A closer look into the CMB anisotropies

A simple test of cosmological principle

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CMB anisotropies

Microwave sky:
\[ \frac{\Delta T(\hat{n})}{T_0} \]

CMB: Density fluctuations
380,000 yr after the big-bang

Expand with the spherical harmonics
\[ \frac{\Delta T(\hat{n})}{T_0} = \sum_{\ell m} a_{\ell m} Y_{\ell m} \]

Power spectrum: Calc the variance of the expanded coefficients
\[ C_\ell = \langle a_{\ell m}^* a_{\ell m} \rangle = \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2 \]

WMAP power spectrum
statistical anomalies?

- large-scale anomalies
  - low quadrupole, lack of correlations (COBE, WMAP, PLANCK)
  - multipole alignment (e.g., Copi+, '06)
  - dipole modulation (e.g., Eriksen+, '04)

- small-scale anomalies
  - Features in the primordial power spectrum
    - cold spot

Credit: WMAP team/NASA

Power-law OK

oscillatory feature?

KI & Yokoyama, in preparation

\[ P(k) \times 10^{10} \]

\[ k_d = \ell \]

- low quadrupole

- octopole

- sky mask
  - 5y9 mask

- 310.3831
Let's look for a signal beyond the standard cosmological model.
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• summary
Inflation will produce a primordial fluctuation that will be transferred to the CMB anisotropy. The CMB angular power spectrum is given by:

$$\frac{\ell (\ell + 1) C_\ell}{2\pi} = \int d\ln k k^3 P(k) |T_\ell(\eta_0, k)|^2$$

where $C_\ell$ is the CMB angular power spectrum, $P(k)$ is the primordial fluctuation power spectrum, and $|T_\ell(\eta_0, k)|^2$ is the transfer function.
Reconstruction of primordial fluctuation from data

\[ C_\ell \quad P(k) \]

- Let's find the initial condition of our own Universe
- Simplest inflation models predict a smooth, power law \( P(k) \)
  - search for fine and global structures beyond sample variance
A brute force reconstruction

Planck reports a power deficit at $\ell \approx 20$
- divide a range of wavenumber into many bins
- we find hints for the departure from the power law at $\ell \approx 120$
- constraints are weaker for PLANCK
A close look at around $l=120$

- A simple 3 parameter feature model: (S-type in the figure)

\[
k^3 P(k) = A \left( \frac{k}{k_0} \right)^{n-1} + B \left( \frac{k}{k_0} \right)^{n-1} \exp \left( -\frac{(k - k_{\ast})^2}{\kappa^2} \right) \cos \left( \pi \frac{k - k_{\ast}}{\kappa} \right)
\]

- Power-law
- Oscillatory feature on top of the power-law

- Posterior probability to find $B=0$ is $8 \times 10^{-4}$ including looking-elsewhere effect
Implications

- It is interesting that our featured spectrum improved the fit not only the TT spectrum, but also the TE (WMAP)
  \[ \Delta \chi_{\text{eff}} = -21 \quad \text{breakdown: } -12.5 \, (\text{TT}) \quad -8.5 \, (\text{TE}) \]
- Standard cosmological parameters, esp. \( \Omega_b h^2, n_s \) can shift comparable to the statistical fluctuation depending on whether we incorporate this feature or not  
  (KI and Yokoyama, in preparation)
- E-mode polarization will be more suitable for the P(k) reconstruction  
  (Mortonson+, PRD, '09)
  - Let's wait for the PLANCK's polarization data next year
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Today's question

- The variance ($C_\ell$) contains most of the cosmological information

$$C_\ell = \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2$$

- Why should we divide the squares by $(2\ell + 1)$, and not by $2\ell$? (textbooks say we should divide by (d.o.f - 1) to get an unbiased estimate)

- This is because we have *implicitly* assumed that the mean of $a_{\ell m}$ is zero.
We believe that, according to the cosmological principle, we can write any perturbation variables as:
\[ \phi(t, x) = \phi_0(t) + \delta\phi(t, x) \]

\[ \langle \delta\phi(t, x) \rangle = 0 \quad \text{Independent of position } (x) \]

This is possible when \( \phi(t, x) \) is statistically homogeneous:

\[ \langle \phi(\vec{x}) \rangle = \langle \phi(\vec{x} + \vec{T}) \rangle \quad \vec{T} \text{: arbitrary vector} \]

The condition should be tested by observations!
Another motivation

- In the analysis of CMB anisotropies, zero mean is usually assumed implicitly.
  - Any higher order statistics, such as variance, skewness, kurtosis etc... are affected by this assumption.
  - Non-zero mean have been indicated by LSS (e.g., Labini, arXiv:1103.5974)
    - However, LSS suffers from bias, selection rules, galaxy evolution,...

Let's look for in the CMB anisotropies!
Mean of CMB anisotropies (1)

- CMB fluctuations $\delta T(x, \hat{n})$ are related with the primordial fluctuations (random variable) $\phi(\vec{k})$ through transfer function $T(\vec{k}, \hat{n})$ as

$$\delta T(x, \hat{n}) = \int \frac{d^3 k}{(2\pi)^3} T(\vec{k}, \hat{n}) \phi(\vec{k}) e^{i \vec{k} \cdot \vec{x}}$$

- Expanded coefficients of CMB fluctuations:

$$a_{\ell m} = \int d^2 \hat{n} \delta T(\hat{n}) Y_{\ell m}(\hat{n})$$

$$= 4\pi (-i)^\ell \int \frac{d^3 k}{(2\pi)^3} T_{\ell}(k) \phi(\vec{k}) Y_{\ell m}(\vec{k})$$

- Therefore, $\langle \phi \rangle = 0 \Rightarrow \langle a_{\ell m} \rangle = 0$

- Furthermore, if $\phi(\vec{k})$ are Gaussian, so are $a_{\ell m}$
Meaning of the zero mean

Fluctuations such that

\[
\int d\hat{n} \frac{\Delta T(\hat{n})}{T} = \sum_{\ell m} \int d\hat{n} a_{\ell m} Y_{\ell m}(\hat{n}) = 0
\]

do not necessarily mean

\[
\langle a_{\ell m} \rangle = 0
\]

\[a_{3m} = (1, 1) \text{ for } m \geq 0\]

\[a_{31} = (1, 0), \ a_{3,3} = (-1, 0)\]

\[a_{30}, \ a_{3,2} = (0, 0)\]
Difficulty...Foreground, Noise, Mask

- Some of the CMB photons are not primordial origin
  - Dust emission, synchrotron, free-free...
  - They have non-Gaussian dist., *non-zero mean*
- Cleaning should not be perfect
  - masking the galactic disk
  - *induces unwanted correlations*
- Instrumental noises
  - they have zero-mean

*Bennett et al., 2003*
CMB MAP in practice

- Putting the mask $M(\hat{n})$

$$\delta T(\hat{n})_{\text{obs}} \equiv M(\hat{n})\delta T_{\text{CMB}}(\hat{n})$$

Going to the spherical harmonic space

$$(\delta T_{\text{obs}})_{\ell m} \equiv c_{\ell m} = \sum M_{\ell m; \ell' m'}(a_{\ell' m'} + N_{\ell' m'})$$

observed signal

- Zero mean still holds true if $\langle N_{\ell m} \rangle = 0$, however $c_{\ell m}$ and $c_{\ell' m'}$ are not independent due to the mask coupling

We cannot use a simple statistical test (such as the student's t-test)
Beating the mask

- Mask introduces unwanted correlations between the sample $a_{\ell m}$s
- Simple statistical tests rely on the independence... what would you do?
  - Do a test including the correlations
    - Monte Carlo simulation (Kashino, KI, Takeuchi, PRD '12)
  - Construct a de-correlated variable
    - V-vector method (Armendariz-Picon, JCAP '11)
    - Principal component analysis
v-vector method (Armendariz-Picon, JCAP, '11)

- Goal: to remove the effect of the mask from the observed spherical harmonic coefficients

\[ c_{\ell m} = \sum M_{\ell m; \ell' m'} a_{\ell' m'} \]

- Let us use a vector notation:
  \[ \vec{c}_m = M \cdot \vec{a}_m \]

- Find m-independent v-vectors that satisfy
  \[ \vec{v}^t = \vec{v}^t M \]
  \[ v_{\ell m} = \begin{cases} v_{\ell} & \text{for } |m| \leq m_{\text{max}} \text{ and } m_{\text{max}} \leq \ell \leq \ell_{\text{max}} \\ 0 & \text{(otherwise)} \end{cases} \]

- Construct \( d_m \) as a dot product of \( \vec{v} \) and \( \vec{c}_m \)
  \[ d_m \equiv \vec{v} \cdot \vec{c}_m = \sum_{\ell} v_{\ell m} c_{\ell m} = \vec{v} \cdot (M \vec{a}_m) = \vec{v} \cdot \vec{a}_m \]
  for \( |m| \leq m_{\text{max}} \)
v-vector method (summary)

• Construct $d_m$ as a dot product of $\vec{v}$ and $\vec{c}_m$

$$
d_m \equiv \vec{v} \cdot \vec{c}_m = \vec{v} \cdot \vec{a}_m = \sum_{\ell=m_{\text{max}}}^{\ell_{\text{max}}} v_{\ell m} c_{\ell m} \quad (|m| \leq m_{\text{max}})
$$

• The new stochastic variable $d_m$ have following properties:
  
  – Foreground insensitive (because we work on $c_{\ell m}$)
  
  – Statistically independent samples (because $\vec{v}$ is constant)
  
  – zero-mean Gaussian if $a_{\ell m}$ are zero mean Gaussian
  
  – have m-independent variance

$$
\sigma^2 = \sum_{\ell=m_{\text{max}}}^{\ell_{\text{max}}} K_{\ell}^2 (B_{\ell}^2 C_{\ell} + N_0) v_{\ell}^2
$$
Visualization of v-vectors

\[(\ell_{\text{max}}, m_{\text{max}}) = (212, 177)\]

銀河面が除かれた重み付けがなされる。
RESULTS
Distribution of the stochastic variable from PLANCK and CMB

$l_{\text{max}}=38, m_{\text{max}}=19$

$l_{\text{max}}=86, m_{\text{max}}=61$

$l_{\text{max}}=60, m_{\text{max}}=39$

$l_{\text{max}}=112, m_{\text{max}}=87$
Distribution of the stochastic variable from PLANCK and CMB

\( l_{\text{max}}=142, \ m_{\text{max}}=113 \)

\( l_{\text{max}}=212, \ m_{\text{max}}=177 \)

\( l_{\text{max}}=176, \ m_{\text{max}}=143 \)

\( l_{\text{max}}=256, \ m_{\text{max}}=213 \)
Noise levels

- black – signal, blue – noise
- noise becomes significant on smaller angular scales
- noise contributes up to 40% for WMAP @ lmax=256
Result

KI, in preparation
Monte-Carlo simulation results

Kashino, KI, Takeuchi, PRD, '12

We found the same tendency!  99.93% anomaly
summary

• In the analysis of CMB anisotropies, “zero mean” has been assumed implicitly (or by the Cosmo. Principle)
  – Zero mean should be confirmed by observation data themselves!
• We test this hypothesis using recent WMAP and PLANCK temperature anisotropies maps
• We find a hint of deviation from the zero-mean hypothesis at $\ell \approx 230$ in both WMAP and PLANCK