# Double/Exceptional Field Theory and Generalized Supergravity

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String: T-duality, Integrability and Geometry
4–8 March 2019

# Solution generating transformations

**Recently, various techniques of solution generating transformations in SUGRA are developed.**

- Yang–Baxter deformation

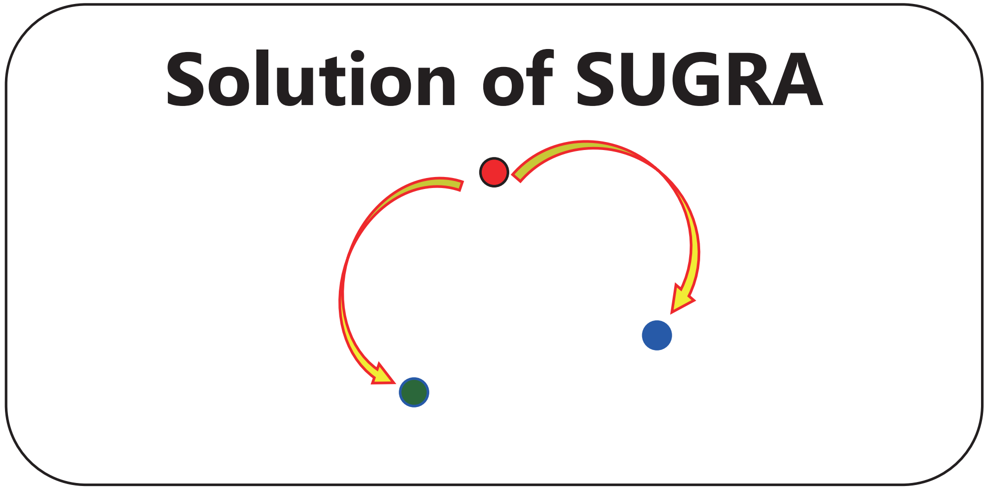
[Klimcik, Delduc, Magro, Vicedo, Kawaguchi, Matsumoto, Yoshida, Arutyunov, Borsato, Frolov, Hoare, Roiban, Tseytlin, Kyono, Sakamoto, Orlando, Reffert, Wulff, van Tongeren, Osten, Thompson, Araujo, Bakhmatov, Ó Colgáin, Sheikh–Jabbari, ···]

- Non–Abelian T–duality

[de la Ossa, Quevedo, Giveon, Rocek, Alvarez, Alvarez–Gaume, Lozano, Klimcik, Severa, Sfetsos, Thompson, Ó Colgáin, Hoare, Tseytlin, Borsato, Wulff, ······]

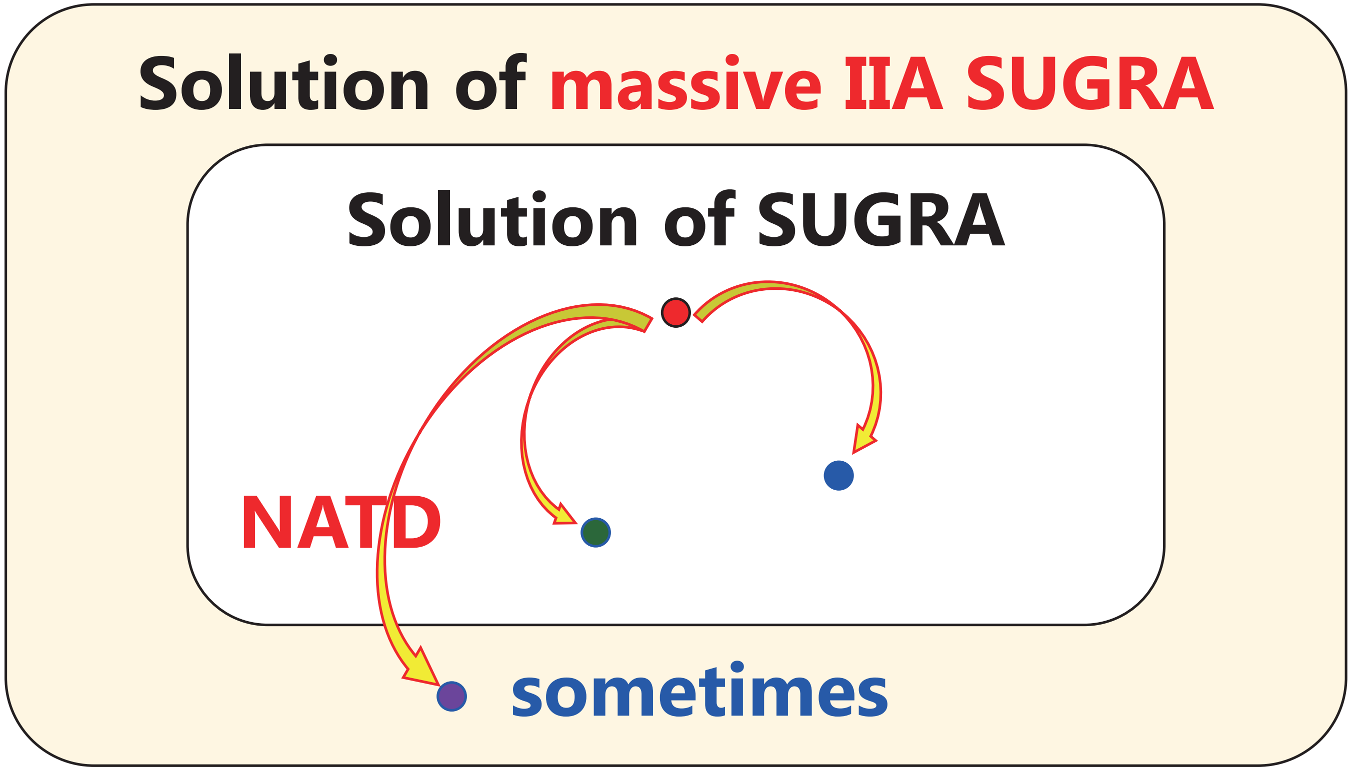
# Solution generating transformations

**Usually,**



**Yang-Baxter deformation**
**or Non-Abelian T-duality**

# Solution generating transformations



[Sfetsos, Thompson, 1012.1320;
Lozano, Ó Colgáin, Sfetsos, Thompson, 1104.5196; ⋯]

# Solution generating transformations

**Sometimes, solutions of "Generalized SUGRA" are produced.**

## Yang–Baxter deformation

[Arutyunov, Borsato, Frolov, 1507.04239;
Arutyunov, Frolov, Hoare, Roiban, Tseytlin, 1511.05795;
Kyono–Yoshida, 1605.02519;
Orlando, Reffert, Sakamoto, Yoshida, 1607.00795;
Fernandez–Melgarejo, Sakamoto, YS, Yoshida, 1710.06849; ···]

## Non-Abelian T–duality

[Gasperini, Ricci, Veneziano, hep-th/9308112;
Fernandez–Melgarejo, Sakamoto, YS, Yoshida, 1710.06849;
Hong, Kim, Ó Colgáin, 1801.09567; ···]

# (Type IIB) Generalized Supergravity

[Arutyunov–Frolov–Hoare–Roiban–Tseytlin ’15;

(NS–NS sector) Hull–Townsend ’86 ]

**Generalized Supergravity Equations of motion (GSE)**

$$R_{mn}-\frac{1}{4}H_{mkl}H_{n}{}^{kl}-T_{mn}+D_{m}X_{n}+D_{n}X_{m}=0\,,$$

$$\frac{1}{2}D^{k}H_{kmn}+\frac{1}{2}\hat{\mathcal{F}}^{k}\hat{\mathcal{F}}_{kmn}+\frac{1}{12}\hat{\mathcal{F}}_{mnklp}\hat{\mathcal{F}}^{klp}=X^{k}H_{kmn}+D_{m}X_{n}-D_{n}X_{m}\,,$$

$$R-\frac{1}{12}H^{2}+4D_{m}X^{m}-4X_{m}X^{m}=0\,,$$

$$D^{m}\hat{\mathcal{F}}_{m}-X^{m}\hat{\mathcal{F}}_{m}-\frac{1}{6}H^{mnk}\hat{\mathcal{F}}_{mnk}=0\,,$$

$$D^{k}\hat{\mathcal{F}}_{kmn}-X^{k}\hat{\mathcal{F}}_{kmn}-\frac{1}{6}H^{kpq}\hat{\mathcal{F}}_{kpqmn}-(I\wedge\hat{\mathcal{F}}_{1})_{mn}=0\,,$$

$$D^{k}\hat{\mathcal{F}}_{kmnpq}-X^{k}\hat{\mathcal{F}}_{kmnpq}+\frac{1}{36}\epsilon_{mnpqrstuvw}H^{rst}\hat{\mathcal{F}}^{uvw}-(I\wedge\hat{\mathcal{F}}_{3})_{mnpq}=0\,.$$

$$\pounds_{I}g_{mn}=0\,,\qquad \pounds_{I}B_{mn}=0\,,$$
$$\pounds_{I}\Phi=0\,,\qquad \pounds_{I}\hat{\mathcal{F}}_{p}=0\,.$$

$$T_{mn}\equiv\frac{1}{2}\hat{\mathcal{F}}_{m}\hat{\mathcal{F}}_{n}+\frac{1}{4}\hat{\mathcal{F}}_{mkl}\hat{\mathcal{F}}_{n}{}^{kl}+\frac{1}{4\times4!}\hat{\mathcal{F}}_{mpqrs}\hat{\mathcal{F}}_{n}{}^{pqrs}-\frac{1}{4}g_{mn}\Big(\hat{\mathcal{F}}_{k}\hat{\mathcal{F}}^{k}+\frac{1}{6}\hat{\mathcal{F}}_{pqr}\hat{\mathcal{F}}^{pqr}\Big)\,.$$

$$X_{m}\equiv\partial_{m}\Phi+(g_{mn}-B_{mn})\,I^{n}$$

**Killing vector**

# Generalized Supergravity

**If a target space is a solution of Generalized SUGRA, superstring theory has the kappa-invariance**

[Tseytlin, Wulff 1605.04884]

**and the rigid scale-invariance.**

[Hull, Townsend ’86;
Arutyunov, Frolov, Hoare, Roiban, Tseytlin ’15]

**However, the Weyl invariance seems to be broken.**

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[Fernandez−Melgarejo, Sakamoto, YS, Yoshida 1811.10600]

**Weyl invariance may not be broken*!***

**talk by** Kentaroh Yoshida

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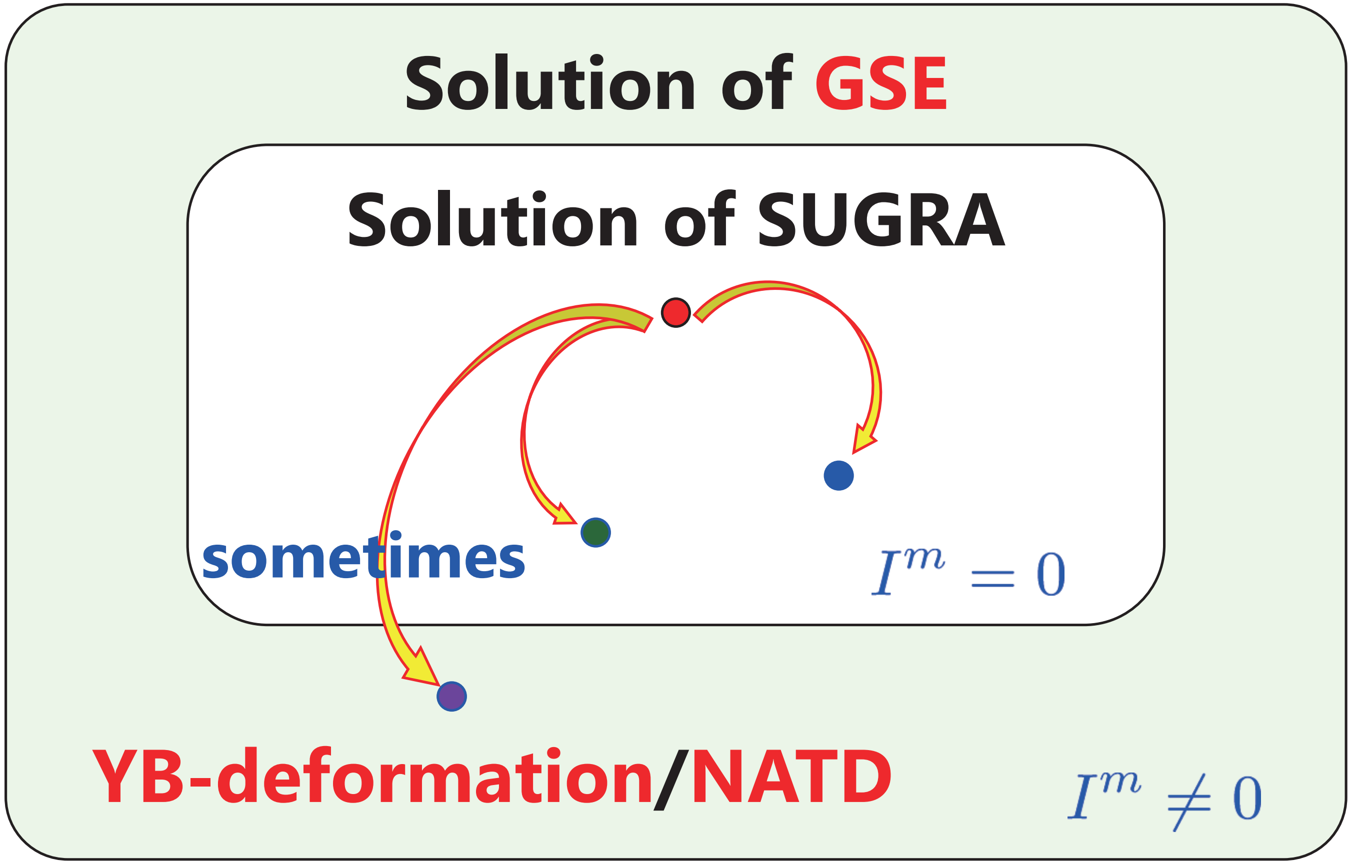
**String theory may be consistently defined*!***

# Solution generating transformations



**Solution of GSE**

**Solution of SUGRA**

**sometimes**

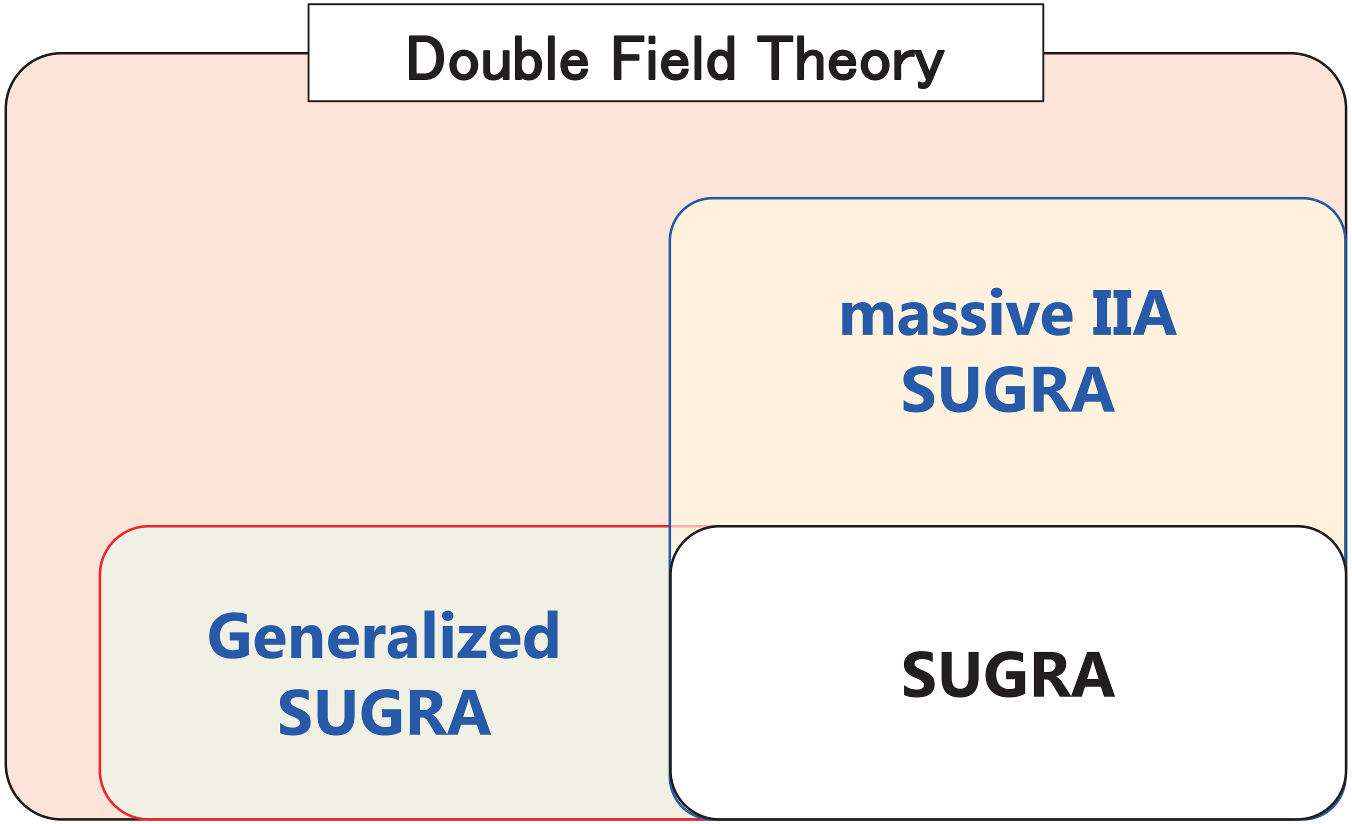
$I^m = 0$

**YB-deformation/NATD**

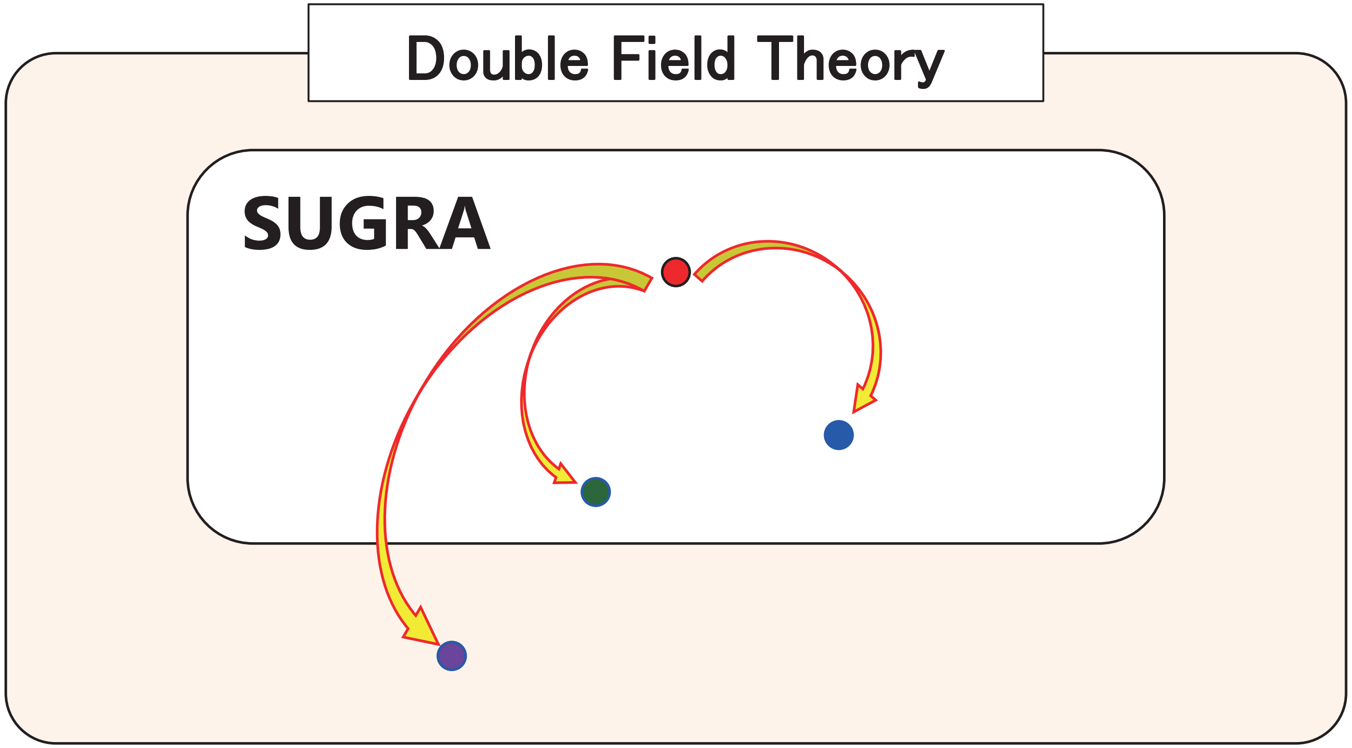
$I^m \neq 0$

# In this talk

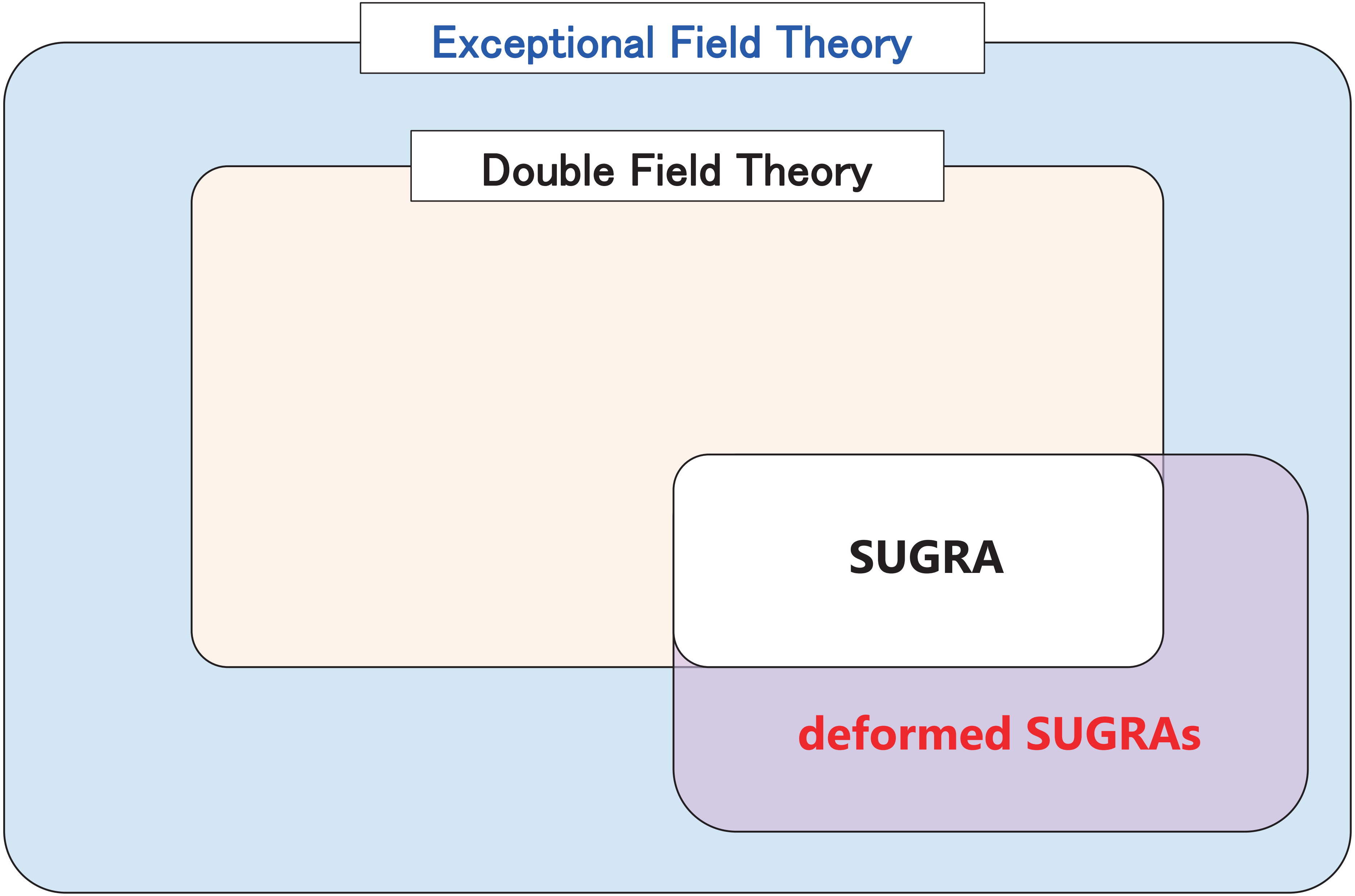
**I will explain that**



# YB-deformation/NATD are solution generating transformations of DFT



# If time allows,



Exceptional Field Theory

Double Field Theory

**SUGRA**

**deformed SUGRAs**

# Plan

1. **Review of DFT**

2. **How to derive GSE from DFT?**
   **How to derive massive IIA SUGRA?**

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**67 pages**

3. **Exceptional Field Theory (EFT) (*U*-duality)**

   **more deformed SUGRAs can be derived**

# 1. Review of DFT

10 years ago!

[Siegel '93;

Hull, Zwiebach, 0904.4664;
Hohm, Hull, Zwiebach, 1006.4823;
I. Jeon, K. Lee, J.-H. Park, 1011.1324; ⋯]

# Several formulations of DFT

- Generalized metric formulation: toroidal DFT

  [Hohm, Hull, Zwiebach; I. Jeon, K. Lee, J.-H. Park; ⋯]

- Flux formulation

  [Geissbuhler, Marques, Nunez, Penas, 1304.1472]

- DFT on group manifold ← **talk by Falk Hassler**

  [Blumenhagen, Hassler, Lust, 1410.6374]

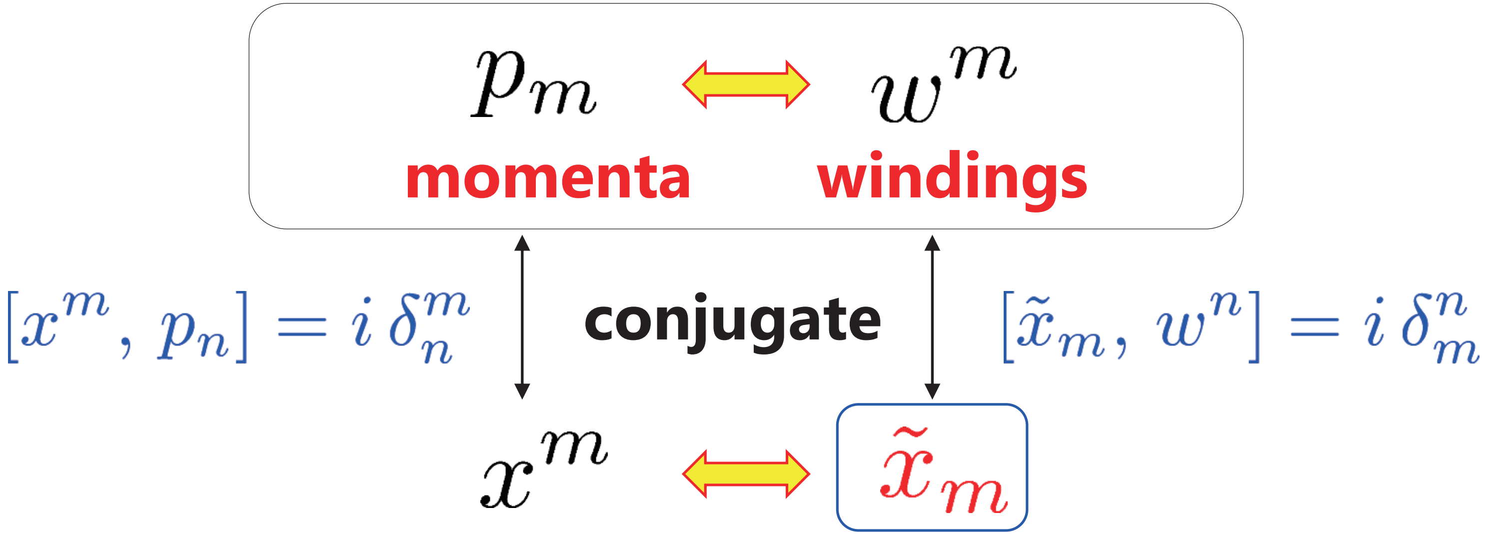
- DFT in supermanifold formulation

  [Carow-Watamura, Ikeda, Kaneko, Watamura, 1812.03464]

⋮

**talk by Noriaki Ikeda**

# T−duality



$p_m \Longleftrightarrow w^m$

**momenta** **windings**

$[x^m, p_n] = i\,\delta^m_n$ **conjugate** $[\tilde{x}_m, w^n] = i\,\delta^n_m$

$x^m \Longleftrightarrow \boxed{\tilde{x}_m}$

**winding coordinates**

**(dual coordinates)**

**Suggested in** [Duff ’90; Tseytlin ‘91;
Kugo, Zwiebach ’92; Siegel ’93; ⋯]

# Doubled space

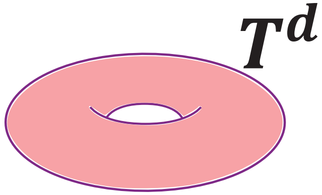
**Generalized coordinates:**

$$\underset{\substack{\uparrow\\ 2d\text{-dim.}}}{(x^{M})} = \Big(\overset{\substack{d\text{-dim.}\\ \swarrow}}{x^{m}}\,,\ \underset{\substack{\nwarrow\\ d\text{-dim.}}}{\tilde{x}_{m}}\Big)$$

**On the doubled space, there is a natural metric, known as the generalized metric:**

$$(\mathcal{H}_{MN}) = \begin{pmatrix} (g - B\,g^{-1}\,B)_{mn} & B_{mp}\,g^{pn} \\ -g^{mp}\,B_{pn} & g^{mn} \end{pmatrix}.$$

# Doubled space



**String theory / $T^d$**

$$M^2 = \frac{2}{\alpha'}\left(z^M\,\mathcal{H}_{MN}\,z^N + N + \tilde{N} - 2\right)$$

$$(z^M) \equiv \begin{pmatrix} w^m \\ p_m \end{pmatrix}$$

**winding**

**momenta**

**O(*d*,*d*) T-duality symmetry:**

$$z^M \to (\Lambda^{-1})^M{}_N\, z^N\,, \qquad \mathcal{H}_{MN} \to (\Lambda^{\mathrm{T}}\,\mathcal{H}\,\Lambda)_{MN}\ .$$

# O(*d*,*d*) metric

**O(*d*,*d*) matrix is defined by**

$$(\Lambda^{\mathrm{T}}\,\eta\,\Lambda)_{MN} = \eta_{MN}\,.$$

$$\eta_{MN} = \begin{pmatrix} 0 & \delta_n^m \\ \delta_m^n & 0 \end{pmatrix}, \qquad \eta^{MN} = \begin{pmatrix} 0 & \delta_n^m \\ \delta_m^n & 0 \end{pmatrix}$$

**O(*d*,*d*) metric**

# Standard convention

**We raise/lower the O(*d*,*d*) indices *M, N, …* by using**

$$\eta^{MN} = \begin{pmatrix} 0 & \delta^m_n \\ \delta^n_m & 0 \end{pmatrix}$$ **or** $$\eta_{MN} = \begin{pmatrix} 0 & \delta^m_n \\ \delta^n_m & 0 \end{pmatrix}$$

---

$$x^M = (x^m,\, \tilde{x}_m) \qquad x_M \equiv \eta_{MN}\, x^N = (\tilde{x}_m,\, x^m)$$

$$\partial_M = (\partial_m,\, \tilde{\partial}^m) \qquad \partial^M \equiv \eta^{MN}\, \partial_N = (\tilde{\partial}^m,\, \partial_m)$$

# Diffeomorphism

**In the usual spacetime,**
**diffeomorphisms are generated by the Lie derivative:**

$$\pounds_v w_m = v^n \, \partial_n w_m + \partial_m v^n \, w_n \, .$$

**The diffeomorphism-invariant gravitational theory**
**is the Einstein gravity:**

$$\mathcal{L}_{\mathrm{GR}} = \sqrt{-g} \, R \, .$$

# Generalized diffeomorphism

**In the doubled space,**
**diffeomorphisms are generated by**
**the generalized Lie derivative:**

$$\hat{\pounds}_V W_M = V^N \partial_N W_M + (\partial_M V^P - \partial^P V_M) W_P$$

**"generalized"**

$$\pounds_v w_m = v^n \partial_n w_m + \partial_m v^n w_n$$

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## Derivations

**Hamiltonian formulation of string:** [Siegel hep-th/9305073]

**Gauge symmetry of CSFT:** [Hull, Zwiebach 0904.4664]

# Double Field Theory

**Double Field Theory is**
**the generalized diffeomorphism-invariant theory.**

**Fundamental fields:**

**DFT dilaton**

$$\boxed{\mathcal{H}_{MN}(x^M)\,, \qquad d(x^M)\,.}$$

$$(\mathcal{H}_{MN}) = \begin{pmatrix} (g - B\,g^{-1}\,B)_{mn} & B_{mp}\,g^{pn} \\ -g^{mp}\,B_{pn} & g^{mn} \end{pmatrix}\,, \qquad \mathrm{e}^{-2d} = \mathrm{e}^{-2\Phi}\,\sqrt{|g|}\,.$$

***T*-duality invariant.**

# DFT action

**Lagrangian of DFT (NS-NS sector):**

[Hohm, Hull, Zwiebach, 1006.4823]

$$
\begin{aligned}
\mathcal{L}_{\mathrm{DFT}} \equiv \mathrm{e}^{-2d}\Big(&\mathcal{H}^{MN}\,\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d \\
&+ 4\partial_M\mathcal{H}^{MN}\,\partial_N d + \frac{1}{8}\,\mathcal{H}^{MN}\,\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\,\mathcal{H}^{MN}\,\partial_M\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{NL}\Big)
\end{aligned}
$$

**This combination is invariant under the generalized diffeomorphism.**

# Consistency condition

**In fact, the generalized diffeo.-invariance of the action requires a condition, called the section condition.**

$$\boxed{\eta^{MN}\,\partial_M A\,\partial_N B = 0\,.}$$

**This is also required for**
**the closure of the generalized Lie derivative.**

$$[\hat{\pounds}_{V_1},\,\hat{\pounds}_{V_2}]\,W^M = \hat{\pounds}_{[V_1,\,V_2]_C}W^M$$

# Section condition

$$\eta^{MN}\,\partial_M A\,\partial_N B = 0\,.$$

$$\eta^{MN} = \begin{pmatrix} 0 & \delta^m_n \\ \delta_m^n & 0 \end{pmatrix}$$

$$\mathcal{H}_{MN}(x^M)\,,\quad d(x^M)\,,\quad V^M(x^M)$$

**fields or diffeomorphism parameter**

**This is trivially satisfied if all fields are independent of the dual coordinates** $\tilde{x}_m$ .

$$\tilde{\partial}^m = 0$$

$$\eta^{MN}\,\partial_M A\,\partial_N B = \partial_m A\,\underline{\tilde{\partial}^m B} + \underline{\tilde{\partial}^m A}\,\partial_m B = 0\,.$$

# Section condition

$$\eta^{MN}\,\partial_M A\,\partial_N B = \partial_m A\,\tilde{\partial}^m B + \tilde{\partial}^m A\,\partial_m B = 0\,.$$

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**Another solution is** $\partial_m = 0$

$$\mathcal{H}_{MN}(\cancel{x^m},\,\tilde{x}_m)\,,\quad d(\cancel{x^m},\,\tilde{x}_m)\,,\quad V^M(\cancel{x^m},\,\tilde{x}_m)$$

---

**Section condition always removes the dependence on a half of the doubled coordinates.**

$$(x^m,\,\cancel{\tilde{x}_m})$$

$$(\cancel{x^m},\,\tilde{x}_m)$$

---

**In fact, depending on the choice of the coordinates we can reproduce the SUGRA or GSE.**

# Conventional SUGRA

**canonical section**

$$\partial_M = (\partial_m ,\, \tilde{\partial}^m) = (\partial_m ,\, 0)$$

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**Parameterization**

$$(\mathcal{H}_{MN}) = \begin{pmatrix} (g - B\, g^{-1}\, B)_{mn} & B_{mp}\, g^{pn} \\ -g^{mp}\, B_{pn} & g^{mn} \end{pmatrix} , \qquad \mathrm{e}^{-2d} = \mathrm{e}^{-2\Phi} \sqrt{|g|}\, .$$

---

$$\mathcal{L}_{\mathrm{DFT}} \equiv \mathrm{e}^{-2d}\Big(\mathcal{H}^{MN}\, \partial_M \partial_N d - \partial_M \partial_N \mathcal{H}^{MN} - 4\mathcal{H}^{MN}\, \partial_M d\, \partial_N d$$
$$+\, 4\partial_M \mathcal{H}^{MN}\, \partial_N d + \frac{1}{8}\, \mathcal{H}^{MN}\, \partial_M \mathcal{H}^{KL}\, \partial_N \mathcal{H}_{KL} - \frac{1}{2}\, \mathcal{H}^{MN}\, \partial_M \mathcal{H}^{KL}\, \partial_K \mathcal{H}_{NL}\Big)$$

$$\Downarrow$$

$$\mathcal{L} = \sqrt{|g|}\, \mathrm{e}^{-2\Phi} \left(R + 4\, |\partial\Phi|^2 - \frac{1}{12}\, |H|^2\right) + \partial_m \big(4\sqrt{|g|}\, \mathrm{e}^{-2\Phi}\, g^{mn} \partial_n \Phi\big)\, .$$

# Generalized diffeomorphism

$$\delta_V \mathcal{H}_{MN} = \hat{\pounds}_V \mathcal{H}_{MN} = V^P \, \partial_P \mathcal{H}_{MN} + (\partial_M V^P - \partial^P V_M)\, \mathcal{H}_{PN} + (\partial_N V^P - \partial^P V_N)\, \mathcal{H}_{MP} \,.$$

$$(\mathcal{H}_{MN}) = \begin{pmatrix} (g - B\, g^{-1}\, B)_{mn} & B_{mp}\, g^{pn} \\ -g^{mp}\, B_{pn} & g^{mn} \end{pmatrix}$$

$$V^M = (v^m, \, \tilde{v}_m)$$

$$\tilde{\partial}^m = 0$$

$$\boxed{\begin{cases} \delta_V g_{mn} & = \pounds_v g_{mn} \\ \delta_V B_{mn} & = \pounds_v B_{mn} + (\partial_m \tilde{v}_n - \partial_n \tilde{v}_m) \,. \end{cases}}$$

**Generalized diffeomorphism unifies**
**the usual diffeo. and B-field gauge transformation.**

# Short Summary

**Gravitational theory**
**on the doubled space**

$$(x^M) = (x^m,\, \tilde{x}_m)$$

**Fundamental fields (NS-NS sector):**

$$(\mathcal{H}_{MN}) = \begin{pmatrix} (g - B\, g^{-1}\, B)_{mn} & B_{mp}\, g^{pn} \\ -g^{mp}\, B_{pn} & g^{mn} \end{pmatrix}, \qquad \mathrm{e}^{-2d} = \mathrm{e}^{-2\Phi} \sqrt{|g|}\,.$$

$$\mathcal{L}_{\text{DFT}} \xrightarrow[\boxed{\tilde{\partial}^m = 0}]{} \mathcal{L} = \sqrt{|g|}\, \mathrm{e}^{-2\Phi} \left( R + 4\, |\partial\Phi|^2 - \frac{1}{12}\, |H_3|^2 \right).$$

**_2d_ dim. generalized diffeo.** $\underset{\boxed{\tilde{\partial}^m = 0}}{=} \left\{ \begin{array}{l} \textbf{\textit{d} dim. diffeo.} \\ \textbf{gauge sym. of } B_2 \end{array} \right.$

# Side remark

## **differential geometry on the doubled space**

[Siegel ’93;
I. Jeon, K. Lee, J-H. Park ’10;
Hohm, Zwiebach ’12]

# DFT action

**Lagrangian of DFT (NS-NS sector):**

$$\mathcal{L}_{\rm DFT} \equiv {\rm e}^{-2d}\Big(\mathcal{H}^{MN}\,\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d$$
$$+\,4\partial_M\mathcal{H}^{MN}\,\partial_N d + \frac{1}{8}\mathcal{H}^{MN}\,\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\,\partial_M\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{NL}\Big)$$

$$\|$$

**"Einstein-Hilbert" action** ${\rm e}^{-2d}\,\mathcal{S}$

**generalized Ricci scalar curvature**

[Siegel '93; I. Jeon, K. Lee, J–H. Park '10;
Hohm, Zwiebach '12]

# Generalized Ricci tensor

**Generalized connection**

[I. Jeon, K. Lee, J–H. Park ’10;
Hohm, Zwiebach ’12]

$$\nabla_K \eta_{MN} = 0\,, \qquad \nabla_K \mathcal{H}_{MN} = 0\,, \qquad \nabla_M d = 0\,, \qquad \Gamma_{[MNK]} = 0\,.$$

**“torsion” free**

$$R^K{}_{LMN} \equiv \partial_M \Gamma_N{}^K{}_L - \partial_N \Gamma_M{}^K{}_L + \Gamma_M{}^K{}_P \Gamma_N{}^P{}_L - \Gamma_N{}^K{}_P \Gamma_M{}^P{}_L\,.$$

**not covariant**

$$S_{MNKL} \equiv R_{MNKL} + R_{KLMN} - \Gamma_{PMN} \Gamma^P{}_{KL}\,.$$

**not-yet-fully covariant**

**projection**

$$P_M^N = \frac{1}{2}\left(\delta_M^N + \mathcal{H}_M{}^N\right)$$

$$\bar{P}_M^N = \frac{1}{2}\left(\delta_M^N - \mathcal{H}_M{}^N\right)$$

$$\left\{ \begin{aligned} \mathcal{S}_{MN} &\equiv \left(P_M{}^K \bar{P}_N{}^L + \bar{P}_M{}^K P_N{}^L\right) S^P{}_{KLP}\,, \\ \mathcal{S} &\equiv \frac{1}{2}\left(P^{MK} P^{NL} - \bar{P}^{MK} \bar{P}^{NL}\right) S_{MNKL}\,. \end{aligned} \right.$$

**covariant curvatures**

# Generalized Ricci tensor

**Action** $\mathcal{L}_{\mathrm{DFT}} = \mathrm{e}^{-2d}\,\mathcal{S}\,.$

**E.O.M.** $\mathcal{S}_{MN} = 0\,, \qquad \mathcal{S} = 0\,.$

**generalized Ricci flatness**

**manifestly covariant under generalized diffeomorphisms.**

$$\tilde{\partial}^m = 0$$

**E.O.M.**

$$\begin{aligned}
&R_{mn} - \tfrac{1}{4}\,H_{mkl}\,H_n{}^{kl} + 2D_m\partial_n\Phi = 0\,,\\
&-\tfrac{1}{2}\,D^k H_{kmn} + \partial_k\Phi\,H^k{}_{mn} = 0\,,\\
&R + 4\,D^m\partial_m\Phi - 4\,|\partial\Phi|^2 - \tfrac{1}{2}\,|H_3|^2 = 0\,.
\end{aligned}$$

# R–R fields/fermions

# Ramond–Ramond sector 1/2

$$\{\gamma^M, \gamma^N\} = \eta^{MN}$$

[Hohm, Kwak, Zwiebach, ’11]
**based on** [Fukuma, Oota, Tanaka ’99]

$$(\gamma^M) = (\gamma^m, \gamma_m) \Rightarrow \{\gamma^m, \gamma_n\} = \delta^m_n$$

creation annihilation

$$\gamma_m |0\rangle = 0$$

## O(*d,d*) spinor

$$|A\rangle = \sum_p \frac{1}{p!} A_{m_1 \cdots m_p} \gamma^{m_1} \cdots \gamma^{m_p} |0\rangle = 0$$

$$|F\rangle \equiv \partial\!\!\!/ |A\rangle \qquad (\partial\!\!\!/ \equiv \gamma^M \partial_M)$$

$$\boxed{\mathcal{L}_{\text{R-R}} = -\frac{1}{4} \langle F | \mathbb{S} | F \rangle} \Rightarrow \mathcal{L}_{\text{R-R}} = -\frac{1}{4} \sum_p |F_p|^2$$

$$\tilde{\partial}^m = 0$$

# Ramond–Ramond sector 1/2

matter

**E.O.M.**

$$\mathcal{S}_{MN} = \mathcal{E}_{MN}\,, \quad \mathcal{S} = 0\,, \quad \not{\partial}\big[C\,\mathbb{S}\,|F\rangle\big] = 0\,.$$

$$\mathcal{E}_{MN} = \frac{1}{4}\,\mathrm{e}^{2\,d}\Big[\langle F|(\gamma_{(M})^{\mathrm{T}}\,\mathbb{S}\,\gamma_{N)}|F\rangle - \frac{1}{2}\,\mathcal{H}_{MN}\,\langle F|\mathbb{S}|F\rangle\Big]$$

**Energy-momentum tensor**

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O(*d*,*d*) transformation

$$|F\rangle \to \Omega\,|F\rangle\,.$$

**O(*d*,*d*) matrix**

$$\Omega\,\gamma_M\,\Omega^{-1} = \gamma_N\,\Lambda^N{}_M\,.$$

# Ramond–Ramond sector 2/2

**Introduce double vielbein**

[I. Jeon, K. Lee, J.-H. Park ’12]
based on [Hassan ‘01 ]

$$\mathcal{H}^{MN}=V^{M}{}_{A}V^{N}{}_{B}\,\mathcal{H}^{AB}\,.$$

$$\{A\}=\{a,\,\bar{a}\}$$

**two vielbeins**

$$\boxed{E_{mn}=g_{mn}+B_{mn}}$$

$$(V^{M}{}_{a})=\frac{1}{\sqrt{2}}\begin{pmatrix}(e^{-\mathrm{T}})^{m}{}_{a}\\ E_{mn}\,(e^{-\mathrm{T}})^{n}{}_{a}\end{pmatrix}\,,\qquad(\bar{V}^{M}{}_{\bar{a}})=\frac{1}{\sqrt{2}}\begin{pmatrix}(\bar{e}^{-\mathrm{T}})^{m}{}_{\bar{a}}\\ -E^{\mathrm{T}}_{mn}\,(\bar{e}^{-\mathrm{T}})^{n}{}_{\bar{a}}\end{pmatrix}\,.$$

---

**Local flat metric**

$$\mathcal{H}_{AB}=\begin{pmatrix}\eta_{ab}&0\\0&-\bar{\eta}^{\bar{a}\bar{b}}\end{pmatrix},\qquad\eta_{AB}=\begin{pmatrix}\eta_{ab}&0\\0&\bar{\eta}^{\bar{a}\bar{b}}\end{pmatrix}.$$

**double Lorentz symmetry O(*d*) x O(*d*)**

# Ramond–Ramond sector 2/2

**Local flat metric** $$\mathcal{H}_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & -\bar{\eta}^{\bar{a}\bar{b}} \end{pmatrix}, \quad \eta_{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \bar{\eta}^{\bar{a}\bar{b}} \end{pmatrix}.$$

**double Lorentz symmetry O(*d*) x O(*d*)**

**Ramond-Ramond fields (bi-spinor):** $$\mathcal{C}^{\alpha}{}_{\bar{\beta}}$$

---

**In this formulation, there are 2 vielbeins:**

$$(V^{M}{}_{a}) = \frac{1}{\sqrt{2}} \begin{pmatrix} (e^{-\mathrm{T}})^{m}{}_{a} \\ E_{mn}\,(e^{-\mathrm{T}})^{n}{}_{a} \end{pmatrix}, \qquad (\bar{V}^{M}{}_{\bar{a}}) = \frac{1}{\sqrt{2}} \begin{pmatrix} (\bar{e}^{-\mathrm{T}})^{m}{}_{\bar{a}} \\ -E^{\mathrm{T}}_{mn}\,(\bar{e}^{-\mathrm{T}})^{n}{}_{\bar{a}} \end{pmatrix}.$$

**In order to reproduce the usual SUGRA,**

**we should make an identification:** $\bar{e}_{m}{}^{\bar{a}} = e_{m}{}^{a}$ .

# Ramond–Ramond sector 2/2

**Under a general O(*d*,*d*) rotation** $h^{M}{}_{N}=\begin{pmatrix} s^{m}{}_{n} & r^{mn} \\ q_{mn} & p_{m}{}^{n} \end{pmatrix}$

$$V^{M}{}_{a}\rightarrow h^{M}{}_{N}\,V^{N}{}_{a}\,,\qquad \bar{V}^{M}{}_{\bar{a}}=h^{M}{}_{N}\,\bar{V}^{N}{}_{\bar{a}}\,.$$

⇨ **The two vielbeins transform differently:**

$$e_{m}{}^{a}\rightarrow\left[\left(s^{\mathrm{T}}+E^{\mathrm{T}}\,r^{\mathrm{T}}\right)^{-1}\right]_{m}{}^{n}\,e_{n}{}^{a}\,,\qquad \bar{e}_{m}{}^{\bar{a}}\rightarrow\left[\left(s^{\mathrm{T}}-E\,r^{\mathrm{T}}\right)^{-1}\right]_{m}{}^{n}\,\bar{e}_{n}{}^{\bar{a}}\,.$$

---

**In order to keep the identification** $\bar{e}_{m}{}^{\bar{a}}=e_{m}{}^{a}\,,$

**we should combine O(*d*,*d*) rotation and a Lorentz transf.**

$$V_{M}{}^{a}\rightarrow h_{M}{}^{N}\,V_{N}{}^{a}\,,\qquad \bar{V}_{M}{}^{\bar{a}}\rightarrow h_{M}{}^{N}\,\bar{V}_{N}{}^{\bar{b}}\,\Lambda^{\bar{a}}{}_{\bar{b}}\,.$$

$$\Lambda=e^{\mathrm{T}}\,(s+r\,E)^{-1}\,(s-r\,E^{\mathrm{T}})\,e^{-\mathrm{T}}$$

[Hassan ’01; I. Jeon, K. Lee, J.–H. Park ’12 ]

# Ramond–Ramond sector 2/2

**Under a general O(*d*,*d*) rotation** $h^{M}{}_{N} = \begin{pmatrix} s^{m}{}_{n} & r^{mn} \\ q_{mn} & p_{m}{}^{n} \end{pmatrix}$

**only barred index is rotated by the Lorentz transf.**

$$\mathcal{C}^{\alpha}{}_{\bar{\beta}} \to \mathcal{C}^{\alpha}{}_{\bar{\gamma}}\,(\Omega^{-1})^{\bar{\gamma}}{}_{\bar{\beta}}$$

$$\Omega^{-1}\,\bar{\Gamma}^{a}\,\Omega = \Lambda^{\bar{a}}{}_{\bar{b}}\,\bar{\Gamma}^{b}\,.$$

---

**The same idea has been applied in the context of the Non-Abelian T-duality.**

[Sfetsos, Thompson, 1012.1320]

---

**Field strength:** $\mathcal{F} \equiv \mathcal{D}_{+}\mathcal{C}$ **(covariant under generalized diffeo.)**

$$\mathcal{L}_{\text{R-R}} \sim \mathcal{F}^{\alpha\bar{\beta}}\,\mathcal{F}_{\alpha\bar{\beta}}\,.$$

[I. Jeon, K. Lee, J.–H. Park ' 12 ]

# Fermions

**In type II SUGRA, we have**

**gravitino:** $\psi^1_a\,,\quad\psi^2_a$

$$\left\{\begin{array}{l}\Gamma^{11}\,\psi^1_a=-\psi^1_a\\[2ex]\Gamma^{11}\,\psi^2_a=\pm\psi^2_a\end{array}\right.$$

**(IIA/IIB)**

**dilatino:** $\lambda^1\,,\quad\lambda^2$

$$\left\{\begin{array}{l}\Gamma^{11}\,\lambda^1=+\lambda^1\\[2ex]\Gamma^{11}\,\lambda^2=\mp\lambda^2\end{array}\right.$$

**(IIA/IIB)**

$$(\bar{\Gamma}^a\equiv\Gamma^a\,\Gamma^{11})$$

⇓

$$\psi^1_{\bar{a}}\,,\qquad\psi^2_a\,,\qquad\rho^1\equiv\Gamma^a\,\psi^1_a-\lambda^1\,,\qquad\rho^2\equiv\bar{\Gamma}^a\,\psi^2_a-\lambda^2\,.$$

**Covariant combinations**

**Type II DFT action.** [I. Jeon, K. Lee, J.-H. Park , Y. Suh ' 12 ]

c.f. [Coimbra, Strickland-Constable, Waldram '11]

# 2. GSE from DFT

## Neglecting the R-R fields, GSE take the form:

$$\begin{cases} R - \frac{1}{2}\,|H_3|^2 - 4\,D_m X^m - 4\,X^m X_m = 0\,. \\ R_{mn} - \frac{1}{4}\,H_{mpq}\,H_n{}^{pq} + D_m X_n + D_n X_m = 0\,, \\ -\frac{1}{2}\,D^k H_{kmn} + X^k\,H_{kmn} + D_m X_n - D_n X_m = 0\,. \\ \qquad\qquad \boxed{X_m \equiv \partial_m \Phi + (g_{mn} - B_{mn})\,I^n\,.} \end{cases}$$

**non-dynamical vector**

**<u>Constraint :</u>**

$$\pounds_I\, g_{mn} = 0\,, \qquad \pounds_I\, B_{mn} = 0\,, \qquad \pounds_I\, \Phi = 0\,.$$

***I*** **should be a Killing vector of a GSE solution.**

# Example of GSE solution

# Solutions of GSE

**1. Non-unimodular YB deformations**

$$r^{kl}\, f_{kl}{}^{i} \neq 0\,.$$

[Borsato, Wulff, 1608.03570]

**talk by** Junichi Sakamoto

**2. Non-Abelian T-duality**

**for tracefull structure constants**

$$f_{ki}{}^{i} \neq 0\,.$$

# Example from NATD

[Gasperini, Ricci, Veneziano hep-th/9308112]

**Original Background (Type V Bianchi universe):**

$$ds^2 = -dt^2 + t^2\left[dx^2 + e^{2x}(dy^2 + dz^2)\right].$$

**We consider 3 Killing vectors,**

$$v_1 = \partial_x - y\,\partial_y - z\,\partial_z\,, \qquad v_2 = \partial_y\,, \qquad v_3 = \partial_z\,.$$

$$[v_1,\, v_2] = v_2\,, \qquad [v_1,\, v_3] = v_3\,, \qquad [v_2,\, v_3] = 0\,.$$

$$f_{12}{}^{2} = 1\,, \qquad f_{13}{}^{3} = 1 \qquad \Rightarrow \qquad f_{1i}{}^{i} = 2\,.$$

**tracefull!**

# Dual geometry

[Gasperini, Ricci, Veneziano hep–th/9308112]

$$\left\{\begin{aligned}
g_{mn} &= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{t^2}{t^4+y^2+z^2} & 0 & 0 \\ 0 & 0 & \frac{t^4+z^2}{t^2\left(t^4+y^2+z^2\right)} & -\frac{yz}{t^2\left(t^4+y^2+z^2\right)} \\ 0 & 0 & -\frac{yz}{t^2\left(t^4+y^2+z^2\right)} & \frac{t^4+y^2}{t^2\left(t^4+y^2+z^2\right)} \end{pmatrix}, \\
B_{mn} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{y}{t^4+y^2+z^2} & \frac{z}{t^4+y^2+z^2} \\ 0 & -\frac{y}{t^4+y^2+z^2} & 0 & 0 \\ 0 & -\frac{z}{t^4+y^2+z^2} & 0 & 0 \end{pmatrix}, \\
\mathrm{e}^{-2\Phi} &= t^2\left(t^4+y^2+z^2\right).
\end{aligned}\right.$$

[de la Ossa, Quevedo ’92]

$$\Phi' = \Phi - \tfrac{1}{2}\ln\det M_{ij}$$

$$\left(M_{ij} \equiv E^0_{ij} + f_{ij}{}^k\, x_k\right)$$

**This background does not satisfy the SUGRA e.o.m.**

# “A Problem with Non-Abelian Duality?”

[Gasperini, Ricci, Veneziano hep-th/9308112]

$$\left\{\begin{aligned}
g_{mn} &= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{t^2}{t^4+y^2+z^2} & 0 & 0 \\ 0 & 0 & \frac{t^4+z^2}{t^2\left(t^4+y^2+z^2\right)} & -\frac{yz}{t^2\left(t^4+y^2+z^2\right)} \\ 0 & 0 & -\frac{yz}{t^2\left(t^4+y^2+z^2\right)} & \frac{t^4+y^2}{t^2\left(t^4+y^2+z^2\right)} \end{pmatrix}, \\
B_{mn} &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{y}{t^4+y^2+z^2} & \frac{z}{t^4+y^2+z^2} \\ 0 & -\frac{y}{t^4+y^2+z^2} & 0 & 0 \\ 0 & -\frac{z}{t^4+y^2+z^2} & 0 & 0 \end{pmatrix}, \\
\mathrm{e}^{-2\Phi} &= t^2\left(t^4+y^2+z^2\right).
\end{aligned}\right.$$

**They considered a general ansatz for dilaton, but no solution.**

# GSE solution

[Fernandez-Melgarejo, Sakamoto, YS, Yoshida, 1710.06849]

$$\left\{\begin{aligned}
&g_{mn} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{t^2}{t^4+y^2+z^2} & 0 & 0 \\ 0 & 0 & \frac{t^4+z^2}{t^2\left(t^4+y^2+z^2\right)} & -\frac{yz}{t^2\left(t^4+y^2+z^2\right)} \\ 0 & 0 & -\frac{yz}{t^2\left(t^4+y^2+z^2\right)} & \frac{t^4+y^2}{t^2\left(t^4+y^2+z^2\right)} \end{pmatrix}, \\
&B_{mn} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{y}{t^4+y^2+z^2} & \frac{z}{t^4+y^2+z^2} \\ 0 & -\frac{y}{t^4+y^2+z^2} & 0 & 0 \\ 0 & -\frac{z}{t^4+y^2+z^2} & 0 & 0 \end{pmatrix}, \\
&\mathrm{e}^{-2\Phi} = t^2\,(t^4+y^2+z^2)\,, \qquad I = I^m\,\partial_m = 2\,\partial_x\,.
\end{aligned}\right.$$

**Solution of Generalized Supergravity!**

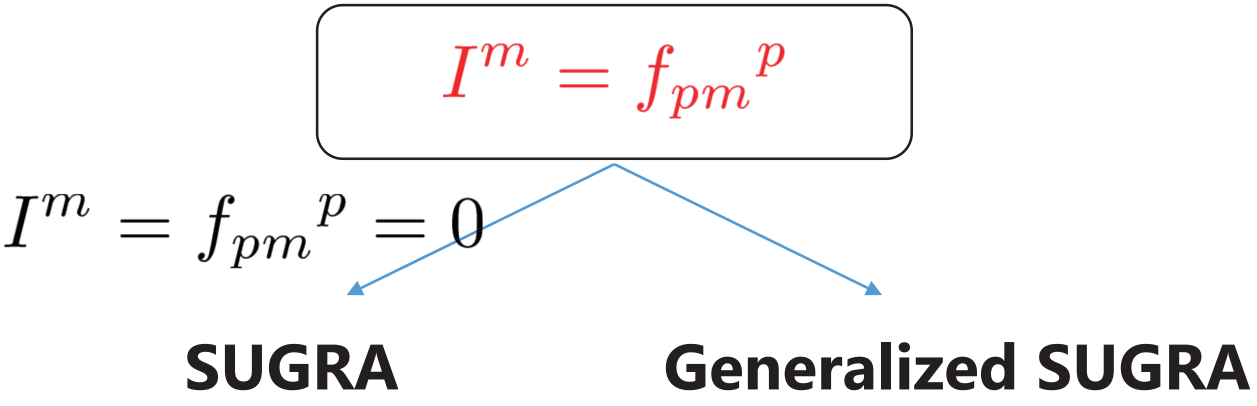
# GSE solutions

**More GSE solutions were obtained from Non-Abelian *T*-dualities of other Bianchi universes.**

[Hong, Kim, Ó Colgáin, 1801.09567]

**General formula:**

**The Killing vector is given by**



$$\boxed{I^m = f_{pm}{}^p}$$

$$I^m = f_{pm}{}^p = 0$$

**SUGRA**

**Generalized SUGRA**

# Generalized SUGRA from DFT

[YS, Uehara, Yoshida, 1611.05856]

[Sakamoto, YS, Yoshida, 1703.09213]

# **Without the R-R fields, GSE have the form:**

$$\begin{cases} R - \frac{1}{2}\,|H_3|^2 - 4\,D_m X^m - 4\,X^m X_m = 0\,. \\ R_{mn} - \frac{1}{4}\,H_{mpq}\,H_n{}^{pq} + D_m X_n + D_n X_m = 0\,, \\ -\frac{1}{2}\,D^k H_{kmn} + X^k\,H_{kmn} + D_m X_n - D_n X_m = 0\,. \\ \boxed{X_m \equiv \partial_m \Phi + (g_{mn} - B_{mn})\,I^n\,.} \end{cases}$$

$$\pounds_I\, g_{mn} = 0\,, \qquad \pounds_I\, B_{mn} = 0\,, \qquad \pounds_I\, \Phi = 0\,.$$

**Due to the Killing property,**

$$\pounds_{I}\, g_{mn} = 0\,, \qquad \pounds_{I}\, B_{mn} = 0\,, \qquad \pounds_{I}\, \Phi = 0\,.$$

**we can always take an adapted coordinates**
**where $I^m$ is constant.**

$$\Longrightarrow \qquad \pounds_{I} = I^m\, \partial_m\,.$$

**Killing equations become**

$$I^m\, \partial_m \mathcal{H}_{MN} = 0\,, \qquad\qquad I^m\, \partial_m d = 0\,.$$

# GSE from DFT

[YS, Uehara, Yoshida, 1611.05856;
Sakamoto, YS, Yoshida, 1703.09213]

**Now, we can derive GSE by introducing a linear dual-coordinate dependence into the DFT dilaton:**

$$d = \bar{d}(x^i) + I^m\, \tilde{x}_m$$

$$I^m\, \partial_m \bar{d} = 0\,.$$

**Section condition is not violated:**

$$\eta^{MN}\, \partial_M d\, \partial_N d = 2\, I^m\, \partial_m \bar{d} = 0\,.$$

$$\eta^{MN}\, \partial_M d\, \partial_N \mathcal{H}_{MN} = I^m\, \partial_m \mathcal{H}_{MN} = 0\,.$$

# GSE from DFT

**According to the ansatz,**

$$d = \bar{d}(x^i) + I^m \, \tilde{x}_m \; ,$$

**derivative of dilaton becomes**

$$\partial^M d = \begin{pmatrix} I^m \\ \partial_m \bar{d} \end{pmatrix}$$

↔

$$\partial^M d = \begin{pmatrix} 0 \\ \partial_m d \end{pmatrix}$$

**canonical section**

$$\tilde{\partial}^m = 0$$

**By substituting the ansatz** [YS, Uehara, Yoshida, 1611.05856]

$$\mathcal{H}_{MN} = \mathcal{H}_{MN}(x^i)\,, \qquad d = \bar{d}(x^i) + I^m\,\tilde{x}_m\,,$$

**into e.o.m. of DFT, we can reproduce the GSE !**

$$\mathcal{S} = R - \frac{1}{2}\,|H_3|^2 - 4\,D_m X^m - 4\,X^m X_m = 0\,.$$

$$\left[\ X_m \equiv \partial_m \Phi + (g_{mn} - B_{mn})\,I^n\ \right]$$

$$\mathcal{S}_{MN} = \begin{pmatrix} 2\,g_{(m|k}\,\mathring{s}^{[kl]}\,B_{l|n)} - \mathring{s}_{(mn)} - B_{mk}\,\mathring{s}^{(kl)}\,B_{ln} & B_{mk}\,\mathring{s}^{(kn)} - g_{mk}\,\mathring{s}^{[kn]} \\ \mathring{s}^{[mk]}\,g_{kn} - \mathring{s}^{(mk)}\,B_{km} & \mathring{s}^{(mn)} \end{pmatrix} = 0\,.$$

$$\Updownarrow$$

$$\mathring{s}_{(mn)} = R_{mn} - \frac{1}{4}\,H_{mpq}\,H_n{}^{pq} + D_m X_n + D_n X_m = 0\,,$$

$$\mathring{s}_{[mn]} = -\frac{1}{2}\,D^k H_{kmn} + X^k\,H_{kmn} + D_m X_n - D_n X_m = 0\,.$$

# Ramond–Ramond sector

[Sakamoto, YS, Yoshida, 1703.09213]

**We make an ansatz**

$$\mathcal{H}_{MN} = \mathcal{H}_{MN}(x^i)\,, \qquad d = \bar{d}(x^i) + I^m\,\tilde{x}_m\,,$$

$$A = \mathrm{e}^{\bar{\Phi}(x^i)+I^m\,\tilde{x}_m}\,\mathcal{A}(x^i)\,,$$

$$F = \mathrm{e}^{\bar{\Phi}(x^i)+I^m\,\tilde{x}_m}\,\mathcal{F}(x^i)\,.$$

$$I^m\,\partial_m\,\mathcal{F}(x^i) = 0\,.$$

$$\Downarrow$$

$$\mathcal{S}_{MN} = \mathcal{E}_{MN}\,, \quad \mathcal{S} = 0\,, \quad \not{\partial}\big[C\,\mathbb{S}\,|F\rangle\big] = 0\,.$$

# Ramond–Ramond sector

[Sakamoto, YS, Yoshida, 1703.09213]

**Type IIA/IIB GSE:**

$$R_{mn} - \tfrac{1}{4} H_{mkl} H_n{}^{kl} + D_m X_n + D_n X_m = T_{mn}\,,$$

$$-\tfrac{1}{2}\, D^k H_{kmn} + \partial_k \Phi\, H^k{}_{mn} = \mathcal{K}_{mn}\,,$$

$$R - \tfrac{1}{2}\, |H_3|^2 + 4 D_m X^m - 4 X_m X^m = 0\,,$$

$$\mathrm{d}\hat{\mathcal{F}}_p + H_3 \wedge \hat{\mathcal{F}}_{p-2} - Z \wedge \hat{\mathcal{F}}_p - \iota_I \hat{\mathcal{F}}_{p+2} = 0\,.$$

$$\left(Z_m \equiv \partial_m \Phi - B_{mn}\, I^n\right)$$

$$T_{mn} \equiv \frac{1}{4} \sum_p \left[\frac{1}{(p-1)!}\, \hat{\mathcal{F}}_{(m}{}^{k_1\cdots k_{p-1}} \hat{\mathcal{F}}_{n)k_1\cdots k_{p-1}} - \frac{1}{2}\, g_{mn}\, |\hat{\mathcal{F}}_p|^2\right],$$

$$\mathcal{K}_{mn} \equiv \frac{1}{4} \sum_p \frac{1}{(p-2)!}\, \hat{\mathcal{F}}_{k_1\cdots k_{p-2}}\, \hat{\mathcal{F}}_{mn}{}^{k_1\cdots k_{p-2}}\,.$$

# Ramond–Ramond sector

[Arutyunov–Frolov–Hoare–Roiban–Tseytlin ’15;
Sakamoto, YS, Yoshida, 1703.09213]

**We have also determined the *T*-duality rule in the presence of the Killing vector:**

$$g'_{ij} = g_{ij} - \frac{g_{iz}\,g_{jz} - B_{iz}\,B_{jz}}{g_{zz}}\,, \qquad g'_{iz} = \frac{B_{iz}}{g_{zz}}\,, \qquad g'_{zz} = \frac{1}{g_{zz}}\,,$$

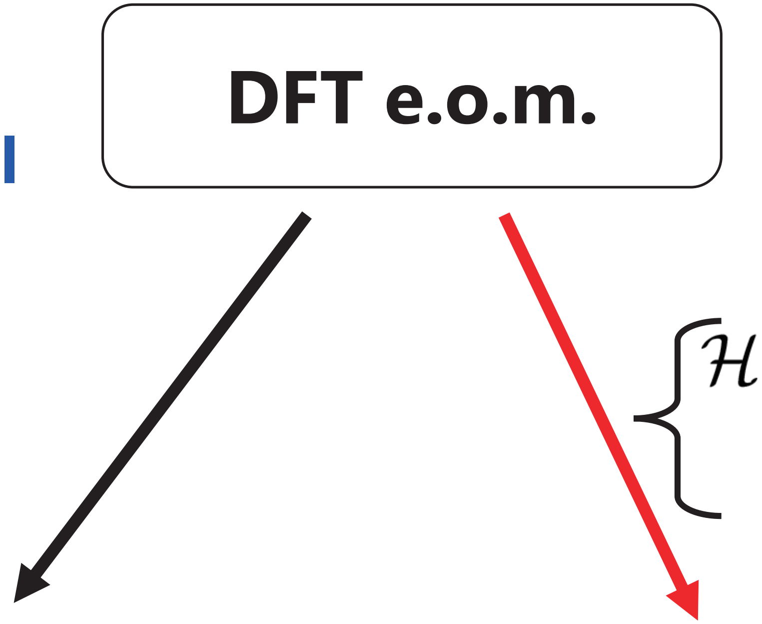
$$B'_{ij} = B_{ij} - \frac{B_{iz}\,g_{jz} - g_{iz}\,B_{jz}}{g_{zz}}\,, \qquad B'_{iz} = \frac{g_{iz}}{g_{zz}}\,,$$

$$\Phi' = \Phi + \frac{1}{4}\,\ln\left|\frac{\det g'_{mn}}{\det g_{mn}}\right| + I^z\,z\,, \qquad I'^i = I^i\,, \qquad I'^z = 0\,,$$

$$C'_{i_1\cdots i_{p-1}z} = \mathrm{e}^{-I^z\,z}\left[C_{i_1\cdots i_{p-1}} - (p-1)\,\frac{C_{[i_1\cdots i_{p-2}|z|}\,g_{i_{p-1}]z}}{g_{zz}}\right],$$

$$C'_{i_1\cdots i_p} = \mathrm{e}^{-I^z\,z}\left[C_{i_1\cdots i_p z} + p\,C_{[i_1\cdots i_{p-1}}\,B_{i_p]z} + p\,(p-1)\,\frac{C_{[i_1\cdots i_{p-2}|z|}\,B_{i_{p-1}|z|}\,g_{i_p]z}}{g_{zz}}\right].$$

# Summary



$$I = I^z\,\partial_z$$

$$(x^i, z)$$

**DFT e.o.m.**

**canonical section**

$$\begin{cases}\mathcal{H}_{MN}(\boxed{x^m}, \cancel{\tilde{x}_m}) \\ d(\boxed{x^m}, \cancel{\tilde{x}_m})\end{cases}$$

$$\begin{cases}\mathcal{H}_{MN}(\boxed{x^i}, \cancel{z, \tilde{x}_i, \tilde{z}}) \\ d(\boxed{x^i}, \cancel{z, \tilde{x}_i}, \boxed{\tilde{z}})\end{cases}$$

**SUGRA e.o.m.**

**GSE with** $I = I^z\,\partial_z$

**Choice of the section is different.**

# Comment

$$\mathcal{L}_{\rm DFT} \equiv {\rm e}^{-2d}\Big(\mathcal{H}^{MN}\,\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d$$
$$+\,4\partial_M\mathcal{H}^{MN}\,\partial_N d + \frac{1}{8}\mathcal{H}^{MN}\,\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\,\partial_M\mathcal{H}^{KL}\,\partial_K\mathcal{H}_{NL}\Big)$$

**DFT action is invariant under a constant O(*d,d*) rotation**

$$\boxed{\begin{aligned}&\mathcal{H}_{MN} \to (\Lambda^{-{\rm T}})_M{}^K\,(\Lambda^{-{\rm T}})_N{}^L\,\mathcal{H}_{KL}\,,\\ &x^M \to \Lambda^M{}_N\,x^N\,, \qquad \partial_M d \to (\Lambda^{-{\rm T}})_M{}^N\,\partial_N d\,.\end{aligned}}$$

**Unlike the Abelian T-duality of string theory, this is a symmetry even without isometries.**

**"formal *T*-duality"**

# GSE solution

**Arbitrary solution of GSE,**

$$\mathcal{H}_{MN} = \mathcal{H}_{MN}(x^i)\,, \qquad d(x) = \bar{d}(x^i) + I^z\,\tilde{z}$$

**Perform a formal T-duality along the z-direction** $z \leftrightarrow \tilde{z}$

$$\mathcal{H}'_{MN} = \mathcal{H}'_{MN}(x^i)\,, \qquad d(x) = \bar{d}(x^i) + I^z\,z$$

**Buscher rule**

$\tilde{\partial}^m = 0$ **Solution of SUGRA**

**A solution of GSE can be mapped to a solution of SUGRA**

[Hoare, Tseytlin, 1508.01150;
Arutyunov–Frolov–Hoare–Roiban–Tseytlin ’15;
YS, Uehara, Yoshida, 1611.05856]

# GSE solution

**Conversely, for an arbitrary linear-dilaton solution**

$$\mathcal{H}_{MN} = \mathcal{H}_{MN}(x^i)\,, \qquad d(x) = \bar{d}(x^i) + I^z\, z\,.$$

$$\Big\Downarrow$$

**a formal T-duality**

$$\mathcal{H}'_{MN} = \mathcal{H}'_{MN}(x^i)\,, \qquad d(x) = \bar{d}(x^i) + I^z\, \tilde{z}\,.$$

**GSE solution**

---

**GSE solution is just a SUGRA solution described in a non-canoical section.**

# Massive IIA SUGRA

**Type II DFT**

↓

**massive type IIA SUGRA**

$$(x^M) = (x^m,\, \tilde{x}_m)$$

[Hohm, Kwak, 1108.4937]

$$A_1 = \bar{A}_1(x^i) + m\,\tilde{z}\,\mathrm{d}z$$

**O(*d,d*) spinor**

$$|A_1\rangle = \big(\bar{A}_m\,\gamma^m + \boxed{m\,\tilde{z}\,\gamma^z}\big)|0\rangle$$

$$|F\rangle = \partial\!\!\!/\,|A\rangle \qquad \partial\!\!\!/ = \gamma^M\,\partial_M = \gamma_z\,\tilde{\partial}^z$$

annihilation operator

R–R 0–form $$|F_0\rangle = m\,|0\rangle$$

# D8−brane solution

$$\mathrm{d}s^2 = H^{-1/2}(x^9)\,\mathrm{d}x^2_{01\cdots 8} + H^{1/2}(x^9)\,\mathrm{d}x^2_9\,,$$

$$\mathrm{e}^{-2\Phi} = H^{5/2}(x^9)\,, \quad H(x) \equiv h_0 + m\,|x|\,.$$

[Bergshoeff, de Roo,
Green, Papadopoulos,
Townsend, '96]

⇓

## D8-brane solution in DFT

$$\mathrm{d}s^2 = H^{-1/2}(x^9)\,\mathrm{d}x^2_{01\cdots 8} + H^{1/2}(x^9)\,\mathrm{d}x^2_9\,,$$

$$A_1 = m\,\tilde{x}_8\,\mathrm{d}x^8\,, \quad \mathrm{e}^{-2\Phi} = H^{5/2}(x^9)\,.$$

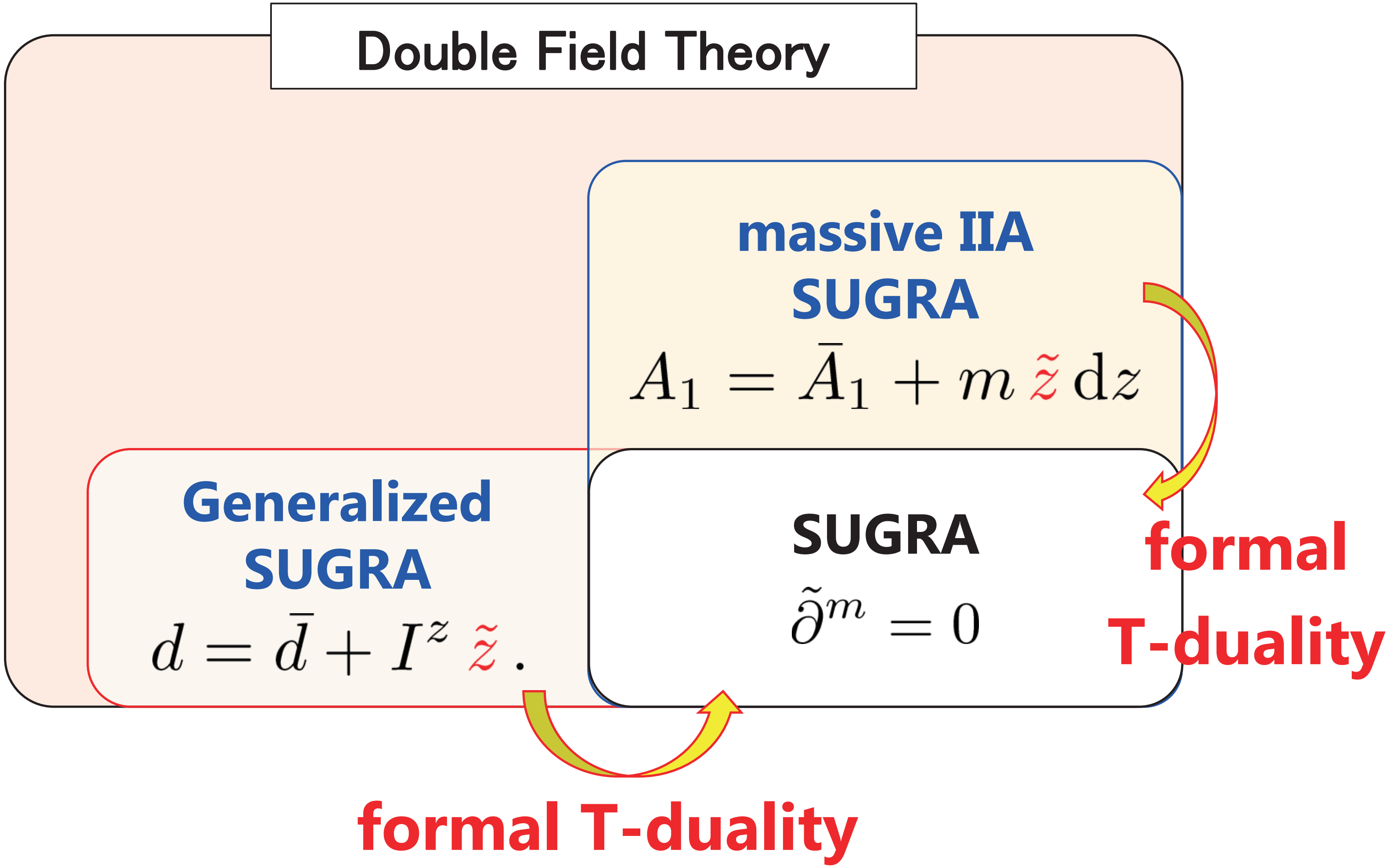
⇓ **formal T-duality** $\tilde{x}_8 \leftrightarrow x^8$

## D7-brane solution

$$\mathrm{d}s^2 = H^{-1/2}(x^9)\,\mathrm{d}x^2_{01\cdots 7} + H^{1/2}(x^9)\,(\mathrm{d}x^2_8 + \mathrm{d}x^2_9)\,,$$

$$A_0 = m\,x^8\,, \quad \mathrm{e}^{-2\Phi} = H^2(x^9)\,.$$

# Summary



**Double Field Theory**

**massive IIA SUGRA**

$$A_1 = \bar{A}_1 + m\,\tilde{z}\,\mathrm{d}z$$

**Generalized SUGRA**

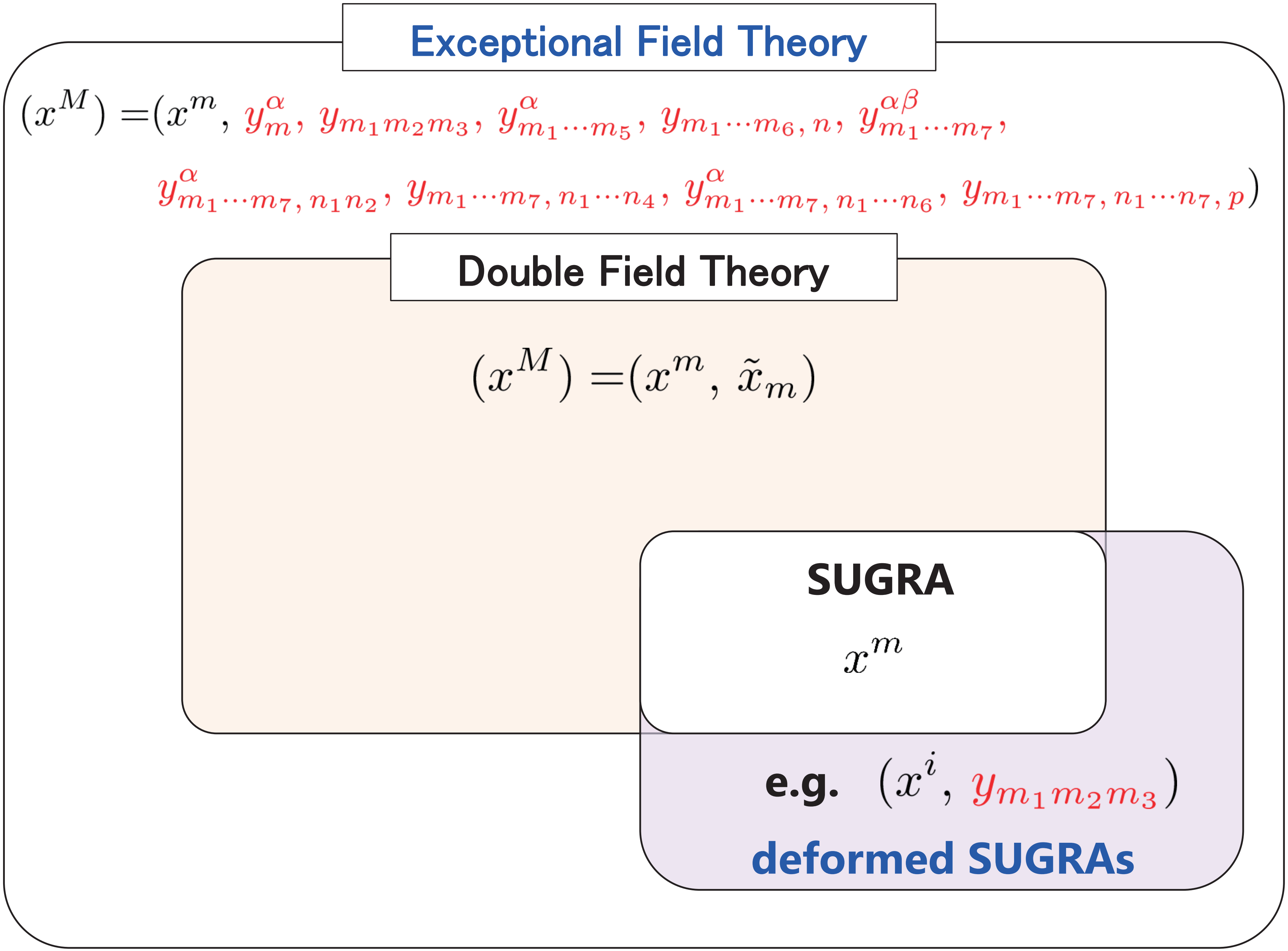
$$d = \bar{d} + I^z\,\tilde{z}\,.$$

**SUGRA**

$$\tilde{\partial}^m = 0$$

**formal T-duality**

**formal T-duality**



**Exceptional Field Theory**

$$(x^M) = (x^m,\, y^\alpha_m,\, y_{m_1m_2m_3},\, y^\alpha_{m_1\cdots m_5},\, y_{m_1\cdots m_6,\,n},\, y^{\alpha\beta}_{m_1\cdots m_7},\, y^\alpha_{m_1\cdots m_7,\,n_1n_2},\, y_{m_1\cdots m_7,\,n_1\cdots n_4},\, y^\alpha_{m_1\cdots m_7,\,n_1\cdots n_6},\, y_{m_1\cdots m_7,\,n_1\cdots n_7,\,p})$$

**Double Field Theory**

$$(x^M) = (x^m,\, \tilde{x}_m)$$

**SUGRA**

$$x^m$$

**e.g.** $(x^i,\, y_{m_1m_2m_3})$

**deformed SUGRAs**

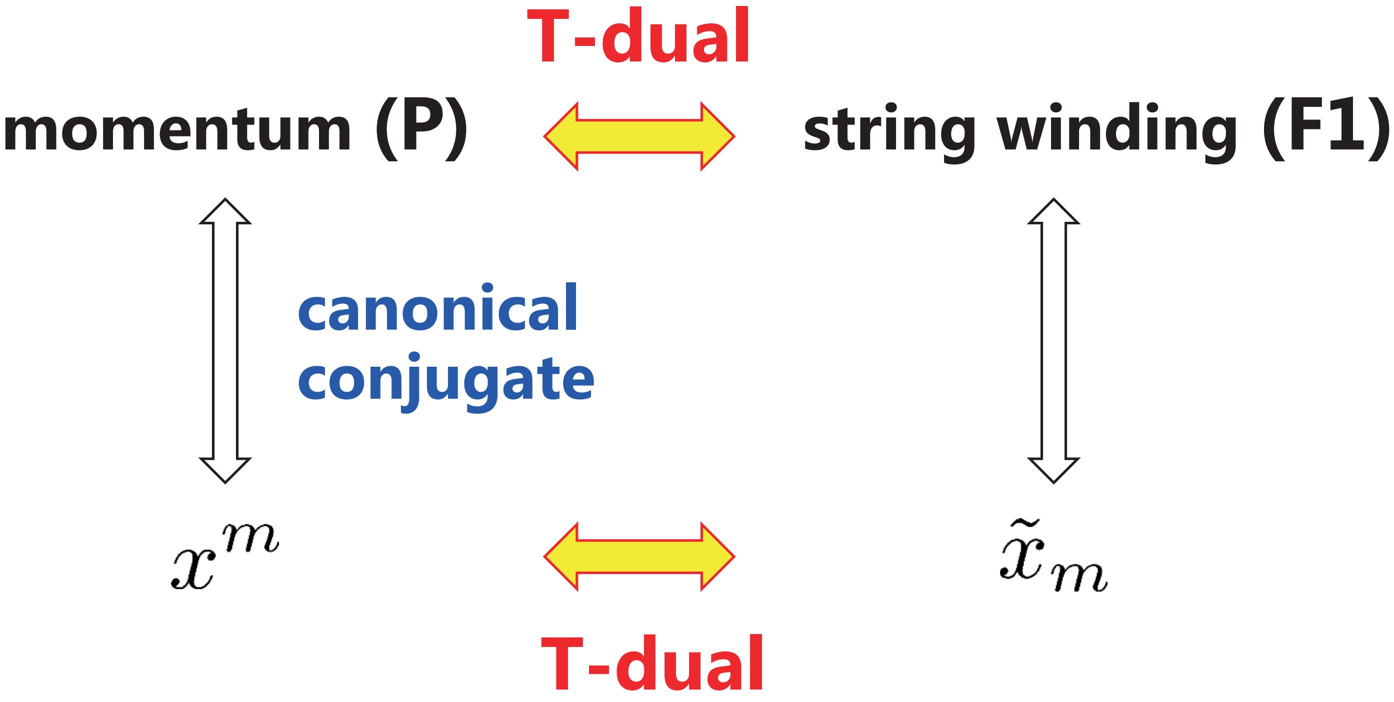
# 3. Exceptional Field Theory (EFT)

*U*-dual version of DFT

[West ’00; Berman, Perry ’11;
Berman, Godazgar, Perry, West ’12;
Hohm, Samtleben ’13; ...]

# In DFT

**We introduced the doubled space.**



**T-dual**

**momentum (P)**

**string winding (F1)**

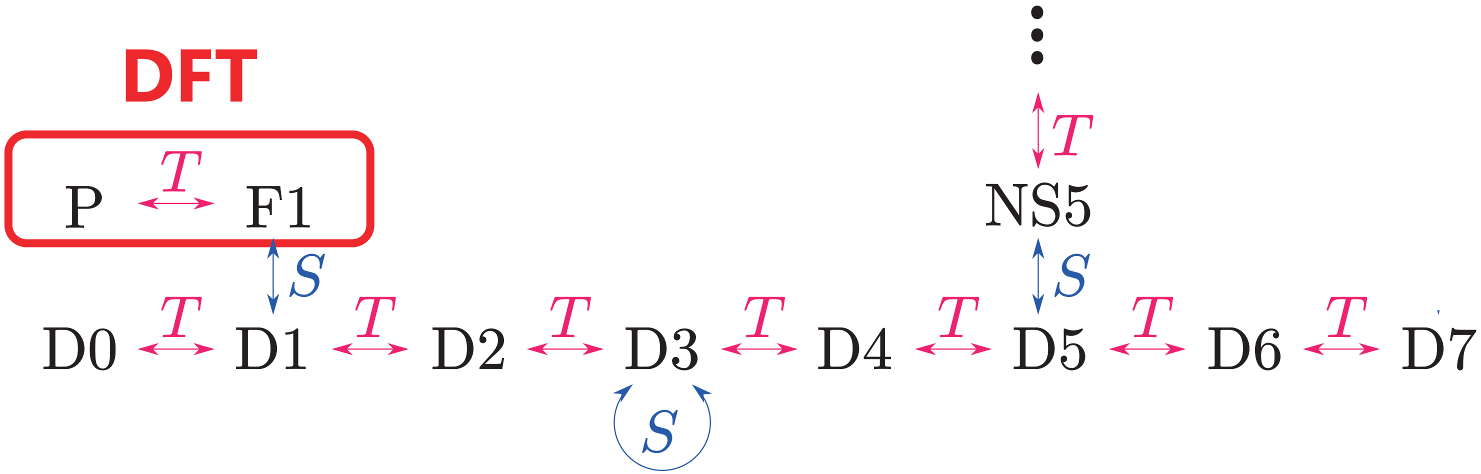
**canonical conjugate**

$x^m$

$\tilde{x}_m$

**T-dual**

# In type II theory



**There are more branes,**
**which are connected by *U*-dualities.**

# How many branes?

**It depends on the dimension of the compactification torus**

**Example : type IIB on $T^3$**

P(1), P(2), P(3)

F1(1), F1(2), F1(3)

D1(1), D1(2), D1(3)

D3(123)

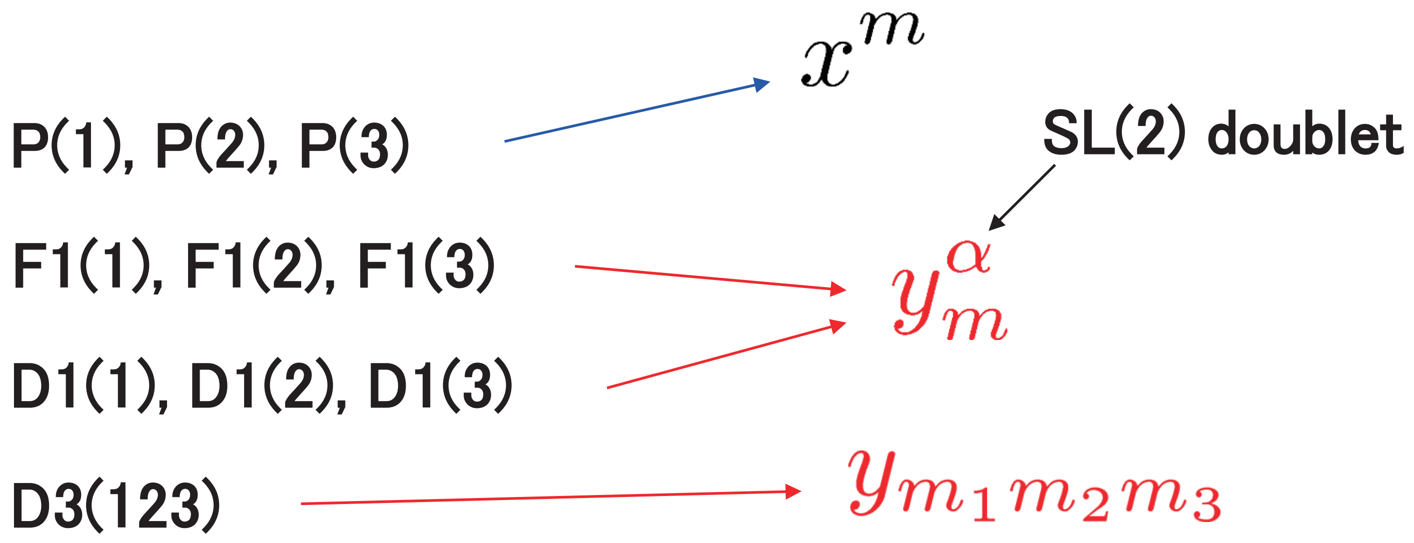
**10 branes are related by *U*-duality**

~~D5/NS5~~ **cannot wrap on $T^3$**

# How many branes?

It depends on the **dimension**
of the compactification **torus**

**Example : type IIB on $T^3$**



P(1), P(2), P(3) → $x^m$

F1(1), F1(2), F1(3) → $y^\alpha_m$

D1(1), D1(2), D1(3) → $y^\alpha_m$

SL(2) doublet → $\alpha$

D3(123) → $y_{m_1 m_2 m_3}$

**Extended space:** $(x^M) = (x^m,\, y^\alpha_m,\, y_{m_1 m_2 m_3})\,.$

# Type IIB theory / $T^{n-1}$

**P** **F1/D1** **D3** **D5/NS5**

$x^m$ $y^\alpha_m$ $y_{m_1 m_2 m_3}$ $y^\alpha_{m_1 \cdots m_5}$

$\underline{n=4}$ $3 + 2 \times 3 + {}_3\mathrm{C}_3$ ✖ $= 10$

**$\mathbf{SL(5)}$**

$\underline{n=5}$ $4 + 2 \times 4 + {}_4\mathrm{C}_3$ ✖ $= 16$

**$\mathbf{SO(5,5)}$**

$\underline{n=6}$ $5 + 2 \times 5 + {}_5\mathrm{C}_3 + 2 \times {}_5\mathrm{C}_5 = 27$

$\vdots$

**$\mathbf{E_6}$**

[Obers, Pioline ’99] “particle multiplet” of **$\mathbf{E}_n$**

# Type IIB theory / $T^6$ or $T^7$

(D7, NS7, **7)

P F1/D1 D3 D5/NS5 KKM triplet of 7–branes

$$(x^M) = (x^m,\ y^\alpha_m,\ y_{m_1m_2m_3},\ y^\alpha_{m_1\cdots m_5},\ y_{m_1\cdots m_6,n},\ y^{\alpha\beta}_{m_1\cdots m_7},$$

$$y^\alpha_{m_1\cdots m_7,n_1n_2},\ y_{m_1\cdots m_7,n_1\cdots n_4},\ y^\alpha_{m_1\cdots m_7,n_1\cdots n_6},\ y_{m_1\cdots m_7,n_1\cdots n_7,p}) .$$

$5^2_2$ / $5^2_3$  $3^4_3$  $1^6_4$ / $1^6_3$  $0^{(1,6)}_4$

**Exotic branes**

[Elitzur, Giveon, Kutasov, Rabinovici ‘97;
Blau, O’ Loughlin ’97; Obers, Pioline ’99;
Eyras, Lozano ’00; Lozano–Tellechea, Ortin ’00]

# Exceptional space

$$
\begin{aligned}
(x^M) =& (x^m,\ y^\alpha_m,\ y_{m_1m_2m_3},\ y^\alpha_{m_1\cdots m_5},\ y_{m_1\cdots m_6,n},\ y^{\alpha\beta}_{m_1\cdots m_7},\\
&\quad y^\alpha_{m_1\cdots m_7,n_1n_2},\ y_{m_1\cdots m_7,n_1\cdots n_4},\ y^\alpha_{m_1\cdots m_7,n_1\cdots n_6},\ y_{m_1\cdots m_7,n_1\cdots n_7,p})
\end{aligned}
$$

$x^\mu$

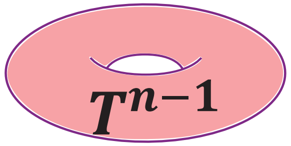
$\mathbb{R}^{1,d-1}$

x

U−duality group

| $E_n$ | SL(5) | SO(5, 5) | $E_6$ | $E_7$ | $E_8$ |
|---|---|---|---|---|---|
| dim $R_1$ | 10 | 16 | 27 | 56 | 248 |

*d*−dim.
external



Exceptional space

# Generalized metric

$$(\mathcal{H}_{MN}) = \begin{pmatrix} (g - B\,g^{-1}\,B)_{mn} & B_{mp}\,g^{pn} \\ -g^{mp}\,B_{pn} & g^{mn} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & B \\ 0 & \mathbf{1} \end{pmatrix} \begin{pmatrix} g & 0 \\ 0 & g^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{1} & 0 \\ -B & \mathbf{1} \end{pmatrix}$$

$$\Downarrow$$

$$\mathcal{M}_{MN} = (\cdots L_6^{\mathrm{T}}\,L_4^{\mathrm{T}}\,L_2^{\mathrm{T}}\,L_0^{\mathrm{T}}\,\underset{\uparrow}{\hat{\mathcal{M}}}\,\underset{\underset{(\Phi,\,C_0)}{\nwarrow}}{L_0}\,\overset{\overset{(B_2,\,C_2)}{\swarrow}}{L_2}\,\underset{\underset{C_4}{\nwarrow}}{L_4}\,\overset{\overset{(B_6,\,C_6)}{\swarrow}}{L_6}\cdots)_{MN}$$

**Einstein-frame metric**

# EFT action

[Hohm, Samtleben ’13; ⋯.]

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{pot}} + \cdots\,,$$

$$\left(\begin{aligned} e &\equiv \sqrt{-\det g_{\mu\nu}} \\ g_{\mu\nu} &\equiv |\det g_{ij}|^{\frac{1}{d-2}}\, \mathsf{g}_{\mu\nu} \end{aligned}\right)$$

$$\begin{aligned}
\mathcal{L}_{\text{EH}} &= eR\,,\\
\mathcal{L}_{\text{scalar}} &= \frac{e}{4\alpha_n}\, g^{\mu\nu}\, \partial_\mu \mathcal{M}_{MN}\, \partial_\nu \mathcal{M}^{MN}\,,\\
\mathcal{L}_{\text{pot}} &= \frac{e}{4\alpha_n}\, \mathcal{M}^{MN}\, \partial_M \mathcal{M}^{PQ}\, \partial_N \mathcal{M}_{PQ} - \frac{e}{2}\, \mathcal{M}^{MN}\, \partial_N \mathcal{M}^{PQ}\, \partial_Q \mathcal{M}_{MP} + e\, \partial_M \ln e\, \partial_N \mathcal{M}^{MN}\\
&\quad + e\, \mathcal{M}^{MN}\, \partial_M \ln e\, \partial_N \ln e + \frac{e}{4}\, \mathcal{M}^{MN}\, \partial_M g^{\mu\nu}\, \partial_N g_{\mu\nu}\,.
\end{aligned}$$

$$\Downarrow$$

## Type IIB SUGRA action

# Section condition in EFT

**$E_n$ EFT** $$\eta^{MN;\,\mathrm{I}}\,\partial_M\,\partial_N=0\,,\qquad \Omega^{MN}\,\partial_M\,\partial_N=0\,.$$

- $$(x^M)=(x^m,\ \overset{\textbf{F1/D1}}{\cancel{y^{\alpha}_{m}}},\ \overset{\textbf{D3}}{\cancel{y_{m_1m_2m_3}}},\ \overset{\textbf{NS5/D5}}{\cancel{y^{\alpha}_{m_1\cdots m_5}}},\ \cancel{\cdots})$$

  $$(m=1,\dots,n-1)$$

  **Type IIB section**

  [Blair, Malek, J.-H. Park ’13; Hohm, Samtleben ‘13]

- $$(x^M)=(x^i,\ \overset{\textbf{M2}}{\cancel{y_{i_1i_2}}},\ \overset{\textbf{M5}}{\cancel{y_{i_1\cdots i_5}}},\ \overset{\textbf{KKM}}{\cancel{y_{i_1\cdots i_7,i}}},\ \cancel{\cdots})$$

  $$(i=1,\dots,n)$$

  **M-theory section**

# EFT action

[Hohm, Samtleben ’13; ⋯.]

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{EH}} + \mathcal{L}_{\mathrm{scalar}} + \mathcal{L}_{\mathrm{pot}} + \cdots\,,$$

$$\left(\begin{array}{c} e \equiv \sqrt{-\det g_{\mu\nu}} \\ g_{\mu\nu} \equiv \left|\det g_{ij}\right|^{\frac{1}{d-2}}\,\mathrm{g}_{\mu\nu} \end{array}\right)$$

$$\begin{aligned}
\mathcal{L}_{\mathrm{EH}} &= eR\,,\\
\mathcal{L}_{\mathrm{scalar}} &= \frac{e}{4\alpha_n}\,g^{\mu\nu}\,\partial_\mu\mathcal{M}_{MN}\,\partial_\nu\mathcal{M}^{MN}\,,\\
\mathcal{L}_{\mathrm{pot}} &= \frac{e}{4\alpha_n}\,\mathcal{M}^{MN}\,\partial_M\mathcal{M}^{PQ}\,\partial_N\mathcal{M}_{PQ} - \frac{e}{2}\,\mathcal{M}^{MN}\,\partial_N\mathcal{M}^{PQ}\,\partial_Q\mathcal{M}_{MP} + e\,\partial_M\ln e\,\partial_N\mathcal{M}^{MN}\\
&\quad + e\,\mathcal{M}^{MN}\,\partial_M\ln e\,\partial_N\ln e + \frac{e}{4}\,\mathcal{M}^{MN}\,\partial_M g^{\mu\nu}\,\partial_N g_{\mu\nu}\,.
\end{aligned}$$

$$\Downarrow$$

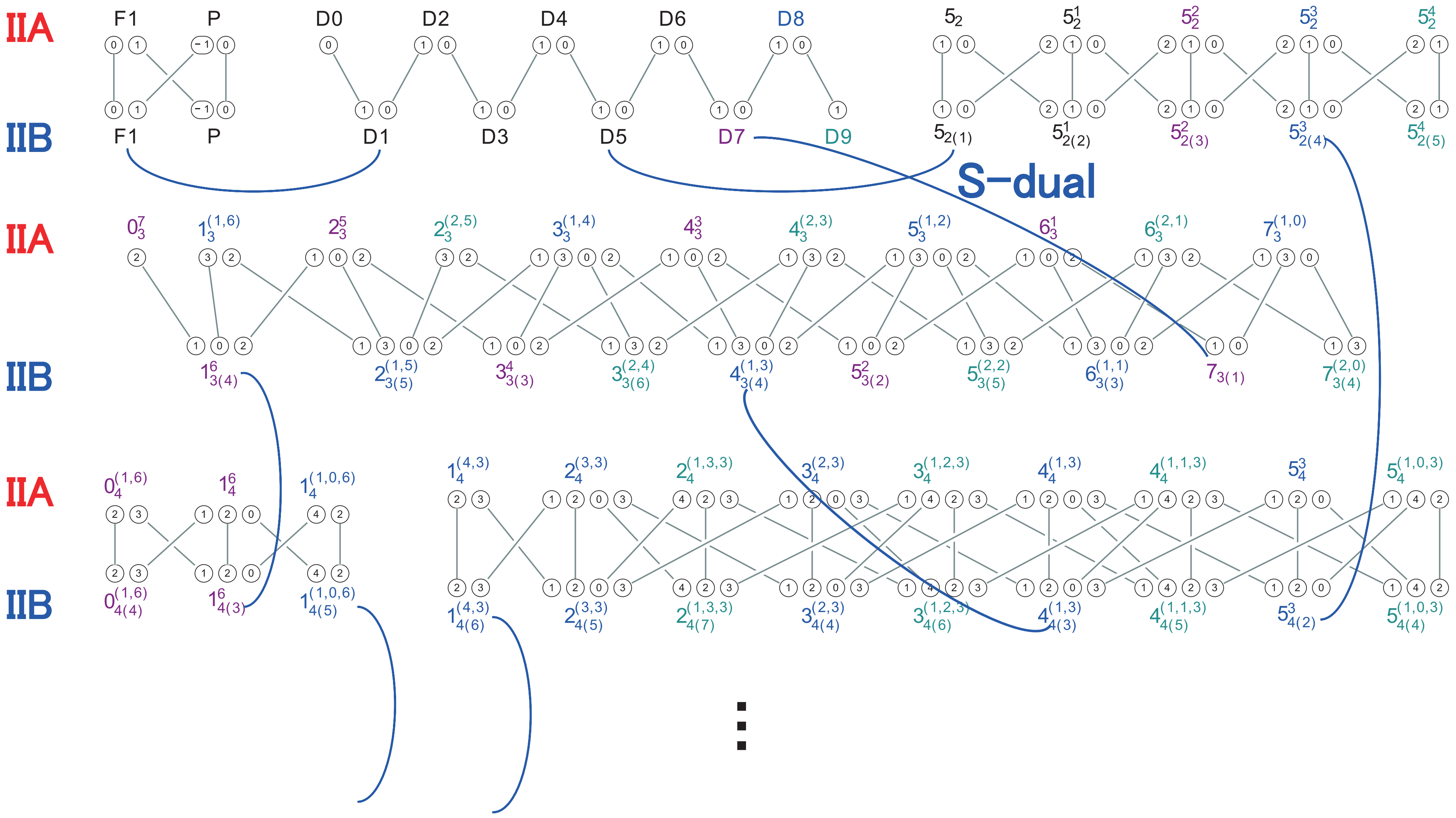
## 11D SUGRA action

$A_3$

$A_6$

$$\mathcal{M}_{MN} = \left(\cdots L_6^{\mathrm{T}}\,L_3^{\mathrm{T}}\,\hat{\mathcal{M}}\,L_3\,L_6\cdots\right)_{MN}$$

# Type II theory, many branes are connected by T-/S-dualities



# Type IIB branes

## Type IIB / $T^7$

[Kimura, Fernandez-Melgarejo, YS '18]

$$
\begin{gathered}
\boxed{\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}\,,\ \mathrm{D7}\,,\ \mathrm{D9}\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}}\,,\ 5^2_2\,,\ 5^3_2\,,\ 5^4_2\,,\ 7_3\,,\ 5^2_3\,,\ 3^4_3\,,\ 1^6_3\,, \\
6^{(1,1)}_3\,,\ 4^{(1,3)}_3\,,\ 2^{(1,5)}_3\,,\ 7^{(2,0)}_3\,,\ 5^{(2,2)}_3\,,\ 3^{(2,4)}_3\,,\ 1^6_4\,,\ 0^{(1,6)}_4\,,\ 1^{(1,0,6)}_4\,,\ 5^3_4\,,\ 4^{(1,3)}_4\,,\ 3^{(2,3)}_4\,, \\
2^{(3,3)}_4\,,\ 1^{(4,3)}_4\,,\ 5^{(1,0,3)}_4\,,\ 4^{(1,1,3)}_4\,,\ 3^{(1,2,3)}_4\,,\ 2^{(1,3,3)}_4\,,\ 9_4\,,\ 7^{(2,0)}_4\,,\ 5^{(4,0)}_4\,,\ 3^{(6,0)}_4\,, \\
2^{(1,5)}_5\,,\ 2^{(3,3)}_5\,,\ 2^{(5,1)}_5\,,\ 1^{(1,0,6)}_5\,,\ 1^{(1,2,4)}_5\,,\ 1^{(1,4,2)}_5\,,\ 1^{(1,6,0)}_5\,,\ 2^{(1,0,0,6)}_5\,,\ 2^{(1,0,2,4)}_5\,, \\
2^{(1,0,4,2)}_5\,,\ 2^{(1,0,6,0)}_5\,,\ 5^4_5\,,\ 5^{(2,2)}_5\,,\ 5^{(4,0)}_5\,,\ 4^{(1,1,3)}_5\,,\ 4^{(1,3,1)}_5\,,\ 3^{(2,0,4)}_5\,,\ 3^{(2,2,2)}_5\,,\ 3^{(2,4,0)}_5\,, \\
2^{(3,1,3)}_5\,,\ 2^{(3,3,1)}_5\,,\ 1^{(4,3)}_6\,,\ 1^{(1,4,2)}_6\,,\ 1^{(2,4,1)}_6\,,\ 1^{(3,4,0)}_6\,,\ 3^{(2,4)}_6\,,\ 3^{(1,2,3)}_6\,,\ 3^{(2,2,2)}_6\,, \\
3^{(3,2,1)}_6\,,\ 3^{(4,2,0)}_6\,,\ 2^{(1,0,2,4)}_6\,,\ 2^{(1,1,2,3)}_6\,,\ 2^{(1,2,2,2)}_6\,,\ 2^{(1,3,2,1)}_6\,,\ 2^{(1,4,2,0)}_6\,,\ 1^{(1,6,0)}_7\,, \\
1^{(3,4,0)}_7\,,\ 1^{(5,2,0)}_7\,,\ 1^{(7,0,0)}_7\,,\ 3^{(6,0)}_7\,,\ 3^{(2,4,0)}_7\,,\ 3^{(4,2,0)}_7\,,\ 3^{(6,0,0)}_7\,,\ 2^{(1,0,1,5,0)}_7\,,\ 2^{(1,0,3,3,0)}_7\,, \\
2^{(1,0,5,1,0)}_7\,,\ 2^{(1,3,3)}_7\,,\ 2^{(3,1,3)}_7\,,\ 2^{(1,0,4,2)}_7\,,\ 2^{(1,2,2,2)}_7\,,\ 2^{(1,4,0,2)}_7\,,\ 2^{(2,1,3,1)}_7\,,\ 2^{(2,3,1,1)}_7\,, \\
2^{(3,0,4,0)}_7\,,\ 2^{(3,2,2,0)}_7\,,\ 2^{(3,4,0,0)}_7\,,\ 1^{(7,0,0)}_8\,,\ 2^{(1,0,6,0)}_8\,,\ 2^{(3,0,4,0)}_8\,,\ 2^{(5,0,2,0)}_8\,,\ 2^{(7,0,0,0)}_8\,, \\
2^{(3,3,1)}_8\,,\ 2^{(1,3,2,1)}_8\,,\ 2^{(2,3,1,1)}_8\,,\ 2^{(3,3,0,1)}_8\,,\ 2^{(1,0,3,3,0)}_8\,,\ 2^{(1,1,3,2,0)}_8\,,\ 2^{(1,2,3,1,0)}_8\,,\ 2^{(1,3,3,0,0)}_8\,, \\
2^{(1,4,2,0)}_9\,,\ 2^{(3,2,2,0)}_9\,,\ 2^{(5,0,2,0)}_9\,,\ 2^{(1,0,5,1,0)}_9\,,\ 2^{(1,2,3,1,0)}_9\,,\ 2^{(1,4,1,1,0)}_9\,,\ 2^{(2,1,4,0,0)}_9\,, \\
2^{(2,3,2,0,0)}_9\,,\ 2^{(2,5,0,0,0)}_9\,,\ 2^{(3,4,0,0)}_{10}\,,\ 2^{(1,3,3,0,0)}_{10}\,,\ 2^{(2,3,2,0,0)}_{10}\,,\ 2^{(3,3,1,0,0)}_{10}\,,\ 2^{(4,3,0,0,0)}_{10}\,, \\
2^{(7,0,0,0)}_{11}\,,\ 2^{(2,5,0,0,0)}_{11}\,,\ 2^{(4,3,0,0,0)}_{11}\,,\ 2^{(6,1,0,0,0)}_{11}\,.
\end{gathered}
$$

defect branes, domain-wall branes, space-filling branes

# Domain–walls

**In [Kimura, Fernandez–Melgarejo, YS '18]**

**all of the domain-wall brane backgrounds**

**were constructed as solutions of EFT.**

**They have a linear dual-coordinate dependence.**

**solution of a certain deformed SUGRA.**

**much like D8-brane solution**

$$\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}\,,\ \mathrm{D7}\,,\ \mathrm{D9}\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5_2^2\,,\ 5_2^3\,,\ 5_2^4\,,\ 7_3\,,\ 5_3^2\,,\ 3_3^4\,,\ 1_3^6\,,$$

$$6_3^{(1,1)}\,,\ 4_3^{(1,3)}\,,\ 2_3^{(1,5)}\,,\ 7_3^{(2,0)}\,,\ 5_3^{(2,2)}\,,\ 3_3^{(2,4)}\,,\ 1_4^6\,,\ 0_4^{(1,6)}\,,\ 1_4^{(1,0,6)}\,,\ 5_4^3\,,\ 4_4^{(1,3)}\,,\ 3_4^{(2,3)}\,,$$

$$2_4^{(3,3)}\,,\ 1_4^{(4,3)}\,,\ 5_4^{(1,0,3)}\,,\ 4_4^{(1,1,3)}\,,\ 3_4^{(1,2,3)}\,,\ 2_4^{(1,3,3)}\,,\ 9_4\,,\ 7_4^{(2,0)}\,,\ 5_4^{(4,0)}\,,\ 3_4^{(6,0)}\,,$$

$$2_5^{(1,5)}\,,\ 2_5^{(3,3)}\,,\ 2_5^{(5,1)}\,,\ 1_5^{(1,0,6)}\,,\ 1_5^{(1,2,4)}\,,\ 1_5^{(1,4,2)}\,,\ 1_5^{(1,6,0)}\,,\ 2_5^{(1,0,0,6)}\,,\ 2_5^{(1,0,2,4)}\,,$$

$$2_5^{(1,0,4,2)}\,,\ 2_5^{(1,0,6,0)}\,,\ 5_5^4\,,\ 5_5^{(2,2)}\,,\ 5_5^{(4,0)}\,,\ 4_5^{(1,1,3)}\,,\ 4_5^{(1,3,1)}\,,\ 3_5^{(2,0,4)}\,,\ 3_5^{(2,2,2)}\,,\ 3_5^{(2,4,0)}\,,$$

$$2_5^{(3,1,3)}\,,\ 2_5^{(3,3,1)}\,,\ 1_6^{(4,3)}\,,\ 1_6^{(1,4,2)}\,,\ 1_6^{(2,4,1)}\,,\ 1_6^{(3,4,0)}\,,\ 3_6^{(2,4)}\,,\ 3_6^{(1,2,3)}\,,\ 3_6^{(2,2,2)}\,,$$

$$3_6^{(3,2,1)}\,,\ 3_6^{(4,2,0)}\,,\ 2_6^{(1,0,2,4)}\,,\ 2_6^{(1,1,2,3)}\,,\ 2_6^{(1,2,2,2)}\,,\ 2_6^{(1,3,2,1)}\,,\ 2_6^{(1,4,2,0)}\,,\ 1_7^{(1,6,0)}\,,$$

$$1_7^{(3,4,0)}\,,\ 1_7^{(5,2,0)}\,,\ 1_7^{(7,0,0)}\,,\ 3_7^{(6,0)}\,,\ 3_7^{(2,4,0)}\,,\ 3_7^{(4,2,0)}\,,\ 3_7^{(6,0,0)}\,,\ \ldots$$

# Example

**$2_5^{(3,3)}$–brane solution :**

$$\mathrm{d}\tilde{s}^2 = H^{3/2}\,\mathrm{d}x_{012}^2 + \tau_2^{-1/2}\,\mathrm{d}x_{345}^2 + H^{1/2}\,\mathrm{d}x_{678}^2 + H^{5/2}\,\mathrm{d}x_9^2\,,$$

$$\mathrm{e}^{-2\tilde{\phi}} = H^{-1}\,, \qquad \beta^{3\cdots 8} = m\,y_{345}\,.$$

dual coordinate

of D3–brane

**Redefinition of $B_6$**

---

$$\mathcal{M}_{MN} = [L^{\mathrm{T}}(y_{345})\,\hat{\mathcal{M}}(x^0, x^1, x^2, \cancel{x^3, x^4, x^5}, x^6, \cdots, x^9)\,L(y_{345})]_{MN}$$

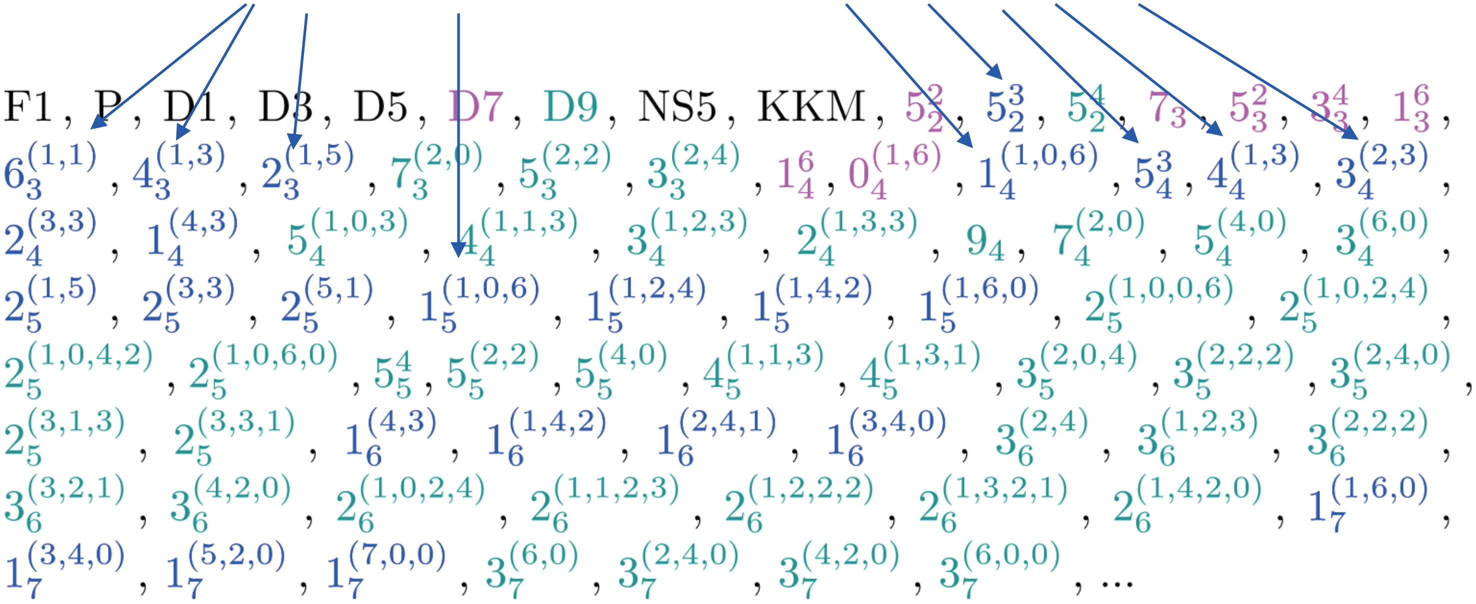
$$\Big\Downarrow\ \mathcal{L}_{\mathrm{EFT}}$$

**We can (in principle) obtain**

**the action of a deformed SUGRA.**

# Domain−walls

**Similarly, we can obtain deformed SUGRAs for all of the domain-wall branes.**



$$\mathrm{F1}\,,\ \mathrm{P}\,,\ \mathrm{D1}\,,\ \mathrm{D3}\,,\ \mathrm{D5}\,,\ \mathrm{D7}\,,\ \mathrm{D9}\,,\ \mathrm{NS5}\,,\ \mathrm{KKM}\,,\ 5_2^2\,,\ 5_2^3\,,\ 5_2^4\,,\ 7_3\,,\ 5_3^2\,,\ 3_3^4\,,\ 1_3^6\,,$$

$$6_3^{(1,1)}\,,\ 4_3^{(1,3)}\,,\ 2_3^{(1,5)}\,,\ 7_3^{(2,0)}\,,\ 5_3^{(2,2)}\,,\ 3_3^{(2,4)}\,,\ 1_4^6\,,\ 0_4^{(1,6)}\,,\ 1_4^{(1,0,6)}\,,\ 5_4^3\,,\ 4_4^{(1,3)}\,,\ 3_4^{(2,3)}\,,$$

$$2_4^{(3,3)}\,,\ 1_4^{(4,3)}\,,\ 5_4^{(1,0,3)}\,,\ 4_4^{(1,1,3)}\,,\ 3_4^{(1,2,3)}\,,\ 2_4^{(1,3,3)}\,,\ 9_4\,,\ 7_4^{(2,0)}\,,\ 5_4^{(4,0)}\,,\ 3_4^{(6,0)}\,,$$

$$2_5^{(1,5)}\,,\ 2_5^{(3,3)}\,,\ 2_5^{(5,1)}\,,\ 1_5^{(1,0,6)}\,,\ 1_5^{(1,2,4)}\,,\ 1_5^{(1,4,2)}\,,\ 1_5^{(1,6,0)}\,,\ 2_5^{(1,0,0,6)}\,,\ 2_5^{(1,0,2,4)}\,,$$

$$2_5^{(1,0,4,2)}\,,\ 2_5^{(1,0,6,0)}\,,\ 5_5^4\,,\ 5_5^{(2,2)}\,,\ 5_5^{(4,0)}\,,\ 4_5^{(1,1,3)}\,,\ 4_5^{(1,3,1)}\,,\ 3_5^{(2,0,4)}\,,\ 3_5^{(2,2,2)}\,,\ 3_5^{(2,4,0)}\,,$$

$$2_5^{(3,1,3)}\,,\ 2_5^{(3,3,1)}\,,\ 1_6^{(4,3)}\,,\ 1_6^{(1,4,2)}\,,\ 1_6^{(2,4,1)}\,,\ 1_6^{(3,4,0)}\,,\ 3_6^{(2,4)}\,,\ 3_6^{(1,2,3)}\,,\ 3_6^{(2,2,2)}\,,$$

$$3_6^{(3,2,1)}\,,\ 3_6^{(4,2,0)}\,,\ 2_6^{(1,0,2,4)}\,,\ 2_6^{(1,1,2,3)}\,,\ 2_6^{(1,2,2,2)}\,,\ 2_6^{(1,3,2,1)}\,,\ 2_6^{(1,4,2,0)}\,,\ 1_7^{(1,6,0)}\,,$$

$$1_7^{(3,4,0)}\,,\ 1_7^{(5,2,0)}\,,\ 1_7^{(7,0,0)}\,,\ 3_7^{(6,0)}\,,\ 3_7^{(2,4,0)}\,,\ 3_7^{(4,2,0)}\,,\ 3_7^{(6,0,0)}\,,\ \ldots$$

# Summary

YB-deformation/NATD can generate solutions of GSE or massive IIA SUGRA.

Deformed SUGRAs can be derived from DFT by taking non-canonical sections.

By considering EFT,
we can consider more non-canonical sections,
and various deformed SUGRAs can be derived.