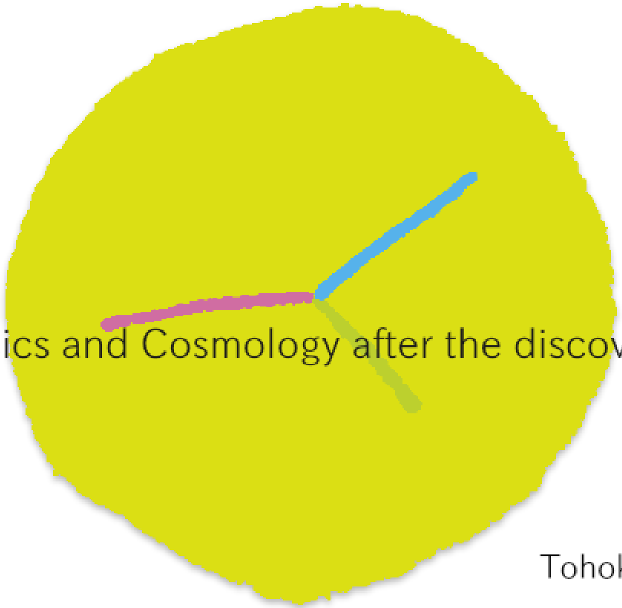
Top-quark Theory at e^+e^- Colliders

Hiroshi Yokoya (University of Toyama)



Particle Physics and Cosmology after the discovery of Higgs boson

Tohoku Forum for Creativity

Outline :

1. Introduction
2. Mass
3. NRQCD calc.
4. New Physics

1. Introduction
2. Mass
3. NRQCD calc.
4. New Physics

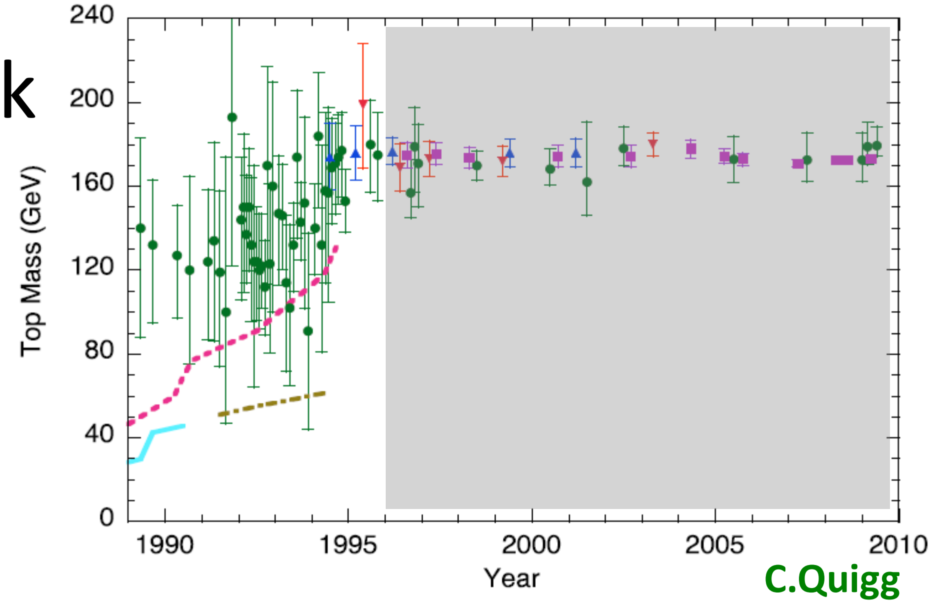
Pre-discovery of Top-quark

- Direct searches
  (…,TRISTAN,…,SppS,Tevatron)

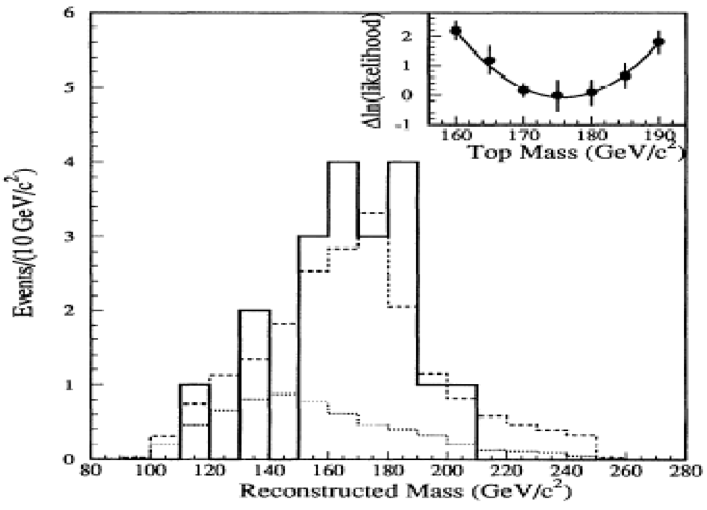
  1977 bottom-quark
  (80’s m_t ~ 15 to 25 GeV ?)
  1990 $m_t > 70$ GeV (SppS)
  $m_t > 89$ GeV (Tevatron)
  1995 discovery (Tevatron)

- Indirect estimate of the mass by
  EW precision data (LEP), GIM violation

  $m_t < 200$ GeV



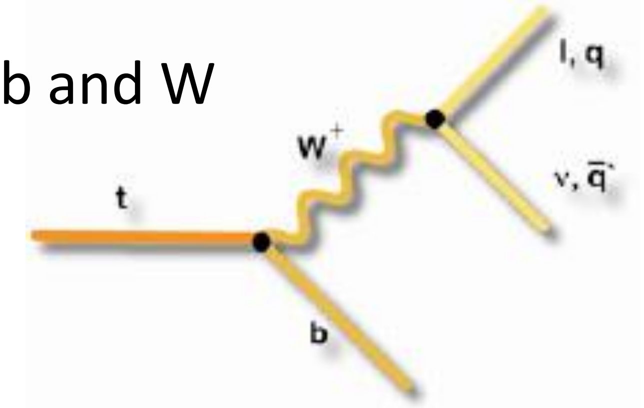
C.Quigg



Top-quark as an ultraheavy quark

Bigi,Dokshitzer,Khoze,
Kuhn,Zerwas 86,,,

- Around '90, it was realized that top-quark is extremely heavy, so that top can decay into b and W



$$\Gamma_t \sim \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 >> \Lambda_{QCD}$$

→ top-quark decays before hadronization,
decay products of single quark are accessible (at least paton-level)

Extraction of mass, spin etc from its decay products

→ top-quark production near threshold (non-relativistic region) can be treated still perturbatively. (Γ_t as IR-cutoff)

Development of NRQCD, Heavy Quark Effective Theory

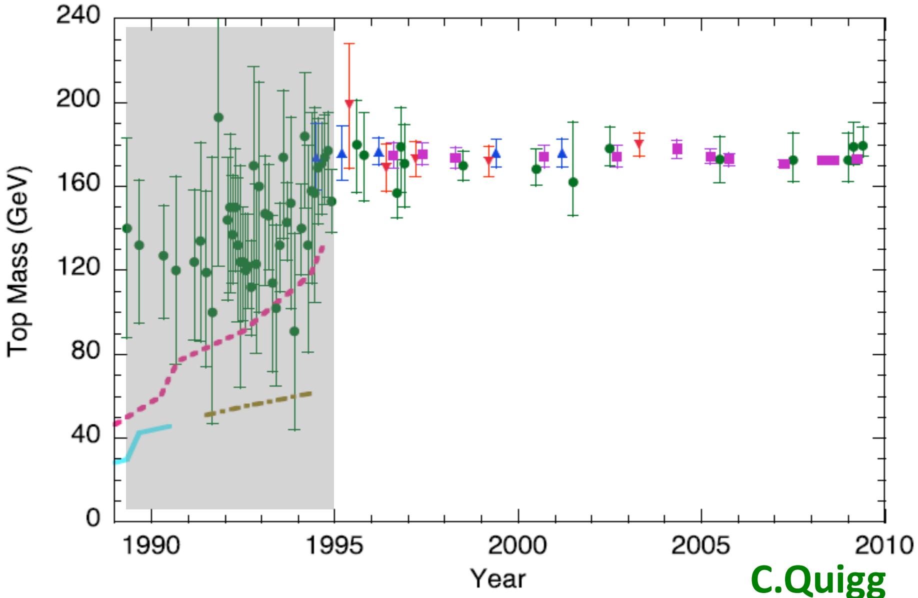
Top-quark mass

- Kinematical measurement

Tevatron : 173.2±0.5±0.7 GeV

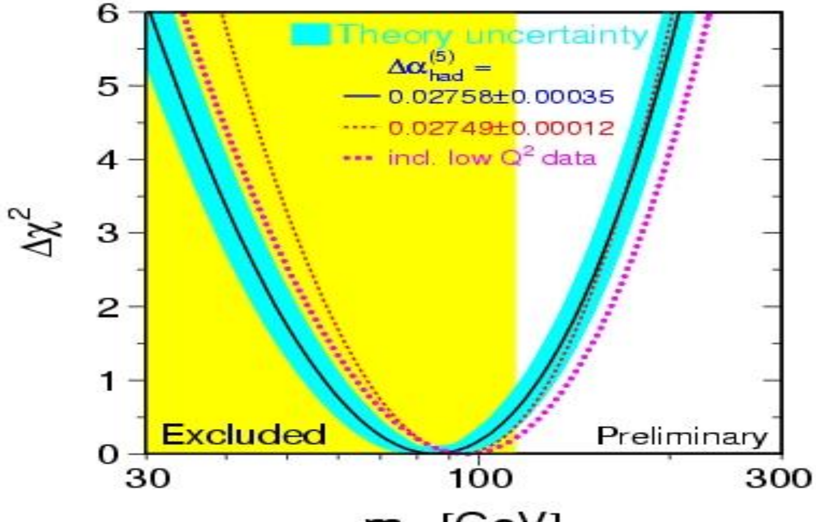
LHC : 173.3±0.3±0.9 GeV

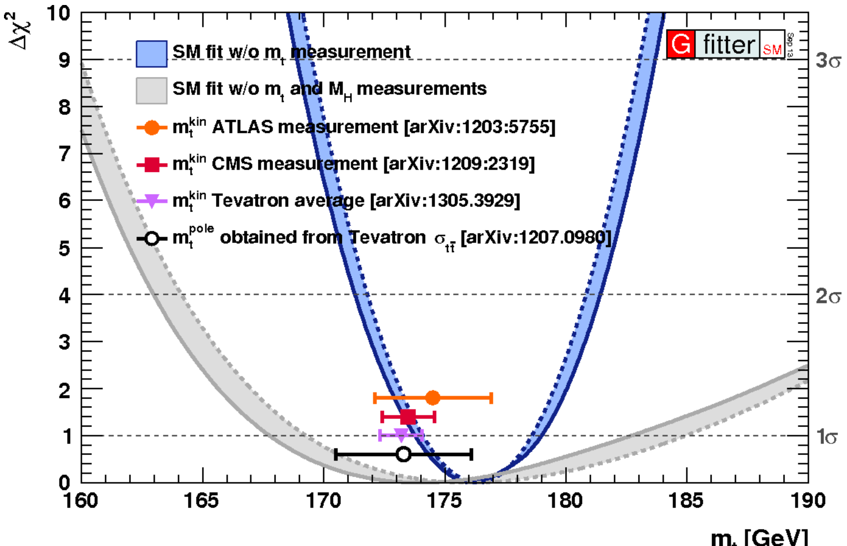
LHC September 2013



C.Quigg

- Consistency with EW precision measurement





Top-quark properties so far

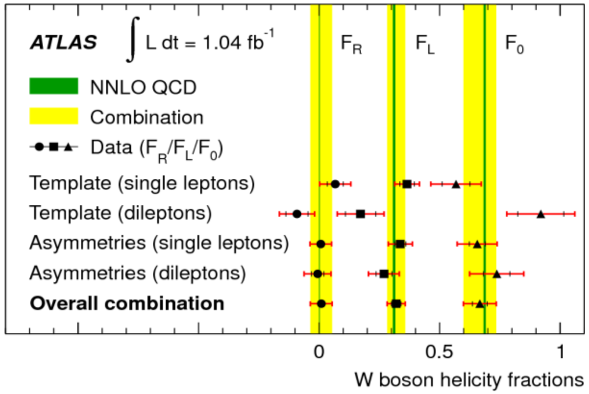
✓mass : 173.2±0.9 GeV

✓width: 2.0±0.5 GeV (theo. pred. ~1.5 GeV)

✓decay: Br(Wq{=d,s,b})~100%,
Br(Wb)/Br(Wq)=0.91±0.04
constraints on Br(γq), Br(Zq), Br(Hc)

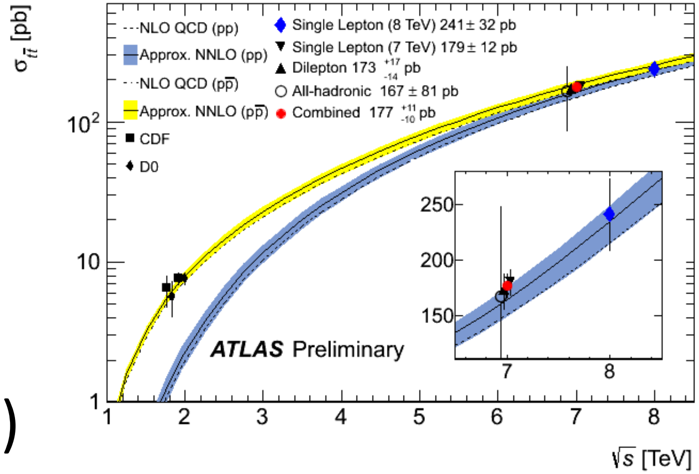
✓charge: +2/3e more preferred than -4/3e

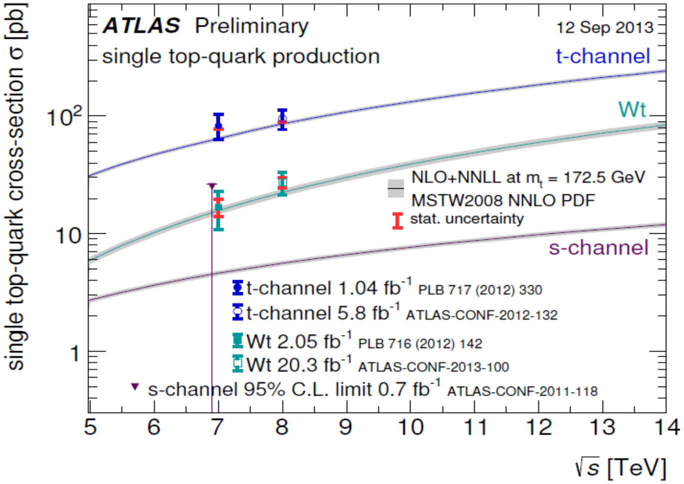
✓W-boson helicity:



F_0~0.7, long. pol.
= NG boson

✓cross-sections:



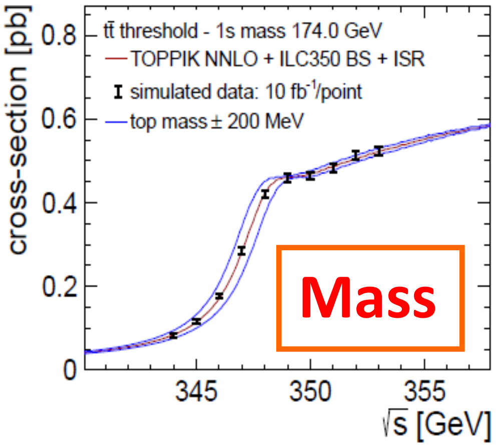


Top-quark production at the ILC

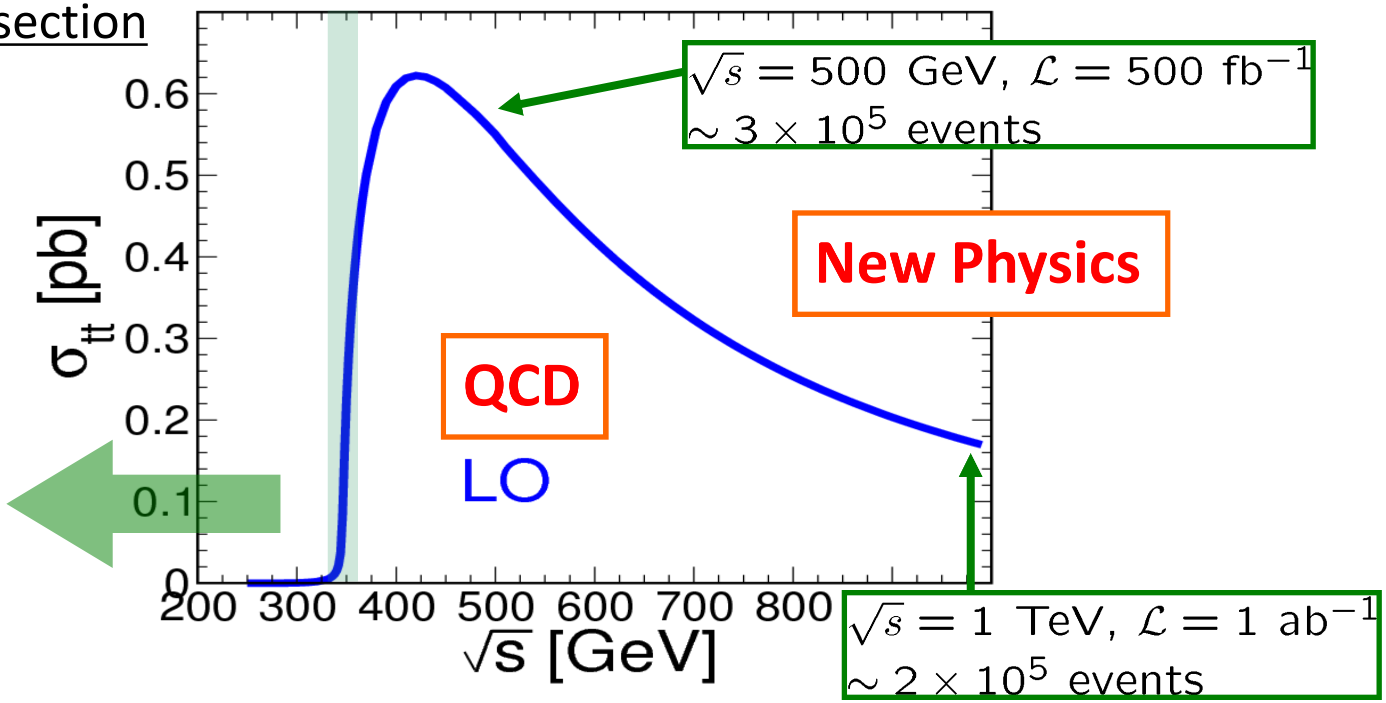
- /s = 250GeV → not allowed in pair, $t\bar{q}$ production?
- /s ~ 350GeV (Threshold scan) → m_t, Γ_t, α_s, y_t precise measurement
- /s = 500GeV, 1TeV → ttH (y_t), couplings measurement, NP search

Pair production cross-section

Threshold region



Mass



$\sqrt{s} = 500$ GeV, $\mathcal{L} = 500$ fb^{-1}
$\sim 3 \times 10^5$ events

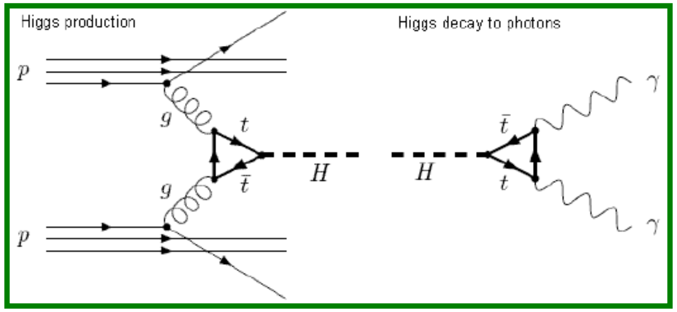
New Physics

QCD

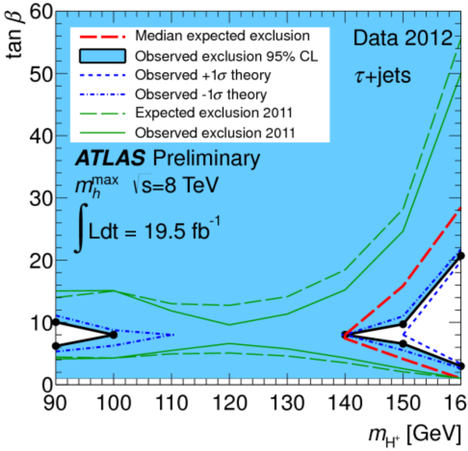
$\sqrt{s} = 1$ TeV, $\mathcal{L} = 1$ ab^{-1}
$\sim 2 \times 10^5$ events

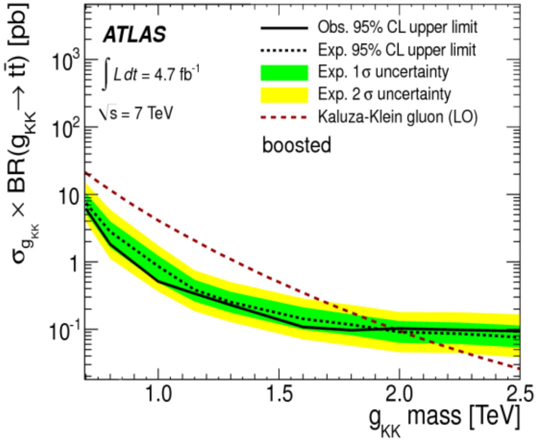
Top-quark as a Window for New Physics

- **Large Yukawa coupling to Higgs-boson** $y_t = \sqrt{2}m_t/v \simeq 1$
- Play an important role in Higgs physics



- Searches for new physics with top-quark
  - ➢ charged Higgs ($t \to bH^+$)
  - ➢ ttbar resonance (KK gluon)
  - ➢ …





Top and Higgs are the promised physics at the ILC

- **We know they exist, therefore these are critically important.**

- Precise mass measurement, ~100 times better accuracy than LHC
- As well, new physics searches can be performed
- Model predictions should better be clarified, like Higgs physics
  - ✓ On which observables the NP effects appear?
  - ✓ Are these large enough compared to experimental accuracy?
  - ✓ Is it possible to discriminate the models?

Mass **NRQCD** **New Physics**

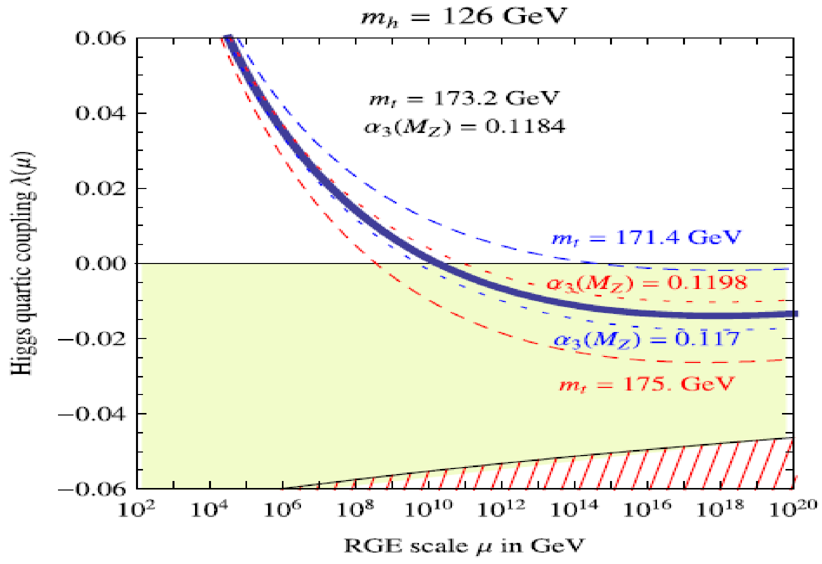
1. Introduction
2. Mass
3. NRQCD calc.
4. New Physics

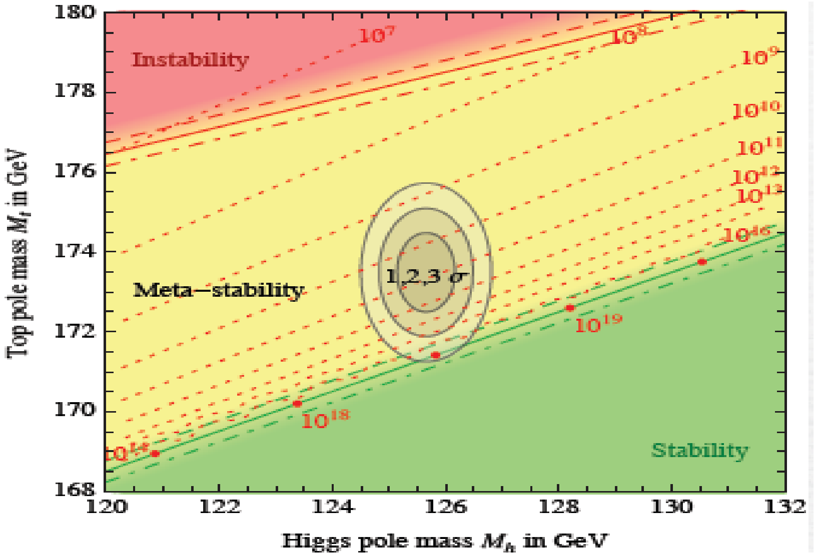
SM vacuum stability

- RGE of Higgs quartic coupling $16\pi^2\mu\frac{d\lambda}{d\mu} = 24\lambda^2 - 6y_t^4 + \cdots$

Top-quark mass is crucial for higher-scale behavior of the SM vacuum

Is it accidental or not? We need more accurate input of the top-quark mass





Top-quark mass

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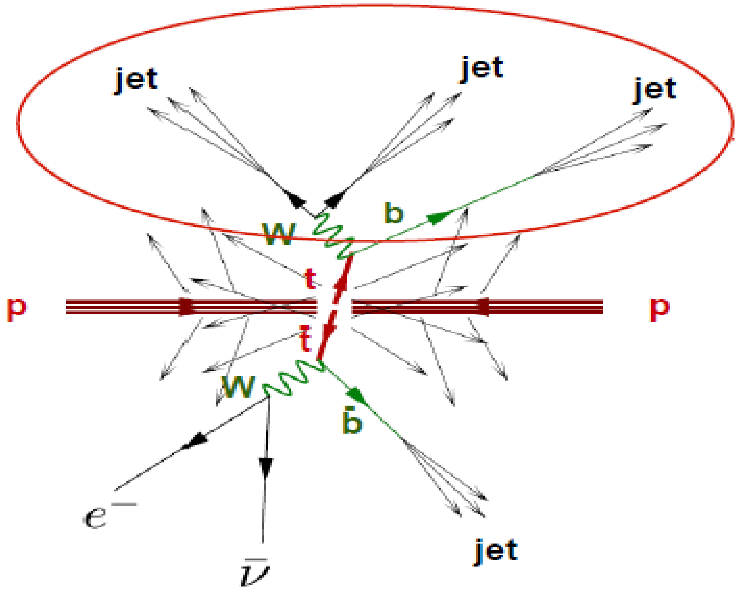
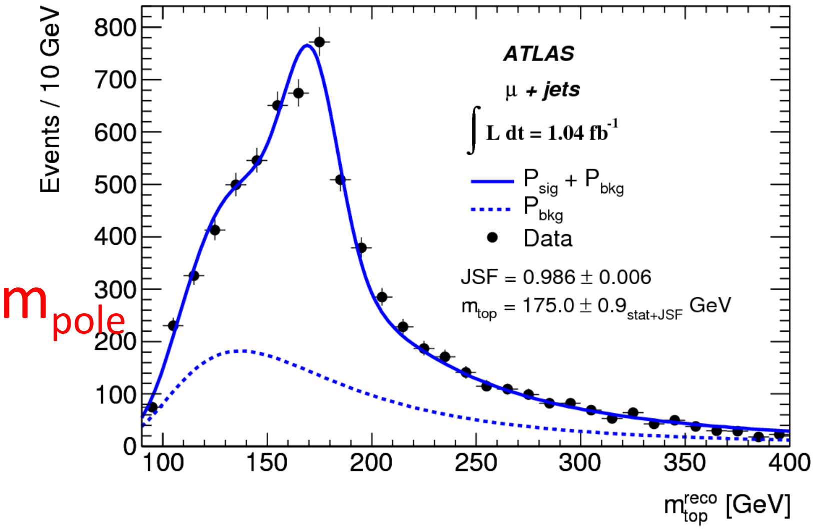


✓ Fit the observed distribution (m_{jjj}) with Templates.

✓ In principal, it could not be the true "pole mass" but "Pythia mass" $m_{jjj} \neq m_{pole}$

✓ Uncontrollable non-pert. QCD effects ~ O(1 GeV)



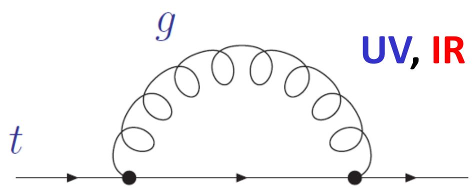
Pole mass vs. $\overline{\text{MS}}$ mass

Pole mass: pole position of the propagator

but, quark can never be free, on-shell pole is not well-defined,

bad convergence (IR sensitive)

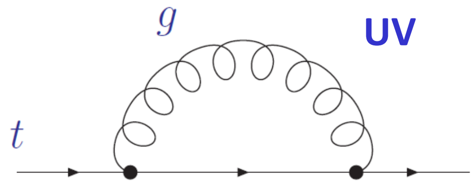
$$S_F(\not{p})^{-1} = \not{p} - m - \Sigma(\not{p}) \simeq \not{p} - m_{pole}$$



Short-distance mass ($\overline{\text{MS}}$,PS,1S):

just a parameter of the Lagrangian, better convergence

$$\delta m_{\overline{\text{MS}}} = \Sigma^{(loop)}(\not{p})|_{\frac{1}{\epsilon}-\gamma_E+\ln 4\pi}$$



Conversion between them is known perturbatively

$$M_{\text{pole}} = m(\mu)\left(1 + \alpha_s(\mu)d_1 + \alpha_s^2(\mu)d_2 + \cdots\right)$$

Hoang,Teubner 99
Kiyo,Sumino 02,,,

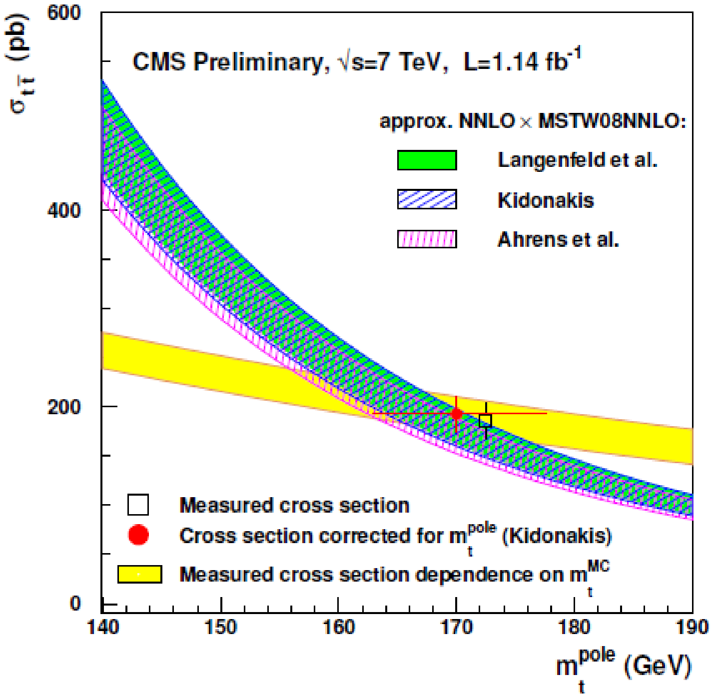
Mass from cross-section:

Compare theoretically calculated cross-section in the MS/pole and the observed cross-section at experiments.

$$\boxed{\frac{\delta m_t}{m_t} \simeq 0.2\,\frac{\delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}}$$

5% measurement

⇒ 1% accuracy in mass



$\overline{\mathrm{MS}}$ scheme is better convergence than pole mass scheme

Langenfeld,Moch,Uwer 09
Alekhin,Djouadi,Moch 13

Tevatron : M_{MS} = 163±3 [GeV]
LHC : M_{MS} = 166±3 [GeV]

(MSTW PDF)

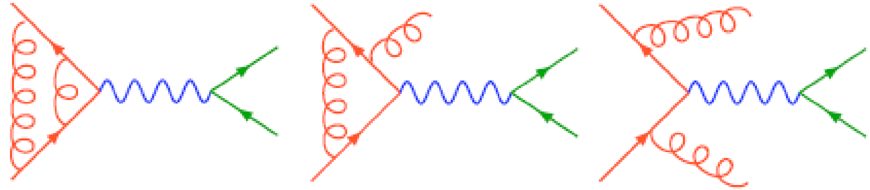
S.Alioli et al. 13, Dawling,Moch 13

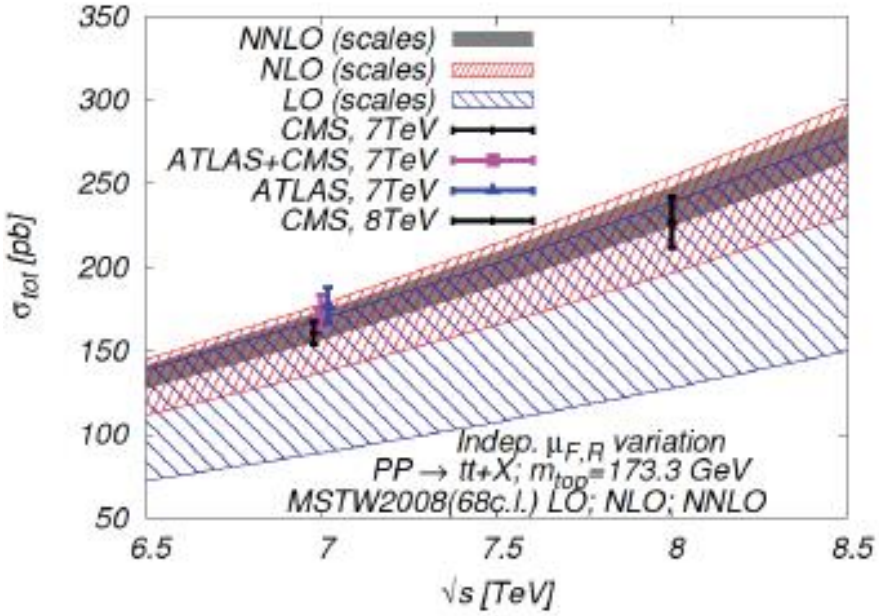
- Similar analysis may be studied by using distributions , ttj process, etc

NNLO cross-section completed at Hadron Colliders

…,Bernreuther,Czakon, Fiedler,Mitov 12, 13

$\hat{\sigma}^{(2)}$





NNLO

Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$+0.259(3.7\%)$ $-0.374(5.3\%)$	$+0.169(2.4\%)$ $-0.121(1.7\%)$
LHC 7 TeV	167.0	$+6.7(4.0\%)$ $-10.7(6.4\%)$	$+4.6(2.8\%)$ $-4.7(2.8\%)$
LHC 8 TeV	239.1	$+9.2(3.9\%)$ $-14.8(6.2\%)$	$+6.1(2.5\%)$ $-6.2(2.6\%)$
LHC 14 TeV	933.0	$+31.8(3.4\%)$ $-51.0(5.5\%)$	$+16.1(1.7\%)$ $-17.6(1.9\%)$

Concurrent uncertainties:

Scales	~ 3%
pdf (at 68%cl)	~ 2-3%
α_s (parametric)	~ 1.5%
m_{top} (parametric)	~ 3%

Alternative methods at Hadron Colliders

- ttj system invariant-mass,
- trilepton inv. mass using J/ψ,
- Endpoint distribution (M_{T2}),
- B-meson decay length

Aiming ~ 1 GeV accuracy, but the problems are always systematics (JES, ISR, PDF,,,)

- Weight function method using lepton energy distribution

Kawabata,Shimizu,Sumino,HY in progress

Define a convolution integral of **lepton energy distribution** and **a weight function**

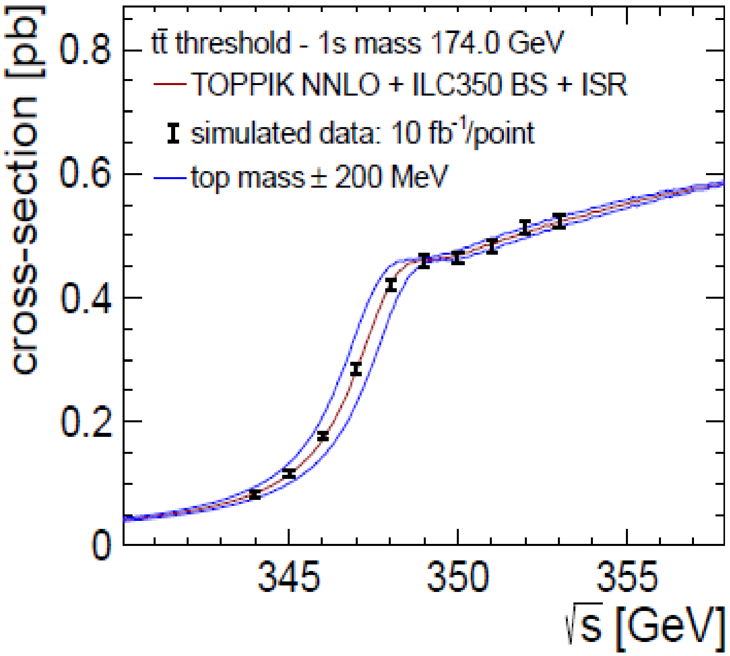
$$I(M) = \int dE_\ell D(E_\ell) W(E_\ell, M)$$

which satisfy $I(M = m_t) = 0$

- Lepton energy is accurately measured
- Suppress systematic uncertainty
- Effects of cuts are studied by MC

Threshold Scan at the ILC

Fujii,Matsui,Sumino 94, Martinez, Miquel 03, Seidel,Simon 12



- ✓ QCD calc. up to NN(N)LO
- ✓ Beam spectrum, ISR effects
- ✓ Beam polarization reduces BG events.
- ✓ Higgs-exchange effects → y_t measurement Horiguchi et al. 13

$$\delta m_t = 16 \text{ MeV}$$ ← PS-mass (can be converted to MS mass)

another δm~40MeV by conversion

$$\delta \Gamma_t = 21 \text{ MeV}$$

$$\delta y_t = 4.2\%$$ ← overall normalization rather uncertain

α_s fixed

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4. New Physics

Top-quark as an ultraheavy quark:

Bigi,Dokshitzer,Khoze,
Kuhn,Zerwas 86,,,

- Around '90, it was realized that top-quark is extremely heavy, $m_t > m_W + m_b$, so that t can decay into b and W

$$\Gamma_t \sim \frac{G_F m_t^3}{8\pi\sqrt{2}}|V_{tb}|^2 >> \Lambda_{\rm QCD}$$

→ top-quark decays before hadronization
decay products of single quark are accessible (at least paton-level)

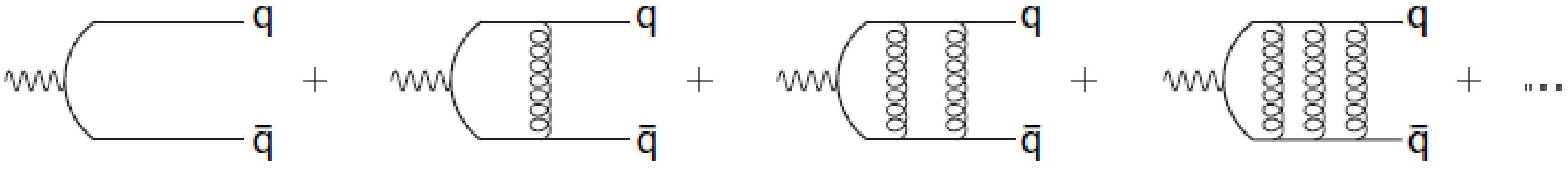
Extraction of mass, spin etc from its decay products

→ top-quark production near threshold (non-relativistic region)
can be treated still perturbatively. (Γ_t as IR-cutoff)

Development of NRQCD, Heavy Quark Effective Theory

Threshold production of top-quark pair:

- At threshold, non-relativistic approximation works



~ 1 $\sim \alpha_s/v$ $\sim (\alpha_s/v)^2$ $\sim (\alpha_s/v)^3$

α_s, v ~ small, but α_s/v ~ finite

- Schrodinger Equation for the 3-point Green function

Fadin,Khoze 87

$$\left[(E+i\Gamma_t)-\left\{-\frac{\nabla^2}{m_t}+V_{QCD}^{(c)}(r)\right\}\right]G^{(c)}(E,\vec{x})=\delta^3(\vec{x})$$

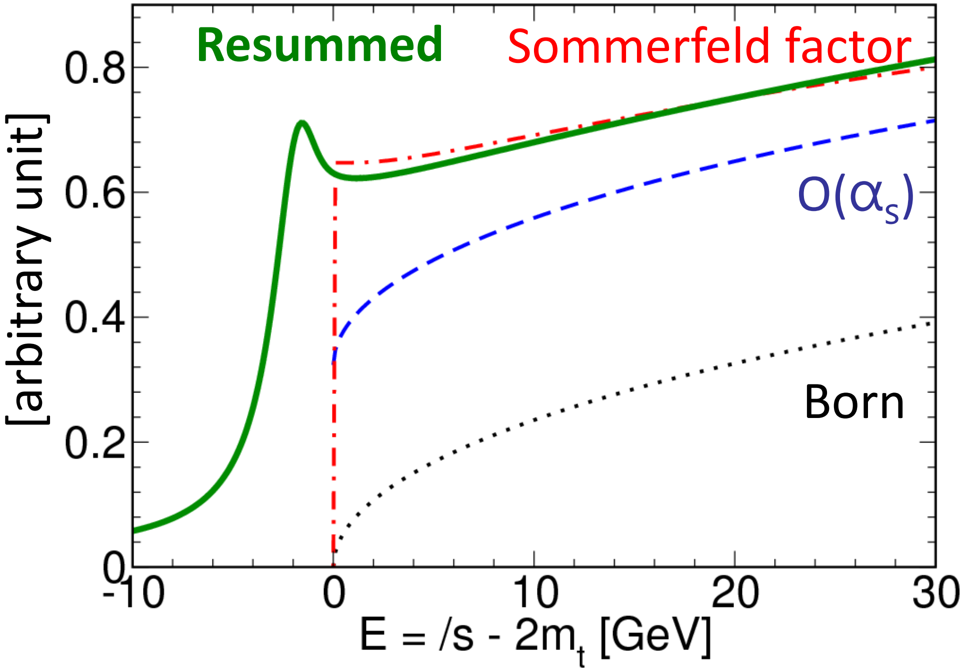
decay width in the imaginary part of the energy

QCD potential in short distance region, r < $1/\Gamma_t$

Threshold production:

$$\sigma_{\text{tot}} \propto \text{Im}[G(\vec{0}, E + i\Gamma_t)]$$

Fadin,Khoze 87, Strassler,Peskin 91, Kwong 91, Sumino et al. 93,,,,



- QCD corr. gives large enhancement near threshold
- One broad resonance remained due to the smearing.
- Peak position, shape are good observables to determine m_t, Γ_t, and also α_s.

Threshold production:

- At $e^+ e^-$ colliders, collision energy is fixed

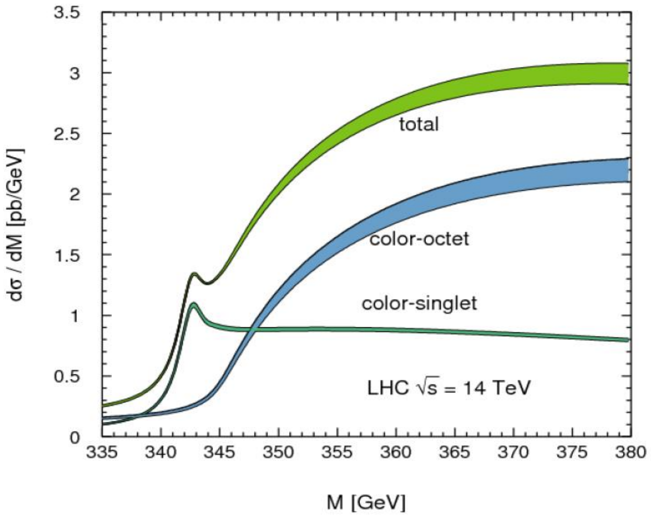
  → Threshold Scan is performed

  → Precise determination of m_t, Γ_t, α_s

Fujii,Matsui,Sumino 94, Martinez, Miquel 03, Seidel,Simon 12, Horiguchi,Ishikawa et al. 13

- At hadron colliders, collision energy is not fixed

  → seen only in the invariant-mass distribution of the ttbar system



- Color **singlet (attractive)** or **octet (repulsive)**
- Combine with ISR/FSR radiation (NLO,NNLL)
- Event generation with Coulomb effect
- ~1% effect in total cross-section

Fadin,Khoze,Sjostrand 90, Hagiwara,Sumino,HY 08, Kiyo et al. 08, Sumino,HY 10, Beneke et al. 11

(v)NRQCD formulation

- (perturbative QCD) $\sigma = \sigma^0(v) + \alpha_s \sigma^1(v) + \cdots$
- Fixed-order in NRQCD

$$\left(\frac{\alpha_s}{v}\right)^n \times \left(1, \{\alpha_s, v\}, \{\alpha_s^2, \alpha_s v, v^2\}, \cdots\right)$$

LO, NLO, NNLO, NNNLO,,

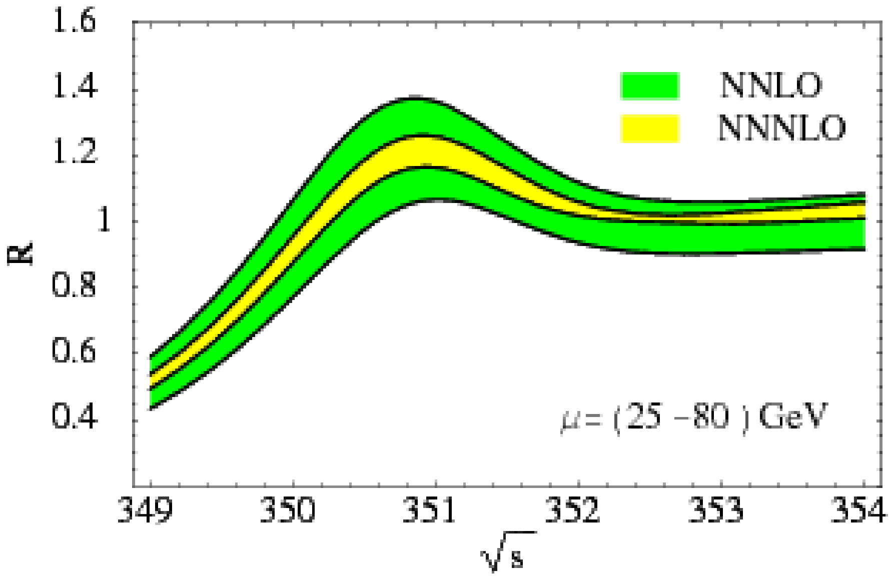
- Resum logarithmic corrections by RGE

$$\times \left([\alpha_s \ln v]^k,\ \alpha_s[\alpha_s \ln v]^k,\ \alpha_s^2[\alpha_s \ln v]^k, \cdots\right)$$

LL, NLL, NNLL,,,

NNLO → NNNLO

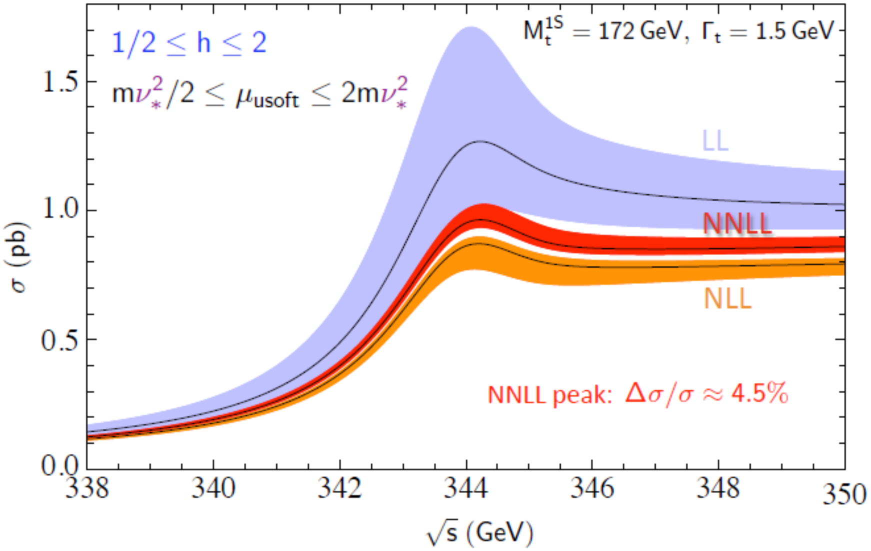
hep-ph/0001286,
Beneke,Kiyo,Schuller 08



peak position is stable,
but uncertainty remains in normalization
(still 10% in NNNLO)

NNLL

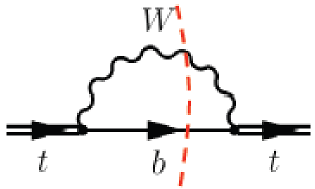
Hoang,Manohar,Stewart,Teubner 02,
Pineda,Signer 06, Hoang,Stahlhofen 13



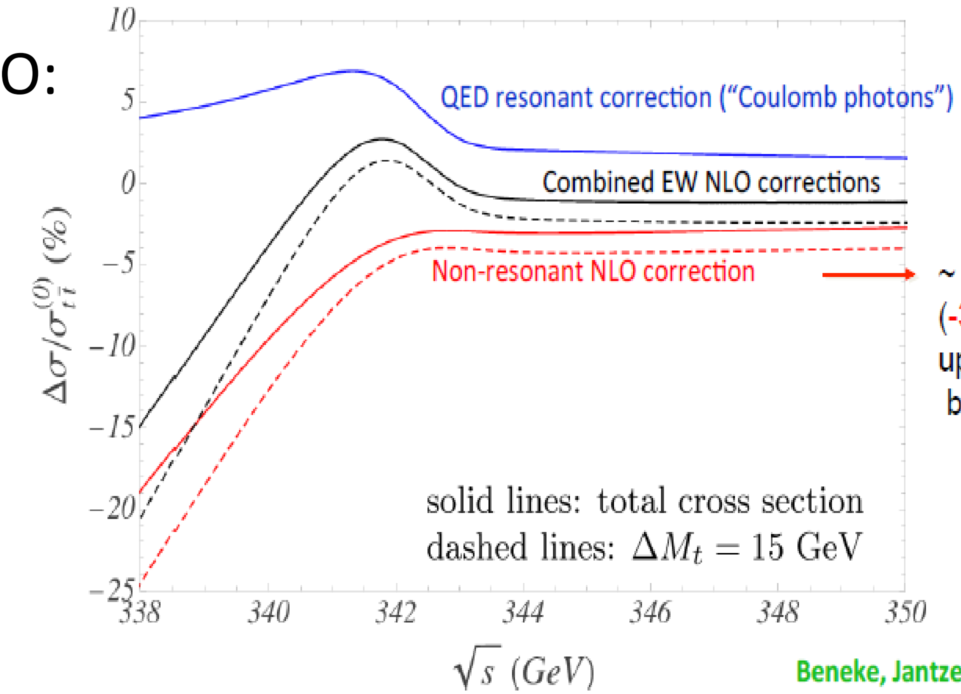
reduce scale uncertainty by RGE ~ 5%

EW (non-resonant) correction

LO: $$\frac{\Gamma_t}{m_t} \sim \alpha_W \sim \alpha_s^2$$



NLO:



~ -30 fb
(-3% above and
up to -20%
below threshold)

Beneke, Jantzen , RF (2010)

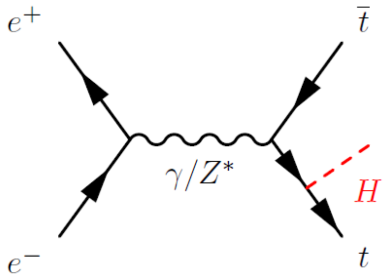
Important correction to the cross-section below the threshold

NNLO: partially available

Jantzen, Ruiz-Femenia 13

ttH production

direct measurement of Yukawa coupling



$$\boxed{\frac{\Delta y_t}{y_t} = \frac{1}{2}\frac{\Delta\sigma}{\sigma}}$$

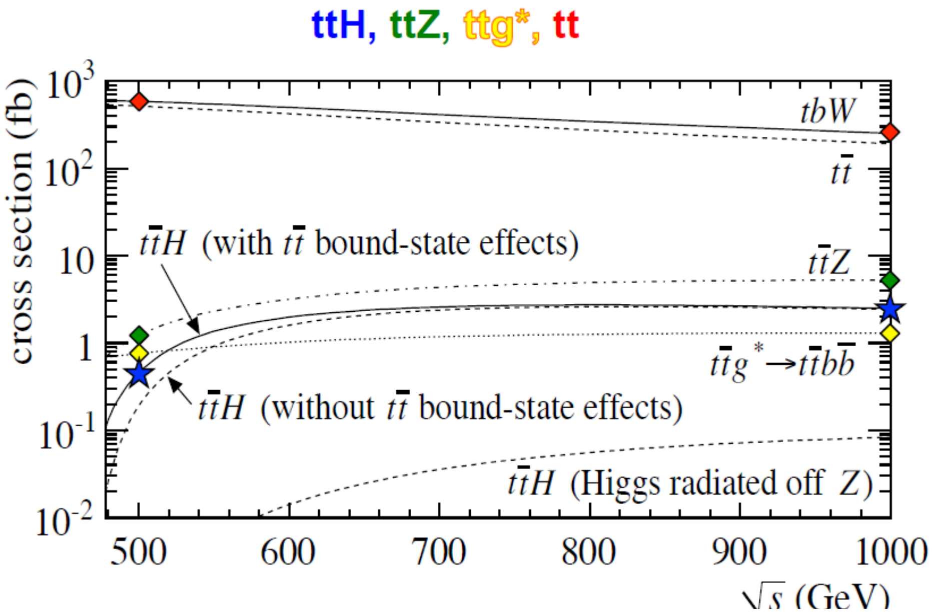
✓ One-loop QCD and EW corr. known

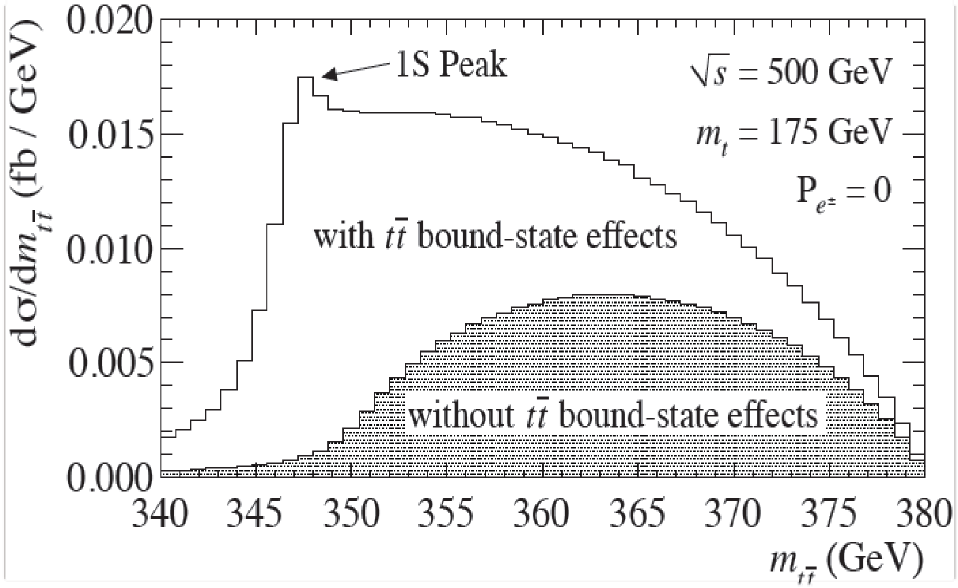
Dittmaier et al. 98, Belanger et al. 03

✓ NLL resummation of Coulomb term

Farrel, Hoang 05, 06

✓ Remaining scale uncertainty ~ a few %





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2. Mass
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4. **New Physics**

Top-quark as a Window for New Physics?

- LHC has great discovery potential for new physics
- What can the ILC still bring to us after the LHC 300(3000) fb^{-1}?

Precision measurements of couplings (to γ,Z,g,h), FCNC, CPV,,,

Degenerated scenario (where LHC may miss) ? e.g. $\tilde{t} \to t + \tilde{\chi}^0$

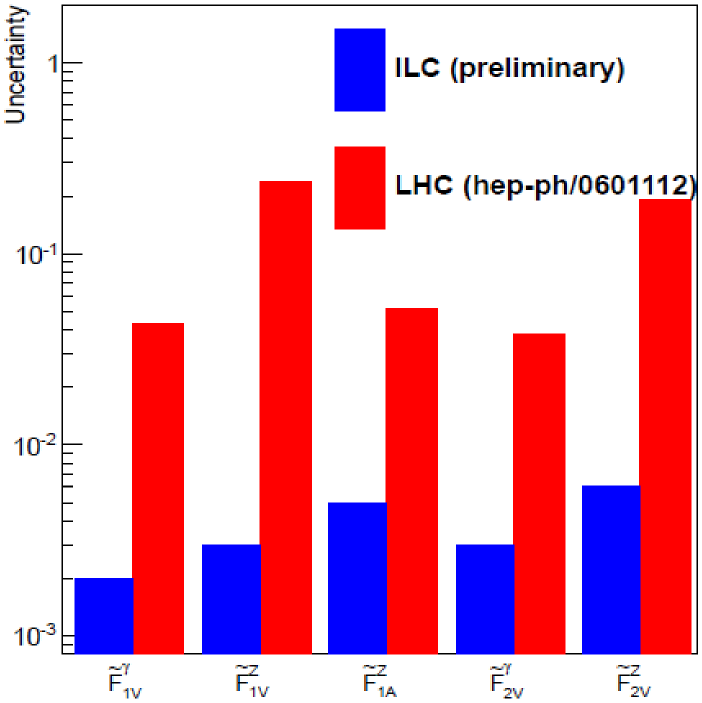
- Clarifying new physics effects in top-quark observables is needed
  - ✓ On which observables the NP effects appear?
  - ✓ Are those large enough to be measured?
  - ✓ Is SM uncertainty well understood?
  - ✓ Is it possible to discriminate models?

Amjad et al. 13

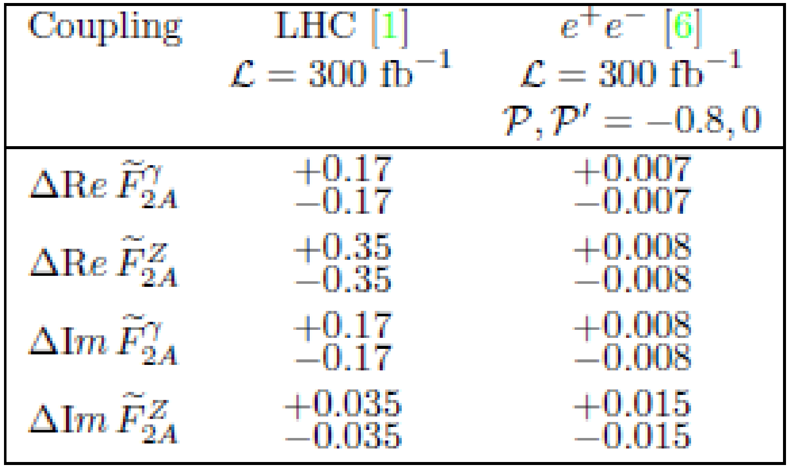
Couplings precision (electroweak bosons):

$$\Gamma_{\mu}^{ttX}(k^2, q, \overline{q}) = ie\left\{\gamma_\mu \left(\widetilde{F}_{1V}^{X}(k^2) + \gamma_5 \widetilde{F}_{1A}^{X}(k^2)\right) + \frac{(q-\overline{q})_\mu}{2m_t}\left(\widetilde{F}_{2V}^{X}(k^2) + \gamma_5 \widetilde{F}_{2A}^{X}(k^2)\right)\right\}$$

10% (LHC) → < 1% (ILC)



CPV couplings ~1% (study for TESLA)



Coupling	LHC [1] $\mathcal{L} = 300$ fb^{-1}	e^+e^- [6] $\mathcal{L} = 300$ fb^{-1} $\mathcal{P}, \mathcal{P}' = -0.8, 0$
$\Delta \mathrm{Re}\, \widetilde{F}^{\gamma}_{2A}$	$+0.17$ -0.17	$+0.007$ -0.007
$\Delta \mathrm{Re}\, \widetilde{F}^{Z}_{2A}$	$+0.35$ -0.35	$+0.008$ -0.008
$\Delta \mathrm{Im}\, \widetilde{F}^{\gamma}_{2A}$	$+0.17$ -0.17	$+0.008$ -0.008
$\Delta \mathrm{Im}\, \widetilde{F}^{Z}_{2A}$	$+0.035$ -0.035	$+0.015$ -0.015

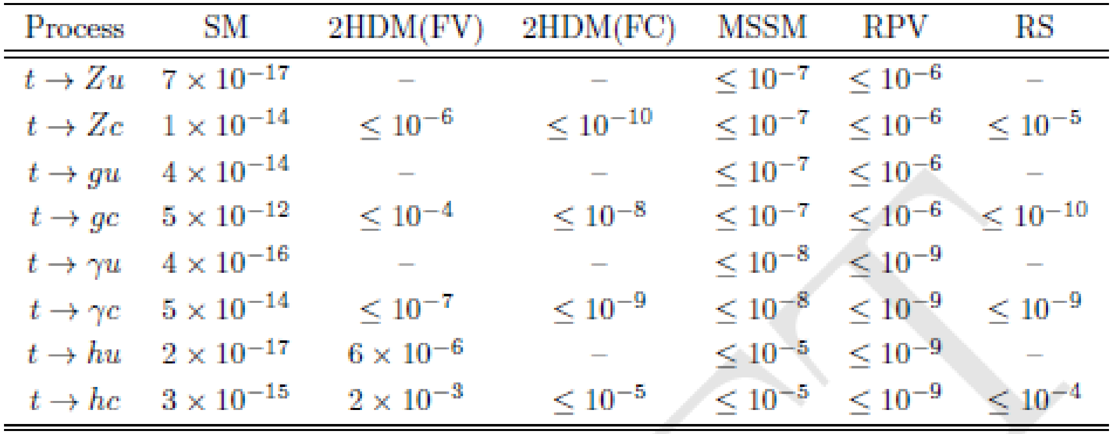
Theory predictions need to be summarized,

extended Higgs models, SUSY, extra dimension,,,

FCNC decay modes

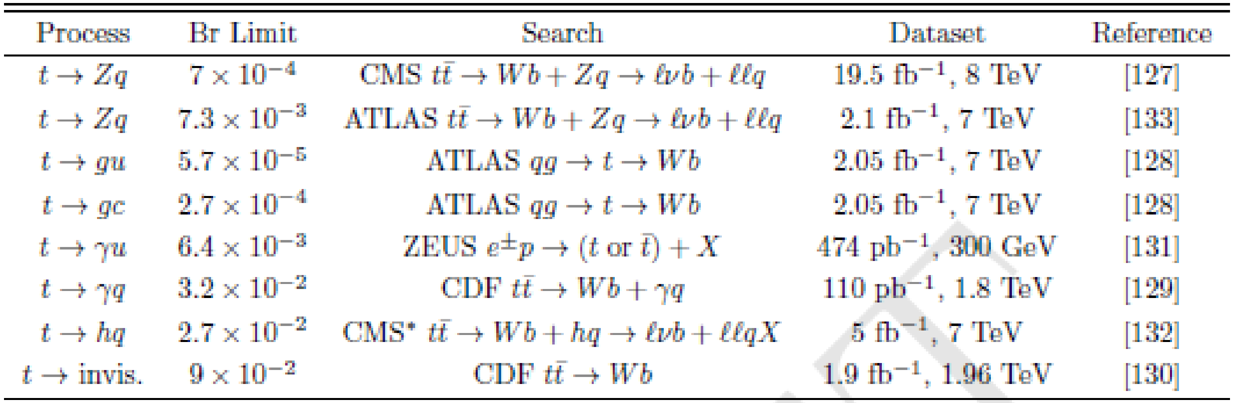
Snowmass report 13

- List of model predictions



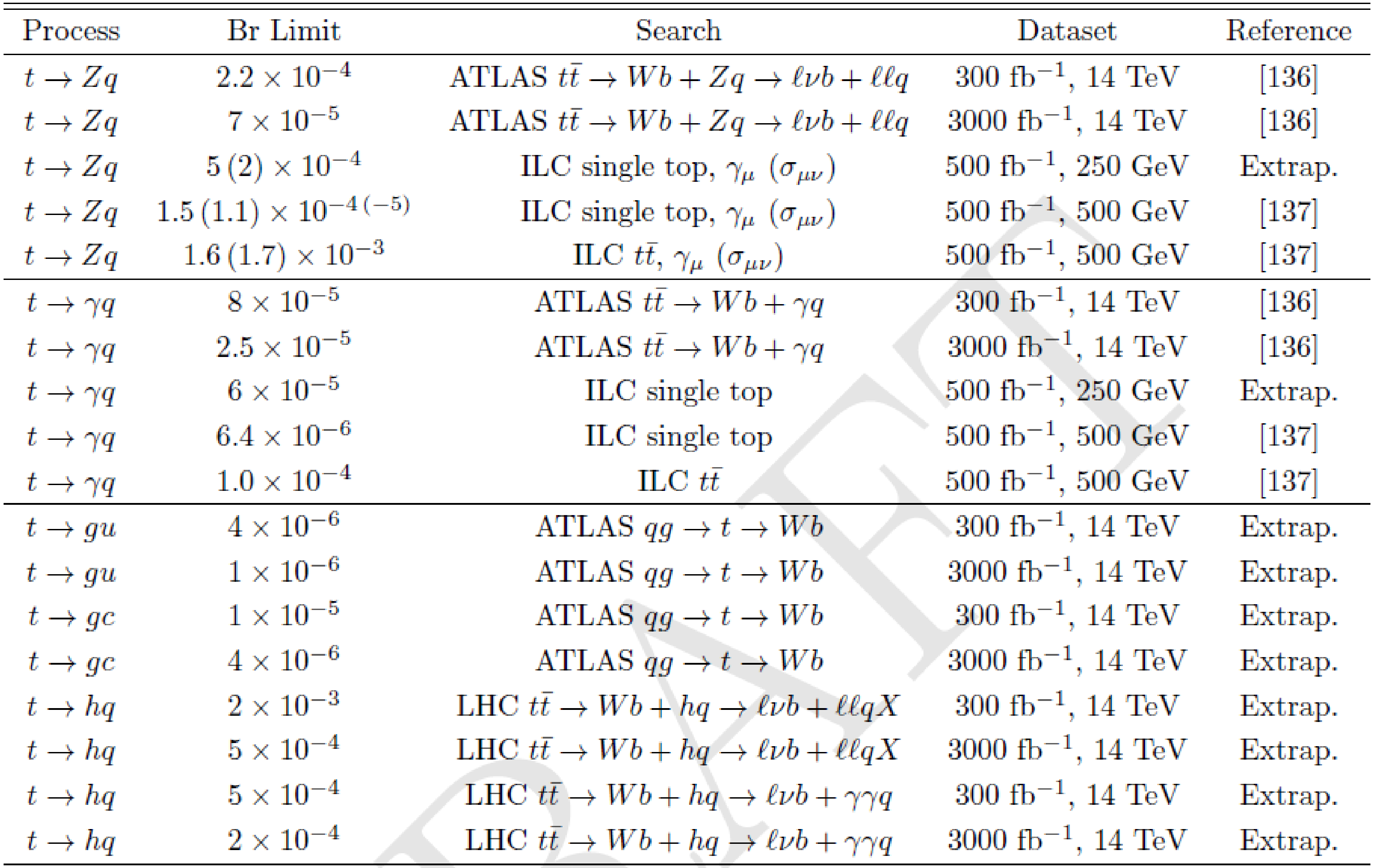
Process	SM	2HDM(FV)	2HDM(FC)	MSSM	RPV	RS
$t \to Zu$	7×10^{-17}	–	–	$\lesssim 10^{-7}$	$\lesssim 10^{-6}$	–
$t \to Zc$	1×10^{-14}	$\lesssim 10^{-6}$	$\lesssim 10^{-10}$	$\lesssim 10^{-7}$	$\lesssim 10^{-6}$	$\lesssim 10^{-5}$
$t \to gu$	4×10^{-14}	–	–	$\lesssim 10^{-7}$	$\lesssim 10^{-6}$	–
$t \to gc$	5×10^{-12}	$\lesssim 10^{-4}$	$\lesssim 10^{-8}$	$\lesssim 10^{-7}$	$\lesssim 10^{-6}$	$\lesssim 10^{-10}$
$t \to \gamma u$	4×10^{-16}	–	–	$\lesssim 10^{-8}$	$\lesssim 10^{-9}$	–
$t \to \gamma c$	5×10^{-14}	$\lesssim 10^{-7}$	$\lesssim 10^{-9}$	$\lesssim 10^{-8}$	$\lesssim 10^{-9}$	$\lesssim 10^{-9}$
$t \to hu$	2×10^{-17}	6×10^{-6}	–	$\lesssim 10^{-5}$	$\lesssim 10^{-9}$	–
$t \to hc$	3×10^{-15}	2×10^{-3}	$\lesssim 10^{-5}$	$\lesssim 10^{-5}$	$\lesssim 10^{-9}$	$\lesssim 10^{-4}$

- Experimental limit so far



Process	Br Limit	Search	Dataset	Reference
$t \to Zq$	7×10^{-4}	CMS $t\bar{t} \to Wb + Zq \to \ell\nu b + \ell\ell q$	19.5 fb^{-1}, 8 TeV	[127]
$t \to Zq$	7.3×10^{-3}	ATLAS $t\bar{t} \to Wb + Zq \to \ell\nu b + \ell\ell q$	2.1 fb^{-1}, 7 TeV	[133]
$t \to gu$	5.7×10^{-5}	ATLAS $qg \to t \to Wb$	2.05 fb^{-1}, 7 TeV	[128]
$t \to gc$	2.7×10^{-4}	ATLAS $qg \to t \to Wb$	2.05 fb^{-1}, 7 TeV	[128]
$t \to \gamma u$	6.4×10^{-3}	ZEUS $e^{\pm}p \to (t \text{ or } \bar{t}) + X$	474 pb^{-1}, 300 GeV	[131]
$t \to \gamma q$	3.2×10^{-2}	CDF $t\bar{t} \to Wb + \gamma q$	110 pb^{-1}, 1.8 TeV	[129]
$t \to hq$	2.7×10^{-2}	CMS* $t\bar{t} \to Wb + hq \to \ell\nu b + \ell\ell qX$	5 fb^{-1}, 7 TeV	[132]
$t \to$ invis.	9×10^{-2}	CDF $t\bar{t} \to Wb$	1.9 fb^{-1}, 1.96 TeV	[130]

- On some of modes, the ILC is expected to put strongest limit



Process	Br Limit	Search	Dataset	Reference
$t \to Zq$	2.2×10^{-4}	ATLAS $t\bar{t} \to Wb + Zq \to \ell\nu b + \ell\ell q$	300 fb^{-1}, 14 TeV	[136]
$t \to Zq$	7×10^{-5}	ATLAS $t\bar{t} \to Wb + Zq \to \ell\nu b + \ell\ell q$	3000 fb^{-1}, 14 TeV	[136]
$t \to Zq$	$5\,(2) \times 10^{-4}$	ILC single top, γ_μ $(\sigma_{\mu\nu})$	500 fb^{-1}, 250 GeV	Extrap.
$t \to Zq$	$1.5\,(1.1) \times 10^{-4\,(-5)}$	ILC single top, γ_μ $(\sigma_{\mu\nu})$	500 fb^{-1}, 500 GeV	[137]
$t \to Zq$	$1.6\,(1.7) \times 10^{-3}$	ILC $t\bar{t}$, γ_μ $(\sigma_{\mu\nu})$	500 fb^{-1}, 500 GeV	[137]
$t \to \gamma q$	8×10^{-5}	ATLAS $t\bar{t} \to Wb + \gamma q$	300 fb^{-1}, 14 TeV	[136]
$t \to \gamma q$	2.5×10^{-5}	ATLAS $t\bar{t} \to Wb + \gamma q$	3000 fb^{-1}, 14 TeV	[136]
$t \to \gamma q$	6×10^{-5}	ILC single top	500 fb^{-1}, 250 GeV	Extrap.
$t \to \gamma q$	6.4×10^{-6}	ILC single top	500 fb^{-1}, 500 GeV	[137]
$t \to \gamma q$	1.0×10^{-4}	ILC $t\bar{t}$	500 fb^{-1}, 500 GeV	[137]
$t \to gu$	4×10^{-6}	ATLAS $qg \to t \to Wb$	300 fb^{-1}, 14 TeV	Extrap.
$t \to gu$	1×10^{-6}	ATLAS $qg \to t \to Wb$	3000 fb^{-1}, 14 TeV	Extrap.
$t \to gc$	1×10^{-5}	ATLAS $qg \to t \to Wb$	300 fb^{-1}, 14 TeV	Extrap.
$t \to gc$	4×10^{-6}	ATLAS $qg \to t \to Wb$	3000 fb^{-1}, 14 TeV	Extrap.
$t \to hq$	2×10^{-3}	LHC $t\bar{t} \to Wb + hq \to \ell\nu b + \ell\ell qX$	300 fb^{-1}, 14 TeV	Extrap.
$t \to hq$	5×10^{-4}	LHC $t\bar{t} \to Wb + hq \to \ell\nu b + \ell\ell qX$	3000 fb^{-1}, 14 TeV	Extrap.
$t \to hq$	5×10^{-4}	LHC $t\bar{t} \to Wb + hq \to \ell\nu b + \gamma\gamma q$	300 fb^{-1}, 14 TeV	Extrap.
$t \to hq$	2×10^{-4}	LHC $t\bar{t} \to Wb + hq \to \ell\nu b + \gamma\gamma q$	3000 fb^{-1}, 14 TeV	Extrap.

Summary

- Top-quark (as well as Higgs) is the promised physics at the ILC
- Threshold scan gives the precise determination of the mass with a theoretically clean definition.

→ Important input
to the SM vacuum stavility

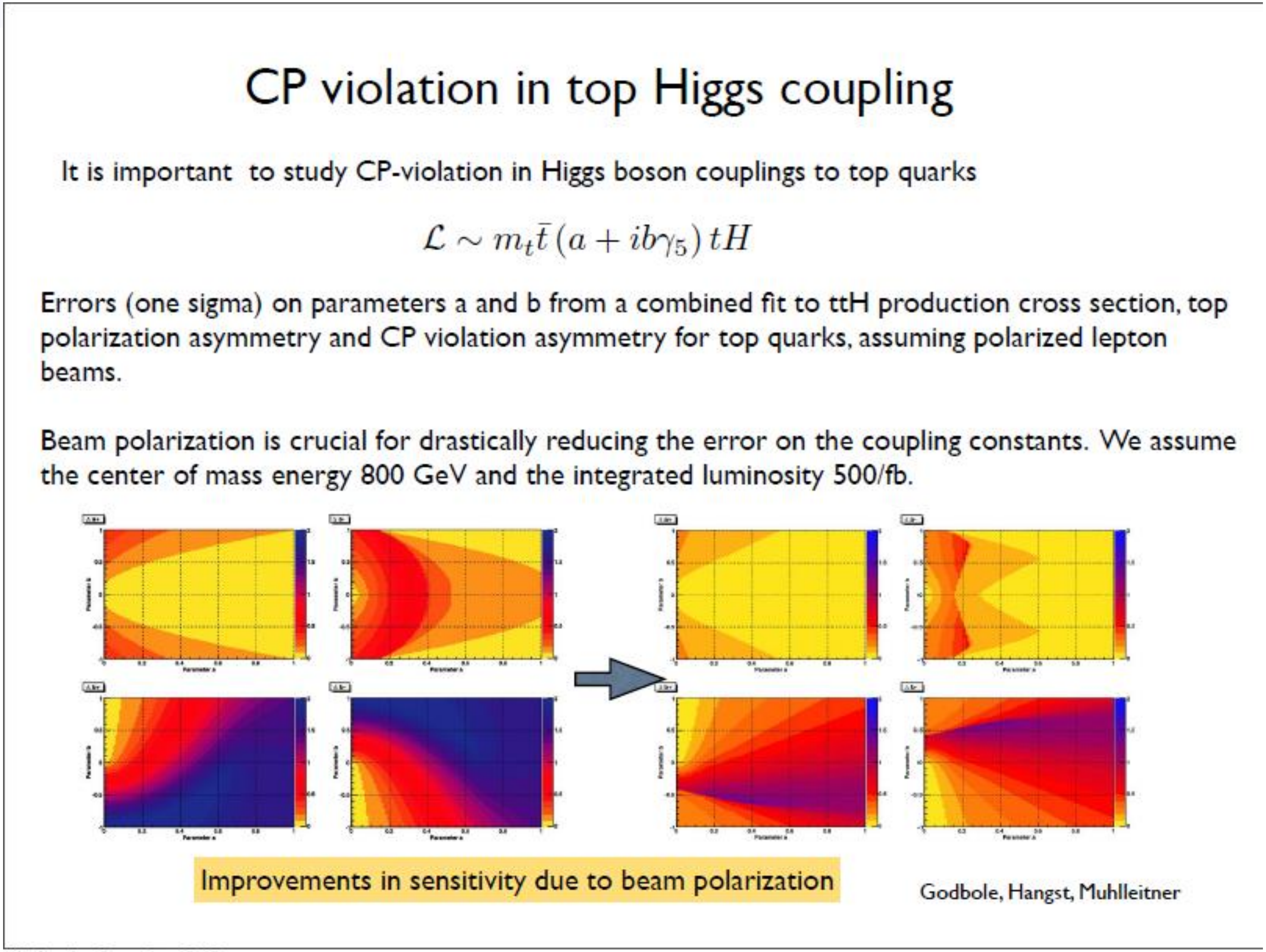
- NRQCD calculations are well-developed

ttbar: NNNLO(NNLL), EW corr. partially NNLO
ttH(ttγ, ttZ): NLO(NLL)

- New physics effects can be probed at the ILC in a better accuracy than at the LHC.

✓ based on the solid NRQCD calc.,
✓ need to clarify the model predictions

- What I have not covered are for example;
  - ✓ A_{FB}, Boosted top, Top partner at hadron colliders,,,



CP violation in top Higgs coupling

It is important to study CP-violation in Higgs boson couplings to top quarks

$$\mathcal{L} \sim m_t \bar{t} \, (a + ib\gamma_5) \, tH$$

Errors (one sigma) on parameters a and b from a combined fit to ttH production cross section, top polarization asymmetry and CP violation asymmetry for top quarks, assuming polarized lepton beams.

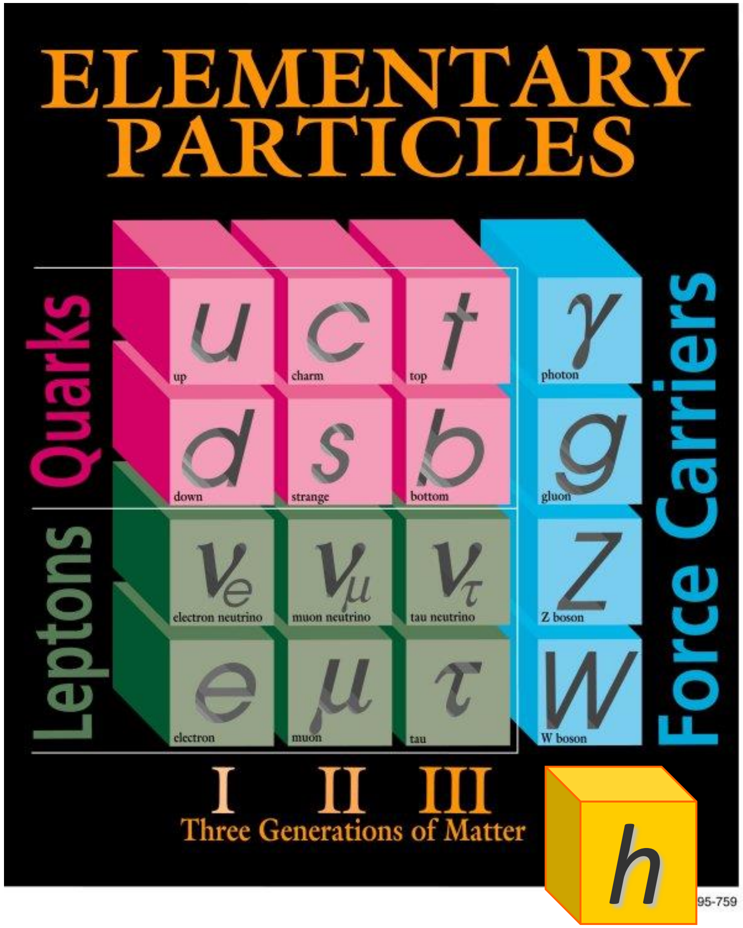
Beam polarization is crucial for drastically reducing the error on the coupling constants. We assume the center of mass energy 800 GeV and the integrated luminosity 500/fb.

Improvements in sensitivity due to beam polarization

Godbole, Hangst, Muhlleitner

History around Top-quark

1973 Kobayashi-Maskawa
Asymptotic free
1974 charm-quark
1975 tau-lepton
1977 bottom-quark
…
1995 top-quark
2000 tau-neutrino



(1979 gluon, 1983 W&Z, 2012 Higgs-boson)