Aperiodic tilings derived from the Ammann-Beenker tiling

Joichiro Nakakura

Department of Physics, Kindai University, Higashi-Osaka 577-8502, Japan
e-mail: jnakakura@kindai.ac.jp

Until recently, the most characteristic feature of quasicrystals is their nontraditional rotational symmetry such as icosahedral, dodecagonal, decagonal, or octagonal one. It has been believed that the symmetry and the aperiodicity of these materials stem from an irrational ratio of two or more length scales controlling their structure, the best-known examples being the Penrose and the Ammann-Beenker tiling as two-dimensional models related to the golden and the silver mean, respectively. However, a notable counter example is the recent discovery of the bronze-mean quasicrystalline tiling because of its hexagonal rotational symmetry, which is usually regarded as a strong indication of "periodic" hexagonal order [1].

In this presentation, we propose a way to construct infinitely many 4-fold quasicrystalline patterns with various inflation factors composed of three types of tiles: small squares, large squares, and parallelograms by extending the Ammann-Beenker tiling. Their patterns include a sequence of quasiperiodic tilings with inflation factors of metallic means of multiples of 2: $(k+\sqrt{k^2+4})/2$ $(k=2,4,6,\dots)$ as shown in Fig.1. Remarkably, it converges to a square lattice in the limit of the metallic mean at infinity.

These even-numbered metallic-mean tilings can be regarded as "aperiodic approximants" of a square lattice. In addition to aperiodic approximants with 6-fold symmetry [2], the results show that two fundamental crystalline (hexagonal and square) lattices are recovered from these aperiodic approximants, complementing the well-established notion of crystalline approximants.

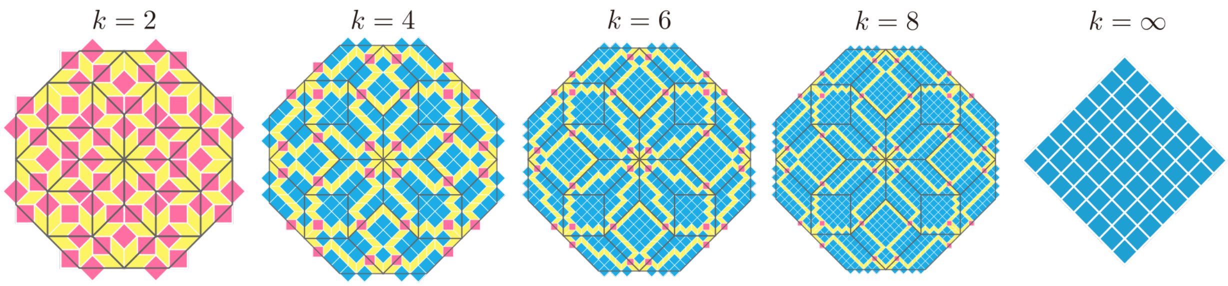

Fig.1 Metallic-mean aperiodic approximants. k-th metallic-mean quasicrystalline patterns with $k=2$ (Ammann-Beenker tiling), $k=4$, $k=6$, $k=8$ and $k\to\infty$, respectively.

[1] T. Dotera, S. Bekku, P. Ziherl, Nat. Mater. **16**, 987 (2017).
[2] T. Dotera, J. Nakakura, J. Matsuzwawa, P. Ziherl in ICQ14 (2019).