Escaping the branched polymer phase in dynamical triangulations

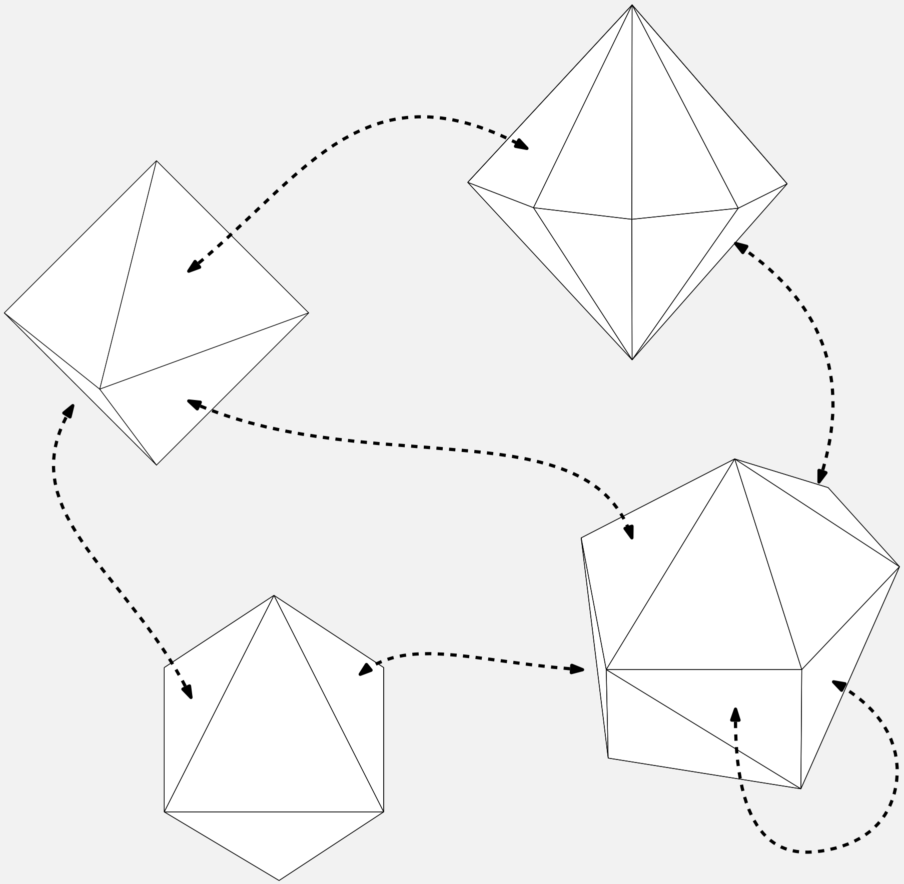
Luca Lionni
YITP – Kyoto U

10/09/18 – Tohoku U – Risan workshop

A simple combinatorial problem in discrete geometry

A simple combinatorial problem in discrete geometry

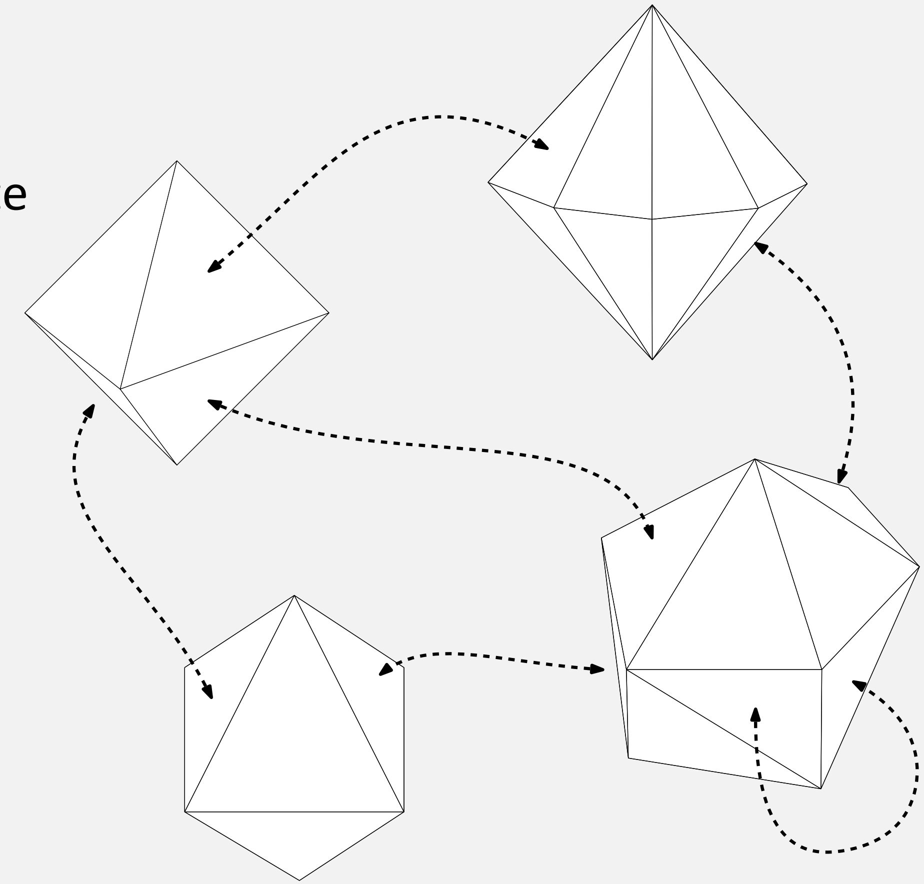
In any dimension, take a collection of “rigid” building blocks of your choice.
Glue them together in every possible way.



A simple combinatorial problem in discrete geometry

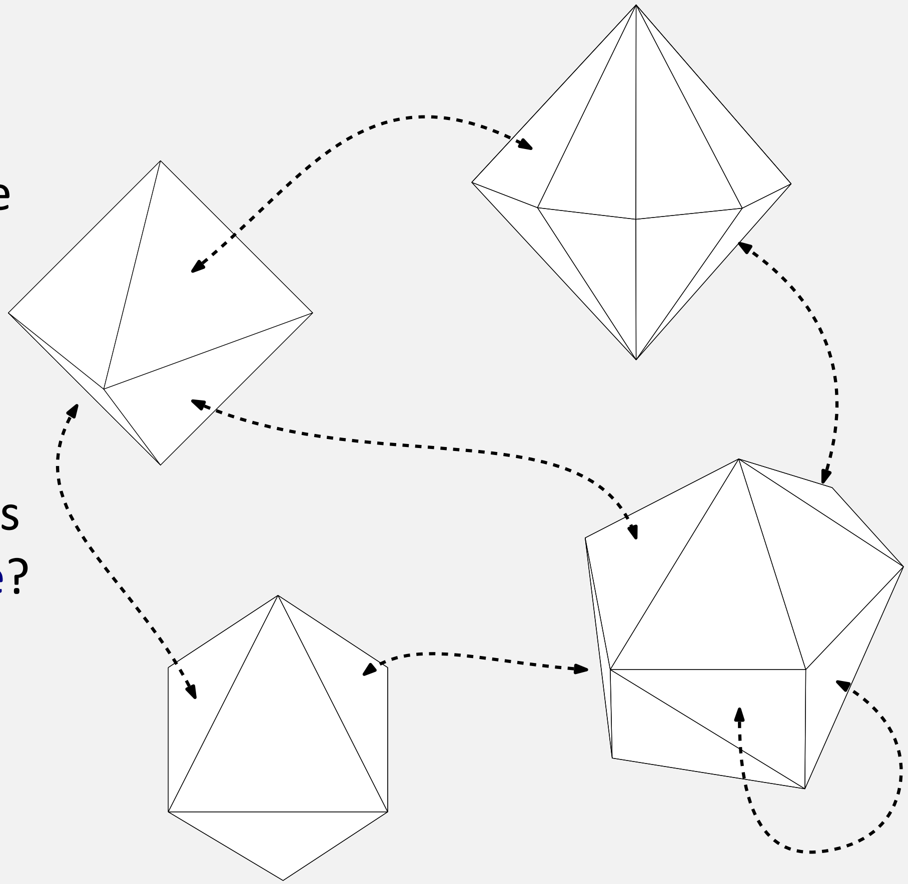
In any dimension, take a collection of “rigid” building blocks of your choice.
Glue them together in every possible way.

→ Can we count the resulting discrete spaces according to their global curvature?



A simple combinatorial problem in discrete geometry

In any dimension, take a collection of "rigid" building blocks of your choice.
Glue them together in every possible way.



→ Can we count the resulting discrete spaces according to their global curvature?

→ What are the asymptotic properties of the spaces of maximal curvature? What do they look like in the continuum?

1 – Curvature…?

2 –Discrete quantum gravity (Dynamical triangulations)

3 – Previously known results

4 – Some recent results

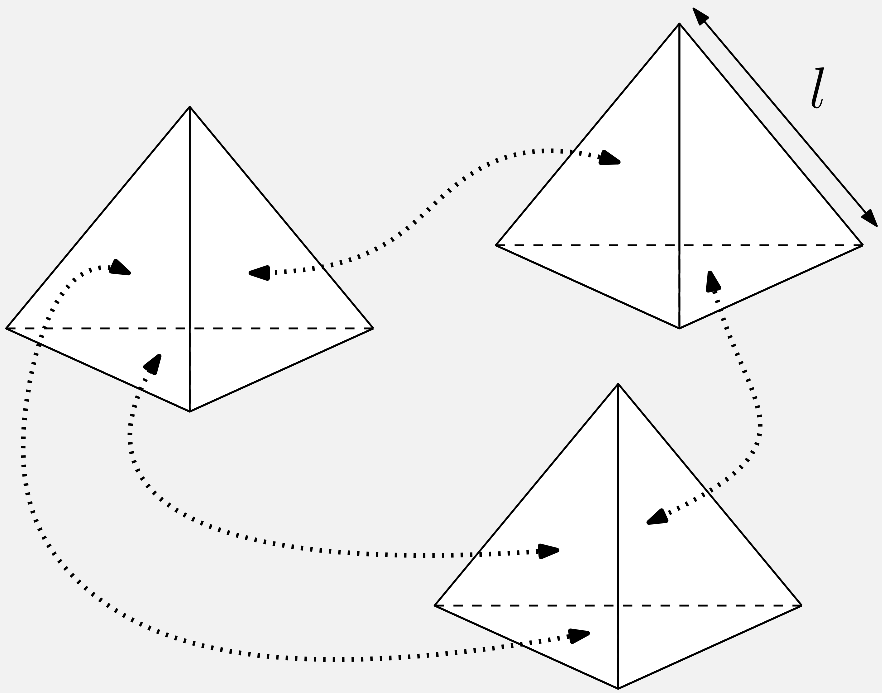
5 – Random tensor models

6 – Ongoing work

1 – Curvature…?

1 – Curvature…?

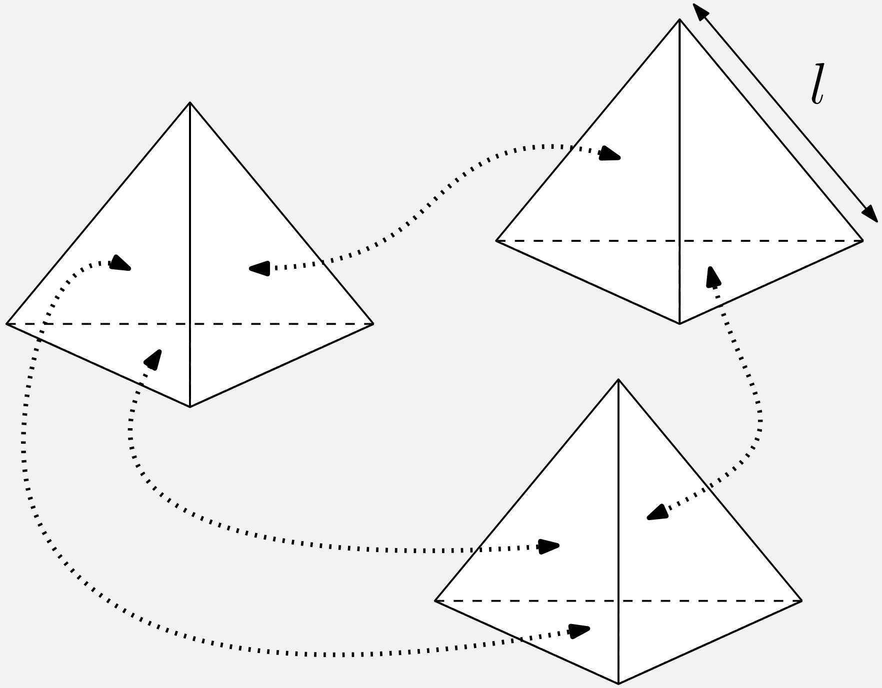
Start with the simplest case: triangulations



1 – Curvature…?

In a triangulation, suppose that all edges have the same length

→ "canonical geometry" (notion of distance, local curvature…)



1 – Curvature…?

<u>Local curvature:</u>

Number of D-simplices around
$(D\text{-}2)$-simplices

1 – Curvature…?

Local curvature:

Number of *D*-simplices around (*D*-2)-simplices

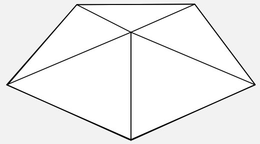
Deficit angle in dimension 2

$$2\pi\left(1 - \frac{\mathcal{N}(v)}{6}\right)$$

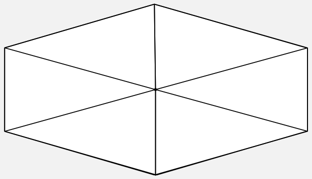
Dimension *D=2*

Number of equilateral triangles around vertices :

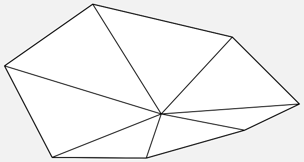
- positive local curvature



- locally flat



- negative local curvature



1 – Curvature…?

Local curvature:

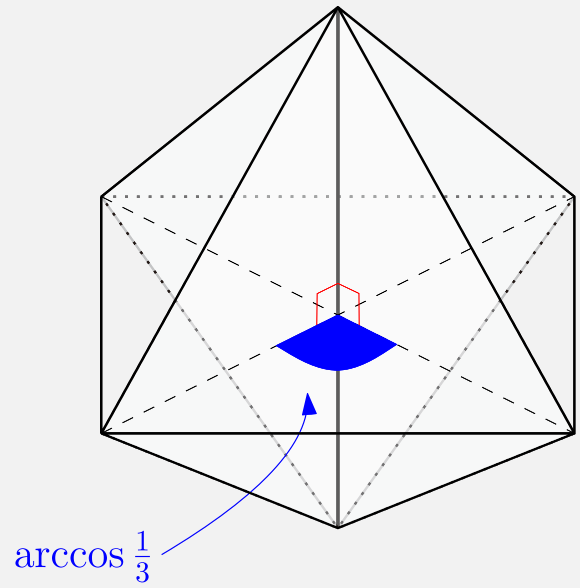
Number of *D*-simplices around (*D*-2)-simplices

Deficit angle in dimension D

$$2\pi - \mathcal{N}_D(v_{D-2}) \times \arccos\frac{1}{D}$$

Dimension *D=3*

Number of tetrahedra around edges :



1 – Curvature…?

Total curvature:

$$\alpha = \arccos \frac{1}{D}$$

$$\mathrm{Curv} \sim \sum_{v_{D-2}} \Big(2\pi - \alpha \, \mathcal{N}_D(v_{D-2}) \Big)$$

$$\sim 2\pi n_{D-2} - \alpha \, \frac{D(D+1)}{2} n_D$$

→ Computed from the number of (*D*-2)-simplices and *D*-simplices (n_{D-2} and n_D)

1 – Curvature…?

Total curvature:

$$\alpha = \arccos \frac{1}{D}$$

$$\mathrm{Curv} \sim \sum_{v_{D-2}} \Big(2\pi - \alpha \, \mathcal{N}_D(v_{D-2}) \Big)$$

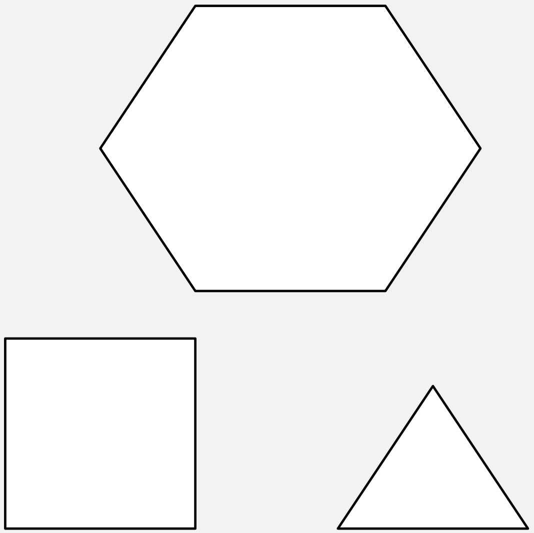
$$\sim 2\pi n_{D-2} - \alpha \, \frac{D(D+1)}{2} n_D$$

→ Computed from the number of (*D*-2)-simplices and *D*-simplices (n_{D-2} and n_D)

→ Maximize the curvature at fixed n_D $\longleftrightarrow$ Maximize n_{D-2} at fixed n_D

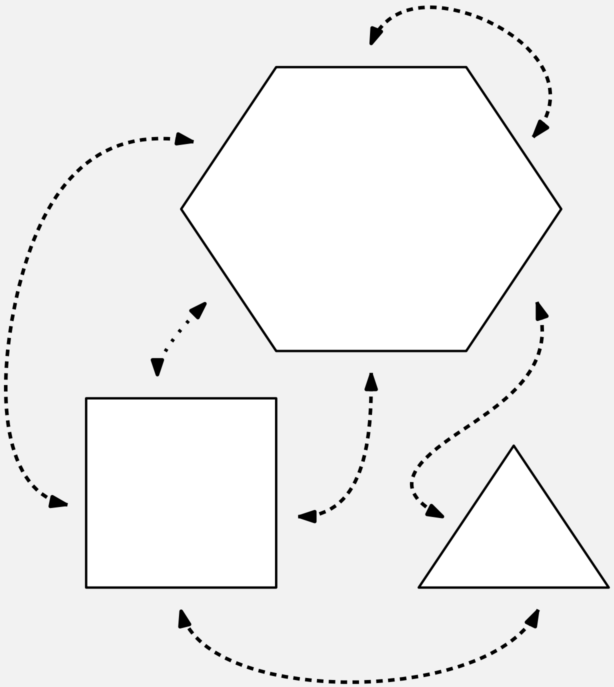
1 – Curvature…?

Dimension *D=2*: Bigger polygons???



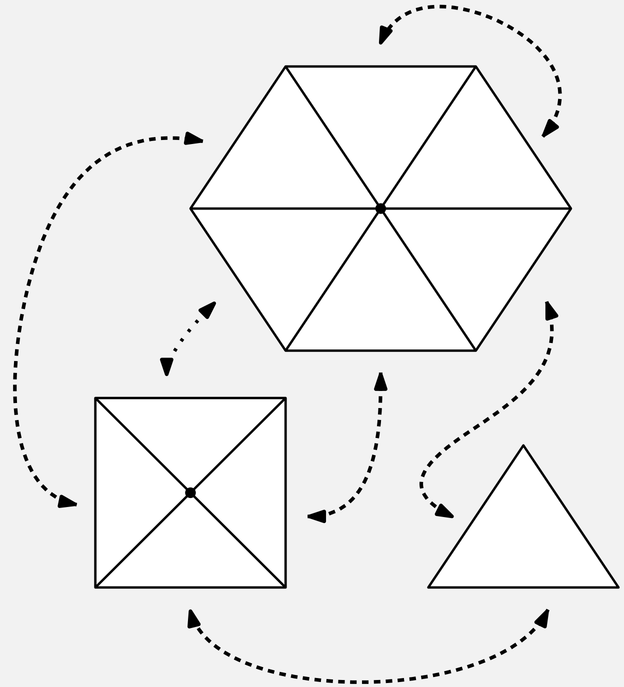
1 – Curvature…?

Dimension *D=2*: Bigger polygons???



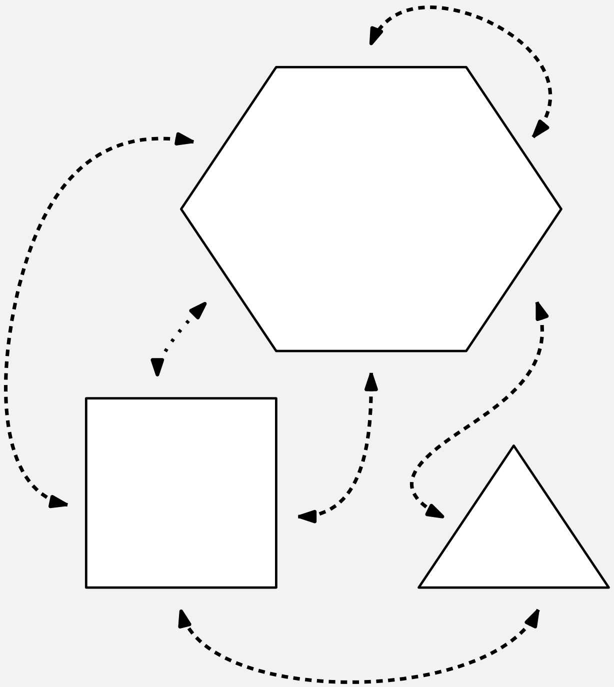
→

A possible choice (only depends on one length)

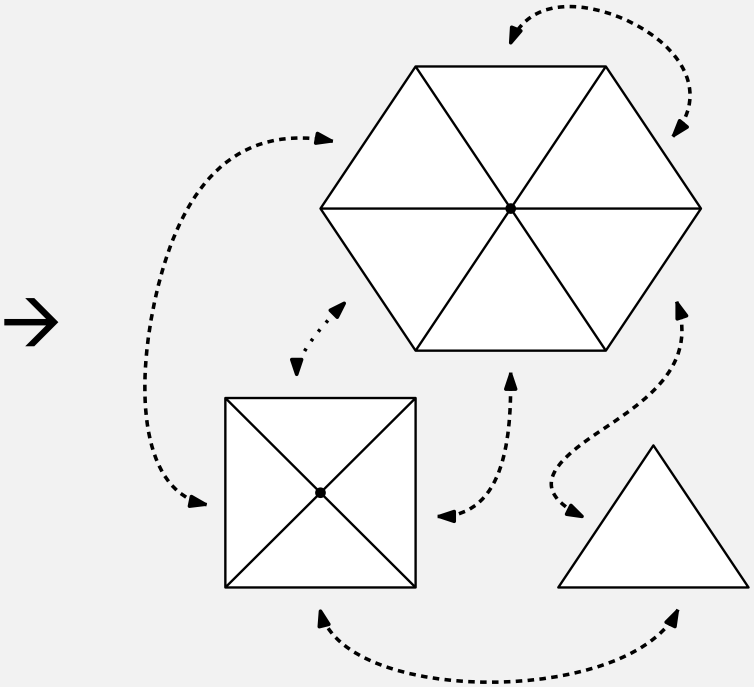


1 – Curvature…?

Dimension *D=2*: Bigger polygons???



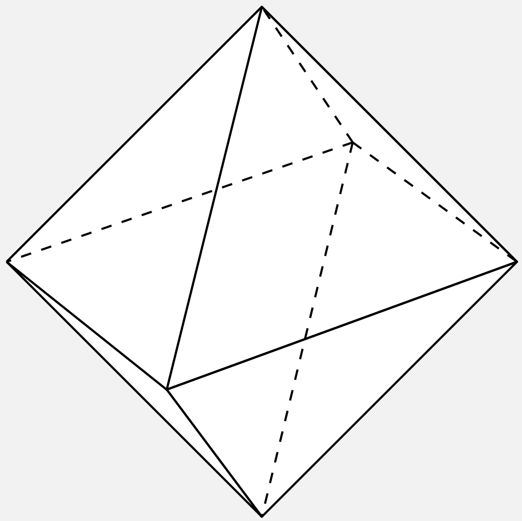
A possible choice (only depends on one length)



Other possible choice which works in what follows: 1 choice of angle for each polygon

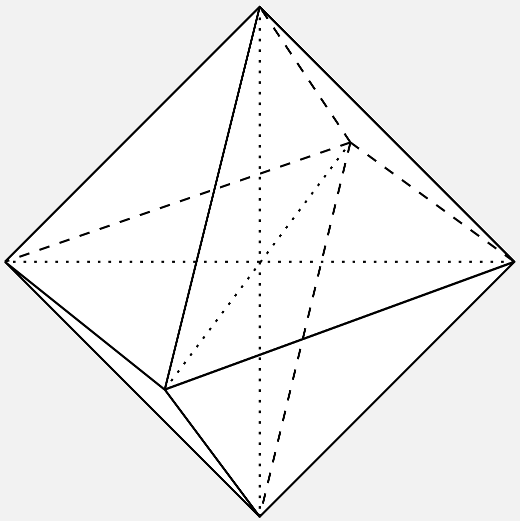
1 – Curvature…?

Dimension *D*: Bigger polygons???



→

A possible choice (only depends on one length): SAME



2 - Discrete quantum gravity:

Dynamical triangulations from bigger building blocks…

2 – Discrete quantum gravity

Einstein-Hilbert partition function for Euclidean pure gravity in dimension *D*

$$F = \int_{\mathcal{M}} D[g]\, e^{-\int d^D x \sqrt{|g|}\left(2\Lambda - \frac{1}{16\pi G} R\right)}$$

Manifold

Metric

Cosmological constant

Ricci scalar curvature

Newton's constant

→ (Euclidean) general relativity without matter from least action principle

2 – Discrete quantum gravity

Einstein-Hilbert partition function for Euclidean pure gravity in dimension *D*

$$F = \int_{\mathcal{M}} D[g] e^{-\int d^D x \sqrt{|g|}(2\Lambda - \frac{1}{16\pi G} R)}$$

- Complicated object
- Not well defined

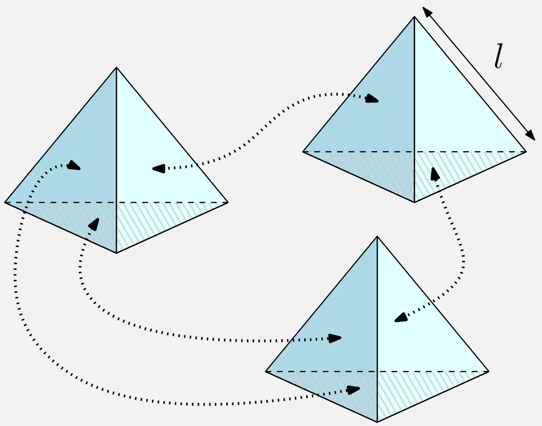
→ Try to make sense of it by considering a discrete analog

2 – Discrete quantum gravity

Einstein-Hilbert partition function for Euclidean pure gravity in dimension *D*

$$F = \int_{\mathcal{M}} D[g] e^{-\int d^D x \sqrt{|g|}(2\Lambda - \frac{1}{16\pi G} R)} \to \sum_{\substack{T \\ \text{connected} \\ \text{triangulation} \\ \text{of } \mathcal{M}}} \frac{1}{C_T} \, e^{-S_{\text{discrete}}}$$

Edges all have same length l
(canonical geometry)



(David, Ambjorn, Kazakov and many more… 80's)

1 – Discrete quantum gravity

Einstein-Hilbert partition function for Euclidean pure gravity in dimension *D*

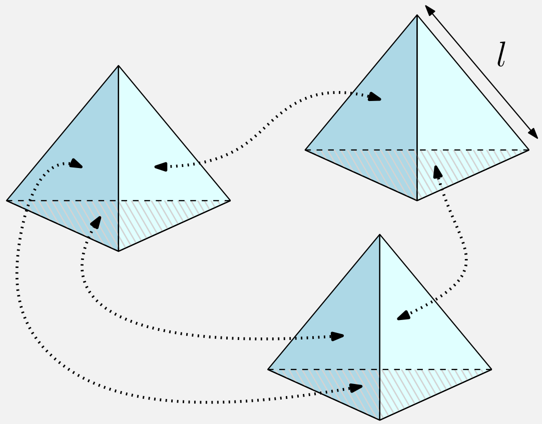
$$F = \int_{\mathcal{M}} D[g]\, e^{-\int d^D x \sqrt{|g|}\left(2\Lambda - \frac{1}{16\pi G} R\right)} \rightarrow \sum_{\substack{T \\ \text{connected} \\ \text{triangulation} \\ \cancel{\text{of } \mathcal{M}}}} \frac{1}{C_T}\, e^{-S_{\text{discrete}}}$$

Allow topology fluctuations at microscopic level

1 – Discrete quantum gravity

Einstein-Hilbert partition function for Euclidean pure gravity in dimension *D*

$$F = \int_{\mathcal{M}} D[g] e^{-\int d^D x \sqrt{|g|}(2\Lambda - \frac{1}{16\pi G} R)} \to \sum_{\substack{T \\ \text{connected} \\ \text{triangulation}}} \frac{1}{C_T} e^{-S_{\text{discrete}}}$$



1 – Discrete quantum gravity

Einstein-Hilbert partition function for Euclidean pure gravity in dimension *D*

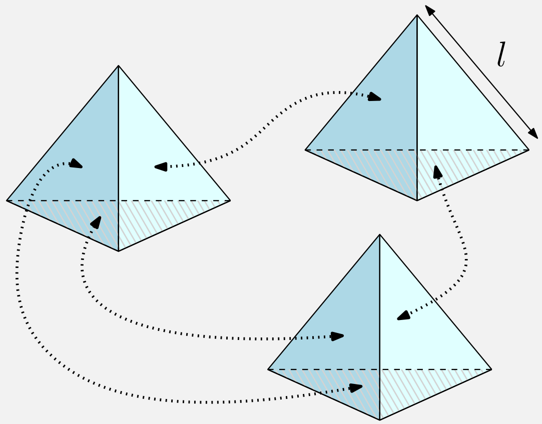
$$F = \int_{\mathcal{M}} D[g] e^{-\int d^D x \sqrt{|g|}(2\Lambda - \frac{1}{16\pi G} R)} \to \sum_{\substack{T \\ \text{connected} \\ \text{triangulation}}} \frac{1}{C_T} \, e^{-\kappa_D n_D} \; e^{\kappa_{D-2} n_{D-2}}$$

(Regge)

$$S_{\text{discrete}} \to \kappa_D \times n_D(T) - \kappa_{D-2} \times n_{D-2}(T)$$

of *D*-simplices

of (*D-2*)-simplices



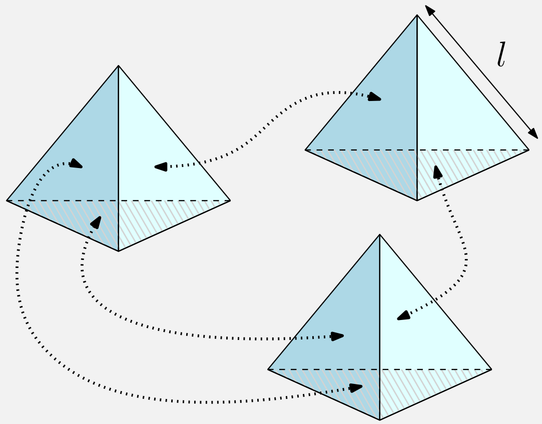
1 – Discrete quantum gravity

Einstein-Hilbert partition function for Euclidean pure gravity in dimension *D*

$$F = \int_{\mathcal{M}} D[g] e^{-\int d^D x \sqrt{|g|}(2\Lambda - \frac{1}{16\pi G} R)} \to \sum_{\substack{T \\ \text{connected} \\ \text{triangulation}}} \frac{1}{C_T} \; e^{-\kappa_D n_D} \; e^{\kappa_{D-2} n_{D-2}}$$

(Regge)

$$S_{\text{discrete}} \;\to\; \kappa_D \times n_D(T) - \kappa_{D-2} \times n_{D-2}(T)$$



$$\int_{\mathcal{M}} d^D x \sqrt{|g|} R \; \propto \; \text{Curvature}$$

Curvature ⟷ number of (*D*-2) and *D*-simplices

1 – Discrete quantum gravity

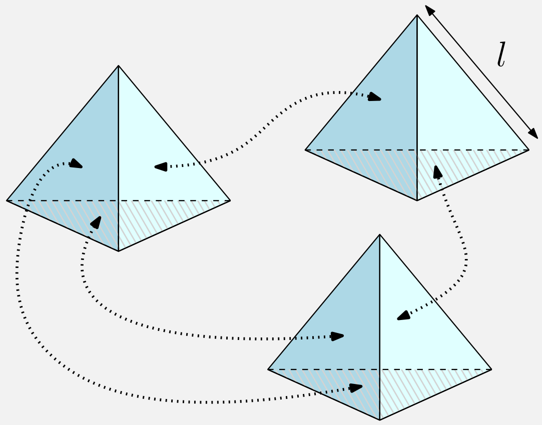
Einstein-Hilbert partition function for Euclidean pure gravity in dimension *D*

$$F = \int_{\mathcal{M}} D[g] e^{-\int d^D x \sqrt{|g|}(2\Lambda - \frac{1}{16\pi G} R)} \to \sum_{\substack{T \\ \text{connected} \\ \text{triangulation}}} \frac{1}{C_T}\, e^{-\kappa_D n_D}\, e^{\kappa_{D-2} n_{D-2}}$$

(Regge)

$$S_{\text{discrete}} \to \kappa_D \times n_D(T) - \kappa_{D-2} \times n_{D-2}(T)$$

$$\boxed{\kappa_{D-2} \propto \frac{1}{G} >> 1}$$



Quantum gravity? Analogy with thermodynamics…

Microscopic description of gaz	**Microscopic description of space-time**
Accessible state S	Triangulation S_T
$n(S)$ particles	$n_D(S_T)$ *D*-simplices
Thermodynamical limit $n \to +\infty$, $\mathrm{Dist} \to 0$	Continuum limit $n_D \to +\infty$, $l \to 0$
Grand-canonical partition function $$\sum_{\text{states } S} e^{\mu n(S)}\, e^{-\beta \mathcal{E}(S)}$$	Discrete Einstein-Hilbert partition function $$\sum_{\substack{S_T \\ \text{connected} \\ \text{triangulation}}} e^{-\kappa_D n_D(S_T)}\, e^{\kappa_{D-2} n_{D-2}(S_T)}$$
Inverse temperature $\beta = \frac{1}{k_B T}$	Inverse of Newton constant $\kappa_{D-2} \propto \frac{1}{G}$
Chemical potential μ	$-\kappa_D?$
Energy $\mathcal{E}(S)$	$-n_{D-2}?$

Quantum gravity? Analogy with thermodynamics…

Microscopic description of gaz	Microscopic description of space-time
Accessible state S	Triangulation S_T
$n(S)$ particles	$n_D(S_T)$ *D*-simplices
Thermodynamical limit $n \to +\infty$, $\text{Dist} \to 0$	Continuum limit $n_D \to +\infty$, $l \to 0$
Grand-canonical partition function $$\sum_{\text{states } S} e^{\mu n(S)}\, e^{-\beta \mathcal{E}(S)}$$	Discrete Einstein-Hilbert partition function $$\sum_{\substack{S_T \\ \text{connected} \\ \text{triangulation}}} e^{-\kappa_D n_D(S_T)}\, e^{\kappa_{D-2} n_{D-2}(S_T)}$$
Inverse temperature $\beta = \frac{1}{k_B T}$	Inverse of Newton constant $\kappa_{D-2} \propto \frac{1}{G}$
Chemical potential μ	$-\kappa_D?$
Energy $\mathcal{E}(S)$	$\cancel{-n_{D-2}}?$ Not bounded from below…

Quantum gravity? Analogy with thermodynamics...

Microscopic description of gaz	Microscopic description of space-time
Accessible state S	Triangulation S_T
$n(S)$ particles	$n_D(S_T)$ *D*-simplices
Thermodynamical limit $n \to +\infty$, $\mathrm{Dist} \to 0$	Continuum limit $n_D \to +\infty$, $l \to 0$
Grand-canonical partition function $$\sum_{\text{states } S} e^{\mu n(S)} \; e^{-\beta \mathcal{E}(S)}$$	Discrete Einstein-Hilbert partition function $$\sum_{\substack{S_T \\ \text{connected} \\ \text{triangulation}}} e^{(a\kappa_{D-2}-\kappa_D)n_D} \; e^{-\kappa_{D-2}(an_D-n_{D-2})}$$
Inverse temperature $\beta = \frac{1}{k_B T}$	Inverse of Newton constant $\kappa_{D-2} \propto \frac{1}{G}$
Chemical potential μ	$a\kappa_{D-2} - \kappa_D$
Energy $\mathcal{E}(S)$	$an_D - n_{D-2}$

a ??

2 – Discrete quantum gravity

Discrete Einstein-Hilbert partition function

$$\lambda = e^{a\kappa_{D-2} - \kappa_D}$$

$$N = e^{-\beta} = e^{\kappa_{D-2}}$$

$$F(\lambda, N) = \sum_{\substack{\text{connected}\\ \text{triangulations}}} \frac{1}{C}\, \lambda^{n_D}\, N^{-(a n_D - n_{D-2})}$$

① Large *N* limit (Physical limit of small Newton constant) :

a has to be chosen such that $a n_D - n_{D-2}$ bounded from below

2 – Discrete quantum gravity

Discrete Einstein-Hilbert partition function

$$\lambda = e^{a\kappa_{D-2}-\kappa_D}$$

$$N = e^{-\beta} = e^{\kappa_{D-2}}$$

$$F(\lambda, N) = \sum_{\substack{\text{connected}\\\text{triangulations}}} \frac{1}{C}\, \lambda^{n_D}\, N^{-(an_D - n_{D-2})}$$

① Large *N* limit (Physical limit of small Newton constant) :

a has to be chosen such that $an_D - n_{D-2}$ bounded from below

→ well-defined *1/N*-expansion:

$$\frac{1}{N^k}F(\lambda, N) = F_0(\lambda) + \frac{1}{N}F_1(\lambda) + \frac{1}{N^2}F_2(\lambda) + \cdots$$

"Generating function" of connected triangulations that minimize $an_D - n_{D-2}$

(*k = D*?)

2 – Discrete quantum gravity

Discrete Einstein-Hilbert partition function

$$\lambda = e^{a\kappa_{D-2}-\kappa_D}$$

$$N = e^{-\beta} = e^{\kappa_{D-2}}$$

$$F(\lambda, N) = \sum_{\substack{\text{connected}\\ \text{triangulations}}} \frac{1}{C} \lambda^{n_D} N^{-(an_D - n_{D-2})}$$

① Large *N* limit (Physical limit of small Newton constant) :

a has to be chosen such that $an_D - n_{D-2}$ bounded from below

② Continuum limit $l \to 0$ → should be non-trivial

a has to be chosen such that infinitely many spaces minimize $an_D - n_{D-2}$

2 – Discrete quantum gravity

Discrete Einstein-Hilbert partition function

$$\lambda = e^{a\kappa_{D-2}-\kappa_D}$$

$$N = e^{-\beta} = e^{\kappa_{D-2}}$$

$$F(\lambda, N) = \sum_{\substack{\text{connected} \\ \text{triangulations}}} \frac{1}{C}\, \lambda^{n_D}\, N^{-(a n_D - n_{D-2})}$$

① Large *N* limit (Physical limit of small Newton constant) :

a has to be chosen such that $a n_D - n_{D-2}$ bounded from below

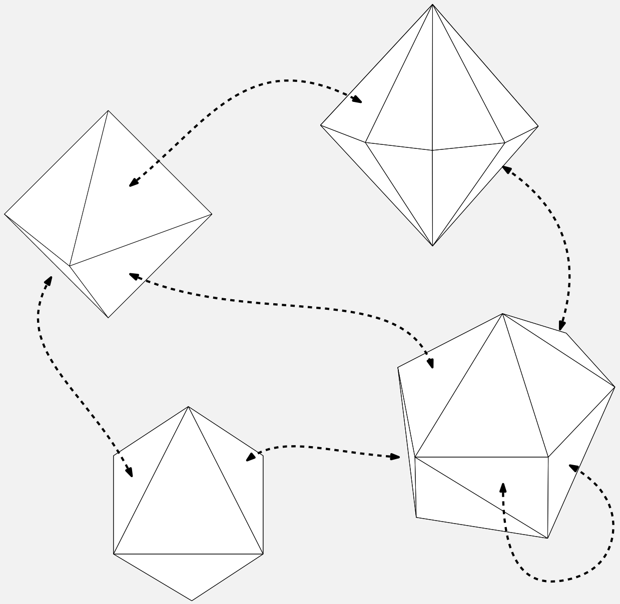
② Continuum limit $l \to 0$ → should be non-trivial

a has to be chosen such that infinitely many spaces minimize $a n_D - n_{D-2}$

Continuum limit <-> singularity of F_0 → asymptotics of F_0 (string susceptibility…)

More general setting: glue any kind of building blocks

- Can we find a satisfying conditions ① and ② ?
- What do we recover in the large *N* limit?



→ *Identify triangulations which maximize* n_{D-2} *at fixed* n_D
(= maximize the curvature at fixed n_D = minimize the energy)

If they behave as $an_D - n_{D-2} = -k$, then we can choose this a.

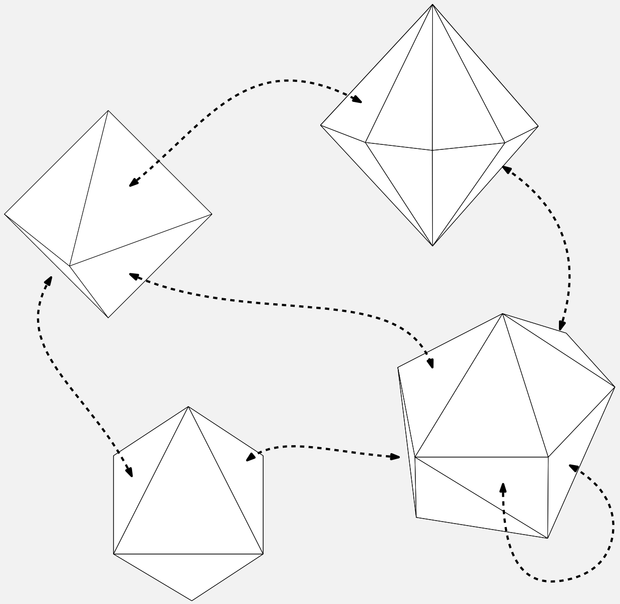
→ *Enumerate the corresponding triangulations*
(to obtain the large N correlation functions)

- What do we recover in the continuum limit?

→ *Properties of large such triangulations?*
What are the Hausdorff dimension, fractal dimension…?

More general setting: glue any kind of building blocks

- Can we find a satisfying conditions ① and ② ?
- What do we recover in the large *N* limit?



→ *Identify triangulations which maximize* n_{D-2} *at fixed* n_D
(= maximize the curvature at fixed n_D = minimize the energy)

If they behave as $an_D - n_{D-2} = -k$, then we can choose this a.

→ *Enumerate the corresponding triangulations*
(to obtain the large N correlation functions)

→ The combinatorial problem of the introduction!!!

- What do we recover in the continuum limit?

→ *Properties of large such triangulations?*
What are the Hausdorff dimension, fractal dimension…?

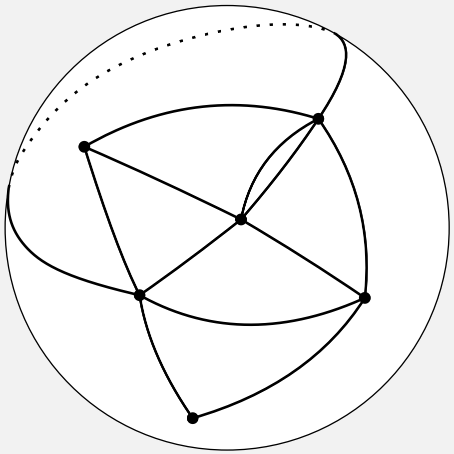
3 – Previously known results: D=2

3 – Previously known results: D=2

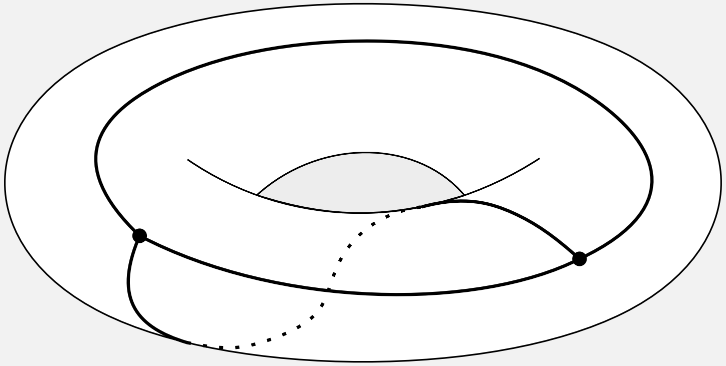
In dimension *D=2*, gluings of polygons are combinatorial maps (=ribbon graphs)

Combinatorial maps are discrete *D=2* surfaces

The curvature of a map only depends on its genus! (Genus = number of holes)



Discrete sphere

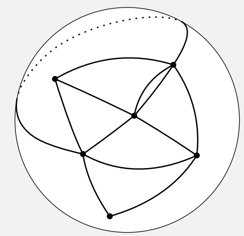


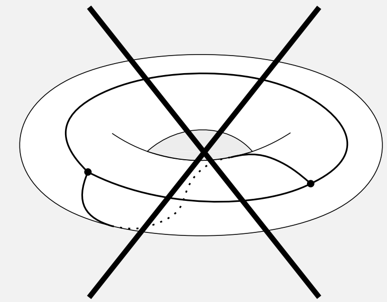
Discrete torus

→ The enumeration of maps of a given genus is a very active domain of research since the 60's (Tutte, Bender, Canfield, and so many more …)

3 – Previously known results: D=2

→ $a = 1/2$





→ Large *N* limit selects all discrete spheres

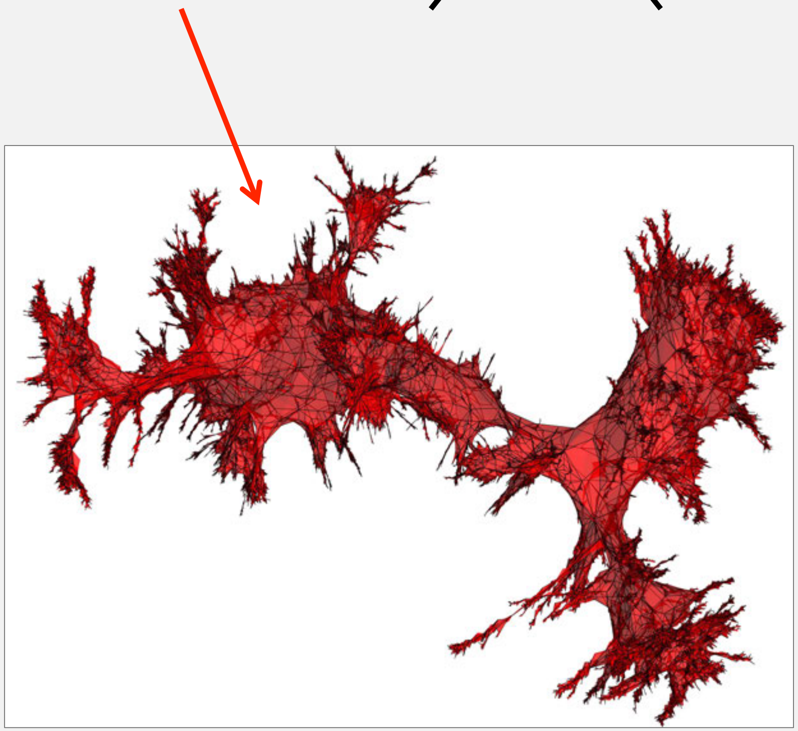


Fig : J. Bettinelli

→ Continuum limit is the ***Brownian sphere***,

A random continuous metric space with Hausdorff dimension 4 (Marckert, LeGall, Miermont… 2006…)

Equivalent to *Liouville D=2 quantum Gravity*

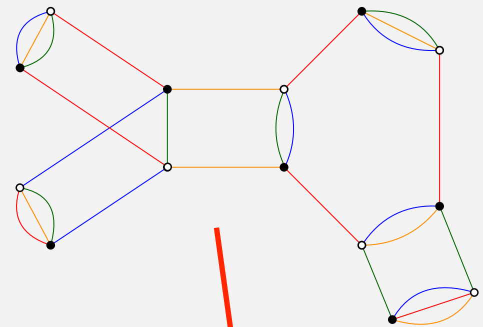
(Conjectured by physicists in 80's, 90's…

…math. proof by Miller & Sheffield 2016…)

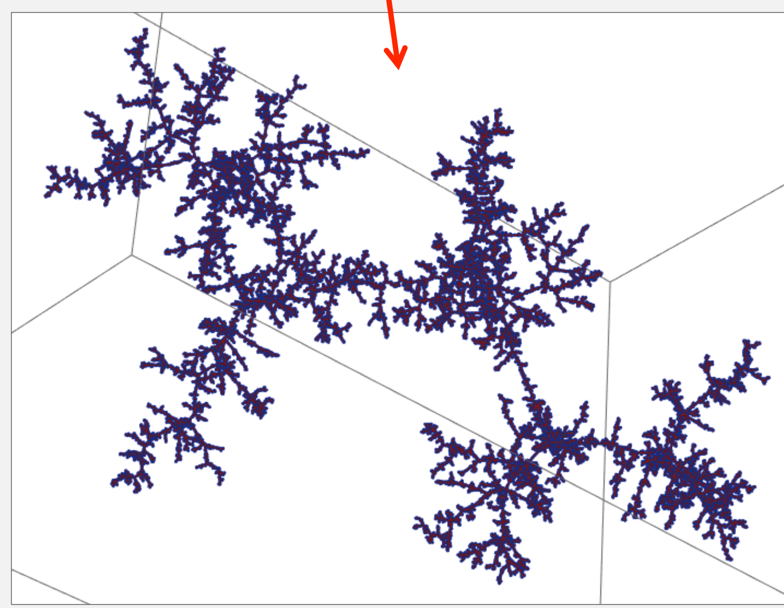
3 – Previously known results: colored triangulations in any D

3 – Previously known results: triangulations in any D

→ $a = D(D-1)/4$



→ Large *N* limit selects melonic triangulations



→ Continuum limit is ***branched polymers*,**

A random continuous tree with Hausdorff dimension 2

(…Aldous… 1990…Gurau & Ryan 2014)

…Disappointing limit from the geometric point of vue …

4 – Some recent results

4 – Some recent results

Dimension 3

Octahedra → *Branched polymers* (Bonzom, L.L. 2016) $a = 11/8$

Simplest torus → "*Branched polymers*" (Bonzom, L.L., Rivasseau 2015, L.L., Thürigen 2017) $a = 1$

All colored-triangulated spheres (balls) → "*Branched polymers*" (Bonzom 2018) (a is known)

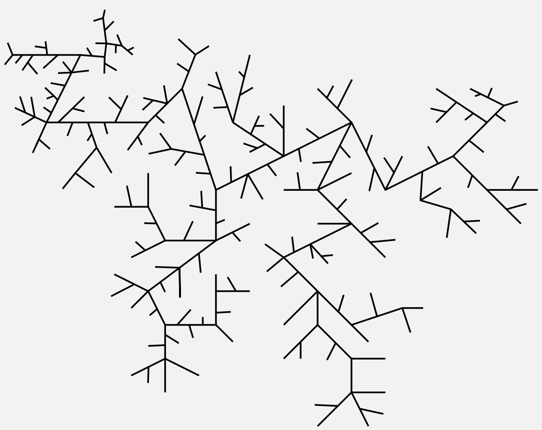
…To be continued, but quite disappointing…

4 – Some recent results

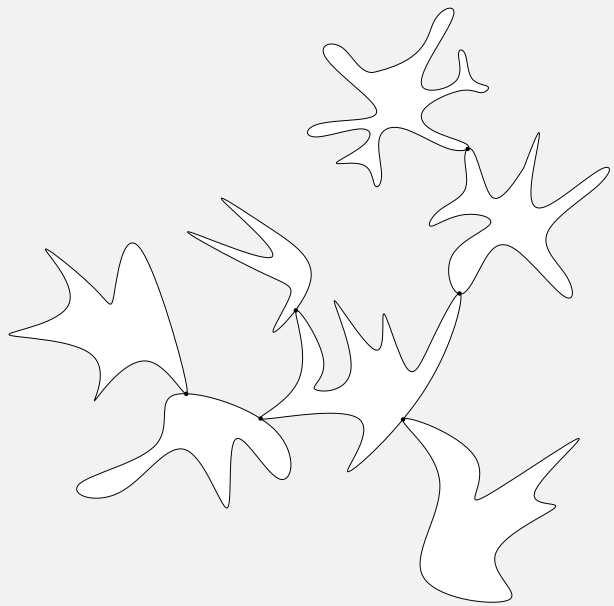
Dimension 4

Building blocks of size 4 → **3 critical regimes** (Bonzom, Delepouve, Rivasseau 2015) $a = 3/2$

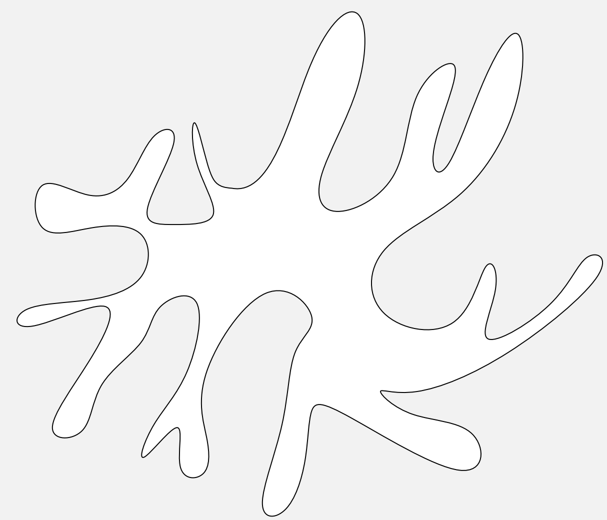
Building blocks of size 6 → **same 3 critical regimes** (L.L., Thürigen 2017) (a is known)



"Branched polymers"
(= continuum tree)



"Proliferation of baby universes"
→ *Cacti of BS* ? $\gamma = 1/3$



"2D quantum gravity"
(= Brownian sphere BS)

→ In *D=2*, the critical behavior of large *N* surfaces does not depend on the discretization of the boundary, it is universal (*2D quantum gravity*).

→ In *D=3*, as far as we know, it also seems universal (*branched polymers*).

→ In *D=4*, the critical behavior of maximal curvature configurations is NOT universal…

… it depends on the building block… Need to keep exploring!

Can we find new critical regimes this way??

Can we find suitable Brownian continuum volumes *some other way*?

(see ongoing work in the last slides)

5 – Random tensor models

5 – Random tensor models

Introduced in early 90’s by: Ambjorn *et al*, Sasakura, Gross

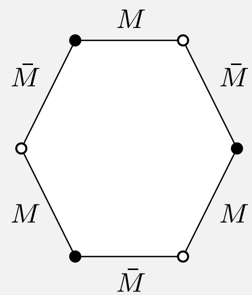
1/N expansion and melonic graphs in 2010-13: Gurau, Rivasseau, Bonzom, Riello, Ryan …

Recent developments presented in this talk: Rivasseau, Bonzom, Delepouve, L.L., Thürigen

Matrix models

Interaction:

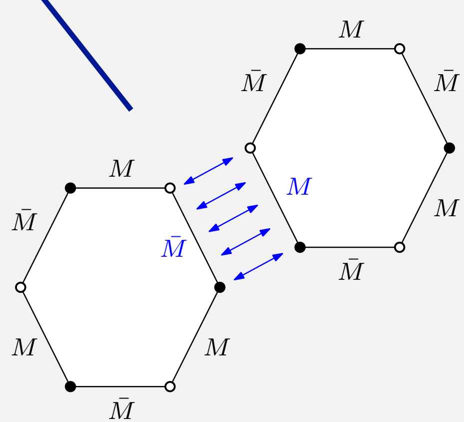
$$\mathrm{Tr}\big((MM^\dagger)^3\big) \longleftrightarrow$$

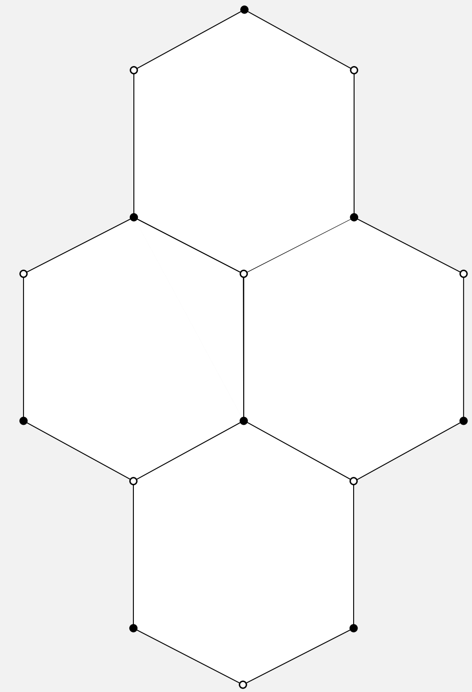


Partition function:

$$Z = \int e^{-N\,\mathrm{Tr}\left[MM^\dagger - \lambda (MM^\dagger)^3\right]} dM dM^\dagger$$

$$= \left[e^{-\frac{1}{N}\,\mathrm{Tr}\frac{\partial}{\partial M}\frac{\partial}{\partial M^\dagger}} e^{\lambda N \mathrm{Tr}(MM^\dagger)^3} \right]_{M=0}$$





→ Gluings of hexagons!

Matrix models

<u>***1/N* expansion of 2 point function:**</u>

$$G_p(N,\lambda) = \sum_{g\geq 0} N^{2-2g}\mathcal{G}_{p,g}(\lambda)$$

where the $\mathcal{G}_{p,g}$ are generating functions of connected rooted gluings of *p*-gons of genus *g*

$$\mathcal{G}_{p,g}(\lambda) = \sum_{n\geq 0} c_{p,g,n}\lambda^n$$

the coefficients $c_{p,g,n}$ being the number of rooted surfaces of genus *g* made of *n* *p*-gons

→ Use matrix models to count surfaces!

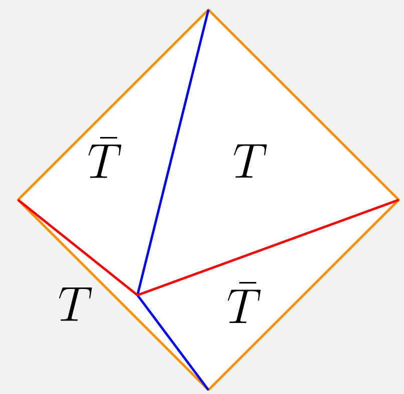
A few names (among so many more): ’t Hooft, Kazakov, David, Itzykson, Zuber, Ginsparg, Di Francesco, Guiter, Bouttier, Eynard…

5 – Random tensor models

Interaction:

Invariant under $U(N)^D \Rightarrow$ Specific colored structure

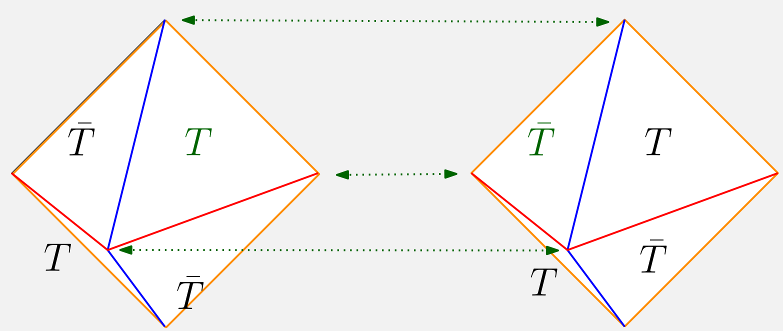
$$\mathrm{Tr}_{octa}(T,\bar{T}) \longleftrightarrow$$



Partition function:

$$Z_{\mathcal{B}} = \int e^{-N^{D-1}\left[T.\bar{T} - \lambda N^{s}\mathrm{Tr}_{\mathcal{B}}(T,\bar{T})\right]} dT d\bar{T}$$

Find the correct value!!
(at most a unique rational… L.L. 2018)



5 – Random tensor models

<u>*1/N* expansion of 2 point function:</u>

$$G_{\mathcal{B}}(\lambda, N) = \sum_{\substack{G \in \mathbb{G}(\mathcal{B}) \\ \text{connected} \\ \text{rooted}}} \lambda^{n_D} \; N^{n_{D-2} - a n_D}$$

Gluings of building blocks $\mathcal{B}$

Where:

$$a = (D-1)\Big(\frac{|\mathcal{B}|}{2} - 1\Big) - s$$

Find the right s $\longleftrightarrow$ Find the right a

…see the discrete QG discussion in early slides

5 – Random tensor models

<u>*1/N* expansion of 2 point function:</u>

$$G_{\mathcal{B}}(\lambda, N) = \sum_{\substack{G \in \mathbb{G}(\mathcal{B}) \\ \text{connected} \\ \text{rooted}}} \lambda^{n_D} \; N^{n_{D-2} - a n_D}$$

Gluings of building blocks $\mathcal{B}$

Where:

$$a = (D-1)\Big(\frac{|\mathcal{B}|}{2} - 1\Big) - s$$

Find the right s $\longleftrightarrow$ Find the right a

…see the discrete QG discussion in early slides

If s is well chosen, we have a well-defined *1/N* expansion, with infinitely many terms per (non-empty) order.

As for matrix models, the tensor models count gluings of building blocks, according to some well chosen generalization of the genus.

→ The conclusions from last section also apply for tensor models!

Conclusions

Conclusions

- Recent exact results identify 3 universality classes for some simple Euclidean dynamical triangulations / random tensor models

  → escaped the branched polymer phase in DT

  → new classes from Euclidean DT? Not excluded but still open question

- We can identify and count exactly the large *N* spaces for many building blocks in D=3, very few in D=4 + continuum limit (?)

  We can identify and count the large spaces contributing at any order in 1/N for triangulations and for a few others (=double scaling)

- These combinatorial techniques apply to the identification of graphs contributing to the SYK model (and SYK-like tensor models), for which we can also identify the graphs contributing at any order…

Conclusions

Very different needs!!

Large N graphs

Discrete QG models:

Need higher dimensional random geometry to emerge at large N in the continuum (NOT branched polymers)

… Currently unknown (and hard to find…)

SYK-like models:

Need solvability at large N

→ Tree-like graphs

→ Branched polymers in the continuum

… The large majority of models!!

Some ongoing work

1 – Random tensor models.
(e.g… Description at any order for all the tree-like theories (SYK-like). [with S. Dartois])

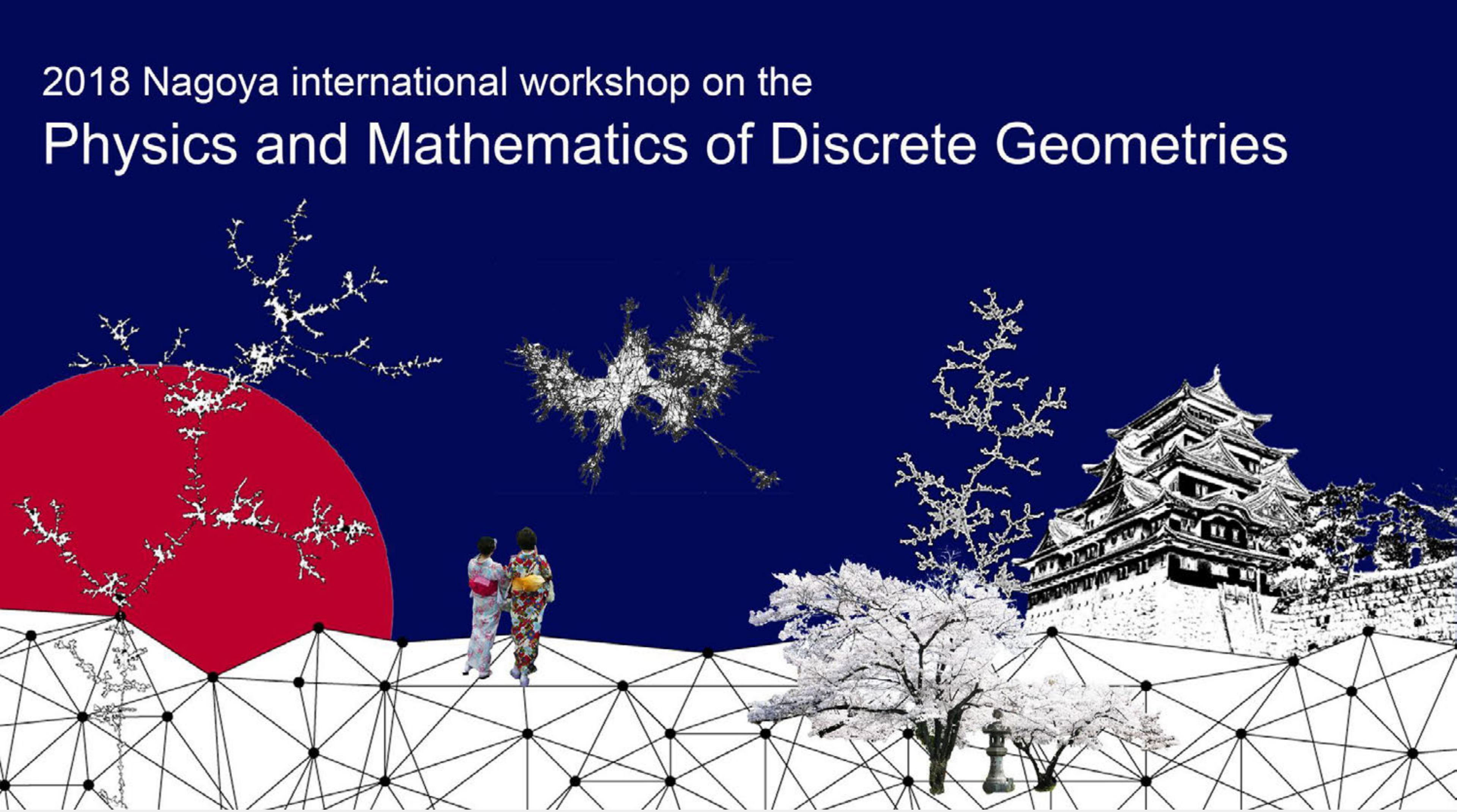
2 –New continuum limits (Brownian volumes) from more direct approaches?
[one project with S. Dartois, another one with JF. Marckert]

3 – Enumeration and statistical properties of graphs contributing to the (colored) SYK model at any order in *1/N* [with E. Fusy & A. Tanasa]

4 – Methods apply to ``quantum information'' problems (probability that a multipartite state is entangled) [with S. Dartois & I. Nechita 1808.08554]

5 – Non-linear differential equations involved in turbulences, with random initial conditions and coefficients [with S. Dartois & V. Rivasseau & O. Evnin & G. Valette]

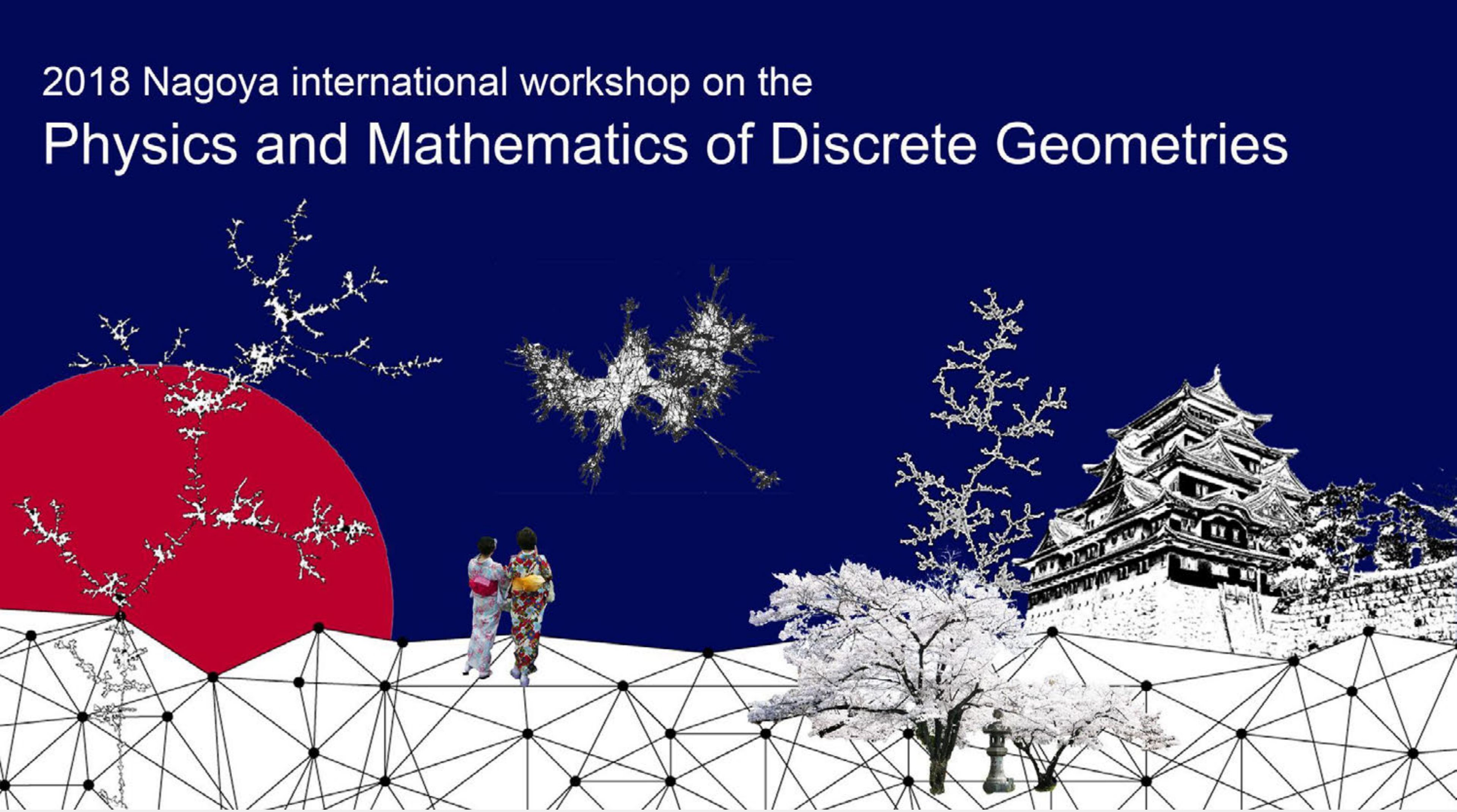
5 – Study of the properties of the wave function of the canonical tensor model
[with N. Sasakura]



November 5-9 // Nagoya University

Organization with Yuki Sato

https://discrete-nagoya.sciencesconf.org



November 5-9 // Nagoya University

Organization with Yuki Sato

https://discrete-nagoya.sciencesconf.org

Thank you for your attention!