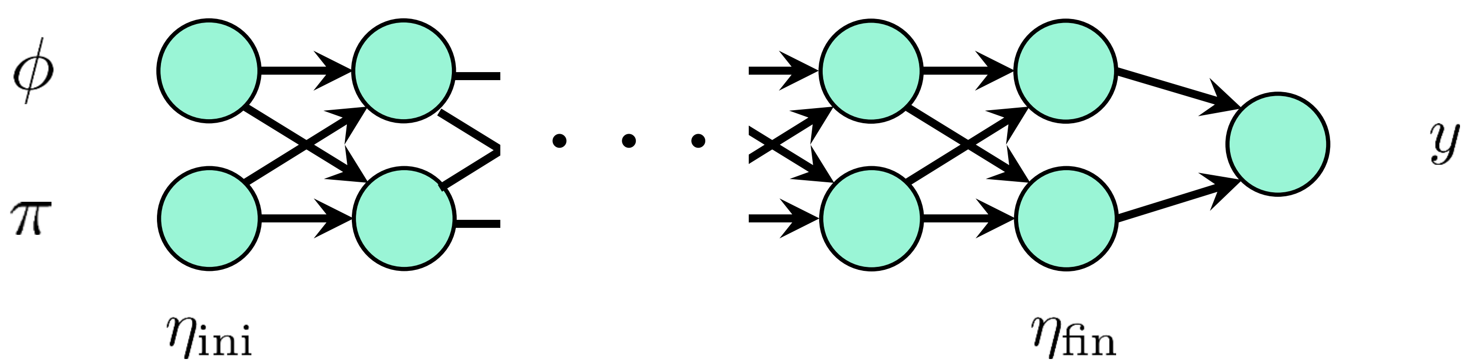


The metric function $h(\eta^{(n)})$ plays the role of weights!

Boundary cond. at the horizon

$$\partial_\eta \pi + h(\eta)\pi - m^2\phi - \lambda\phi^3 = 0\,, \quad \pi = \partial_\eta \phi$$

Near the horizon $\eta \sim 0, \quad h(\eta) \sim 1/\eta$



Boundary cond. at the horizon

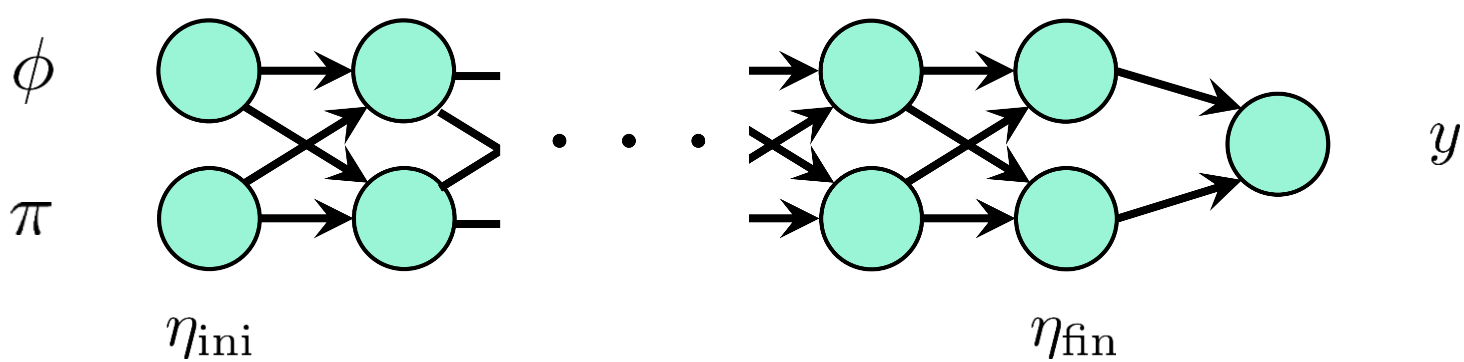
$$\partial_\eta \pi + h(\eta)\pi - m^2\phi - \lambda\phi^3 = 0\,, \quad \pi = \partial_\eta \phi$$

Near the horizon $\eta \sim 0, \quad h(\eta) \sim 1/\eta$

Suppose the field is finite.

$$\eta h(\eta)\pi = \eta(-\partial_\eta \pi + m^2\phi + \lambda\phi^3) \to 0 \quad (\eta \to 0)$$

$$\boxed{\pi\big|_{\eta=\eta_{\rm fin}} = 0}$$



Boundary cond. at the horizon

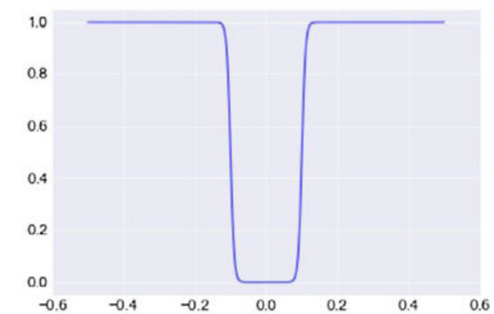
$$\partial_\eta \pi + h(\eta)\pi - m^2\phi - \lambda\phi^3 = 0\,, \quad \pi = \partial_\eta \phi$$

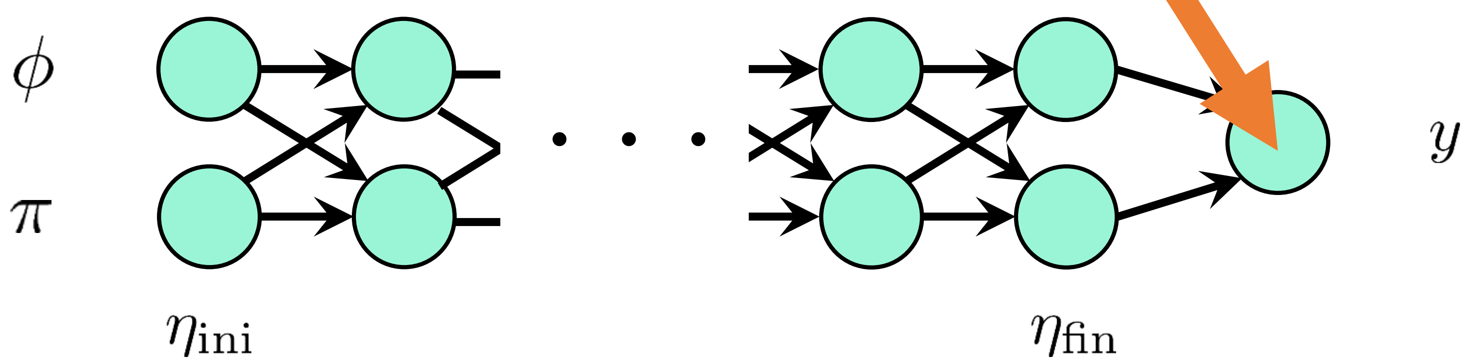
Near the horizon $\eta \sim 0, \quad h(\eta) \sim 1/\eta$

Suppose the field is finite.

$$\eta h(\eta)\pi = \eta(-\partial_\eta \pi + m^2\phi + \lambda\phi^3) \to 0 \quad (\eta \to 0)$$

$$\boxed{\pi|_{\eta=\eta_{\rm fin}} = 0} \longrightarrow \underline{y = \mathrm{tanaka}(\pi|_{\eta=\eta_{\rm fin}})}$$





Data for learning

- Positive data

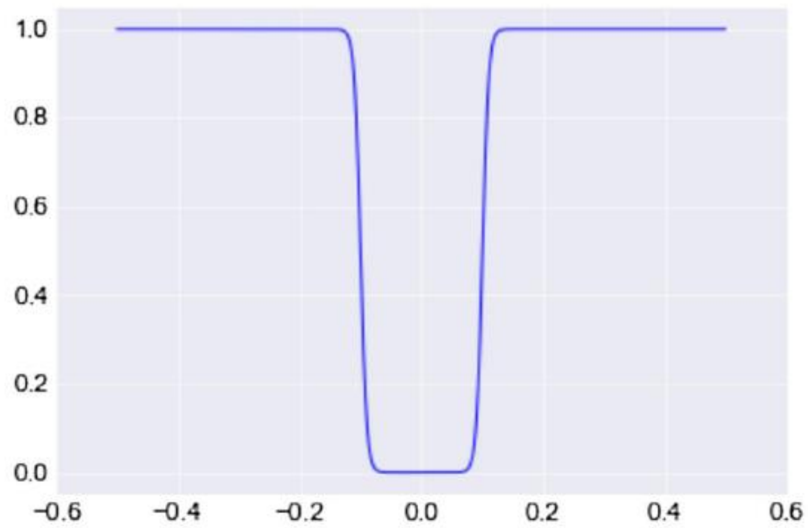
$$\{(\underline{J, \langle \mathcal{O} \rangle_J}), y = 0\}$$

correct response

- Negative data

$$\{(\underline{J, \langle \mathcal{O} \rangle_J}), y = 1\}$$

wrong response



If the metric function is tuned appropriately,

$\pi|_{\eta=\eta_{\mathrm{fin}}} \simeq 0 \longrightarrow y^{\mathrm{output}} \simeq 0$ for positive data

$\pi|_{\eta=\eta_{\mathrm{fin}}} \not\simeq 0 \longrightarrow y^{\mathrm{output}} \simeq 1$ for negative data

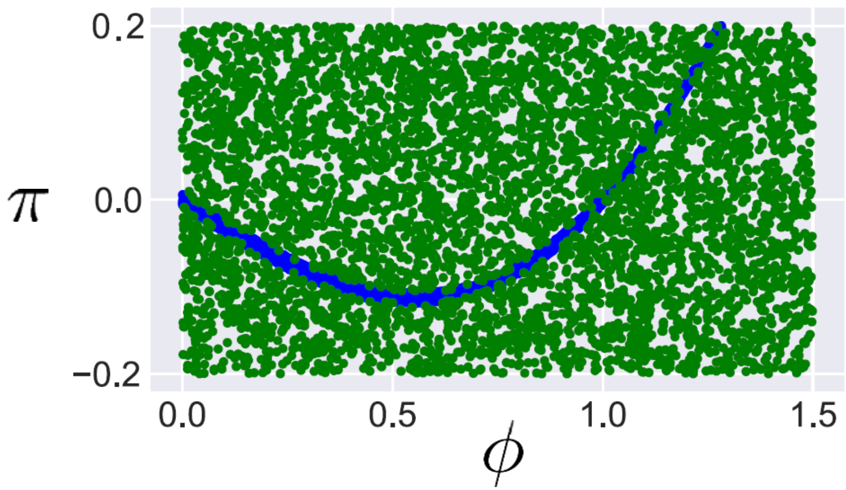
Test learning

Can the machine reproduce the black hole metric?

- 4d AdS Schwarzschild

$$h(\eta) = 3\coth(3\eta)$$

- Generate positive and negative data randomly



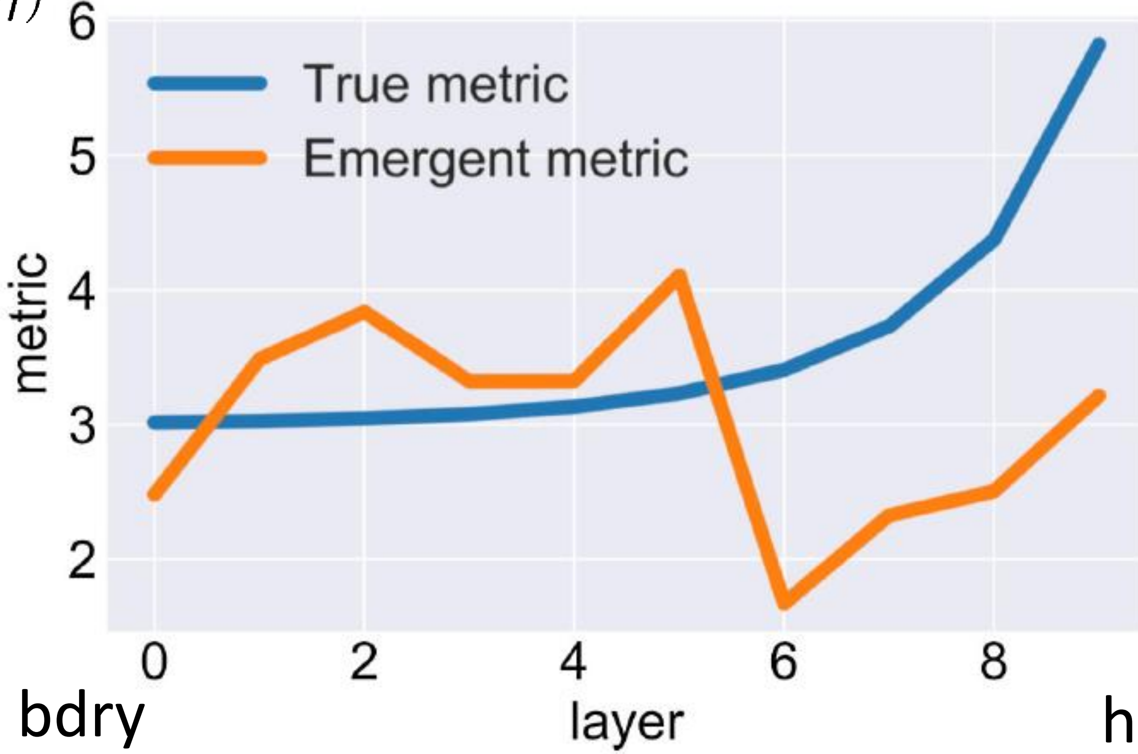
- Learn $h(\eta)$ from the data.

$$h^{\text{learn}}(\eta) \overset{?}{=} 3\coth(3\eta)$$

Test learning

loss function $E = \sum_i |y(x^{(i)}) - y^{(i)}|$

$h(\eta)$



bdry

horizon

loss function < 0.0002

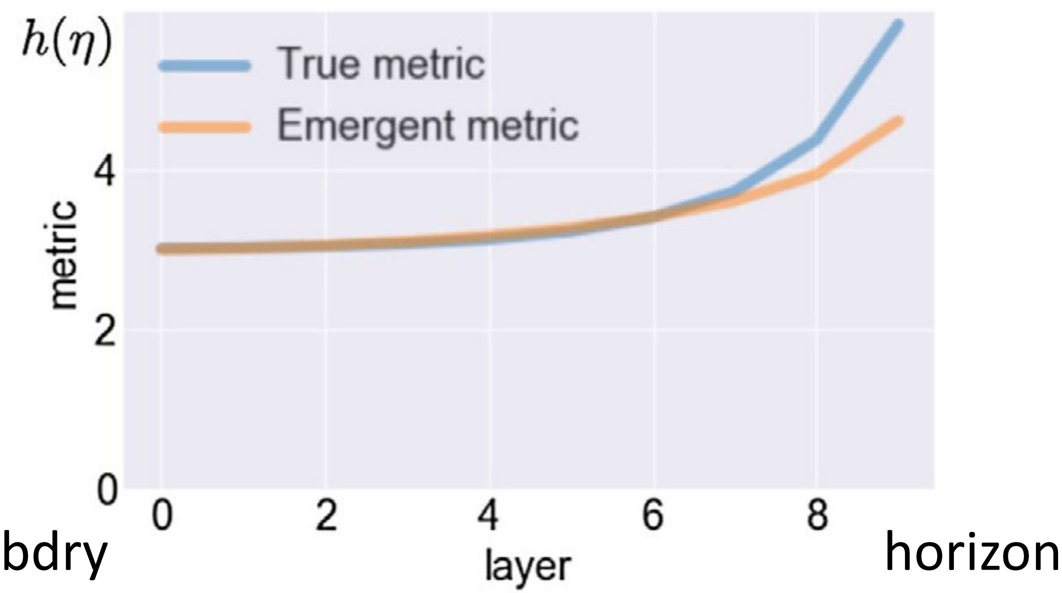
Test learning

- Introduce a regularization

$$E \;\to\; E + c_{\text{reg}} \underline{\sum (\eta^{(n)})^4 \left[h(\eta^{(n+1)}) - h(\eta^{(n)}) \right]^2}$$

$$\propto \int d\eta (\eta^2 h'(\eta))^2 \qquad (c_{\text{reg}} = 10^{-3})$$

Smooth function is preferred.



Deep learning and holographic QCD

[Hashimoto, SS, Tanaka, Tomiya (in preparation)]

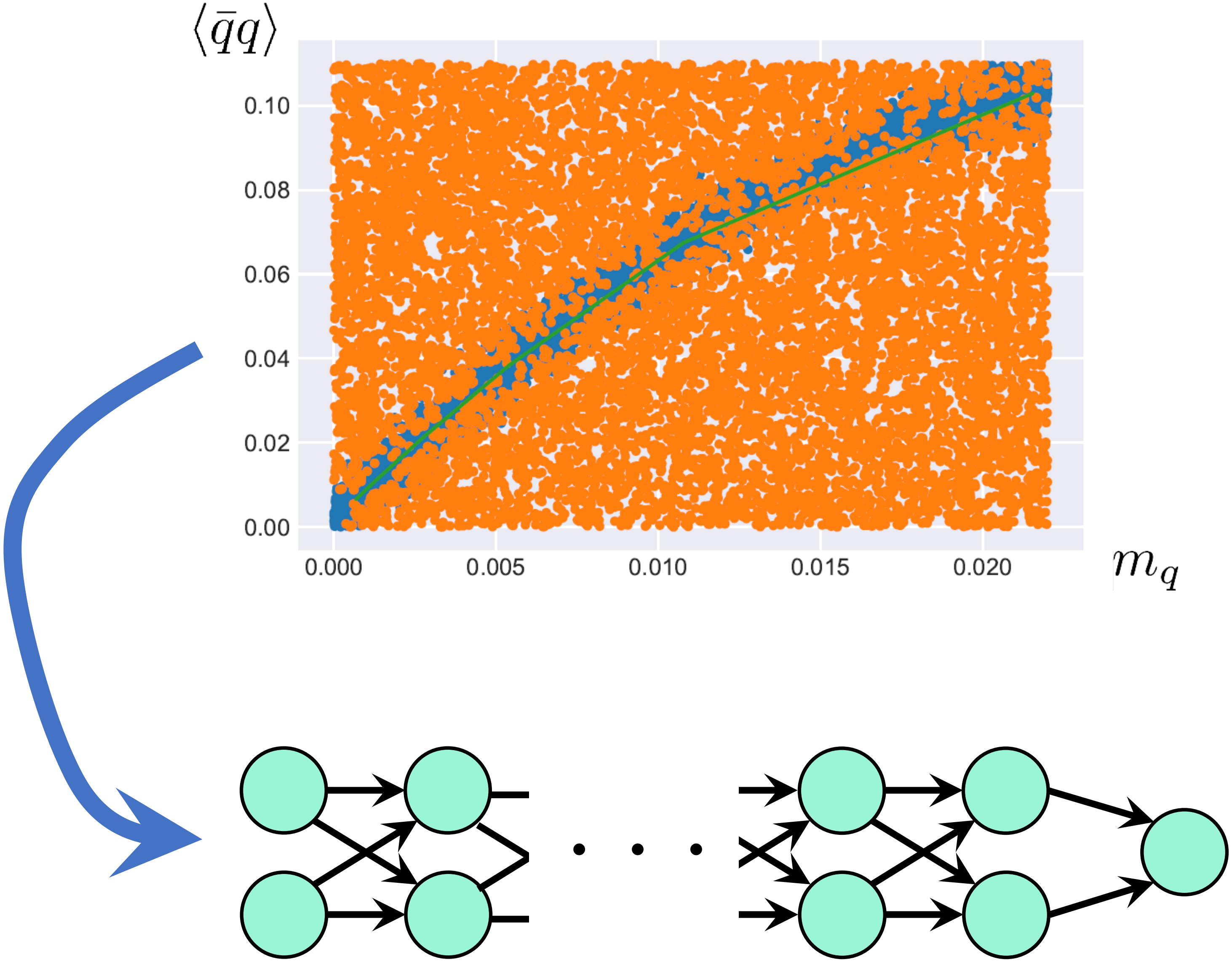
- Use the result of lattice QCD.
  - **quark mass** vs **chiral condensate** [Unger (2010)]

m_q[GeV]	$\langle\bar{\psi}\psi\rangle$ [(GeV)^3]
0.00067	0.0063
0.0013	0.012
0.0027	0.021
0.0054	0.038
0.011	0.068
0.022	0.10

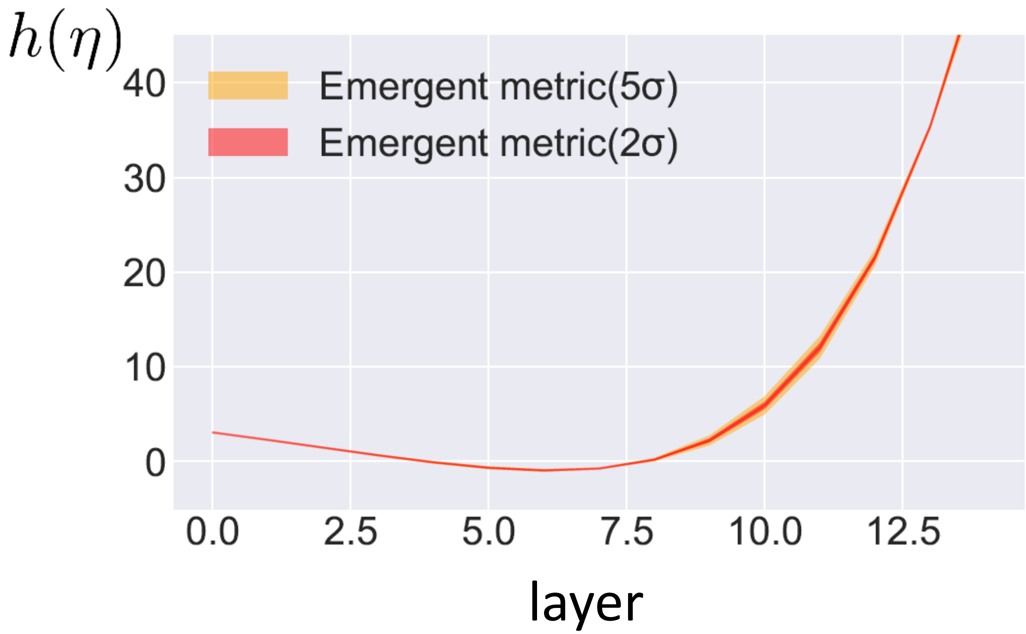
$T \sim 0.2[\mathrm{GeV}]$

- Learn the metric, and compute other quantities

Data for learning



Learned metric



L_{AdS} is also learned. $\longrightarrow$ $L_{\mathrm{AdS}} \sim 4.2[\mathrm{GeV}^{-1}]$

In this approach,
only a combination of metric compos. can be obtained.

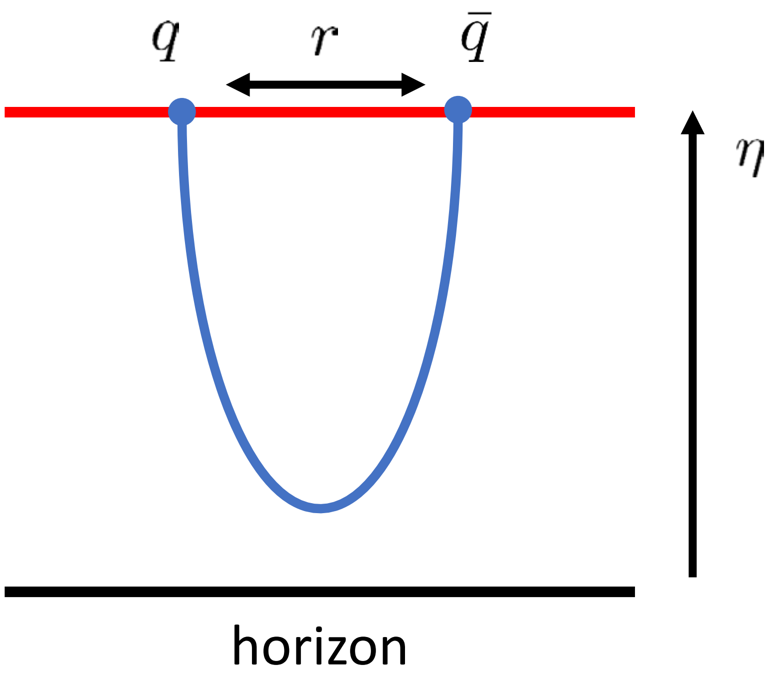
$$h(\eta) = \partial_\eta \log \sqrt{f(\eta) g(\eta)^{d-1}}$$

Holographic $Q\bar{Q}$ potential

$$ds^2 = d\eta^2 - f(\eta)dt^2 + g(\eta)dx^i dx^i$$

If $f(\eta)$ and $g(\eta)$ are obtained, we can compute quark-antiquark potential holographically.

Evaluate a classical string action. [Maldacena (1998), Rey & Yee (1998)]



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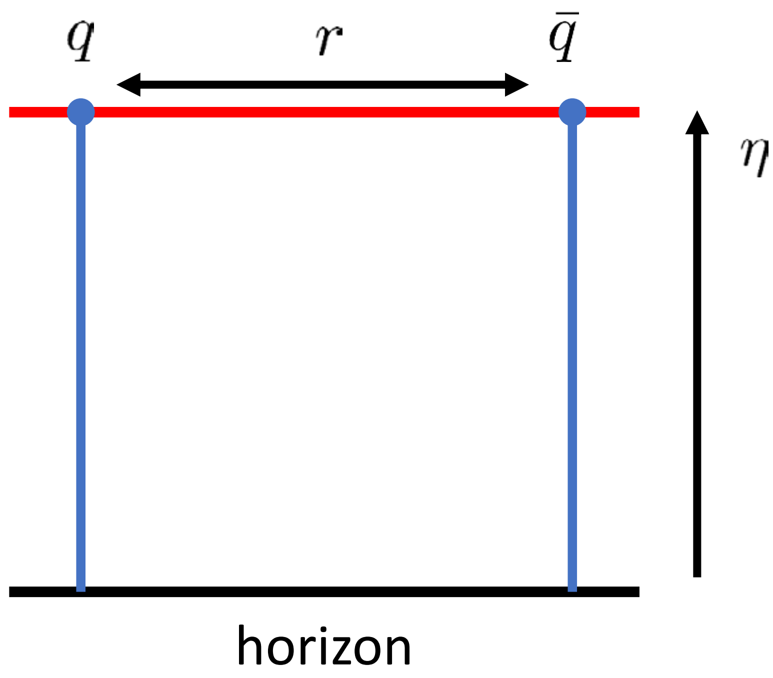


Compute $Q\bar{Q}$ potential

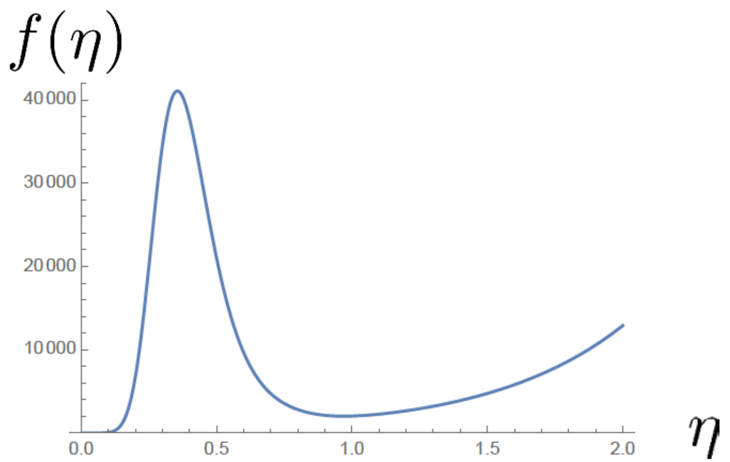
$h(\eta) = \partial_\eta \log \sqrt{f(\eta) g(\eta)^3}$ has been obtained.

$f(\eta)$ can be determined if $g(\eta)$ is fixed.

- Ansatz

$$g(\eta) = \alpha e^{2\eta}$$

(pure AdS)



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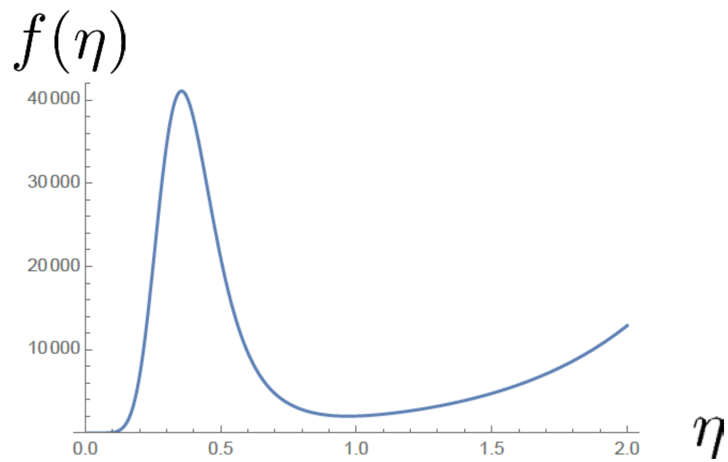
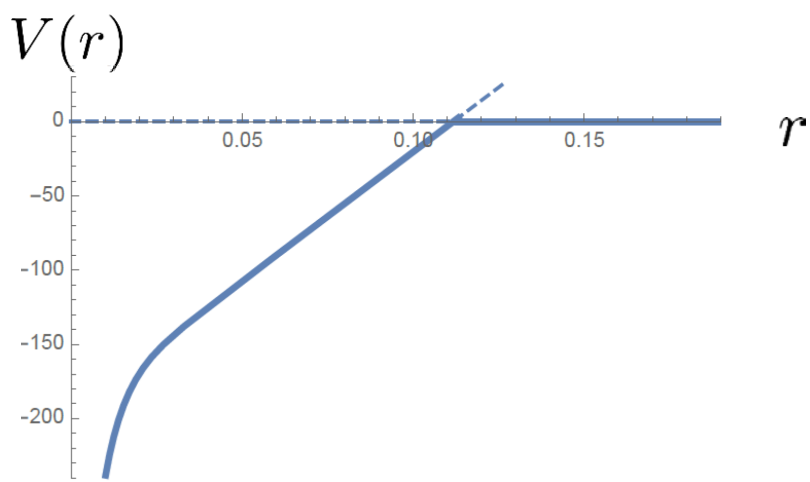
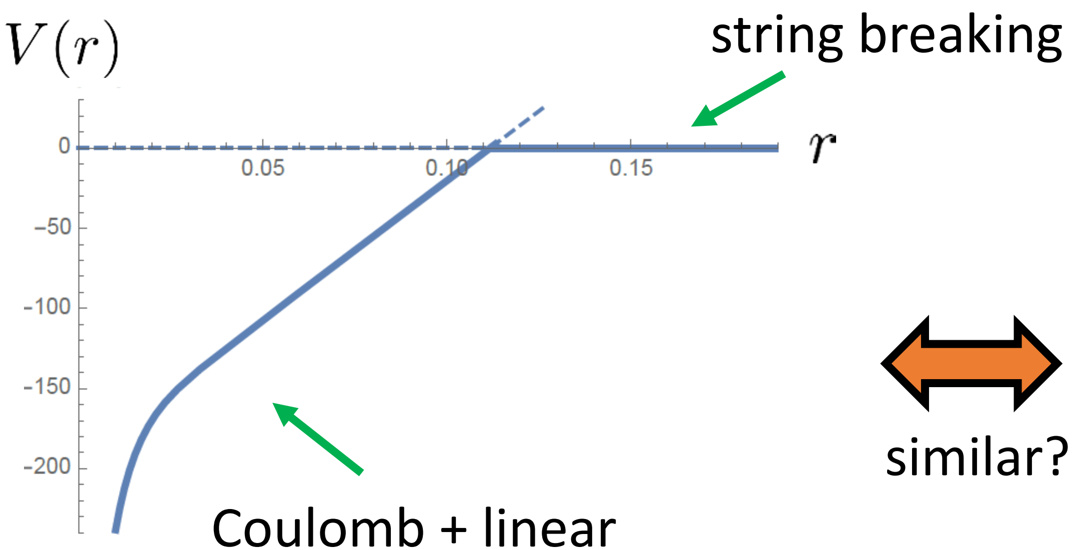




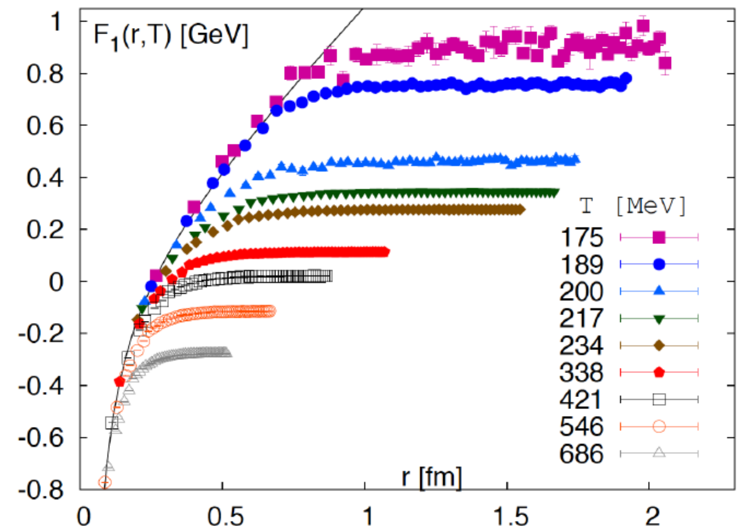
(another ansatz gives a similar shape)

$$g(\eta) = \alpha \cosh(2\eta)$$

Obtained $Q\bar{Q}$ potential



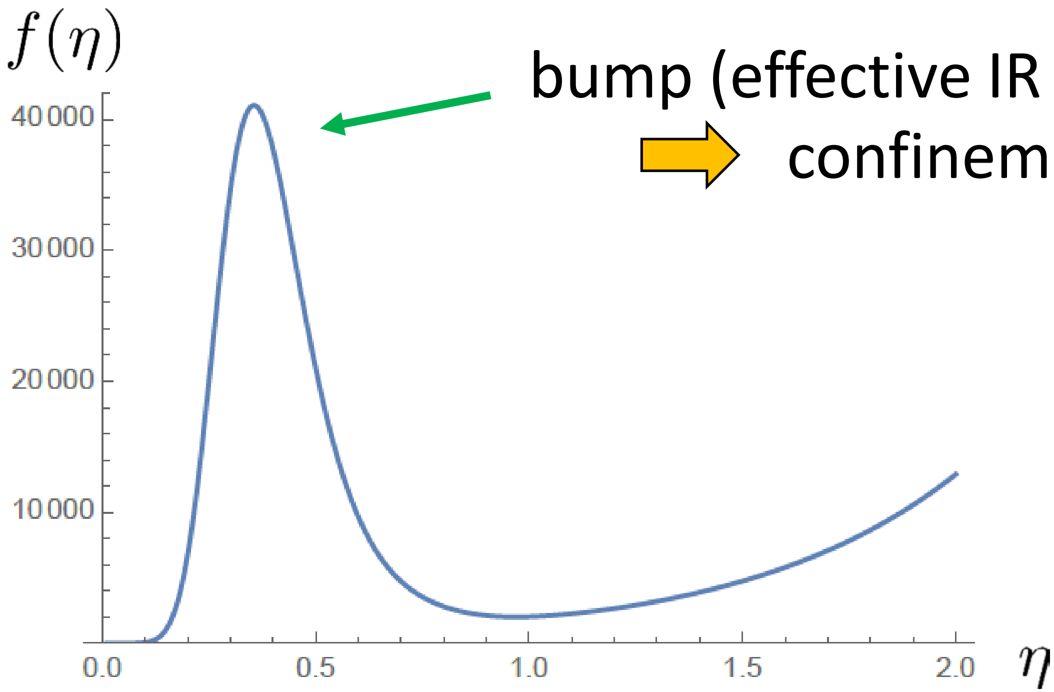
similar?



lattice result

Fig from [Petreczky (2010)]

The first holographic model which shows both of confinement and string breaking (as far as we know)



bump (effective IR wall)

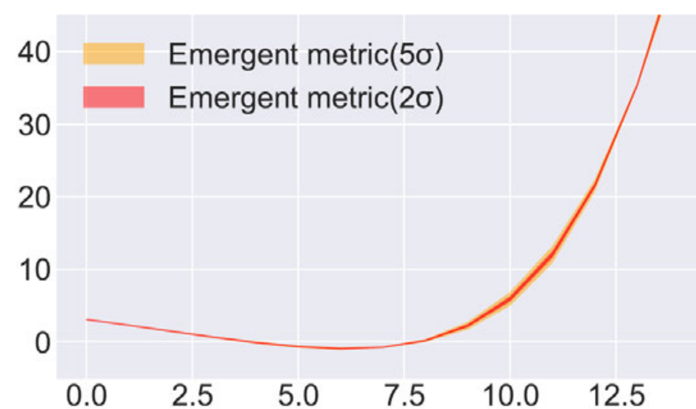
⇨ confinement

horizon

⇨ string breaking

a region $h(\eta) < 0$

⇨ bump



Outline

- ✓ Introduction
- ✓ Deep learning
- ✓ AdS/CFT
- ✓ Deep learning and AdS/CFT
- ■ Summary

Summary

- Presented a deep neural network representation of the propagation to the bulk direction.
  - metric is weight and updated in the process of learning. **Spacetime is a neural network?**
- Applied it to a holographic QCD model.
  - obtained a $Q\bar{Q}$ potential.
  - there are many points which should be improved.
- Machine learning techniques may be also useful in other regions in physics.