Non-Abelian T-duality and the AdS/CFT correspondence (I)

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1. Introduction & motivation: NATD in AdS/CFT

Non-Abelian T-duality (NATD) has proved to be very useful as a solution generating technique in AdS/CFT

Many interesting AdS backgrounds (some of which evading existing classifications) have been constructed

What NATD does to the CFT remains less understood

Examples so far show that NATD may change the CFT dual to the AdS backgrounds in which it is applied

This may happen because, contrary to its Abelian counterpart, NATD has not been proved as a String Theory symmetry

In this talk we will discuss different examples for which the CFT realization of NATD has been studied

In all of them the NATD background will be associated to QFT living on (Dp,NS5) Hanany-Witten brane set-ups

$\rightarrow$ Novel way of describing (Dp,NS5) CFTs holographically

These examples have been studied in:

- Y.L., Carlos Núñez, 1603.04440
- Y.L., Niall Macpherson, Jesús Montero, Carlos Núñez, 1609.09061
- Georgios Itsios, Y.L., Jesús Montero, Carlos Núñez, 1705.09661
- Y.L., Niall Macpherson, Jesús Montero, 1810.08093

Outline:

1. Introduction and motivation: NATD in AdS/CFT
2. Basics of NATD: i) NATD vs Abelian T-duality
   ii) NATD as a solution generating technique
3. The NATD of $AdS_5 \times S^5$
4. The NATD of $AdS_5 \times S^5$ as a Gaiotto-Maldacena geometry
5. The NATD of Klebanov-Witten
6. Conclusions

2. Basics of NATD: i) NATD vs Abelian T-duality

Using the string sigma-model Rocek and Verlinde proved that Abelian T-duality is a symmetry to all orders in g_s and α'

(Buscher'88; Rocek, Verlinde'92)

The extension to arbitrary wordsheets determines the global properties of the dual variable:

$$\theta \in [0, 2\pi] \quad \xrightarrow{\text{T}} \quad \tilde{\theta} \in [0, 2\pi]$$

In the non-Abelian case neither proof works

Variables living in a group manifold are substituted by variables living in its Lie algebra

$$g \in SU(2) \quad \xrightarrow{\text{NAT}} \quad \chi \in \mathbb{R}^3$$

In the absence of global information the new variables remain non-compact

In more detail:

Rocek and Verlinde's formulation of Abelian T-duality for ST in a curved background (Buscher'88) :

$$\boxed{S = \frac{1}{4\pi\alpha'}\int \Big(g_{\mu\nu}\, dX^\mu \wedge *dX^\nu + B_{\mu\nu}\, dX^\mu \wedge dX^\nu\Big) + \frac{1}{4\pi}\int R^{(2)}\phi}$$

i) Identify an Abelian isometry: $X^\mu = \{\theta, X^\alpha\}$ such that

$\theta \to \theta + \epsilon$ and $\partial_\theta(\text{backgrounds}) = 0$

ii) Gauge the isometry: $d\theta \to D\theta = d\theta + A$

A non-dynamical gauge field / $\delta A = -d\epsilon$

iii) Add a Lagrange multiplier term: $\tilde{\theta}\, dA$, such that

$$\int \mathcal{D}\tilde{\theta} \;\rightarrow\; dA = 0 \;\Rightarrow\; A \text{ exact}$$

(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \rightarrow$ Original theory

iv) Integrate the gauge field

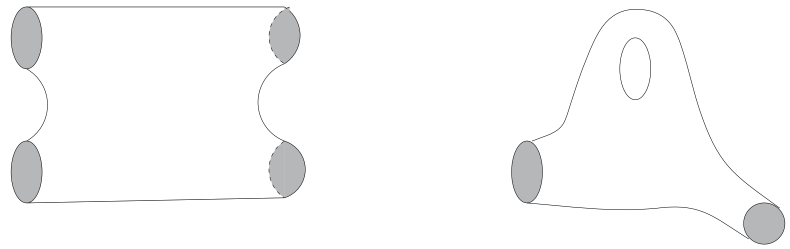
+ fix the gauge: $\theta = 0 \quad \rightarrow$ Dual sigma model:

$\{\theta, X^\alpha\} \rightarrow \{\tilde{\theta}, X^\alpha\}$ and

$(\tilde{g}, \tilde{B}_2, \tilde{\phi})$ given by Buscher's formulae

- Conformal invariance? Original and dual theories can be obtained from the gauged Lagrangian either gauging a vectorial or an axial combination of chiral currents

- Arbitrary worldsheets? (symmetry of string perturbation theory):



(a) (b)

Large gauge transformations: $$\oint_\gamma d\epsilon = 2\pi n\,;\;\; n \in \mathbb{Z}$$

To have invariance:

Multivalued Lagrange multiplier: $$\oint_\gamma d\tilde{\theta} = 2\pi m\,;\;\; m \in \mathbb{Z}$$
such that

$$\int [\text{exact}] \;\rightarrow\; dA = 0 \qquad + \qquad \int [\text{harmonic}] \Rightarrow A \text{ exact}$$

This fixes the periodicity of the dual variable

Non-Abelian T-duality

(De la Ossa, Quevedo'93)

Non-Abelian isometry: $X^m \to g^m_{\ n} X^n \,, g \in G$

i) Gauge it: $dX^m \to DX^m = dX^m + A^m_{\ n} X^n$

$A \in$ Lie algebra of G $\qquad A \to g(A+d)g^{-1}$

ii) Add a Lagrange multiplier term: $\mathrm{Tr}(\chi F)$

$F = dA - A \wedge A$

$\chi \in$ Lie Algebra of G, $\quad \chi \to g\chi g^{-1}$, such that

$\int \mathcal{D}\chi \;\to\; F = 0 \;\Rightarrow\; A$ pure gauge
(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \;\Rightarrow$ Original theory

iii) Integrate the gauge field + fix the gauge $\rightarrow$ Dual theory

However:

- Non-involutive
- Higher genus generalization? Set to zero $W_\gamma = P\, e^{\oint_\gamma A}$
- Global properties?

  For SU(2): $\chi \in \mathbb{R}^3$: Global completion of $\mathbb{R}^3$?
- Conformal invariance not proved in general

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True symmetry in String Theory?

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Some examples:

- New AdS_6 background in IIB

  (Y.L., O Colgain, Rodriguez-Gomez, Sfetsos, PRL (2013))

  Candidate for the holographical description of a 5d fixed point theory

  Motivated classifications of IIB AdS_6 solutions that culminated with the work of D'Hoker, Gutperle, (Karch,) Uhlemann'16,17

- New AdS_3 M-theory solution beyond existing classifications
  Only explicit example in KKK with SU(2) structure

  (Y.L., Macpherson, Montero, O Colgain'15)

ii) NATD as a solution generating technique

Need to know how the RR fields transform

Sfetsos and Thompson (2010) extended Hassan's derivation in the Abelian case:

Implement the relative twist between left and right movers in the bispinor formed by the RR fields

But how do we interpret the non-compact directions in the context of AdS/CFT?

Can we learn something about their global properties from Holography?

3. The NATD of $AdS_5 \times S^5$

$$\begin{tikzcd} AdS_5 \times S^5 \arrow[d, leftrightarrow] \arrow[r, leftrightarrow, "\text{NATD}"] & \text{N=2} \quad AdS_5 \times S^2 \in \text{GM} \arrow[d, leftrightarrow] \\ \text{N=4 SYM} & \text{N=2 SCFT} \end{tikzcd}$$

(Sfetsos, Thompson'10)

(Y.L., Nunez, '16)

- Gaiotto-Maldacena geometries encode the information about the dual CFT
- Useful example to study the CFT realization of NATD

• Take the $AdS_5 \times S^5$ background

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2 + \cos^2\alpha ds^2(S^3)\Big)$$

$$F_5 = 8L^4 \sin\alpha \cos^3\alpha d\alpha \wedge d\beta \wedge \mathrm{Vol}(S^3)$$ + Hodge dual

• Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = ds^2_{AdS_5} + L^2\Big(d\alpha^2 + \sin^2\alpha d\beta^2\Big) + \frac{d\rho^2}{L^2\cos^2\alpha} + \frac{L^2\cos^2\alpha\rho^2}{\rho^2 + L^4\cos^4\alpha} ds^2(S^2)$$

$$B_2 = \frac{\rho^3}{\rho^2 + L^4\cos^4\alpha}\mathrm{Vol}(S^2)\,, \qquad e^{-2\phi} = L^2\cos^2\alpha(L^4\cos^4\alpha + \rho^2)$$

$$F_2 = L^4 \sin\alpha\cos^3\alpha d\alpha \wedge d\beta, \qquad F_4 = B_2 \wedge F_2$$

- New Gaiotto-Maldacena geometry
- What about r ?
  - Background perfectly smooth for all $r \in \mathbb{R}^+$
  - No global properties inferred from the NATD
  - How do we interpret the running of r to infinity in the CFT?
- Singular at $\alpha = \pi/2$ where the original S^3 shrinks (due to the presence of NS5-branes)

This is the tip of a cone with S^2 boundary $\rightarrow$

Large gauge transformations $B_2 \rightarrow B_2 - n\pi \mathrm{Vol}(S^2)$
for $r \in [n\pi, (n+1)\pi]$

This modifies the Page charges such that $N_4 = nN_6$ in each $[n\pi, (n+1)\pi]$ interval

$$\boxed{\begin{array}{lcl} N_4 & \longleftrightarrow & \text{D4-branes on } \mathbb{R}^{1,3}, r \\ N_6 & \longleftrightarrow & \text{D6-branes on } \mathbb{R}^{1,3}, r, S^2 \end{array}}$$

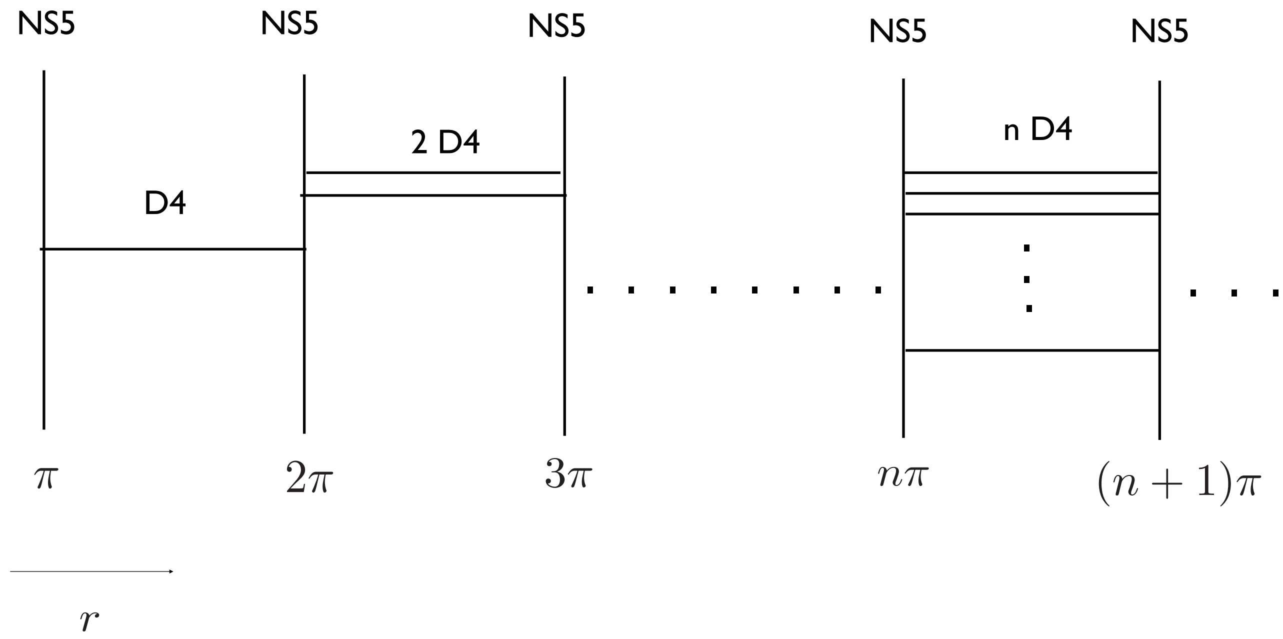
We have also N_5 charge $\quad N_5 = \frac{1}{(2\pi)^2}\int H_3 = \frac{1}{\pi}\int_0^{r_*} dr$

such that every time we cross a π interval one unit of NS5 charge is created. For $r \in [0, n\pi]$, $N_5 = n$.

The picture that arises is that D3 $\rightarrow$ (D4, NS5) which seems to be a general pattern

This is consistent with a D4/NS5 brane set-up:

D4: $\mathbb{R}^{1,3}, r$
NS5: $\mathbb{R}^{1,3}, \alpha, \beta$



(in units of N_6)

These D4/NS5 brane set-ups realize 4d $\mathcal{N}=2$ field theories with gauge groups connected by bifundamentals (Witten'97)

Having the D4 finite extension in the r direction, the field theory living in them is 4d at low energies, with effective gauge coupling:

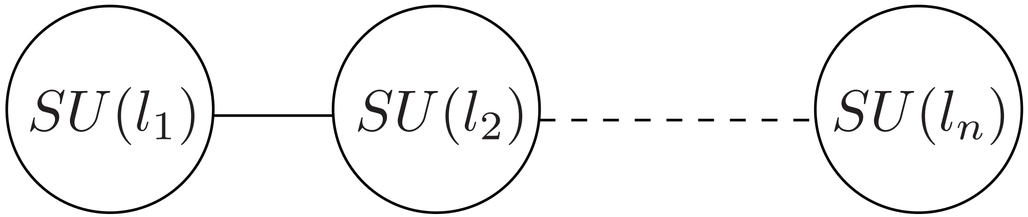
$$\frac{1}{g_4^2} \sim r_{n+1} - r_n$$

Open strings connecting D4's stretched between different NS5 represent bifundamental matter

Strings connecting the D4 to D6 or semi-infinite D4 represent fundamental matter

For l_n D4-branes in $[r_n, r_{n+1}]$ the gauge group is $SU(l_n)$ and there are (l_{n-1}, l_n) and (l_n, l_{n+1}) hypermultiplets.

The field theory is then described by a quiver



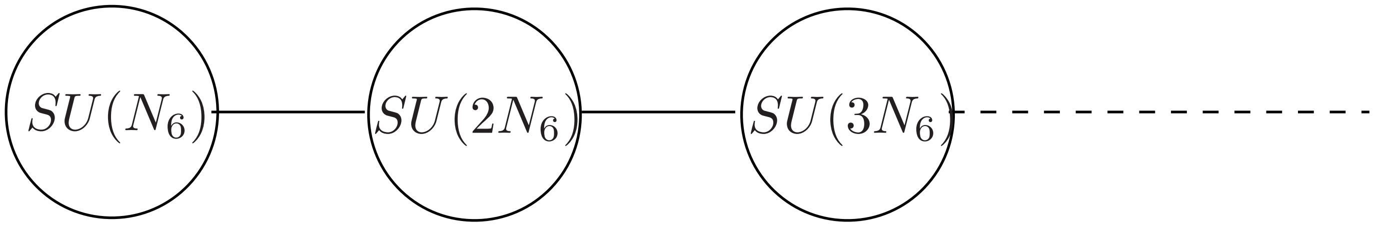
The bifundamentals contribute to the $SU(l_n)$ beta function as $l_{n-1} + l_{n+1}$ flavors.

The beta function thus vanishes at each interval if

$$2l_n = l_{n+1} + l_{n-1}$$

This condition is satisfied by our brane configuration, which has $l_n = nN_6$

It corresponds to an infinite linear quiver:



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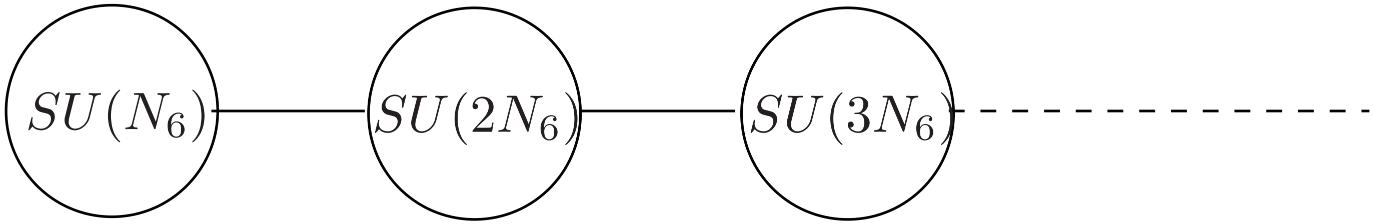


Gaiotto-Maldacena geometries were built to study the CFTs associated to these brane set-ups (Gaiotto, Maldacena'09)

We will now see that, as a GM geometry, the dual quiver associated to the NATD solution is the same above

This brane intersection has also been recently confirmed by the analysis in Terrise, Tsimpis, Whiting'18

4. The NATD as a Gaiotto-Maldacena geometry

i) Gaiotto-Maldacena geometries:

Generic backgrounds dual to 4d N=2 SCFTs.

Described in terms of a function $V(\sigma,\eta)$ solving a Laplace eq. with a given charge density $\lambda(\eta)$ at $\sigma=0$

$$\partial_\sigma[\sigma\partial_\sigma V]+\sigma\partial_\eta^2 V=0, \qquad \lambda(\eta)=\sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0}$$

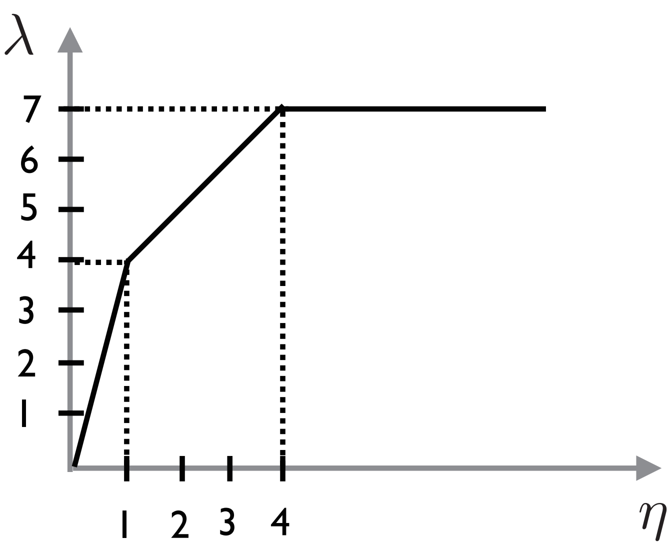
Regularity and quantization of charges impose strong constraints on the allowed form of $\lambda(\eta)$, which encodes the information of the dual CFT:

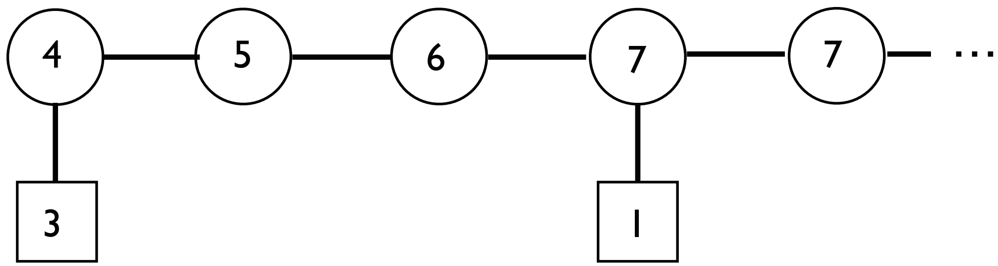
- It must vanish at $\eta=0$
- Its slope must be an integer, always decrease, and it can only change at integer values of η
- Each time the slope changes by k units there is an A_{k-1} singularity

ii) $\mathcal{N}=2$ CFTs from Gaiotto-Maldacena geometries:

- A $SU(n_i)$ gauge group is associated to each integer value of $\eta=\eta_i$, with n_i given by $\lambda(\eta_i)=n_i$
- A kink in the line profile corresponds to extra k_i fundamentals attached to the gauge group at the node n_i

For example:





iii) The NATD as a Gaiotto-Maldacena geometry:

The NATD background is a Gaiotto-Maldacena geometry with

$$\lambda(\eta) = \eta\,, \quad \eta \sim r\,, \; \sigma = \sin\alpha$$

$\lambda(\eta) = \eta \Rightarrow$ Infinite linear quiver, consistent with the brane set-up

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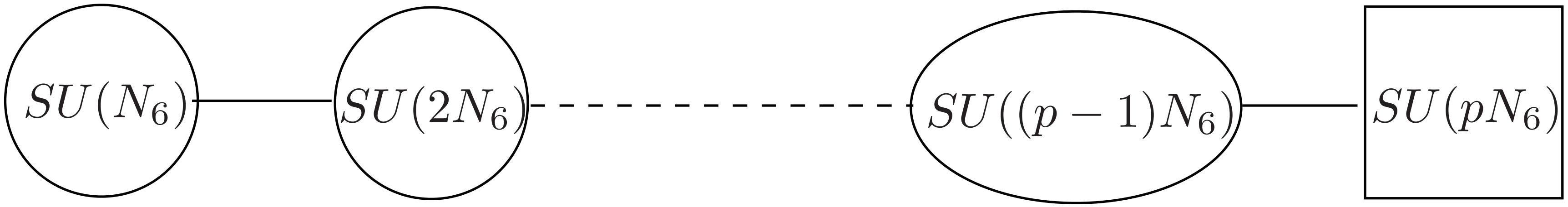
However, the associated CFT is *infinite*.

We will see next that we can complete the quiver to produce a well-defined 4d CFT, and, using holography, complete the geometry

A natural way to complete the quiver is by adding fundamentals:



This completion reproduces correctly the value of the holographic central charge:

From the geometry:

$$c_{NATD} \sim \int_0^{\eta_*} f(\eta) d\eta = \frac{N_6^2 N_5^3}{12}$$

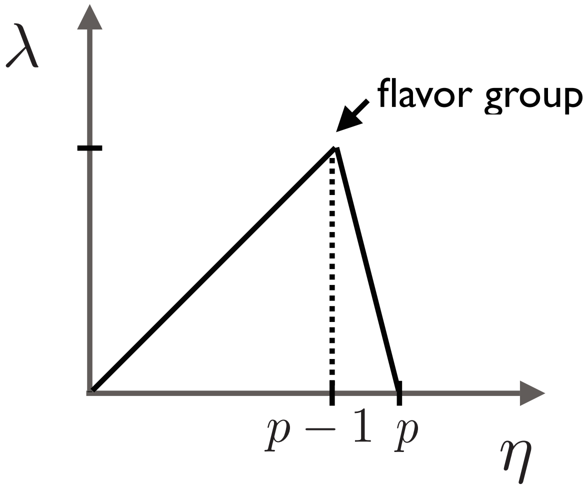
(Klebanov, Kutasov, Murugan'08)

In the field theory we can use: $c = \frac{1}{12}(2n_v + n_h)$ (Shapere, Tachikawa'08)

This gives

$$c = \frac{N_6^2 p^3}{12}\left[1 - \frac{1}{p} - \frac{2}{p^2 N_6^2} + \frac{2}{N_6^2 p^3}\right] \approx \frac{N_6^2 p^3}{12}$$

In the geometry, the completed quiver corresponds to



$$\frac{\lambda(\eta)}{N_6} = \begin{cases} \eta & 0 \leq \eta \leq p-1 \\ (1-p)\eta + (p^2 - p) & (p-1) \leq \eta \leq p \end{cases}$$

The solution to the Laplace equation that gives rise to this charge density *completes* the non-Abelian T-dual geometry, and resolves its singularity

Close to the kink the behaviour of the completed geometry is that of D6-branes

The NATD solution arises when zooming-in away from the kink

This idea also works in other examples

5. The NATD of Klebanov-Witten

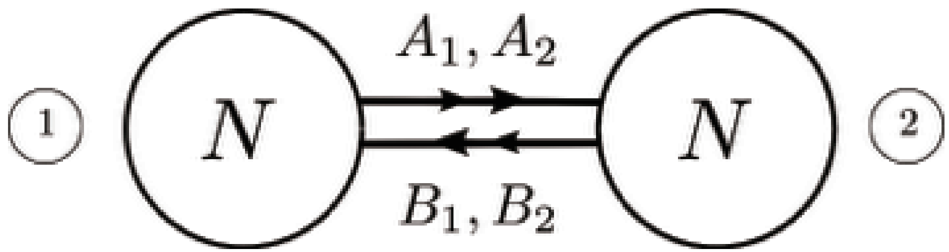
$$\begin{tikzcd}
AdS_5 \times T^{1,1} \arrow[d, leftrightarrow] \arrow[r, leftrightarrow, "\text{NATD}"] & \text{N=1 } AdS_5 \in \text{GMSW} \arrow[d, leftrightarrow] \\
\text{N=1 KW} & \text{N=1 SCFT}
\end{tikzcd}$$

(Itsios, Nunez, Sfetsos, Thompson'13) (Itsios, Y.L., Montero, Nunez, '17)

- GMSW geometries do not encode the information about the dual CFT
- Still, we can extract useful information using the relation with GM

The Klebanov-Witten theory:

$\mathcal{N}=1$ $SU(N)\times SU(N)$ gauge theory with bifundamental matter fields transforming in the $(N,\bar{N})$ and $(\bar{N},N)$ representations of $SU(N)$



Dual to $AdS_5\times T^{1,1}$:

$$ds^2_{T^{1,1}}=\frac{1}{6}(ds^2(S^2_1)+ds^2(S^2_2))+\frac{1}{9}(d\psi+\cos\theta_1 d\phi_1+\cos\theta_2 d\phi_2)^2$$

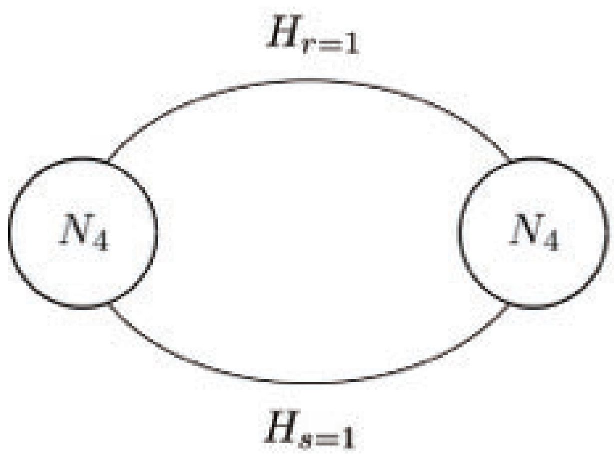
$$F_5=\frac{4}{L}\Big(\mathrm{Vol}(AdS_5)-L^5\mathrm{Vol}(T^{1,1})\Big)$$

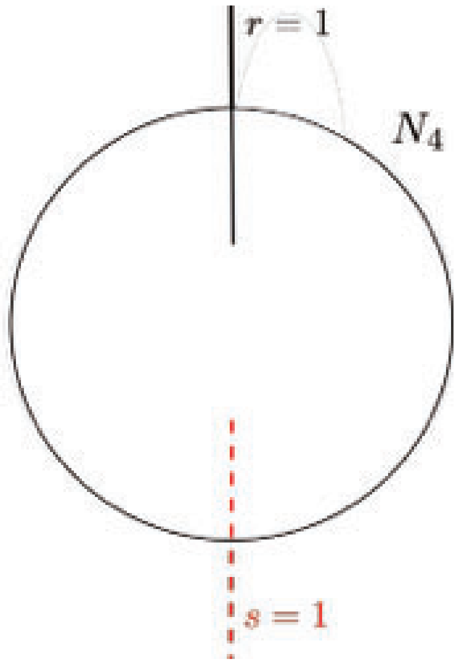
Abelian T-dual realization: (Dasgupta, Mukhi'98; Uranga'98)

IIA background with B_2, F_4 fluxes $\leftrightarrow$ NS5, D4 branes

Two types of, orthogonal, NS5-branes

Mutual rotation equivalent to a mass deformation in the CFT breaking $\mathcal{N}=2$ to $\mathcal{N}=1$



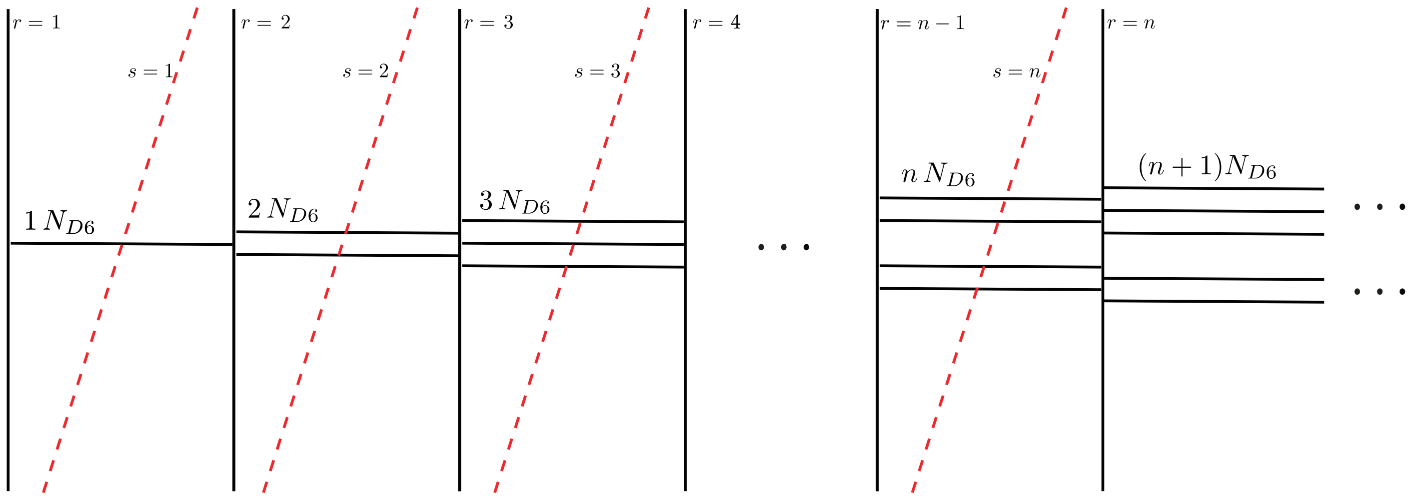


The non-Abelian T-dual solution

IIA background with $B_2\,, F_2\,, F_4 = B_2 \wedge F_2$ fluxes

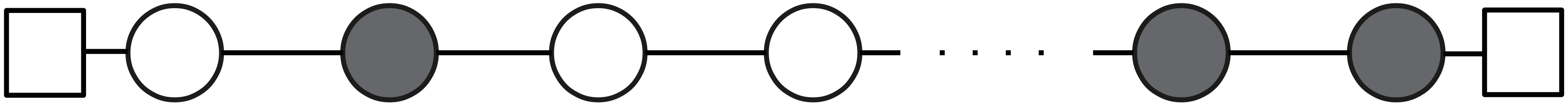
Two types of, orthogonal, NS5 and NS5' branes

Brane set-up:



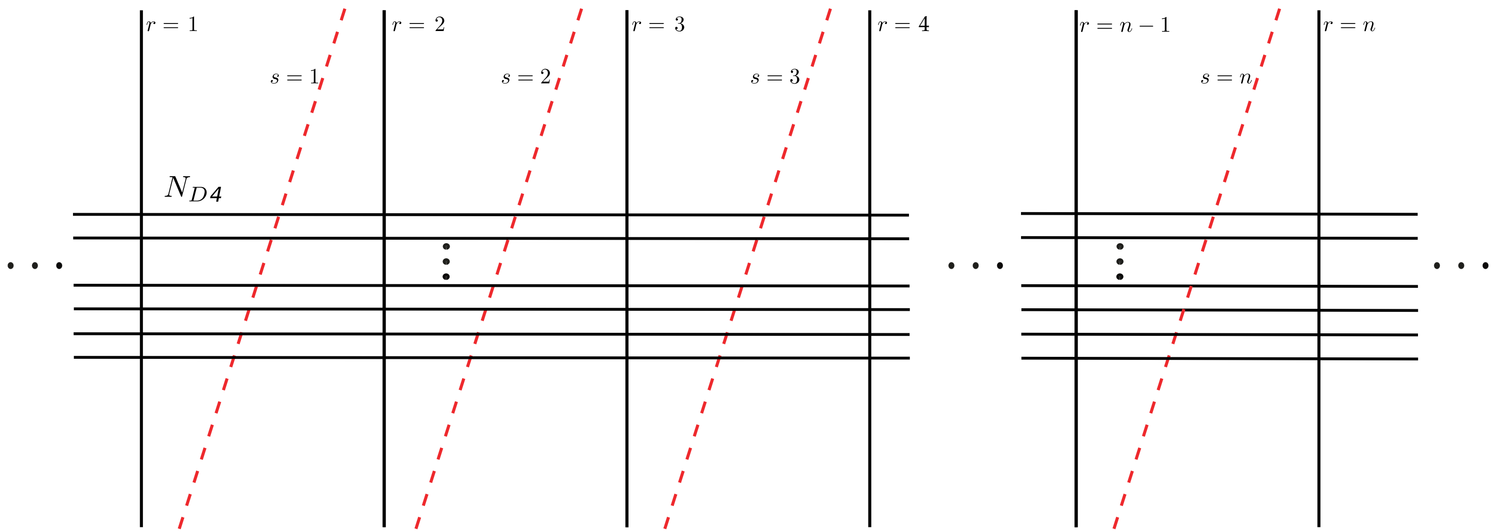
It generalizes the brane set-ups describing the linear quiver gauge theories of Bah-Bobev (Bah, Bobev'13)

Bah-Bobev linear quivers: (Bah, Bobev'13)



$\mathcal{N}=1$ and $\mathcal{N}=2$ $SU(N)$ vector multiplets plus bifundamental matter fields

Realized in brane set-ups with orthogonal NS5 and NS5' branes:



Flow to interacting 4d $\mathcal{N}=1$ CFT in the IR

In the CFT fixed point the central charges can be determined from the 't Hooft anomalies associated to the R-symmetry (Anselmi, Freedman, Grisaru, Johansen'97) :

$$a = \frac{3}{32}(3\mathrm{Tr}\, R^3 - \mathrm{Tr}\, R) \ , \qquad c = \frac{1}{32}(9\mathrm{Tr}\, R^3 - 5\mathrm{Tr}\, R)$$

where R is the R-symmetry, $R = R_0 + \frac{1}{2}\epsilon\mathcal{F}$, determined by a-maximization (Intriligator, Wecht'03)

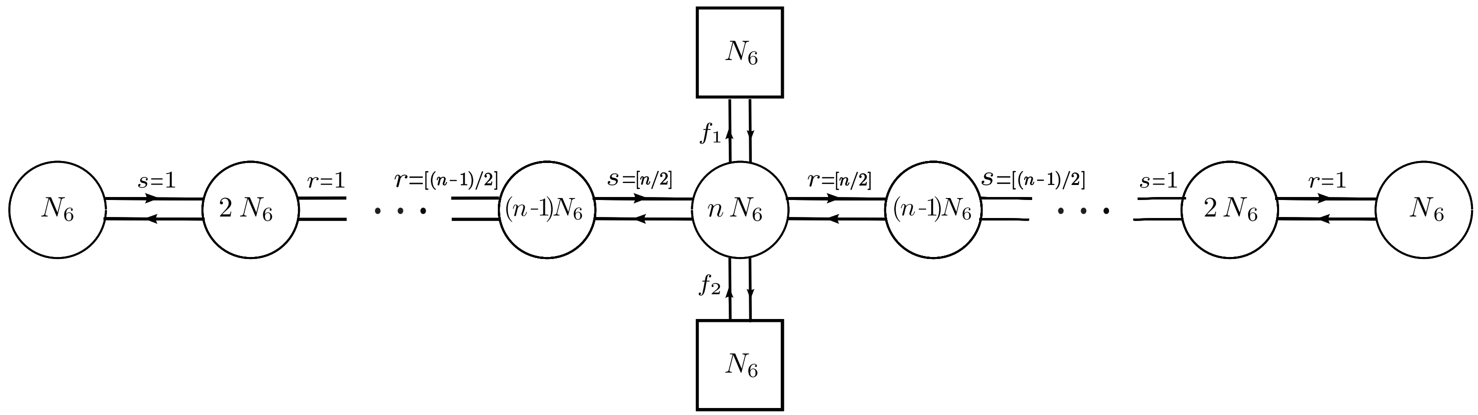
CFT dual to the NATD solution

Infinite linear quiver $\rightarrow$ Complete it, such that:

- Vanishing beta functions and R-symmetry anomalies
- Self-dual under $N_c \to N_f - N_c$
- Mass deformation of the $\mathcal{N}=2$ quiver dual to the NATD of $AdS_5 \times S^5/\mathbb{Z}_2$:

$$\begin{tikzcd} AdS_5 \times S^5/\mathbb{Z}_2 \arrow[r] \arrow[d, "\text{mass}"] & \text{NATD of } AdS_5 \times S^5/\mathbb{Z}_2 \arrow[d, "\text{mass}"] \\ AdS_5 \times T^{1,1} \arrow[r] & \text{NATD of } AdS_5 \times T^{1,1} \end{tikzcd}$$

Our proposed *completed* quiver is:



It is obtained by modding out by $\mathbb{Z}_2$ the $\mathcal{N}=2$ *completed* quiver dual to the NATD of $AdS_5 \times S^5$ and mass deforming it

A non-trivial check is that the central charges satisfy the Tachikawa-Wecht UV/IR relations:

$$a_{\mathcal{N}=1} = \frac{9}{32}(4\,a_{\mathcal{N}=2} - c_{\mathcal{N}=2})\,, \qquad c_{\mathcal{N}=1} = \frac{1}{32}(-12\,a_{\mathcal{N}=2} + 39\,c_{\mathcal{N}=2})$$

(Tachikawa, Wecht'09)

- In the large number of nodes limit: $c_{\mathcal{N}=1} \sim a_{\mathcal{N}=1}$ and $c_{\mathcal{N}=1} = \frac{27}{32} c_{\mathcal{N}=2}$ (Tachikawa, Wecht'09)

- Moreover, $c_{\mathcal{N}=1} \sim a_{\mathcal{N}=1} \sim \frac{9}{64} N_5^3 N_6^2$ matches the holographic result: $c_{NATD} \sim V_{int} \sim \frac{9}{64} N_5^3 N_6^2$

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Open problem: Completion of the NATD solution

Need to introduce flavor D6-branes in the geometry

(Bah'15; Apruzzi, Fazzi, Passias, Tomasiello'15)

The NATD would arise as a result of zooming-in in a particular patch of the completed geometry

6. Conclusions (so far)

- NATD useful as a solution generating technique in AdS/CFT:

  New Gaiotto-Maldacena geometry

  New solution in the class of GMSW

- Significant progress in the CFT interpretation:

  Gaiotto-Maldacena geometry dual to an infinite linear quiver

  GMSW geometry dual to a mass deformation of the field theory dual to $AdS_5 \times S^5/\mathbb{Z}_2$

  Non-trivial check: Tachikawa-Wecht UV/IR relations

$\rightarrow$ Different CFTs after NATD

D3 $\rightarrow$ (D4, NS5)

D3 $\rightarrow$ (D4, NS5, NS5')

AdS solutions constructed through non-Abelian T-duality need to be completed in order to describe well-defined CFTs

In the example discussed the NATD arises away from the kink

- General pattern? :

$AdS_6 \times S^4$: (D4,D8) system $\rightarrow$ (D5,NS5,D7)

$AdS_4 \times S^3 \times S^2$: (D2,D6) system $\rightarrow$ (D3,NS5,D5)

Connection with (Dp,NS5) brane set-ups

7. The NATD of $AdS_6 \times S^4$

The AdS_6/CFT_5 correspondence remains much less understood than its cousins in other dimensions

This is largely due to the fact that in 5d there are no maximally supersymmetric CFTs

Indeed, in 5d the superconformal algebra is unique: $F(4)$
It contains 8 supersymmetries (enhanced to a total of 16)

Accordingly, AdS_6 backgrounds are quite unique:

There is a unique AdS_6 solution to massive Type IIA (the Brandhuber-Oz solution (Brandhuber, Oz'99)), there are no solutions in M-theory, and a kind of no-go theorem existed in Type IIB (Passias'12)

AdS_6/CFT_5 duals

5d gauge theories are non-renormalizable:

$$[g^2] = M^{-1} \quad \rightarrow \quad g^2 E \quad \rightarrow \quad \text{UV completion}$$

Indeed, they can flow to strongly coupled CFTs in the UV for specific gauge groups and matter content

(Seiberg'96; Intriligator, Morrison, Seiberg'97)

String theory realizations are known in some cases that can be used to construct their AdS_6 duals

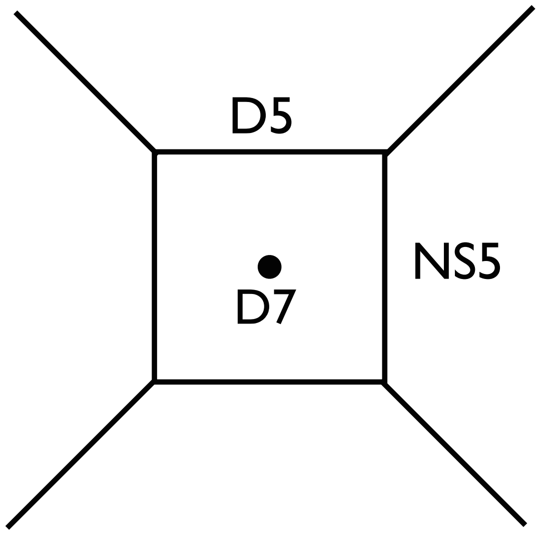
For example, $Sp(N)$ (with specific matter content) can be realized in Type I' in a D4/D8/O8 system (Seiberg'96)

The Brandhuber-Oz solution arises as its near horizon geometry

More general string theory realizations can be given in terms of (p,q) 5-brane webs in Type IIB (Aharony, Hanany'97; Aharony, Hanany, Kol'97)

The 5d field theory lives on D5-branes which are of finite extent on one dimension and are embedded in a web of semi-infinite (p,q) 5-branes as well as optional 7-branes (DeWolfe, Hanany, Iqbal, Katz'99)

These webs are 5d realizations of Hanany-Witten brane set-ups



Their AdS_6 duals remained unknown till the recent work of

D'Hoker, Gutperle, Karch, Uhlemann'16; D'Hoker, Gutperle, Uhlemann'17,18

These works provide a complete classification of AdS_6 solutions to Type IIB SUGRA, which are firm candidates for holographic duals of CFTs living in (p,q) 5-brane webs.

T-duality played a very important role in these developments:

The first AdS_6 solutions in Type IIB were constructed by acting with T-duality (Abelian and non-Abelian) on the Brandhuber-Oz solution

(Y.L., O Colgain, Rodriguez-Gomez, Sfetsos'12; Y.L., O Colgain, Rodriguez-Gomez'13)

The existence of these solutions raised the interest in the study of classifications of AdS6 solutions in Type IIB:

Apruzzi, Fazzi, Passias, Rosa, Tomasiello'14; Kim, Kim, Suh'15; Kim, Kim'16; Gutowski, Papadopoulos'17,

which culminated with the work of

D'Hoker, Gutperle, Karch, Uhlemann'16; D'Hoker, Gutperle, Uhlemann'17,18

The Abelian T-dual solution describes the same 5d $Sp(N)$ fixed point theory as the Brandhuber-Oz solution but now in terms of a D5, NS5, D7/O7 brane system in Type IIB

In Y.L., Macpherson, Montero: 1810.08093 we showed that both the Abelian and non-Abelian T-dual solutions fit in the formalism of DGU and gave a field theory interpretation to the NATD

8. The Brandhuber-Oz solution of massive IIA

The near horizon geometry of the D4, D8/O8 system is a fibration of AdS_6 over half- S^4 with an S^3 boundary at the position of the O8-plane, preserving 16 SUSYs

$$ds^2 = \frac{W^2\,L^2}{4}\left[9\,ds^2(AdS_6) + 4\,ds^2(S^4)\right] \qquad \theta \in [0, \frac{\pi}{2}]$$

$$F_4 = 5\,L^4\,W^{-2}\,\sin^3\theta\,d\theta \wedge \mathrm{Vol}(S^3)$$

$$e^{-\phi} = \frac{3\,L}{2\,W^5}\,, \qquad W = (m\,\cos\theta)^{-\frac{1}{6}} \qquad m = \frac{8 - N_f}{2\pi l_s}$$

- $SO(5)$ symmetry broken to $SO(4) \sim SU(2) \times SU(2)$:

  $SU(2) \leftrightarrow SU(2)_R$ R-symmetry of the field theory

  $SU(2) \leftrightarrow$ global symmetry massless antisym. hyper

9. The Abelian T-dual of Brandhuber-Oz

Take the $AdS_6 \times S^4$ background

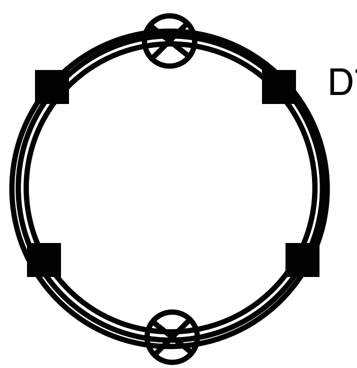
$$ds^2 = \frac{W^2 L^2}{4}\Big[9ds^2(AdS_6) + 4\Big(d\theta^2 + \sin^2\theta ds^2(S^3)\Big)\Big]$$

$$F_4 = 5L^4 W^{-2} \sin^3\theta \, d\theta \wedge \mathrm{Vol}(S^3)$$

and dualize w.r.t. the U(1) fiber

The resulting background preserves all SUSY

It realizes the $Sp(N)$ fixed point theory in a D5,NS5,D7/O7 brane system:



10. The non-Abelian T-dual of Brandhuber-Oz

Take the $AdS_6 \times S^4$ background

$$ds^2 = \frac{W^2 L^2}{4}\Big[9ds^2(AdS_6) + 4\Big(d\theta^2 + \sin^2\theta\, ds^2(S^3)\Big)\Big]$$

$$F_4 = 5L^4 W^{-2}\sin^3\theta\, d\theta \wedge \mathrm{Vol}(S^3)$$

Dualize it w.r.t. one of the $SU(2)$ symmetries

In spherical coordinates adapted to the remaining $SU(2)$:

$$ds^2 = \frac{W^2\, L^2}{4}\left[9\, ds^2(AdS_6) + 4\, d\theta^2\right] + e^{-2\,A}\, dr^2 + \frac{r^2\, e^{2\,A}}{r^2 + e^{4\,A}}\, ds^2(S^2)$$

$$B_2 = \frac{r^3}{r^2 + e^{4\,A}}\,\mathrm{Vol}(S^2) \qquad e^{-\phi} = \frac{3\,L}{2\,W^5}\, e^{A}\sqrt{r^2 + e^{4\,A}}$$

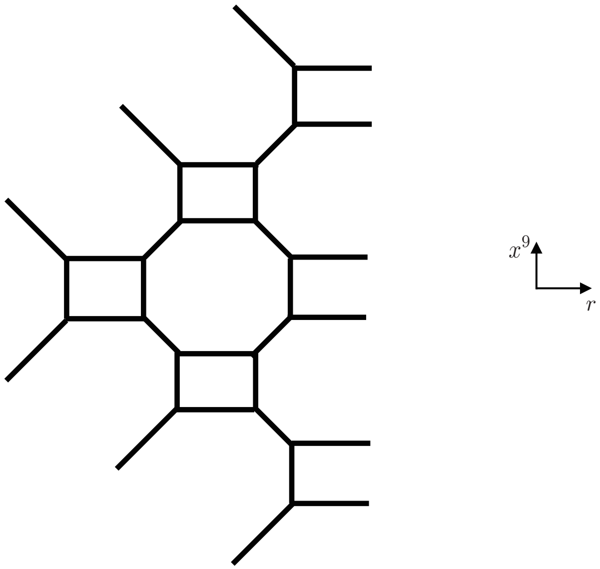
$$F_1 = -G_1 - m\, r\, dr \qquad F_3 = \frac{r^2}{r^2 + e^{4\,A}}\left[-r\, G_1 + m\, e^{4A}\, dr\right] \wedge \mathrm{Vol}(S^2)$$

- It solves the IIB equations of motion, preserving all SUSY
- What about r ?
  - Background perfectly smooth for all $r \in \mathbb{R}^+$
  - No global properties inferred from the NATD
  - How do we interpret the running of r to infinity in the CFT?
- Boundary at $\theta = \pi/2$ inherited
- Singular at $\theta = 0$ where the original S^3 shrinks (due to the presence of NS5-branes)

  This is the tip of a cone with S^2 boundary $\rightarrow$ We have to care about large gauge transformations
- Large gauge transformations modify the quantised charges such that $N_{D5} = nN_{D7}$ in each $[n\pi, (n+1)\pi]$ interval

We have also NS5 charge, such that every time we cross a π interval one unit of NS5 charge is created

This is compatible with a D5/NS5 brane set-up:



The NS5 are bent due to the different number of D5 ending on each side

There is a symmetry under $x^9 \to -x^9$ due to the orientifold action

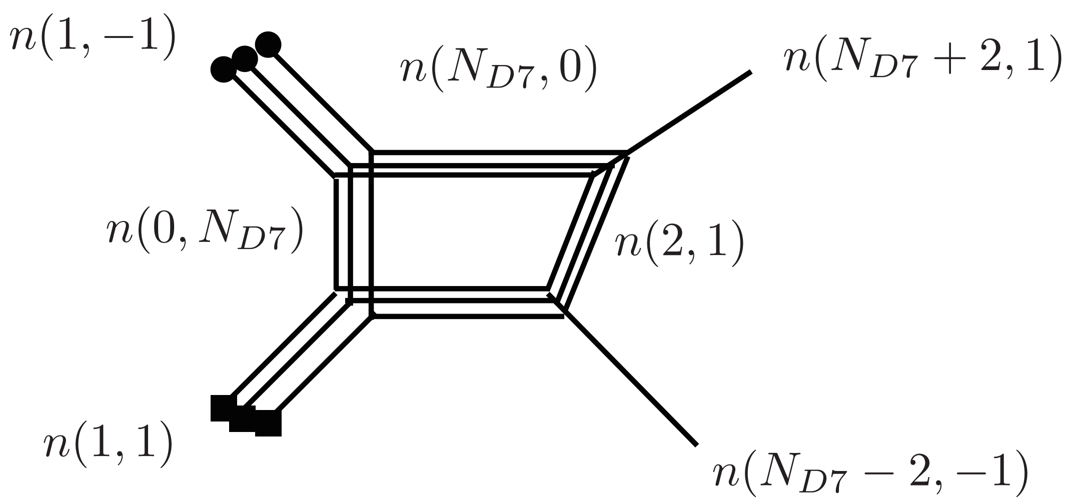
Add the orientifold:

A worldsheet analysis shows that under NATD: $\Omega \to I_\chi \Omega$

$\Rightarrow$ The O8 plane is mapped onto a O5 plane, located at $r = 0$

This O5 is in fact a O7 wrapped on the S^2

Then:



Here:

We have completed the infinite brane set-up with semi-infinite (p,q) 5-branes

$Sp(N_{D7}) \times Sp(2N_{D7}) \times \cdots \times Sp(nN_{D7})$ fixed point theory

At each interval the condition $N_F \leq 2N+4$ for the existence of a $Sp(N)$ UV fixed point theory is satisfied, according to the classification in Intriligator, Morrison, Seiberg'97

This is supported by the central charge and EE, which scale with $N^{5/2}$

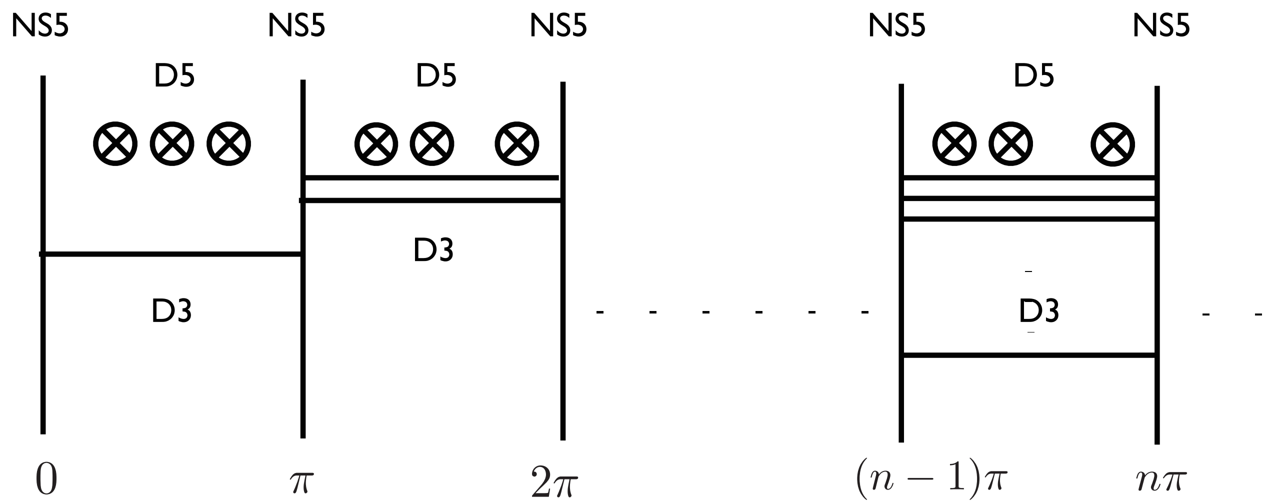
11. The $AdS_4 \times S^2 \times S^2$ example

(Y.L., Macpherson, Montero, Nunez, '16)

Non-Abelian T-duality on a reduction to IIA of $AdS_4 \times S^7/\mathbb{Z}_k$

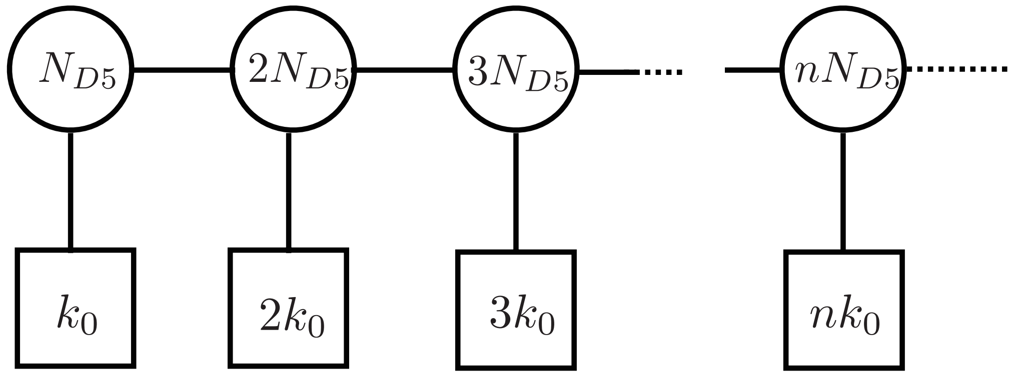
$\rightarrow$ IIB $AdS_4 \times S^2 \times S^2$ background, N=4 SUSY, in the classification of D'Hoker, Estes and Gutperle'07

Analysis of charges: (D3,NS5,D5) brane set-up:



Gaiotto and Witten'08: 3d N=4 $T^{\hat{\rho}}_{\rho}(N)$ theories

$T_{\rho}^{\hat{\rho}}(N)$ field theories flow to CFTs in the infrared if the partitions satisfy certain conditions, that are satisfied by our brane set-up



The holographic duals of these CFTs are known (Assel, Bachas, Estes and Gomis'11)

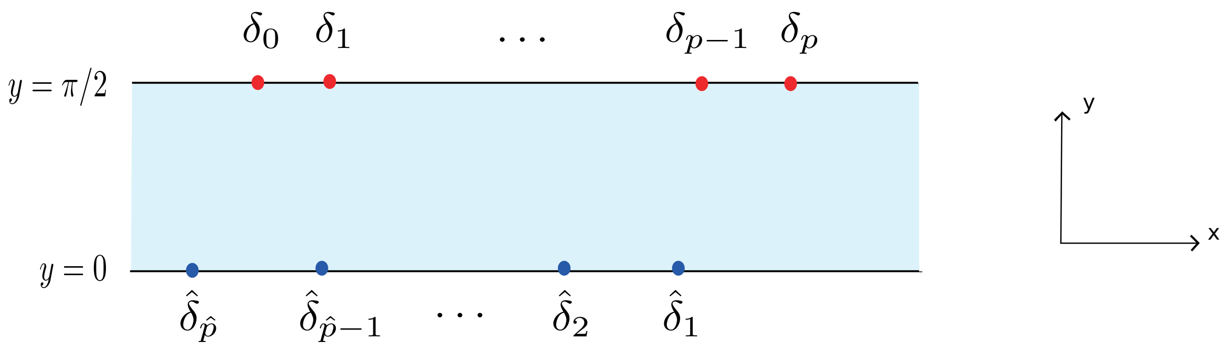
They belong to the general class of $AdS_4 \times S^2 \times S^2$ geometries in D'Hoker, Estes and Gutperle'07

These are fibrations of $AdS_4 \times S^2 \times S^2$ over a Riemann surface that can be completely determined from two harmonic functions $h_1(z,\bar{z}), h_2(z,\bar{z})$

Assel, Bachas, Estes and Gomis'11 showed how to determine these functions from the (D3, NS5, D5) brane set-ups associated to $T_\rho^{\hat\rho}(N)$ theories:

$$h_1 = -\frac{1}{4}\sum_{a=1}^{p} N_5^a \log\tanh\left(\frac{i\frac{\pi}{2}+\delta_a - z}{2}\right) + cc$$

$$h_2 = -\frac{1}{4}\sum_{b=1}^{\hat p} \hat N_5^b \log\tanh\left(\frac{z-\hat\delta_b}{2}\right) + cc$$



The positions of the D5 and NS5 branes are determined, in turn, from the linking numbers of the configuration:

$$\hat\delta_b - \delta_a = \log\tan\left(\frac{\pi}{2}\frac{l_a \hat l_b}{N}\right)$$

The h_1, h_2 functions computed from our *completed* brane set-up agree with those associated to the non-Abelian T-dual geometry far from the location of the added branes

The non-Abelian T-dual arises as a result of zooming-in in a particular region of the *completed* solution

This completion smoothes out the singularities and defines the geometry globally

The ideas about the completion of AdS non-Abelian T-duals are very explicit in this example

The matching between the field theory and holographic central charges is however quite non-trivial in this example

The free energy of specific $T^{\hat{\rho}}_{\rho}(N)$ theories was computed by Assel, Estes and Yamazaki'12 both in the field theory and in the geometry:

- It was shown to exhibit a $F \sim N^2 \log N$ behaviour
- In the geometry the $\log N$ comes from the size of the configuration
- It was argued that this provides an upper bound to the free energy of more general $T^{\hat{\rho}}_{\rho}(N)$ theories

The NATD has a free energy $F \sim \sqrt{k} N_{D5}^{3/2} (n+1)^3$

Quite non-trivially, the *completion* reproduces a $N^2 \log N$ behaviour and satisfies the expected bound

12. Conclusions

The Abelian and non-Abelian T-duals of Brandhuber-Oz were the first AdS6 solutions known in Type IIB SUGRA.

We have shown that they fit within an extension of the $AdS_6 \times S^2$ global solutions with 7-branes recently classified by D'Hoker, Gutperle and Uhlemann

Showing this required extending the formalism of DGU to include D7-branes in the annulus

Non-trivial examples involving smeared NS5 and D7 branes, and O7

The Abelian T-dual solution provides the first known example for the annulus

We have given a candidate CFT dual to the non-Abelian T-dual solution, along the same lines as in previous examples

This 5d fixed point theory belongs to the general classification by Intriligator, Morrison and Seiberg.

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This 5d fixed point theory belongs to the general classification by Intriligator, Morrison and Seiberg.

Open problems:

Work out the properties of this CFT:

- Computation of observables
- Study of global symmetries, enhancement..

Compute the backreaction in the geometry of the external 5-branes added to complete the infinite quiver

Complete the non-Abelian T-dual geometry

More generally,

Non-Abelian T-duality has interesting applications to Holography:

- Generation of new solutions $\rightarrow$ Extension of existing classifications, explicit examples in known classes, construction of new classes of solutions,..

- AdS solutions constructed through NATD are dual to CFTs living on (Dp,NS5) brane intersections $\rightarrow$ First step towards their holographic description

We still need a general prescription for completing the solution

Holography has interesting applications to duality in ST:

- Using holography it is possible to provide the sought for global completion of NATD geometries

The problem remains of course open for backgrounds without a holographic description

Future directions:

NATD and defect CFTs

NATD and BH solutions

NATD and deconstruction

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THANKS!