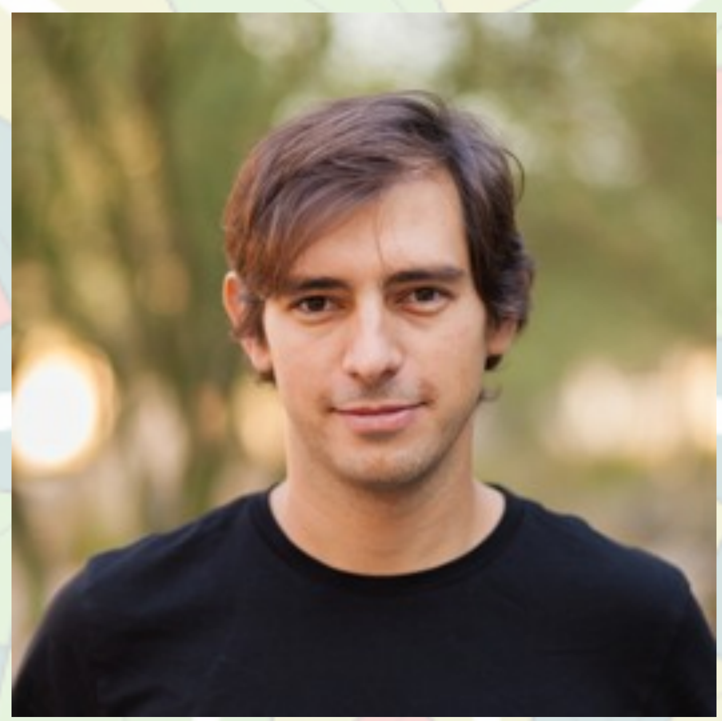
Holographic quantum error-correcting code - exactly solvable toy models for the AdS/CFT correspondence

arXiv:1503.06237

Beni Yoshida (Caltech)

joint work with

Daniel Harlow



Fernando Pastawski

John Preskill

Elementary introduction to our paper !!

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Posted on March 25, 2015 by Beni Yoshida

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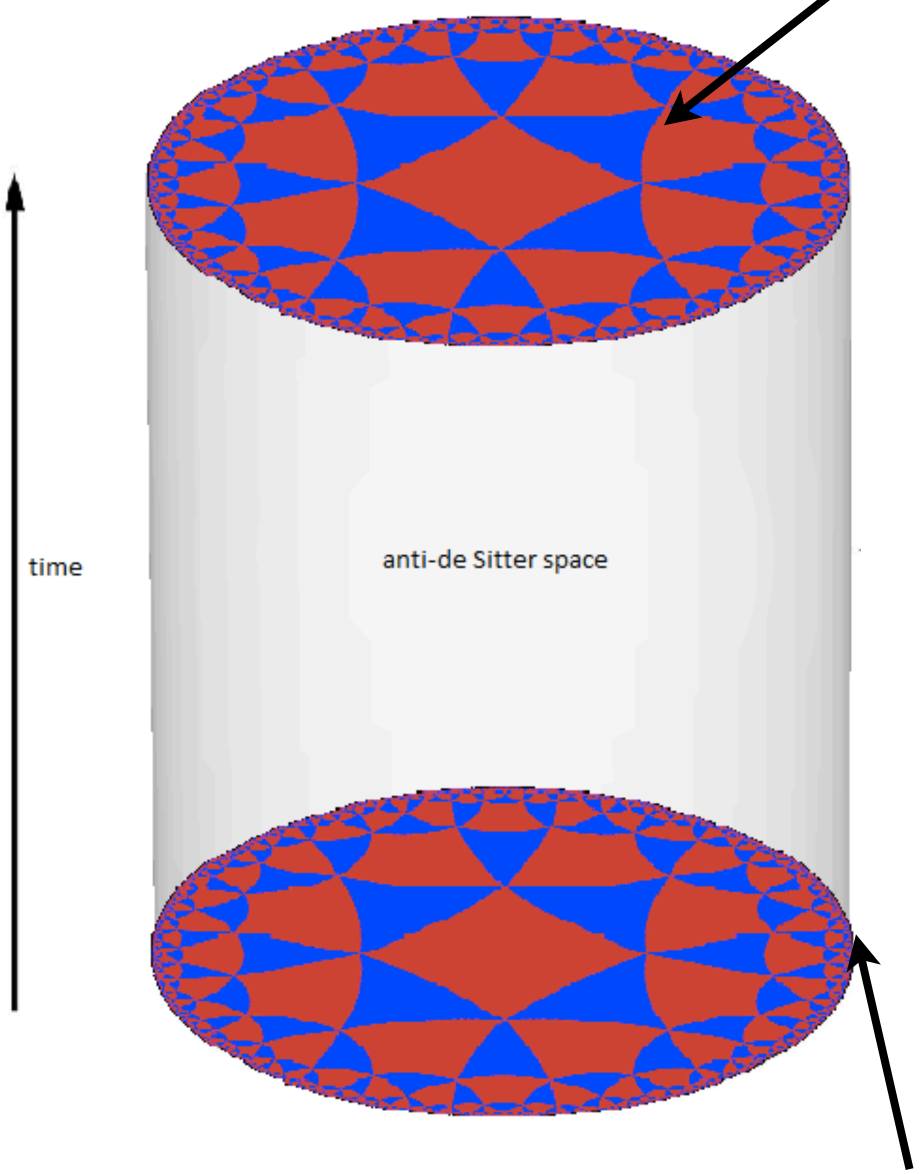
Quantum gravity from quantum error-correcting codes?

The lessons we learned from the Ryu-Takayanagi formula, the firewall paradox and the ER=EPR conjecture have convinced us that quantum information theory can become a

Introduction

AdS/CFT correspondence (Maldacena 98)



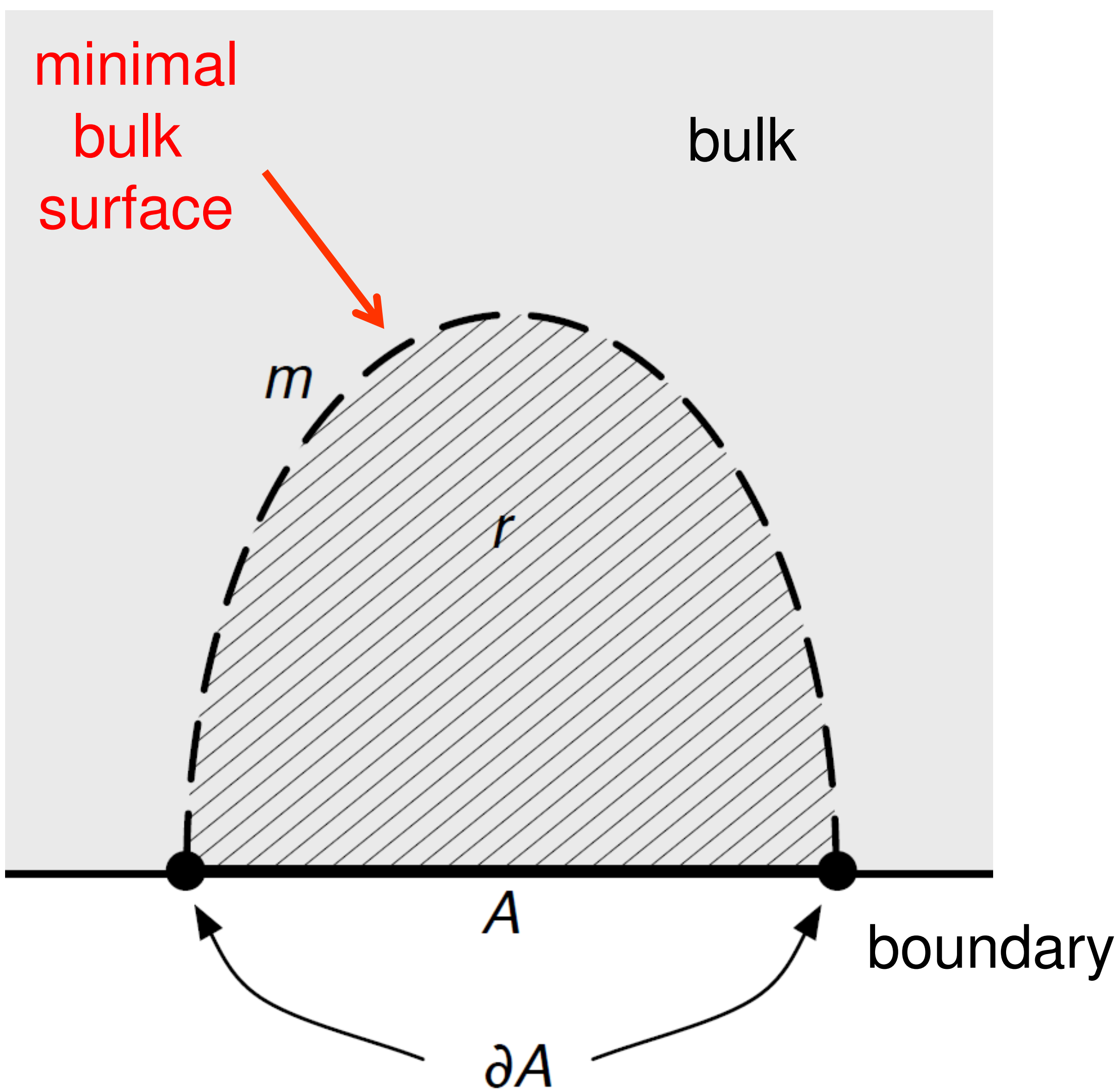
bulk | boundary

$$\phi(x, r) \longleftrightarrow \mathcal{O}(x)$$

- Powerful framework to study strongly-interacting systems
- Advanced our understanding of quantum gravity

Ryu-Takayanagi formula (06)

- Bulk/Boundary duality to Geometry/Entanglement duality



$$S(A) = \frac{1}{4G_N} \min_{\partial m = \partial A} \text{area}(m)$$

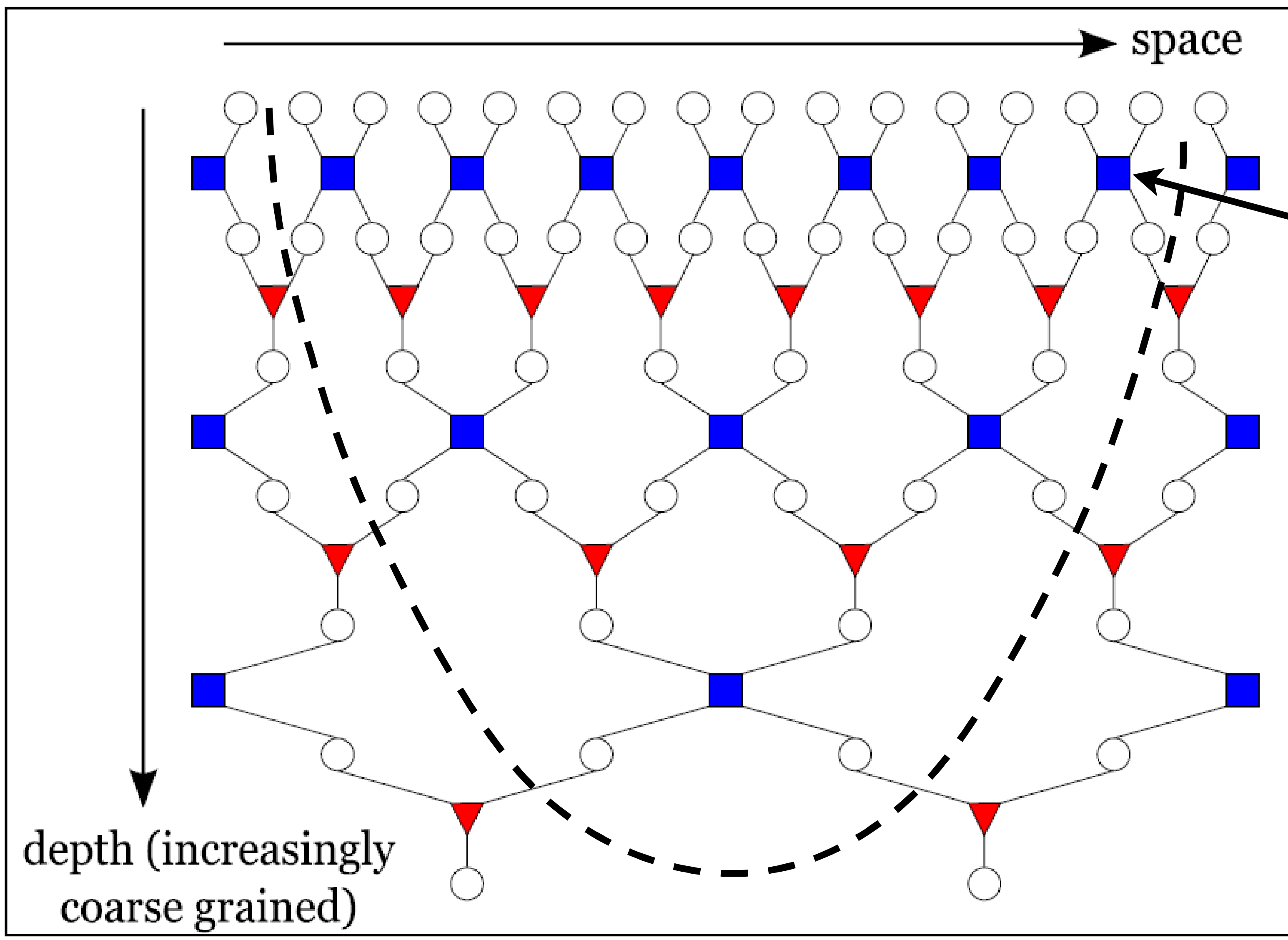
Entanglement

↕

Geometry (Space-time)

MERA (Vidal 07)

- Powerful numerical method to study strongly-correlated systems.

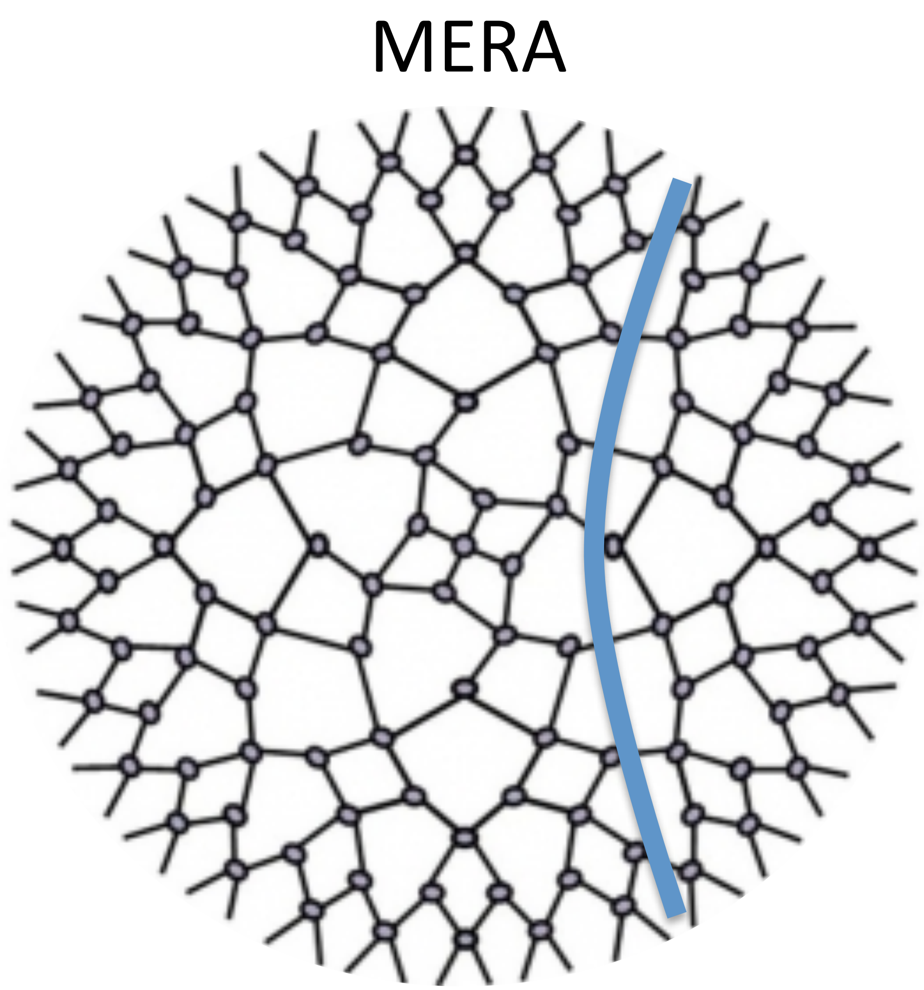


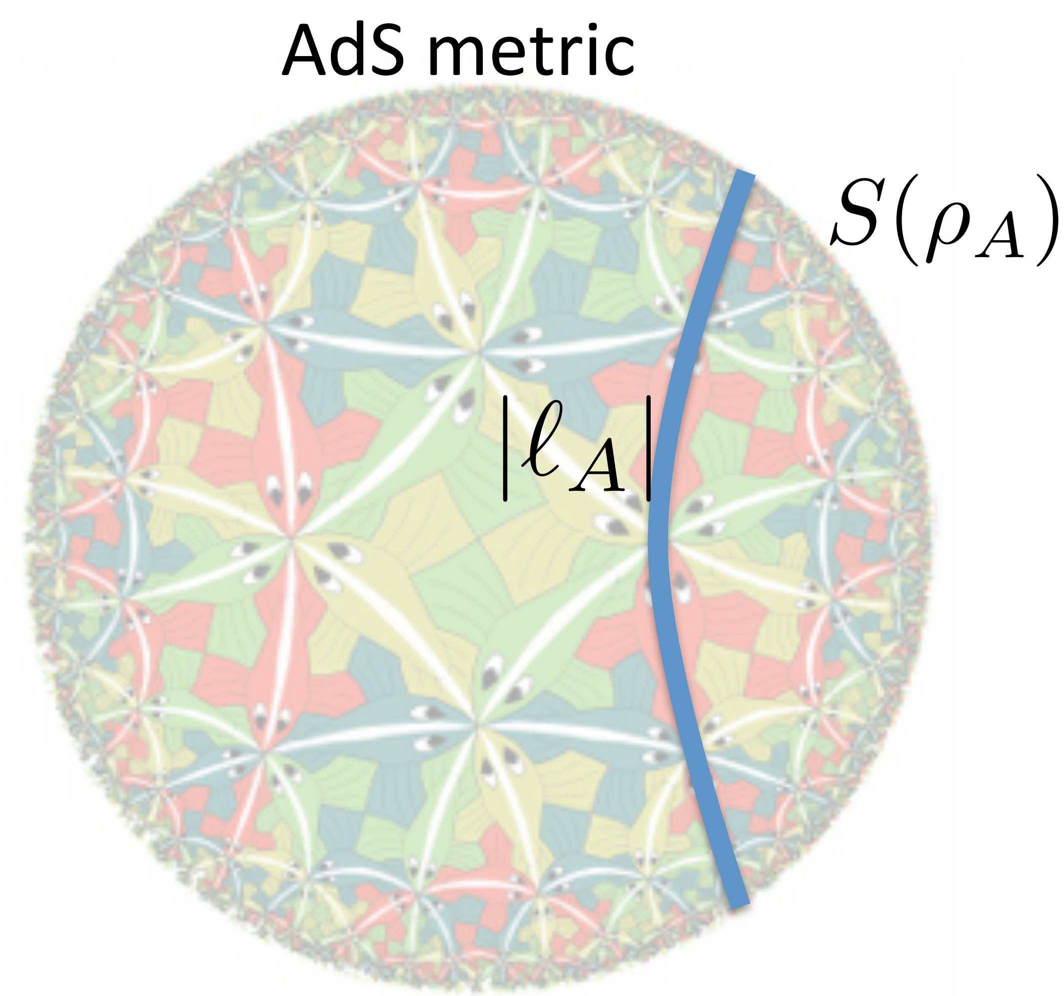
remove short-range entanglement

MERA = Multiscale entanglement renormalization ansatz

AdS/CFT as a tensor network (Swingle 09)

AdS/CFT correspondence can be explained by a tensor network ?



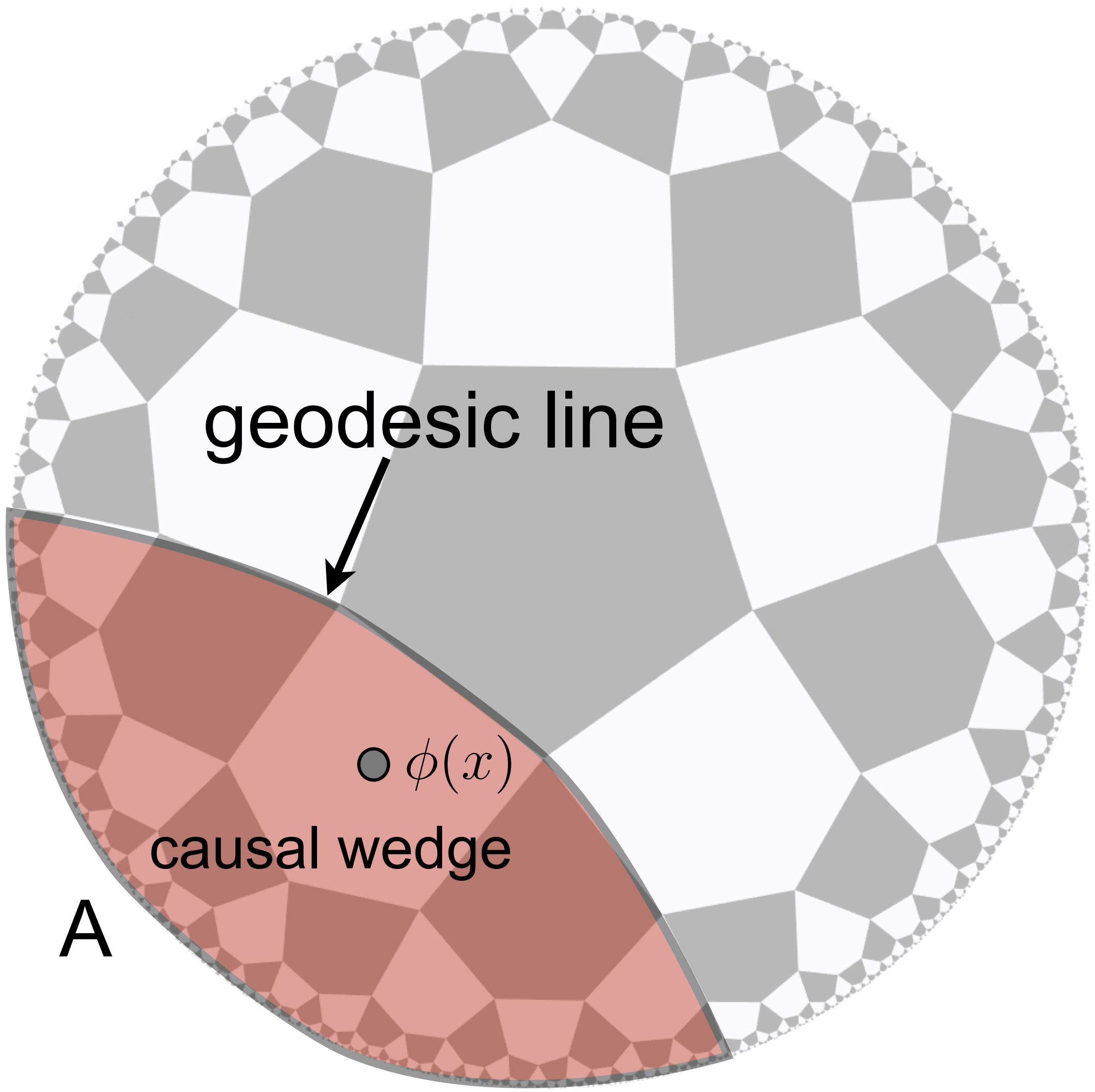


Part I : A simple toy model

The bulk-locality paradox

Rindler-wedge reconstruction

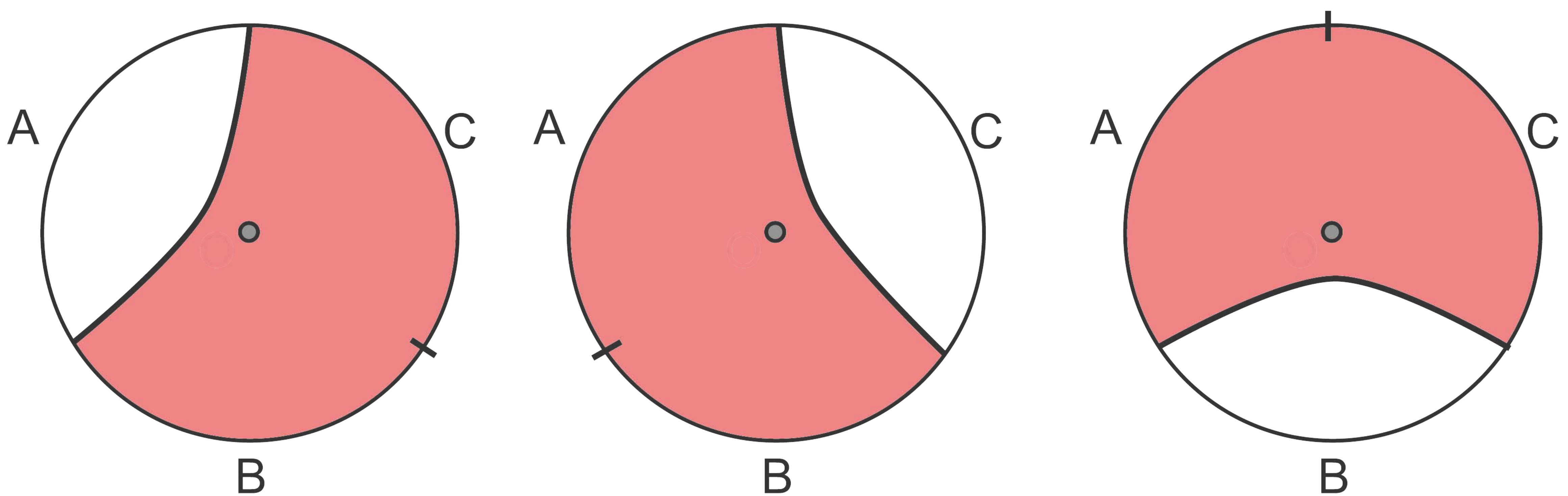
A bulk operator ϕ can be represented by some integral of local boundary operators supported on A if and only if ϕ is contained inside the causal wedge of A.



$$\phi(x) = \int_{\mathbb{S}^{d-1}\times\mathbb{R}} dY\, K(x;Y)\mathcal{O}(Y),$$

Bulk locality paradox

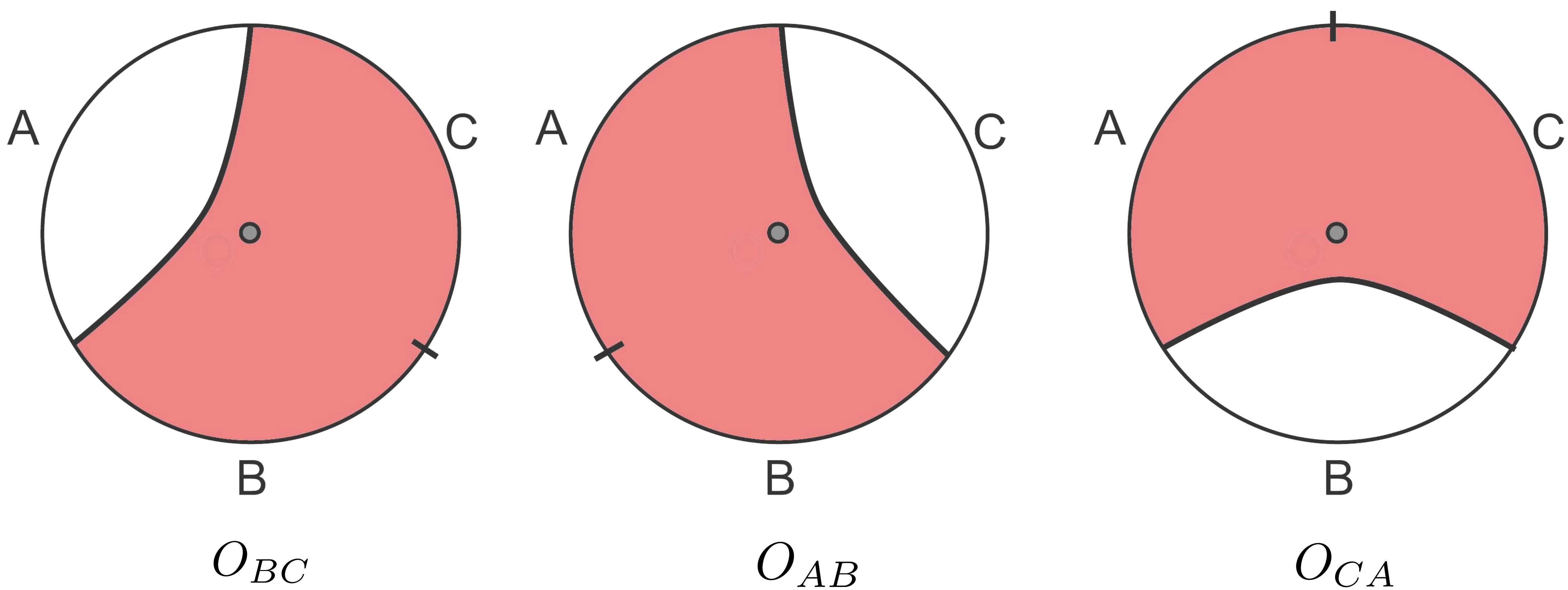
All the bulk operators must correspond to identity operators on the boundary.



If so, the AdS/CFT correspondence seems boring ...

AdS/CFT is a quantum code ?

Solution: T*he AdS/CFT correspondence can be viewed as a quantum error-correcting code*.

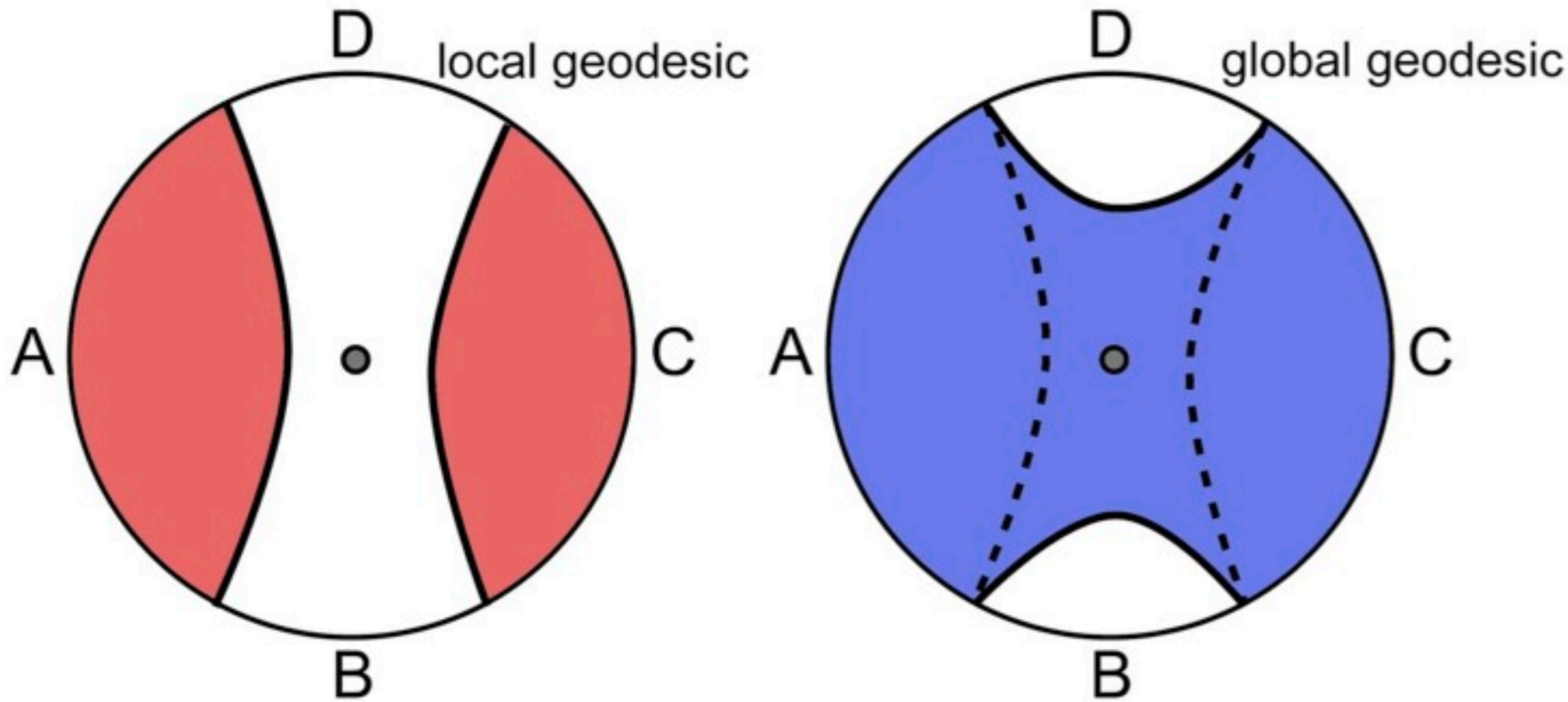


They are different operators, but act in the same manner in a low energy subspace.

cf. Quantum secret-sharing code

Entanglement wedge reconstruction

Operators in the entanglement wedge can be reconstructed (?)



causal wedge of AC

entanglement wedge of AC

* Entanglement wedge may extend over the horizon (vs firewall).

* Along the spirit of the Ryu-Takayanagi formula.

Holographic code

Minimal model

Let us construct the simplest toy model !

- 5 qubits on the boundary
- 1 qubit on the bulk

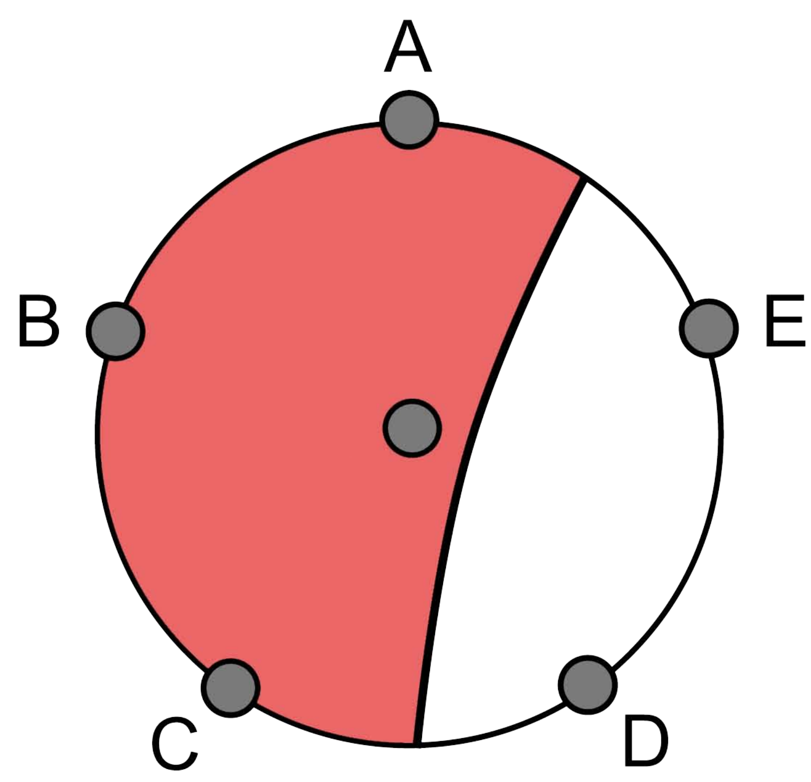
Minimal model

Let us construct the simplest toy model !

- 5 qubits on the boundary
- 1 qubit on the bulk

Causal wedge reconstruction implies

A bulk operator must have representations on ABC, BCD, ...



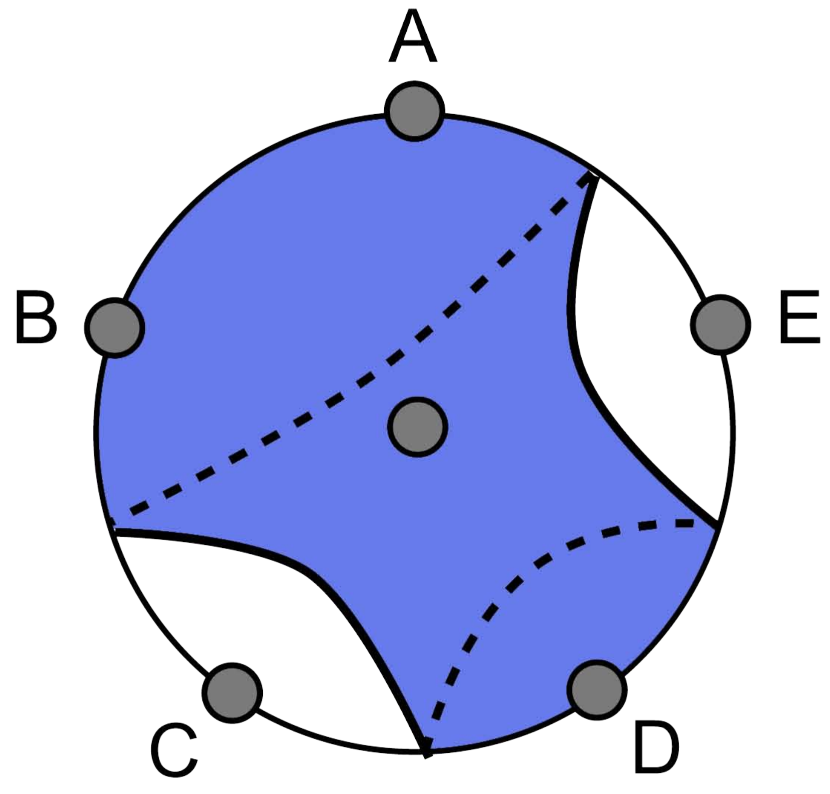
Minimal model

Let us construct the simplest toy model !

- 5 qubits on the boundary
- 1 qubit on the bulk

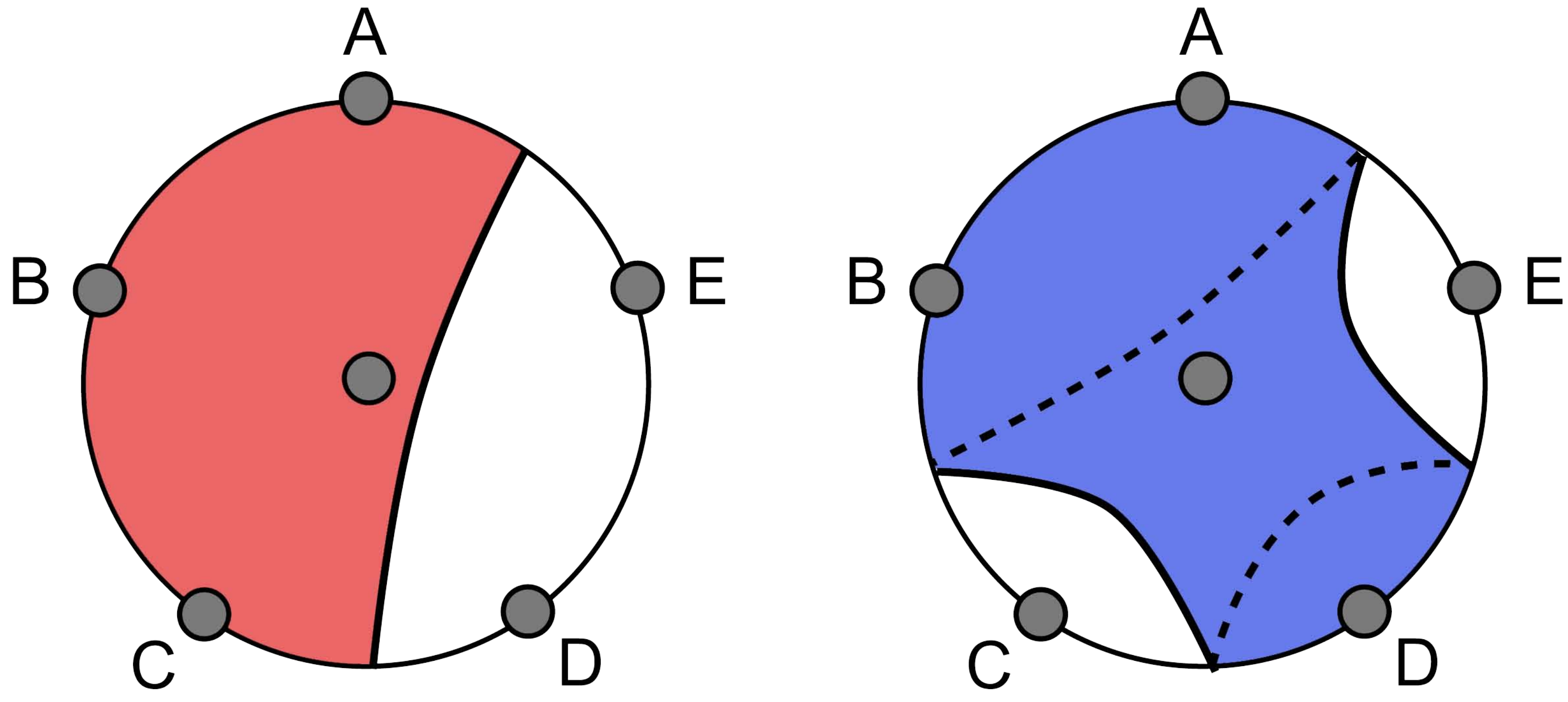
Entanglement wedge reconstruction implies

A bulk operator must have representations on ABD, BCE, ...



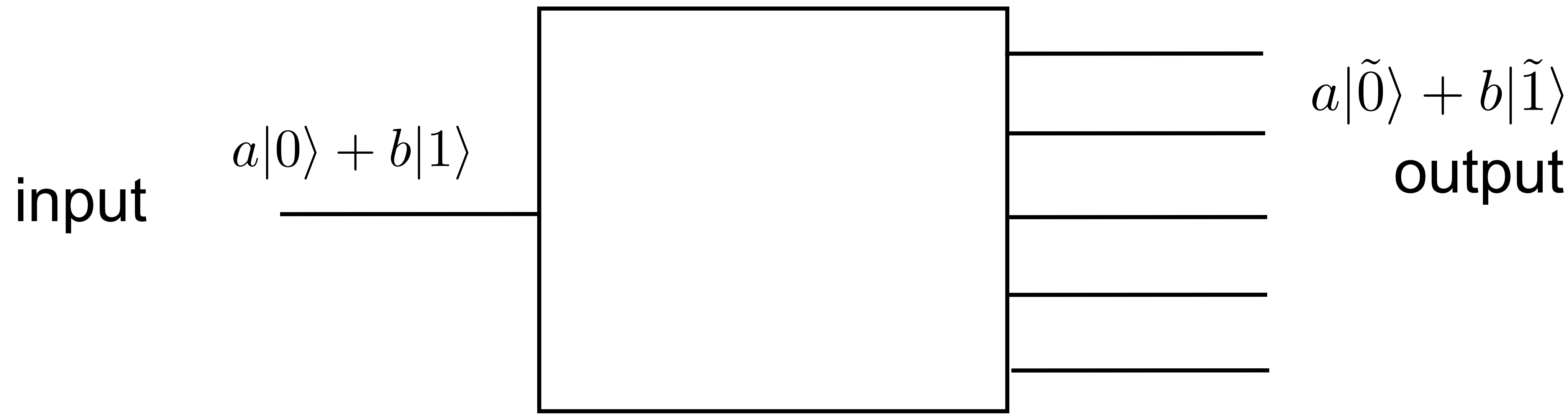
Desired properties

A bulk operator must have representations on any region with three qubits.



Five qubit code

Encode one logical qubit into five physical qubits.



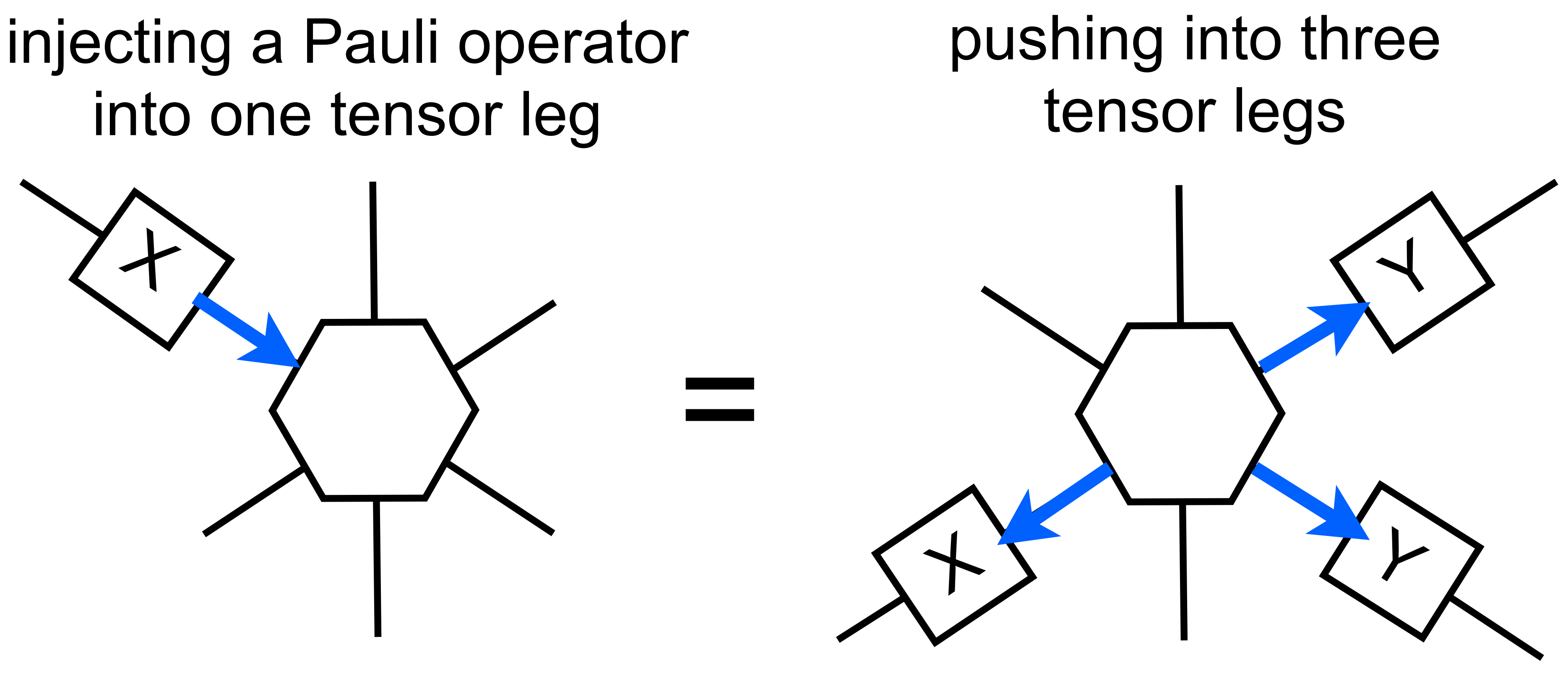
Pauli X, Z ⟶ logical Pauli X, Z

Logical Pauli X,Z have representations on any region with three qubits.

"Distance 3" quantum code

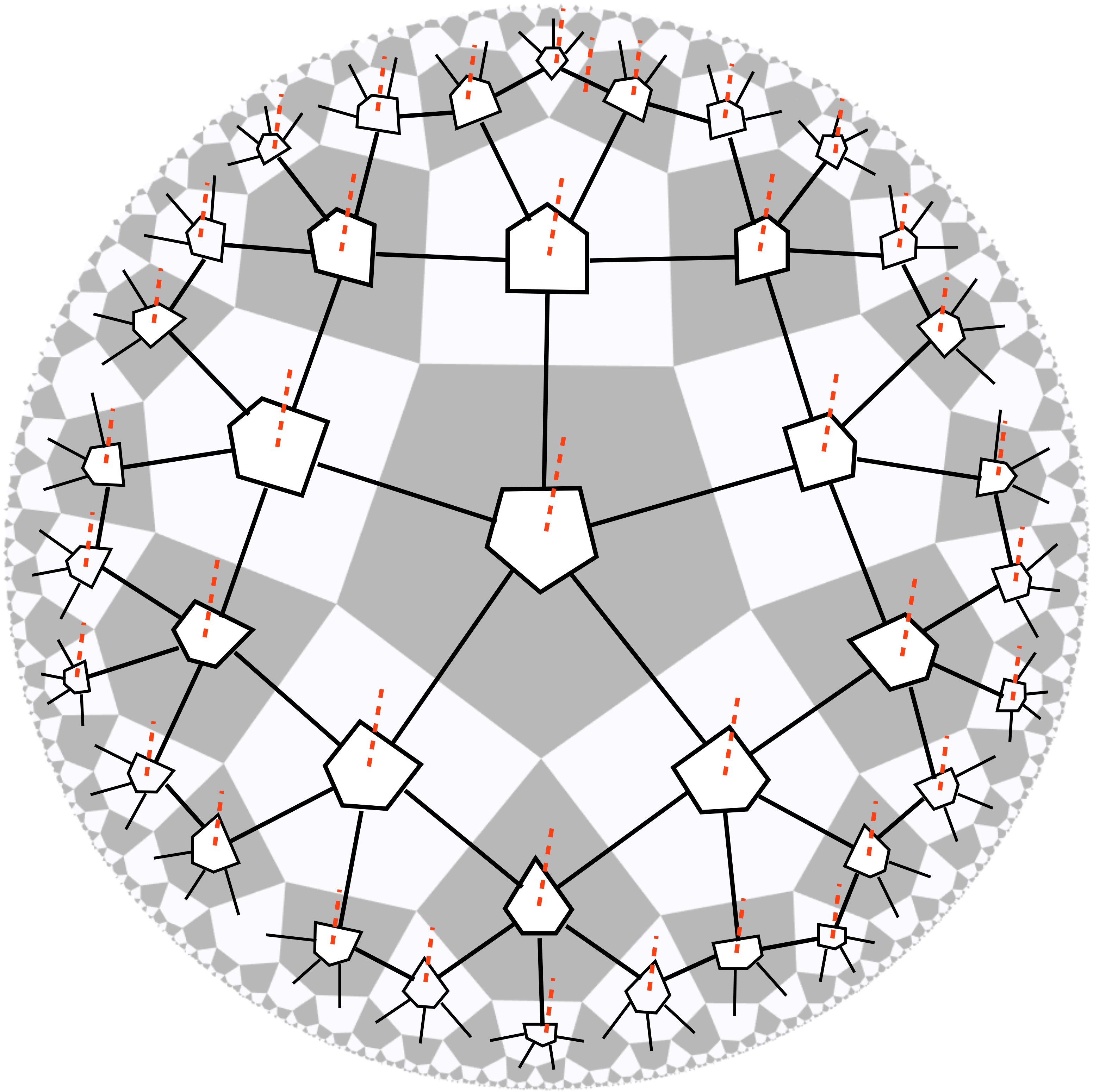
Six-qubit tensor

Isometry from the tensor

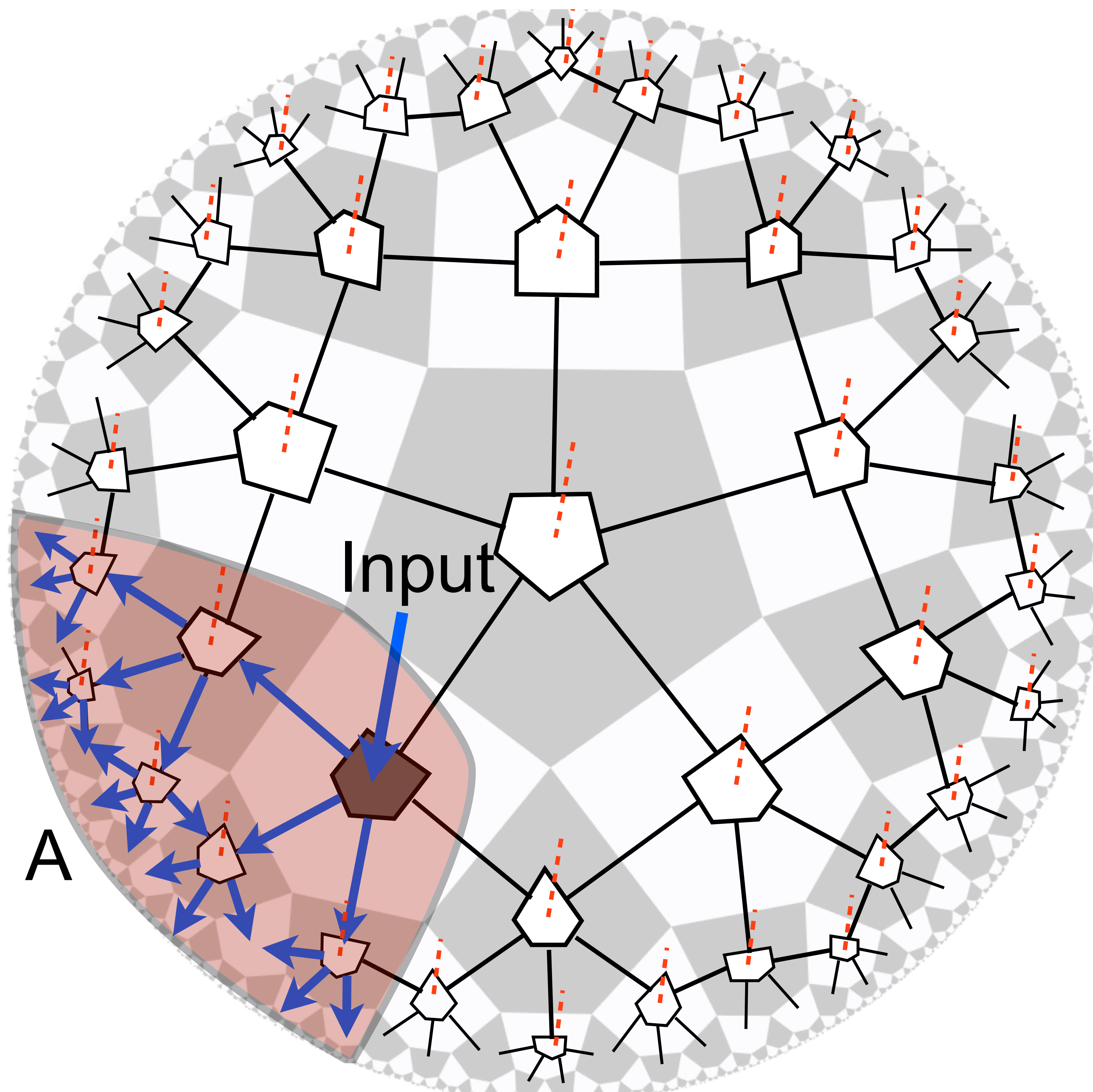


The tensor gives rise to an isometry along any bipartition.

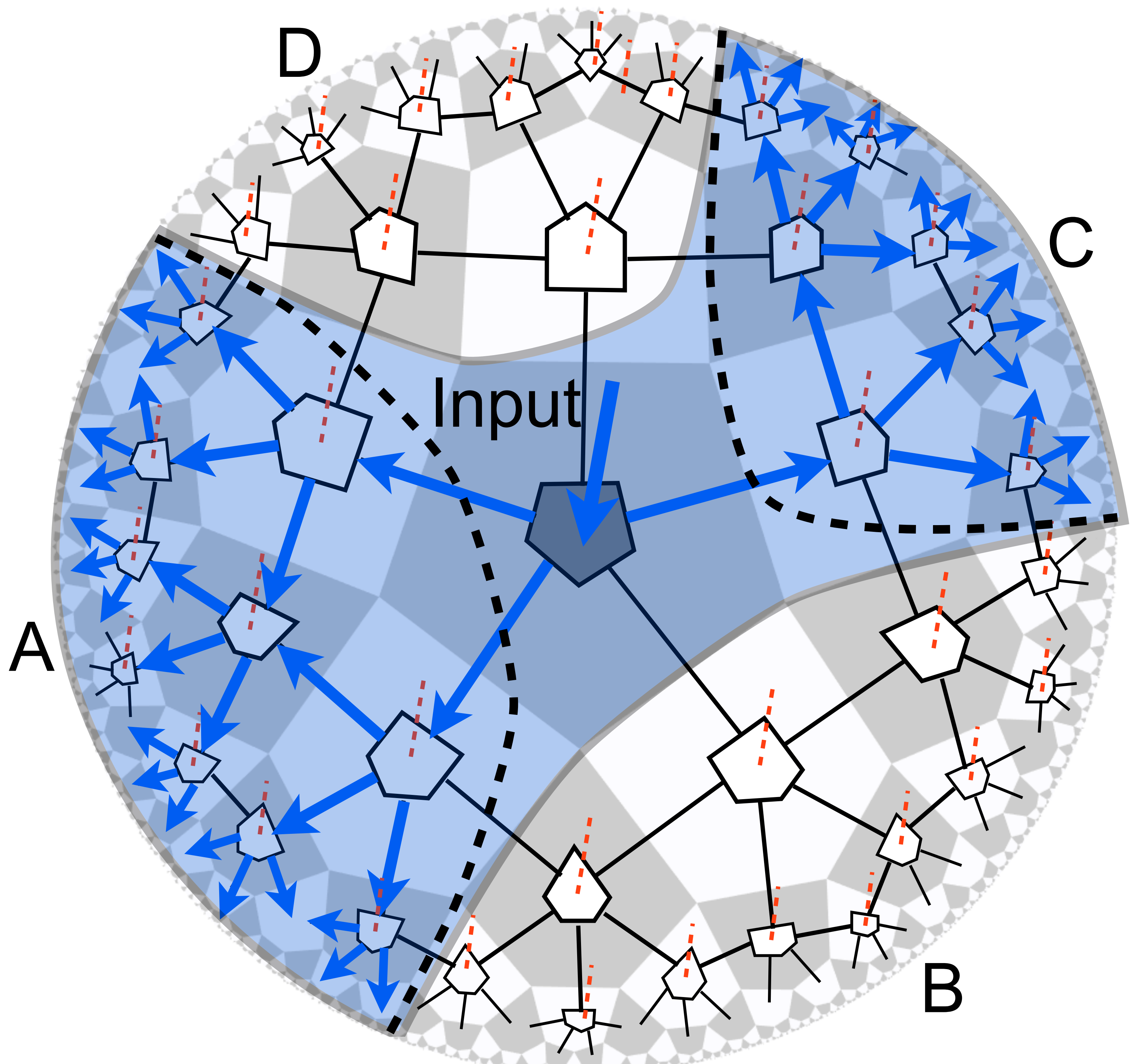
Holographic Code



Causal wedge reconstruction



Entanglement wedge reconstruction



Part II : Generic properties

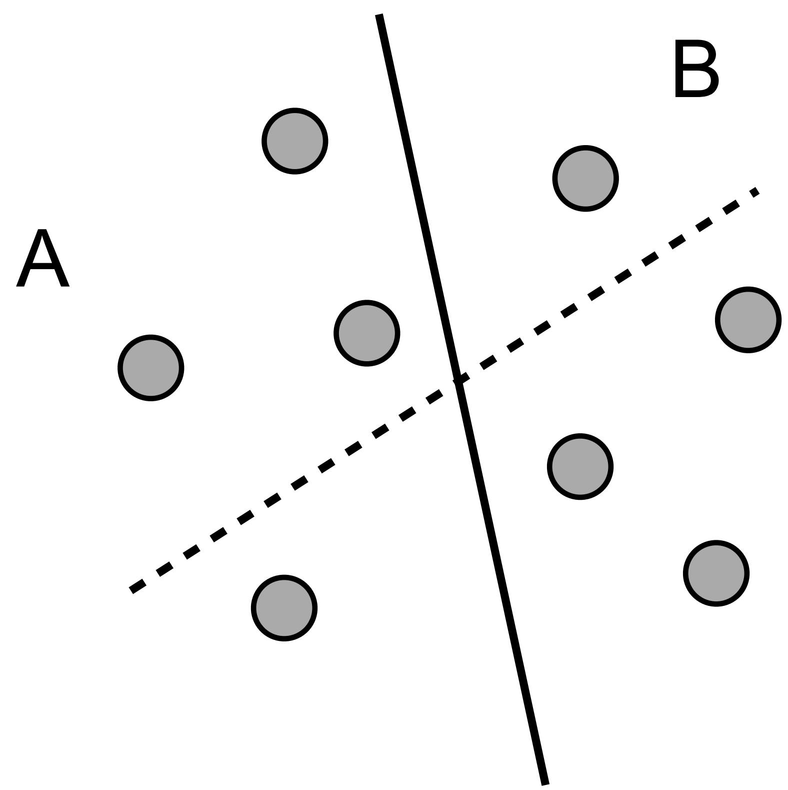
Perfect tensors

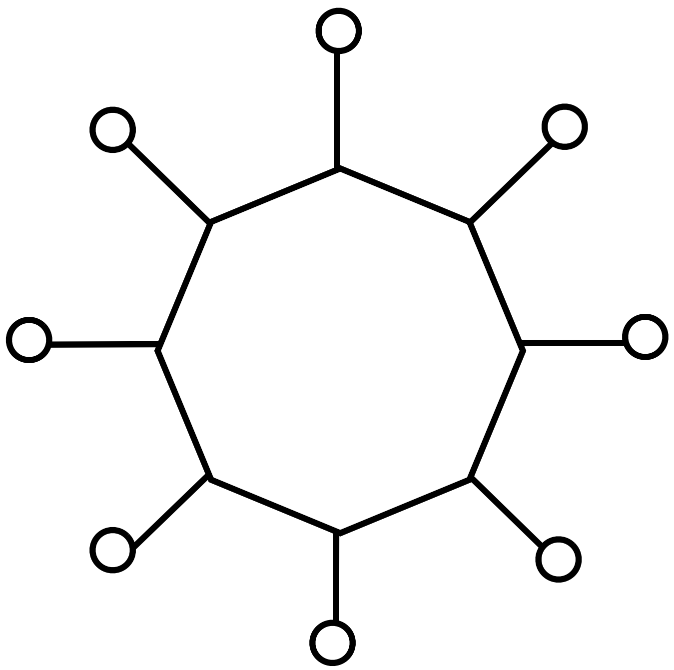
Perfect state / tensor

- A pure state with maximal entanglement in any bipartition

Perfect state (2n spins)

Perfect tensor (2n legs)



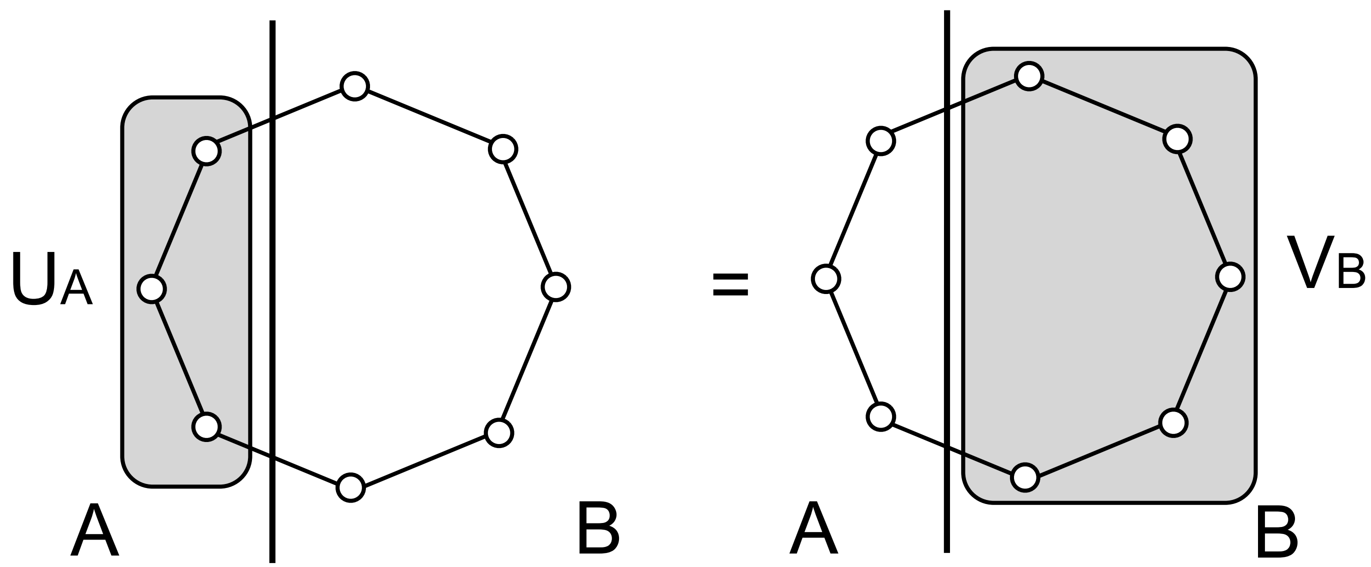


$$\rho_A \propto I_A \qquad \text{for all} \quad |A| \leq n$$

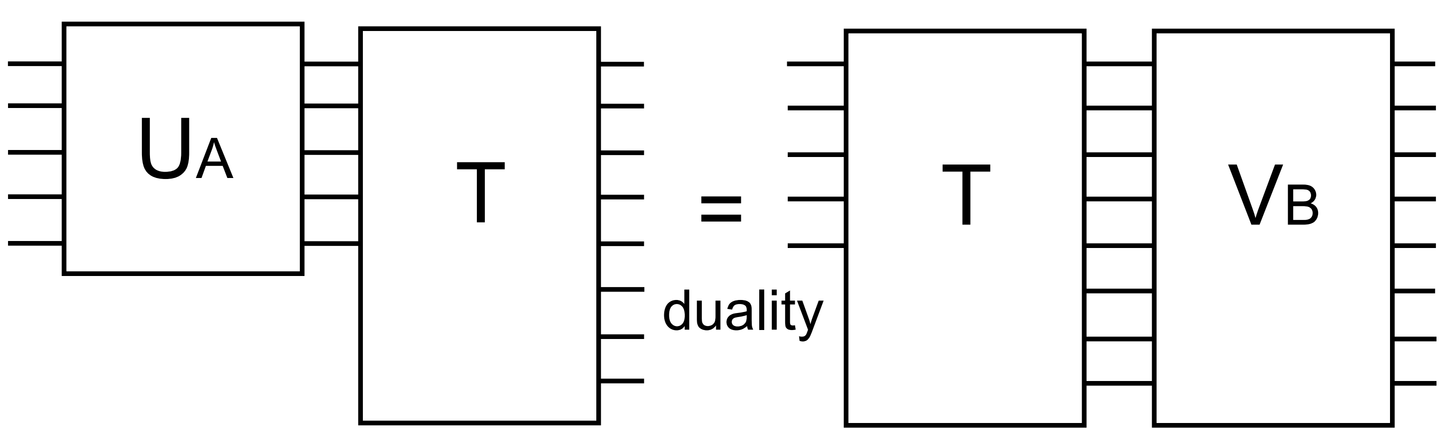
$$|\psi\rangle = \sum_{i_1=0}^{v-1} \sum_{i_2=0}^{v-1} \cdots \sum_{i_n=0}^{v-1} T_{i_1 i_2 \ldots i_n} |i_1 i_2 \ldots i_n\rangle$$

Duality of unitary operators

- Given U_A, there always exists V_B such that $U_A \otimes I|\psi\rangle = I \otimes V_B|\psi\rangle$



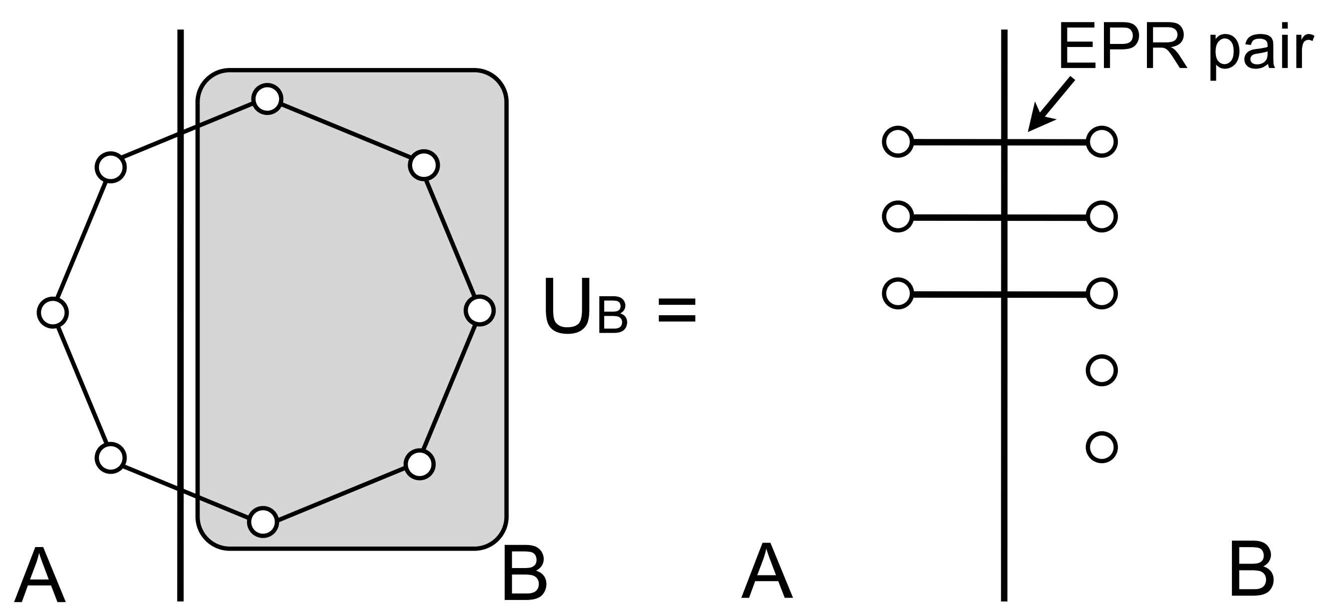
"gate teleportation"



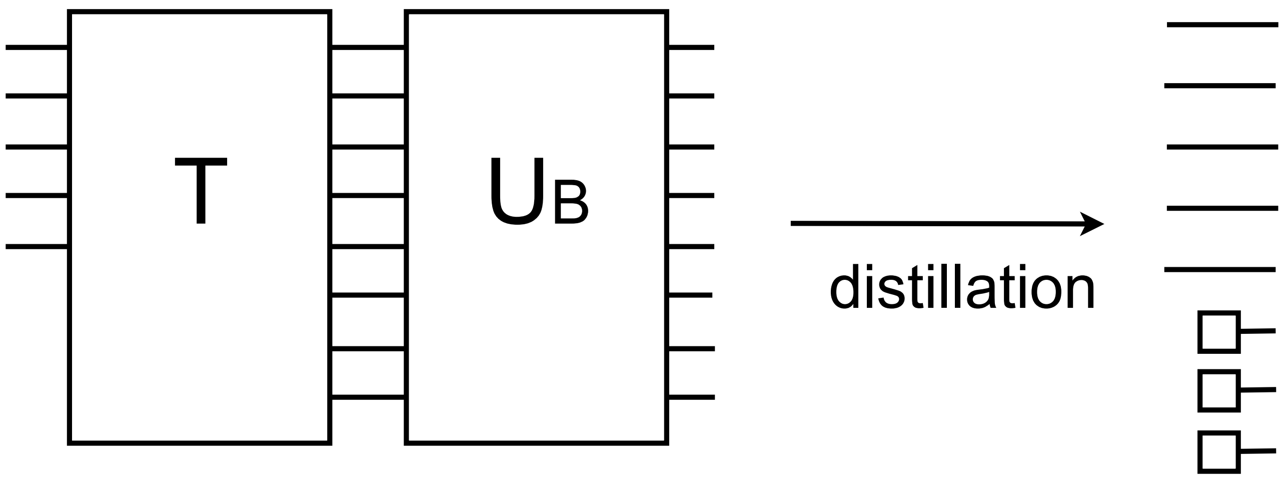
will lead to bulk/
boundary duality...

Distillation of EPR pairs

- By applying a unitary only on B, EPR pairs can be distilled



only on B !



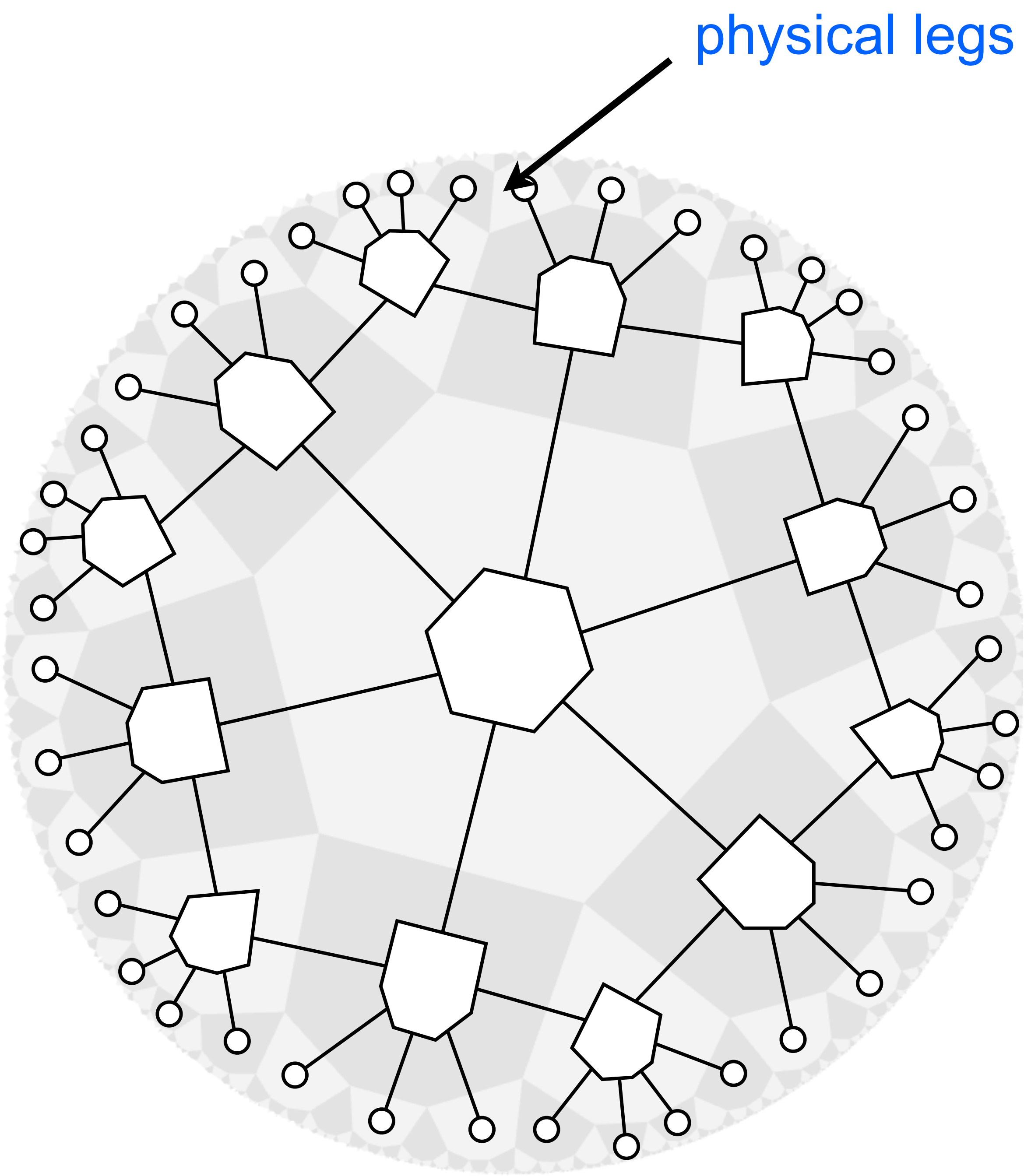
will lead to the RT formula...

Almost perfect tensors

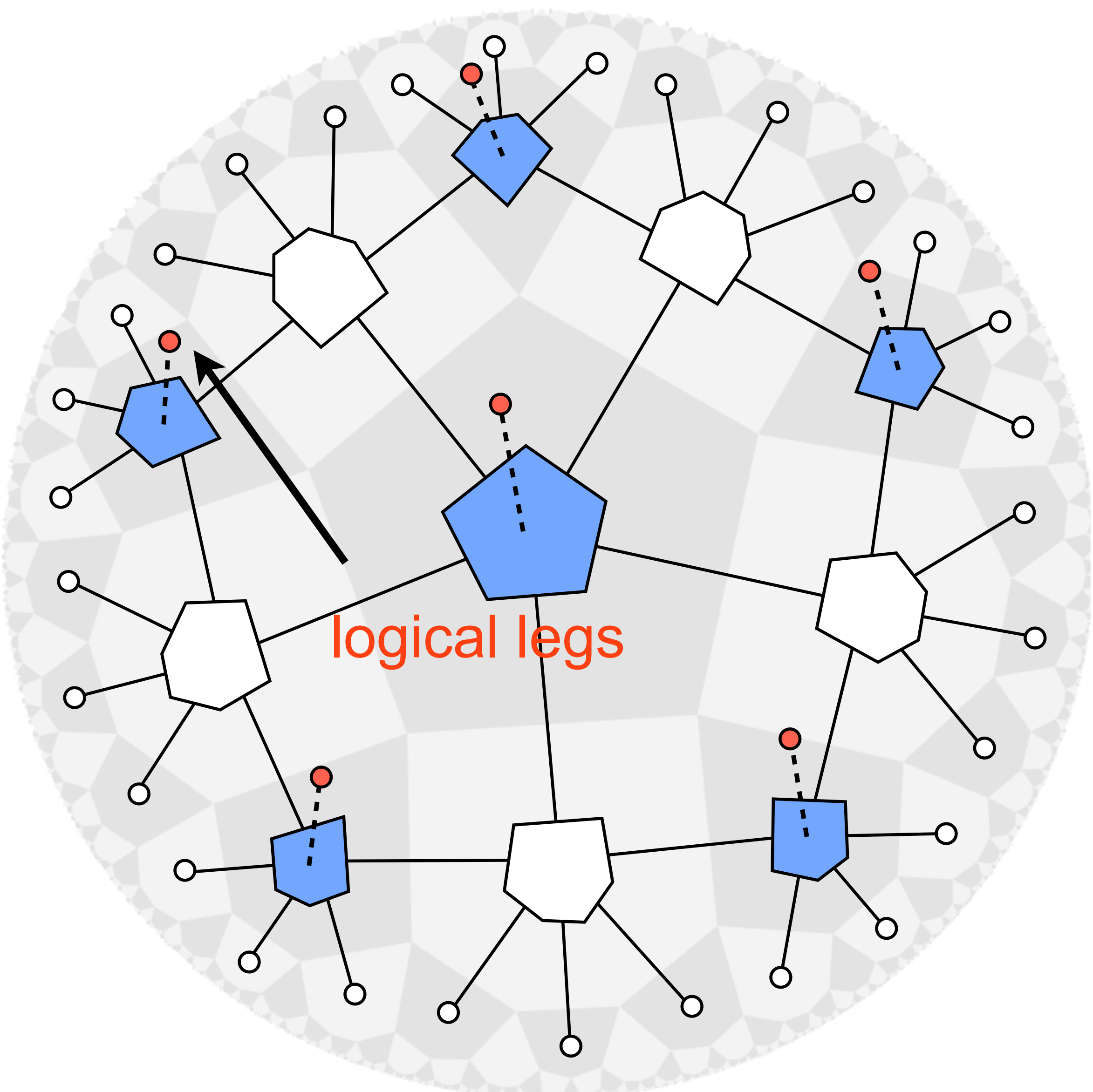
- Perfect tensors are very rare !
  - For qubits, there are only 2-leg and 6-leg perfect tensors.
  - In general, to increase the number of legs, we need to increase the spin (number of internal states).
- But a random tensor does a similar job !
  - Take a random quantum state and construct a tensor
  - It is almost maximally entangled along any cut (canonical typicality, *i.e.* Page's theorem)

Holographic state

Holographic quantum state / code



holographic state



holographic code

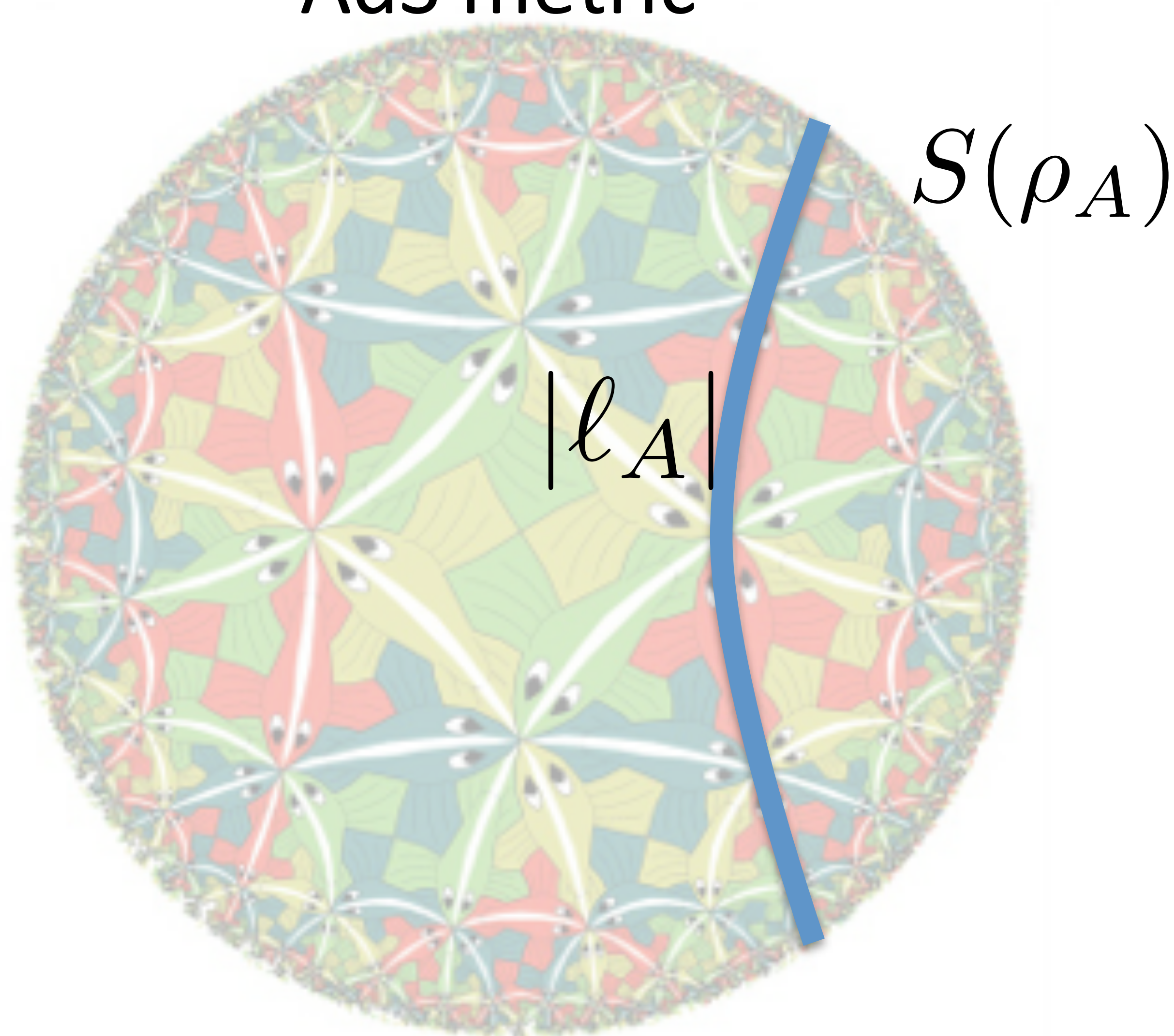
Ryu-Takayanagi formula, it's exact !

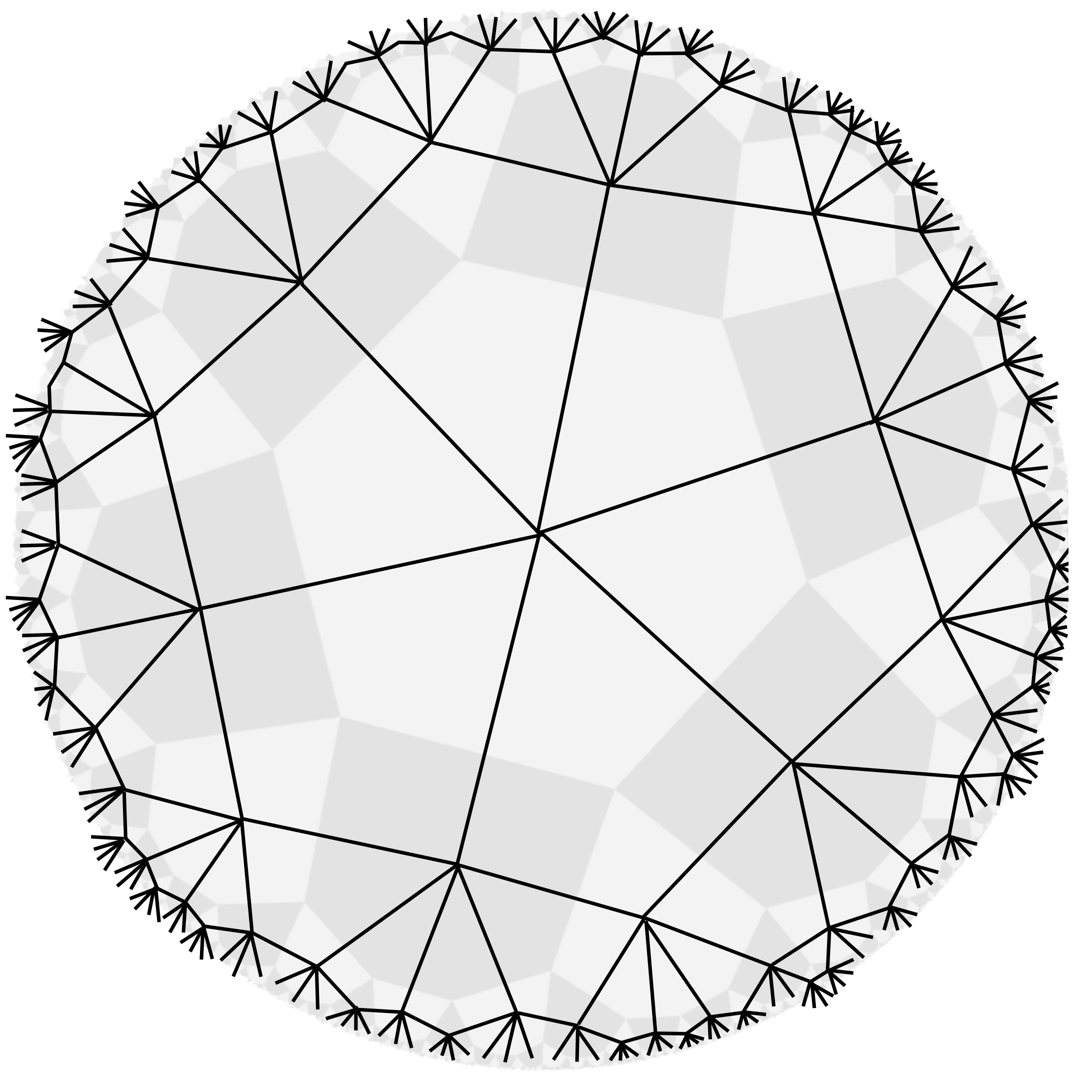
[Claim] Entanglement entropy for A (connected region) is equal to the geodesic length.

$$S_A = \log_2 v \cdot \gamma_A$$

spin dim (v), length (γ_A)

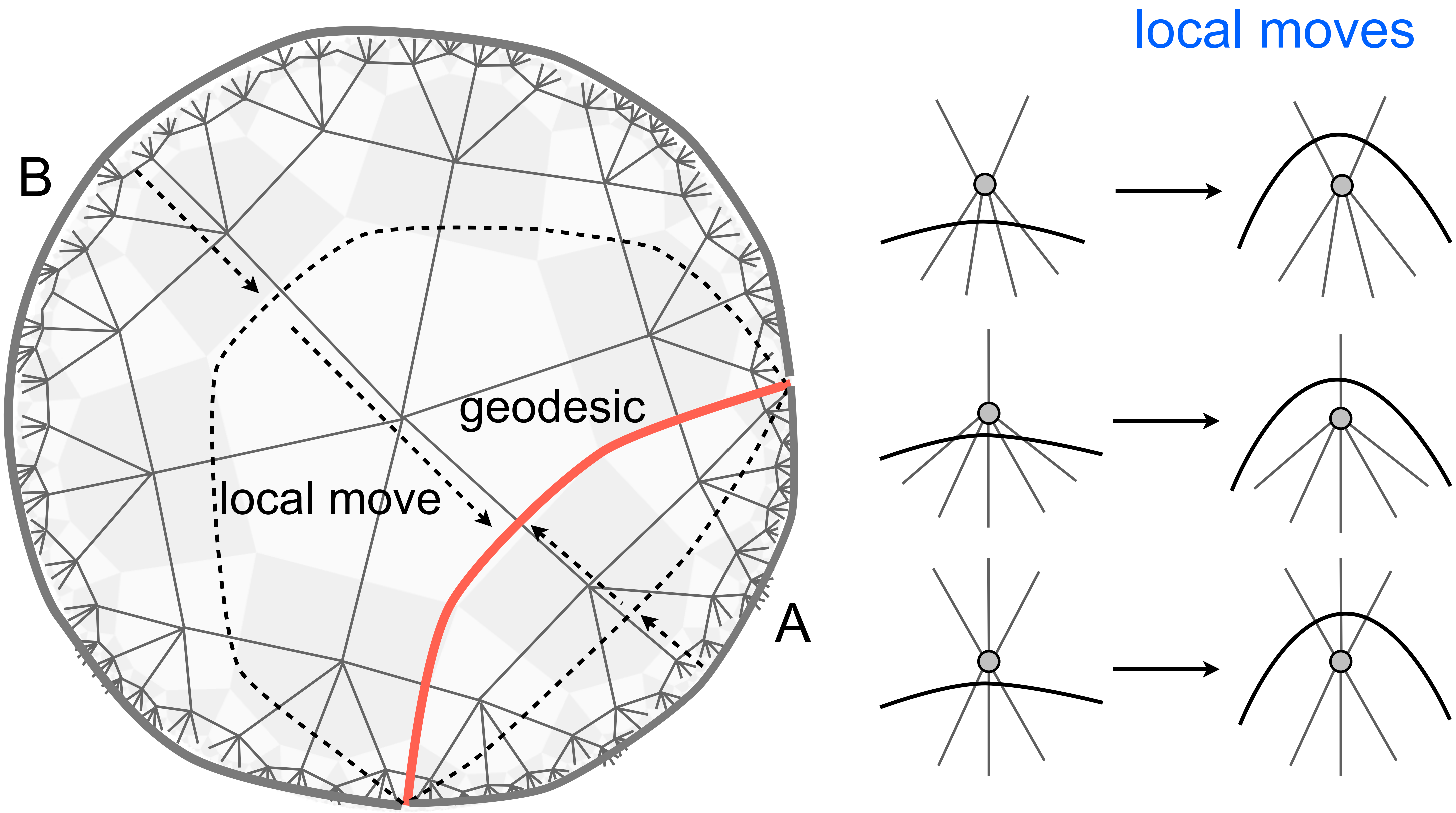
AdS metric





Geodesic line from local moves

Imagine an “algorithm” to find a geodesic line by local updates.



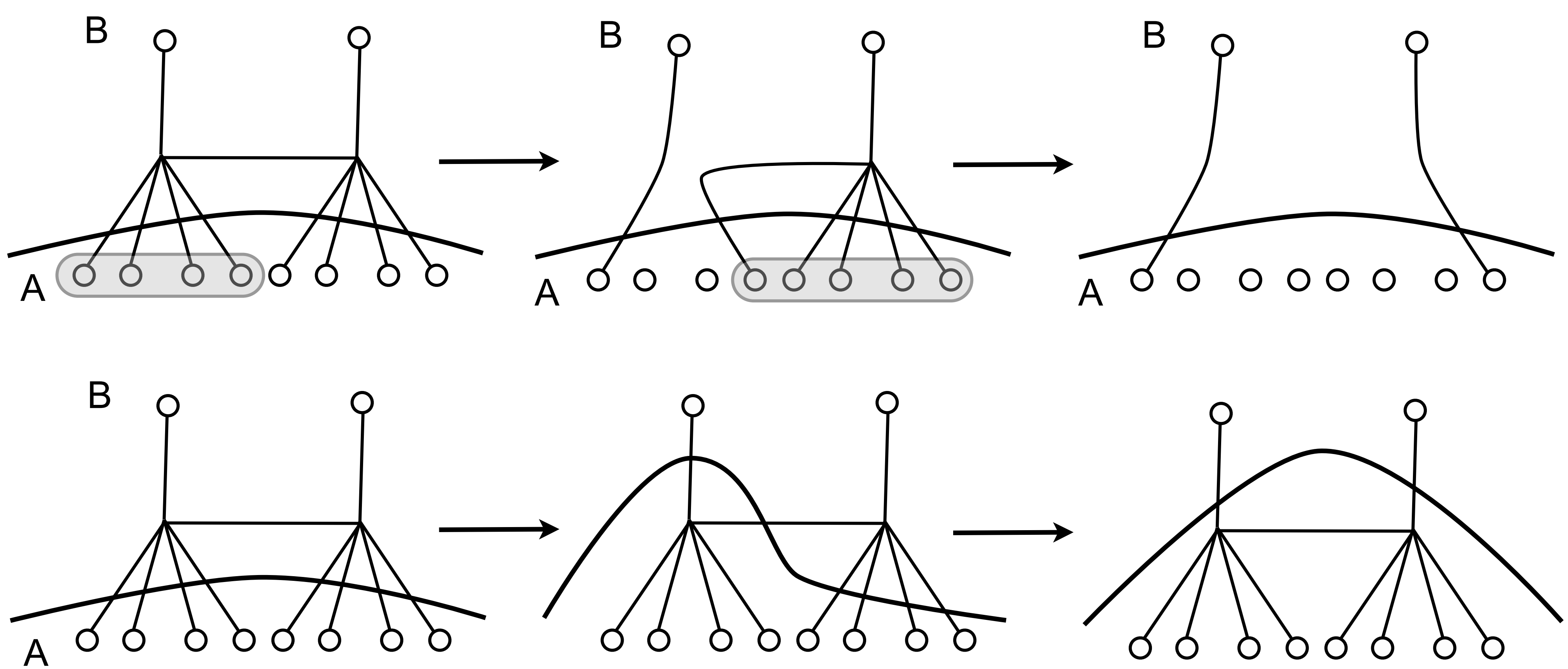
(Even-leg hyperbolic space does not have a local minima).

Local move = distillation of EPR pairs

- Local moves distill EPR pairs and decouple “junks”.

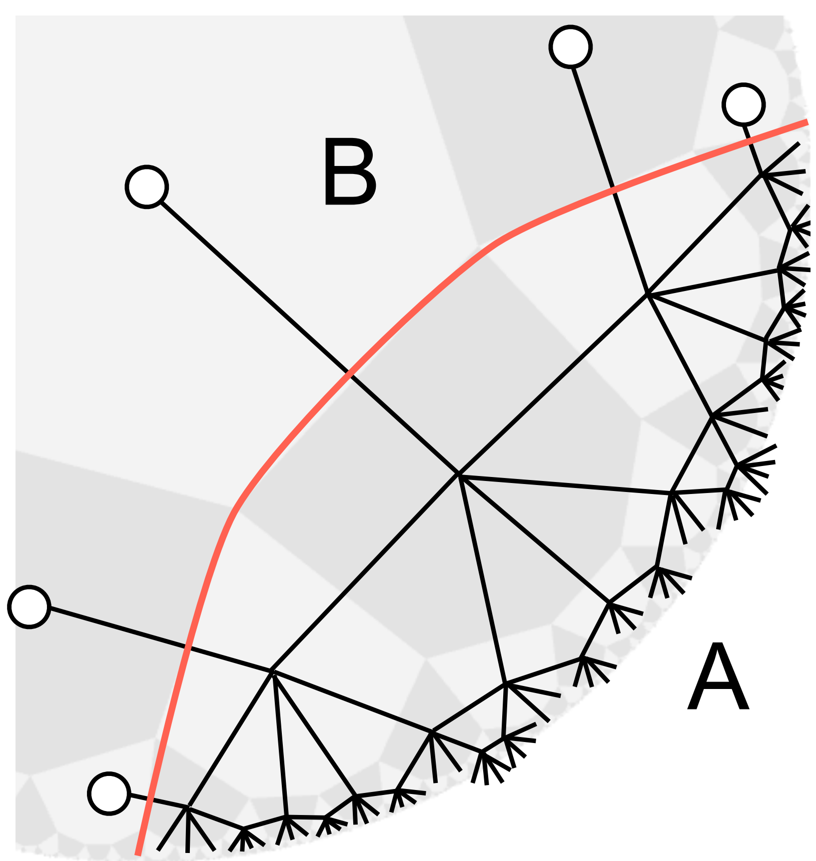
distillation of EPR pairs

local move

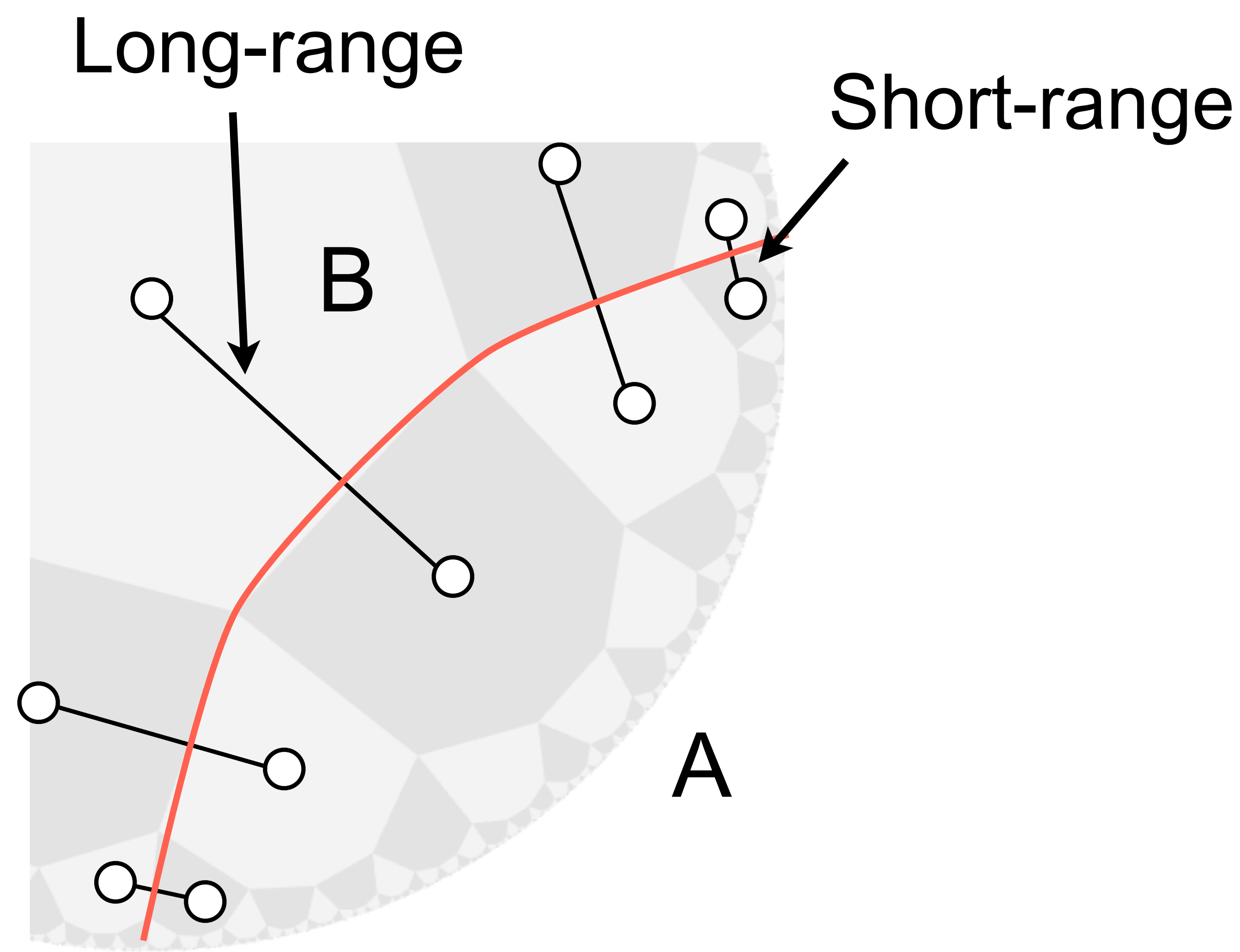


Geometric map of entanglement

- Geodesic line between two regions A and B = EPR pairs



local unitary on B

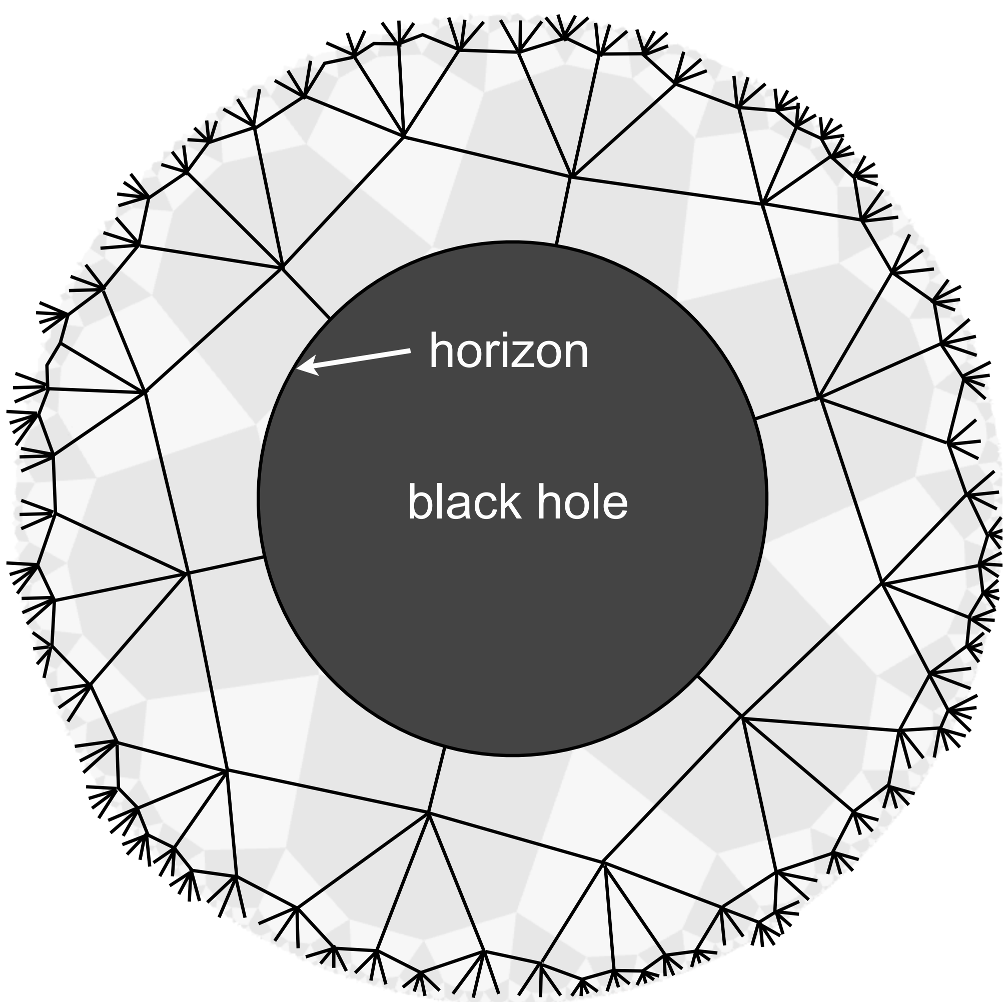


local unitary on A

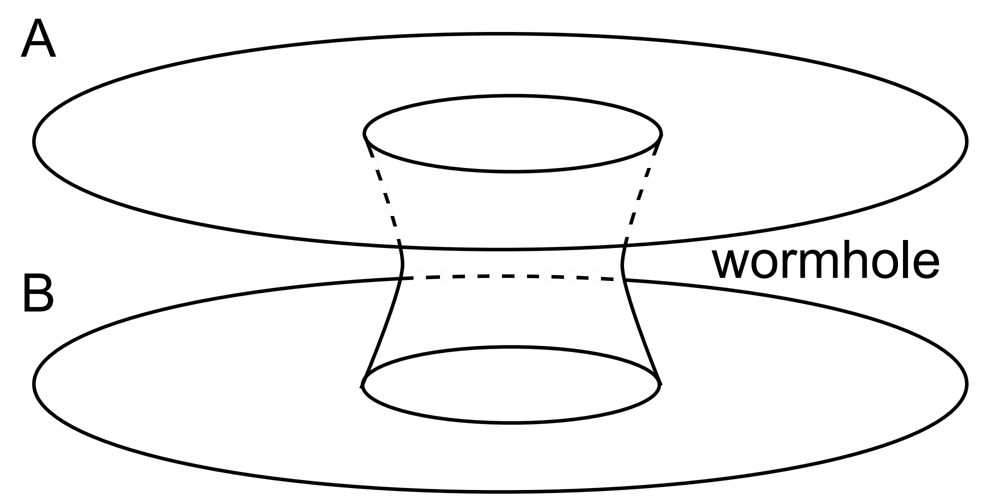
A black hole and wormhole

As a mixed state ρ

As a purified state $|\phi\rangle$



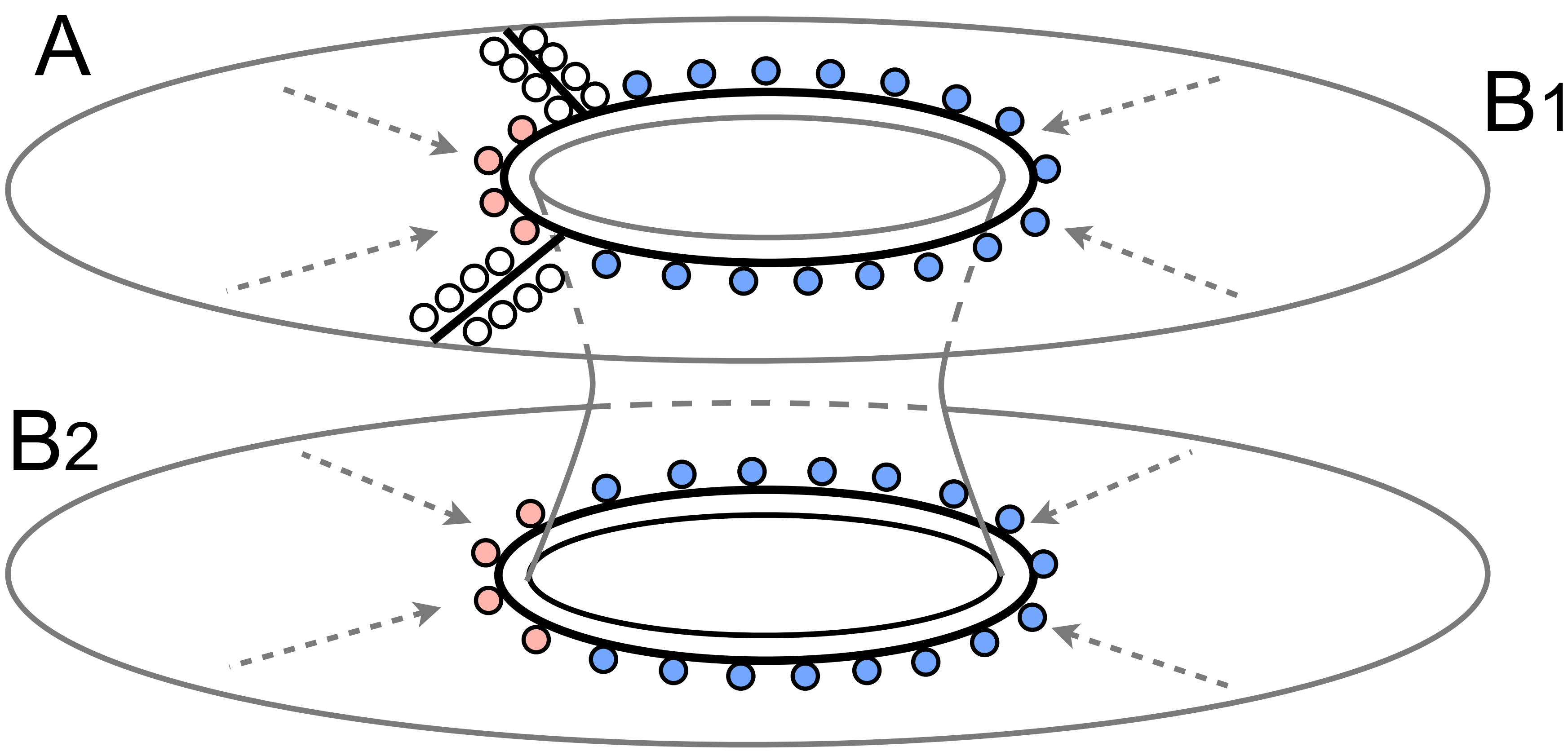
inject maximally mixed state



$$Tr_B(|\phi\rangle\langle\phi|) = \rho$$

Entanglement in a black hole

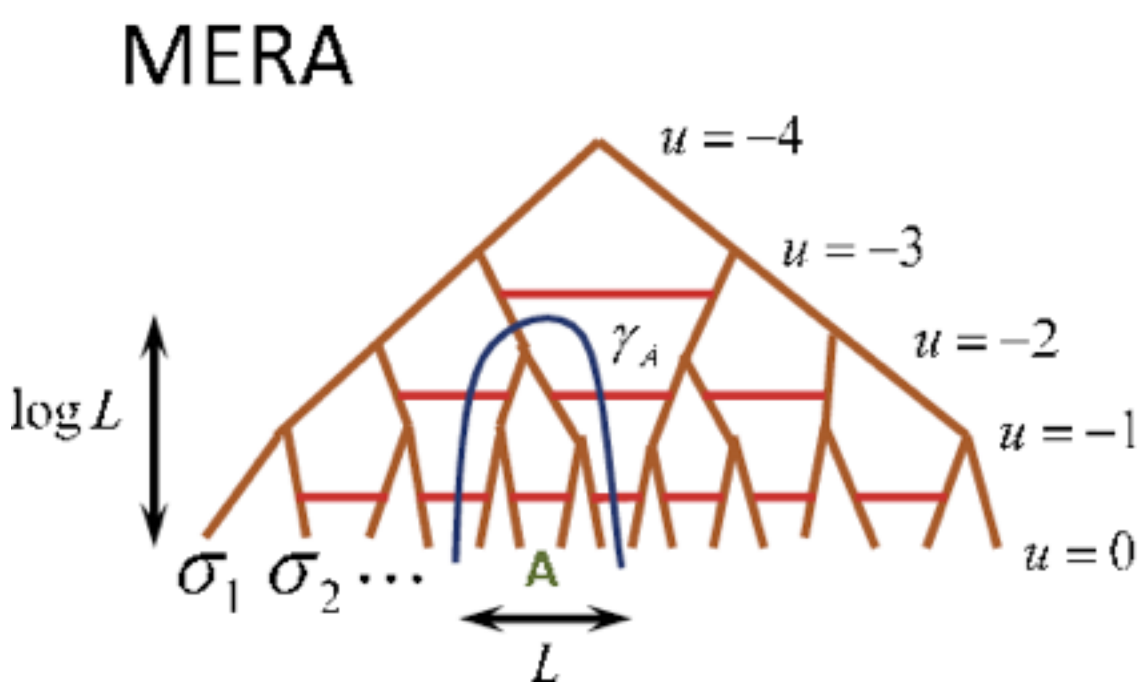
- EPR pairs along the wormhole (ER=EPR ?)
- RT formula with a black hole



Holographic state
(multi-partition)

Multi-partite entanglement

- It is not difficult to create a wavefunction with $S_A \propto \log(L)$, but...



eg) distribute EPR pairs in a tensor tree

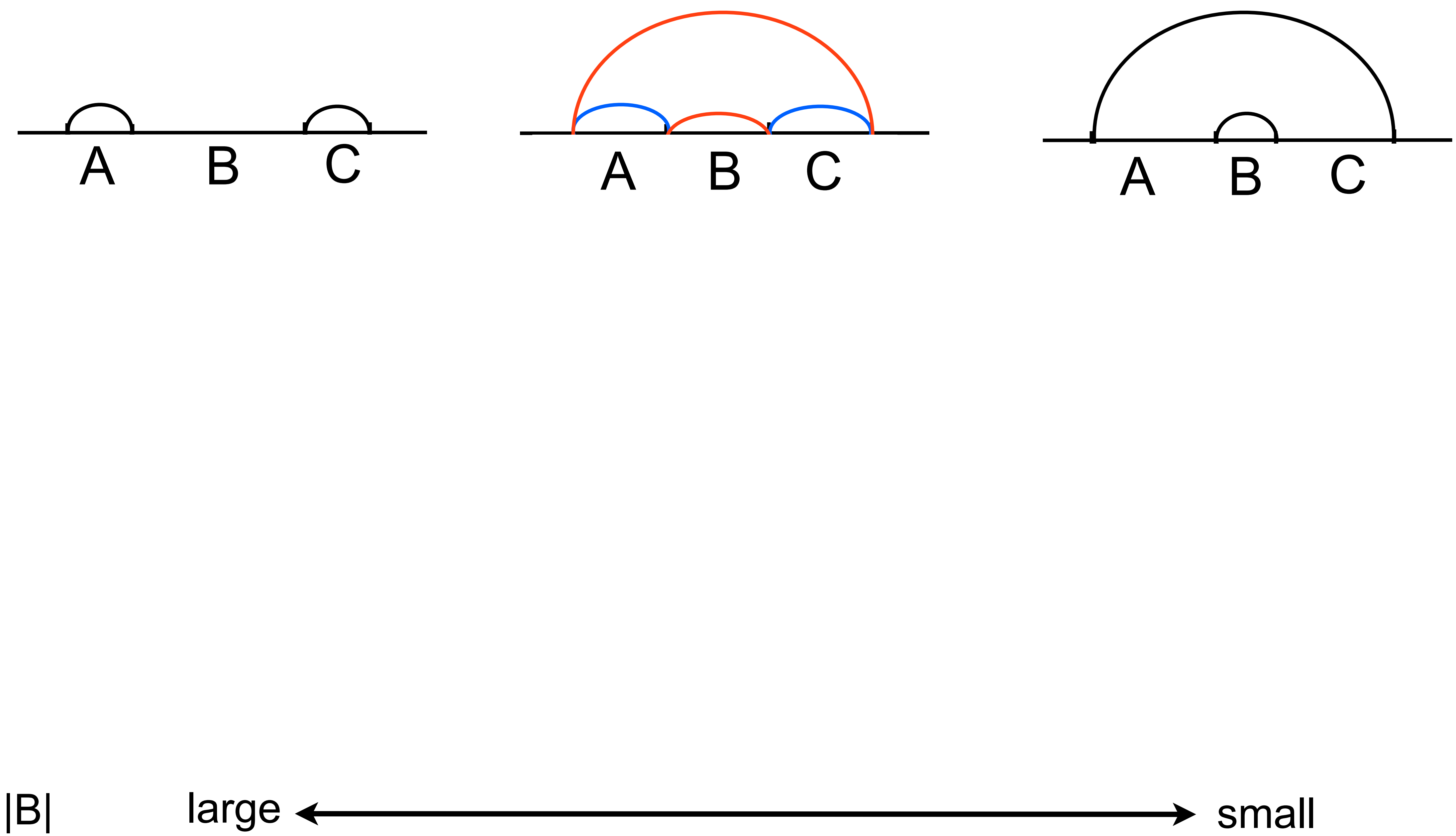
- Negativity of tripartite information (any “holographic state”)

$$I(A,B,C) = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

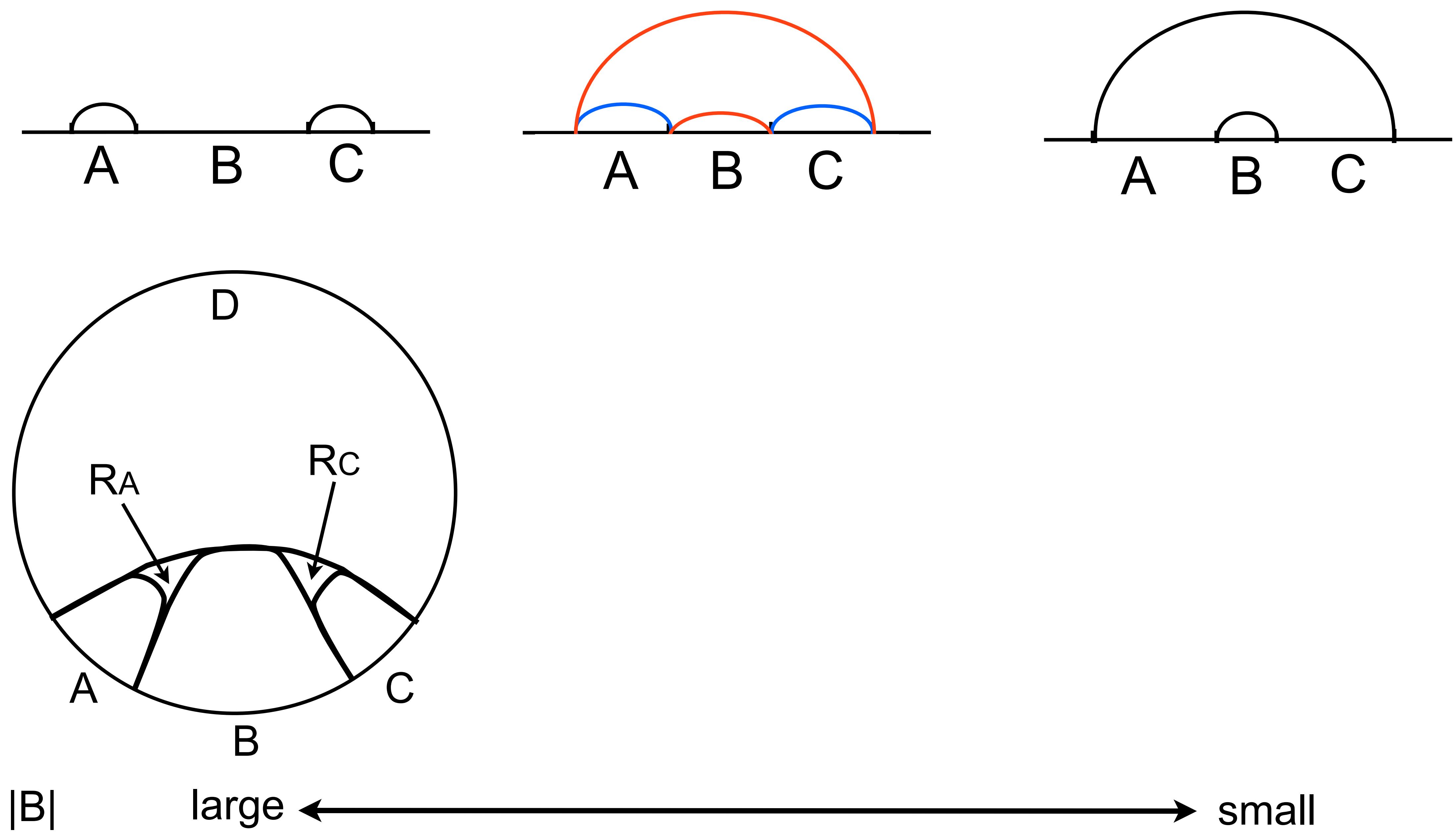
-- EPR pair, then I(A,B,C)=0.

-- GHZ state, then I(A,B,C)>0.

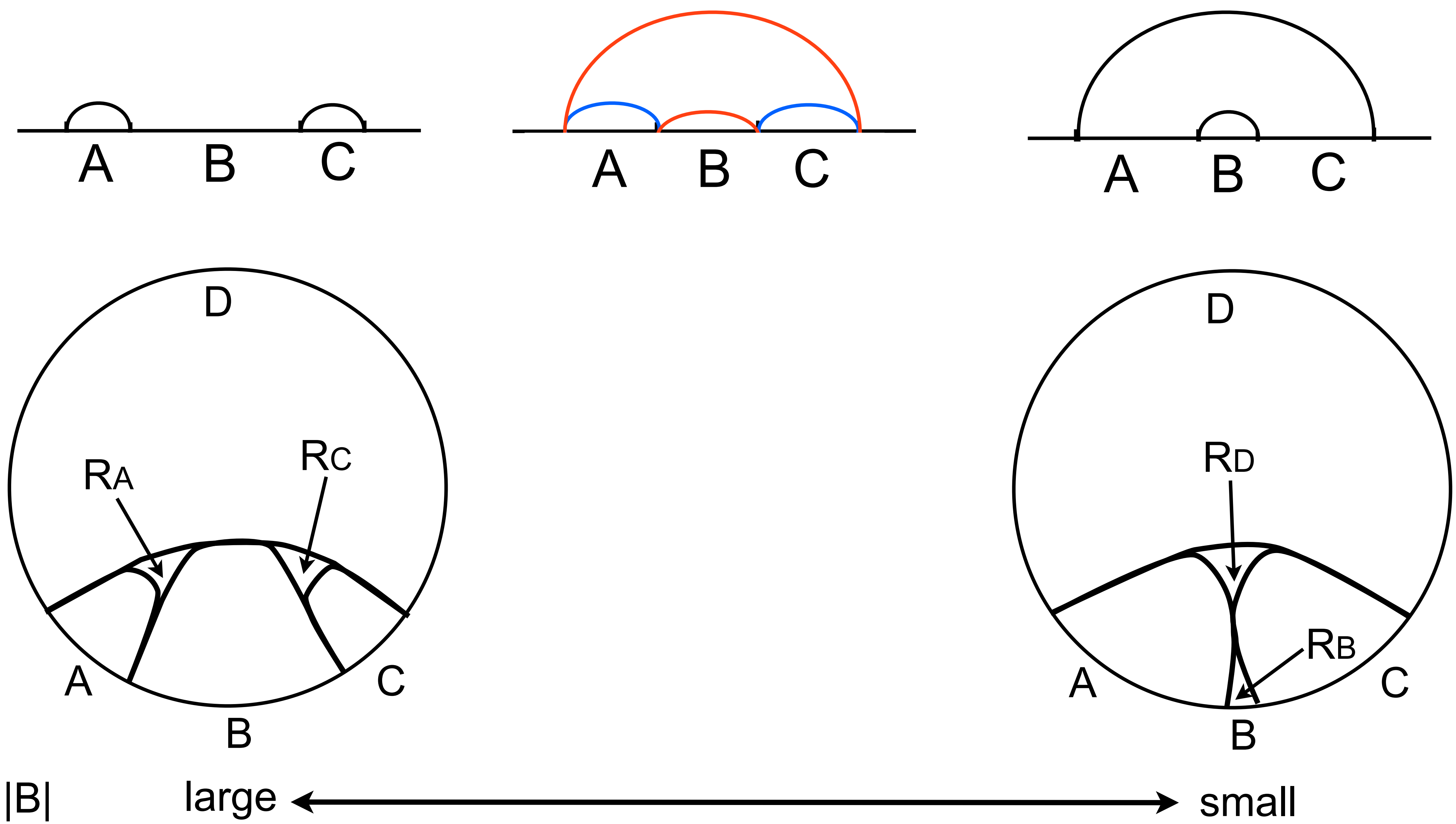
Multi-partition and residual regions



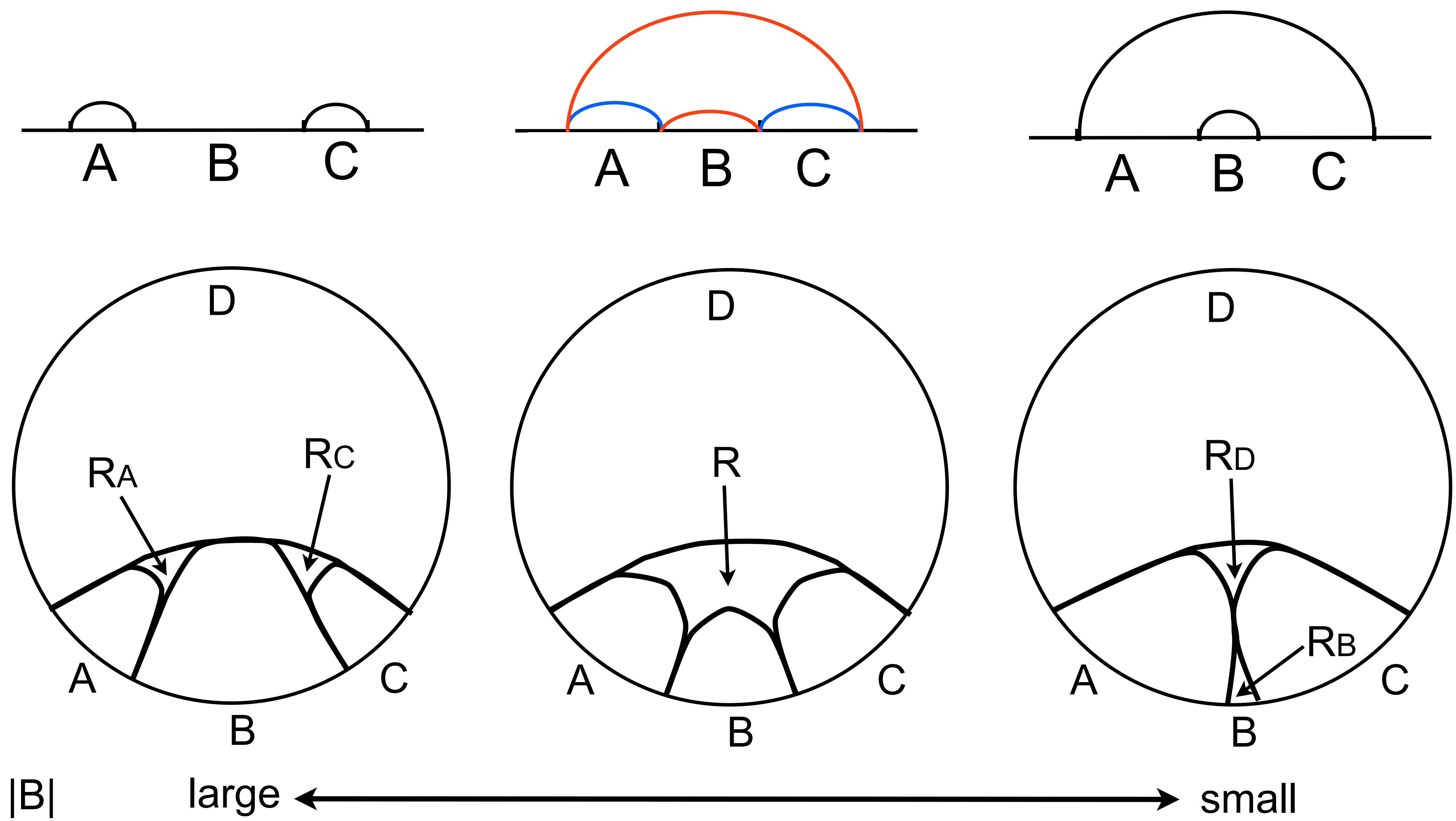
Multi-partition and residual regions



Multi-partition and residual regions

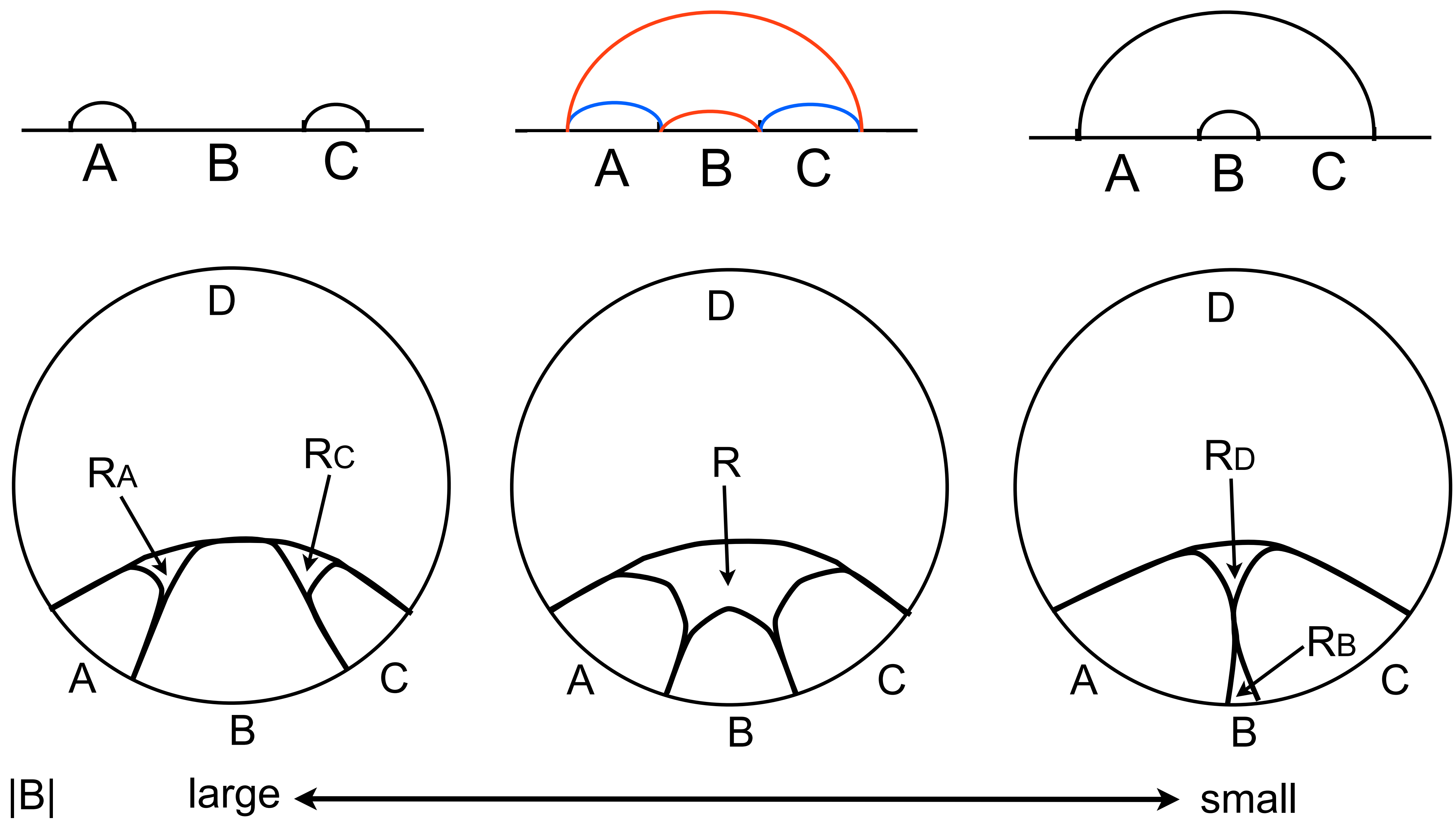


Multi-partition and residual regions

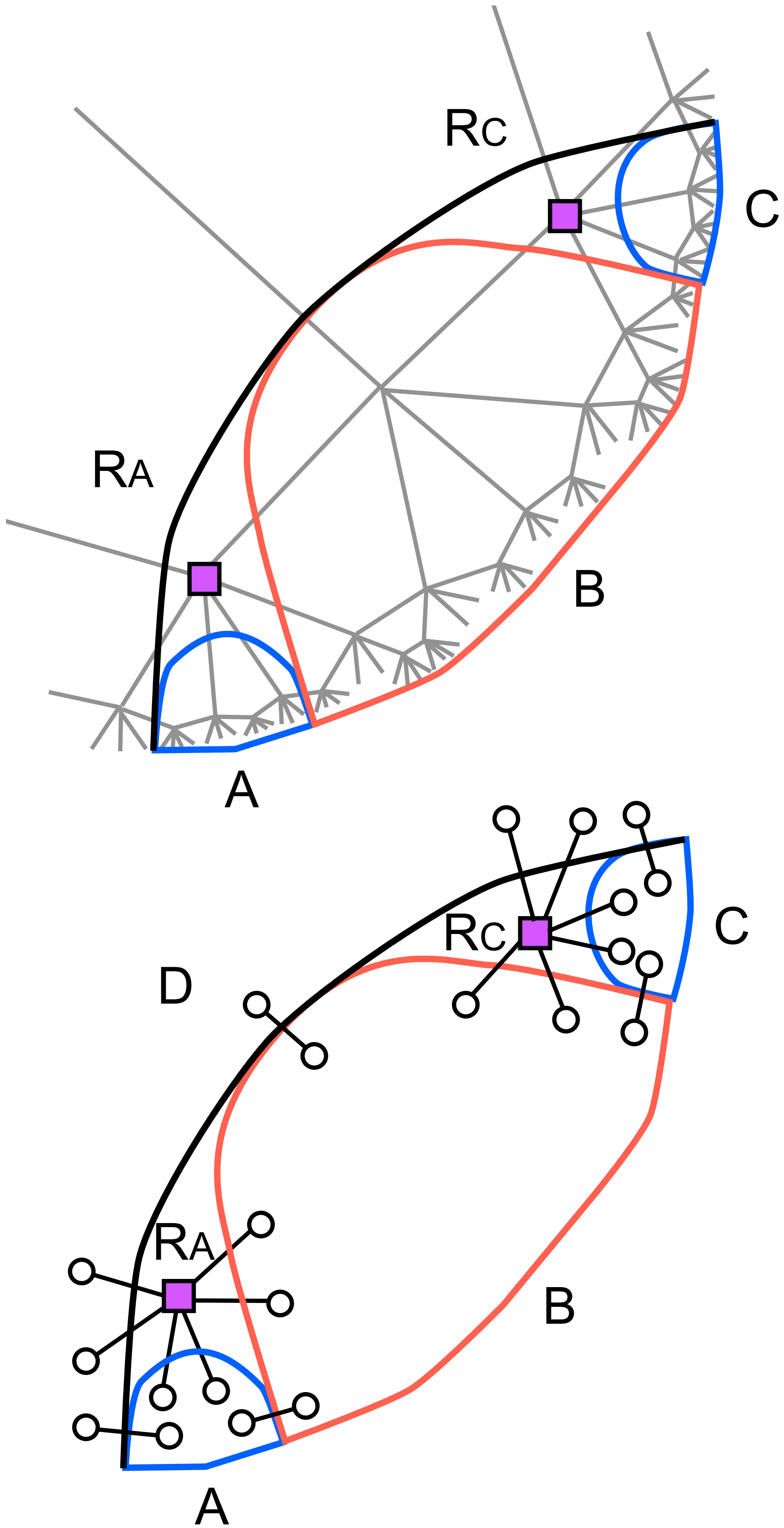


Multi-partition and residual regions

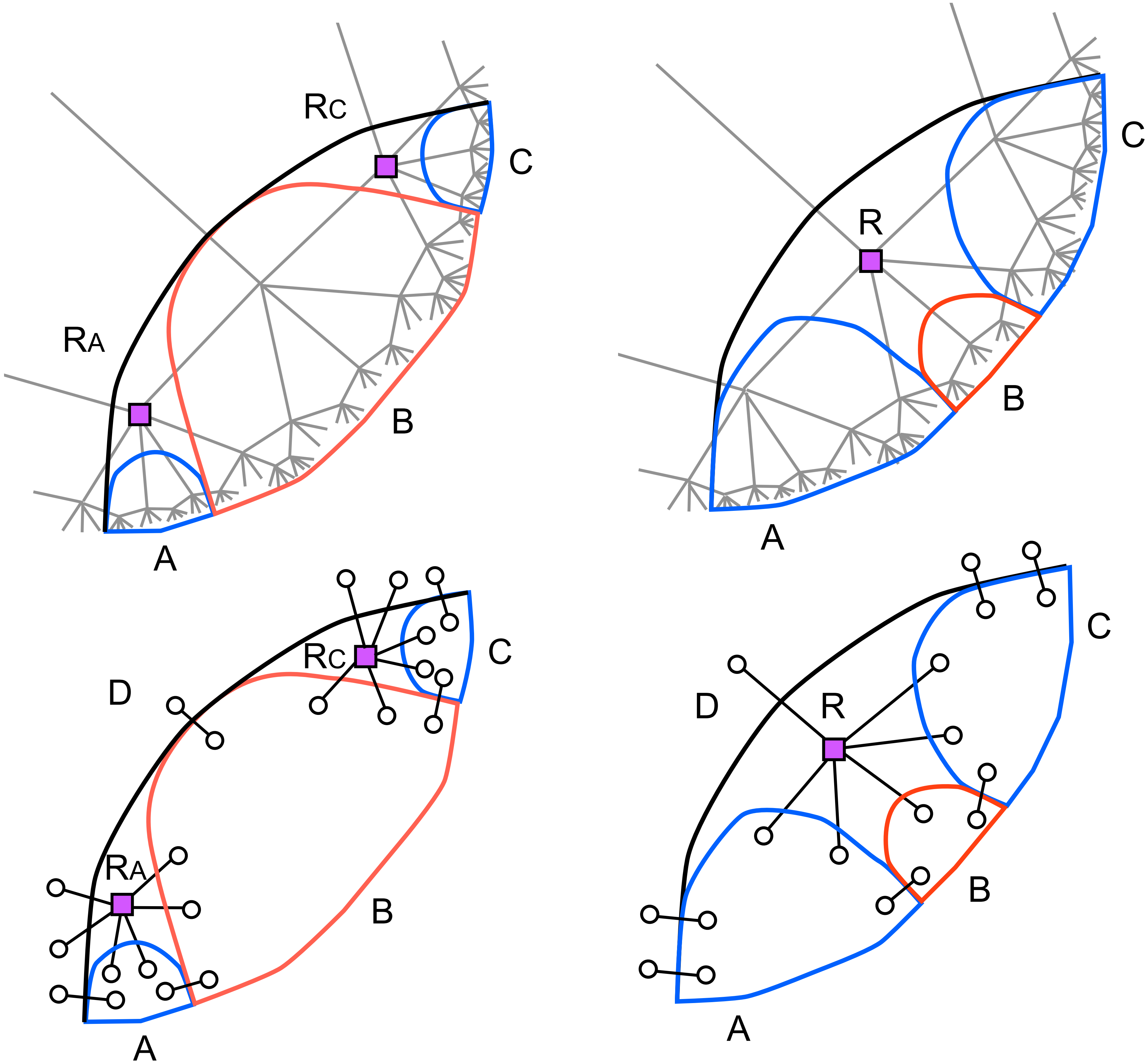
- Identified segment of geodesic lines = EPR pairs
- Residual regions = Multipartite entanglement ?



Residual regions in holographic state

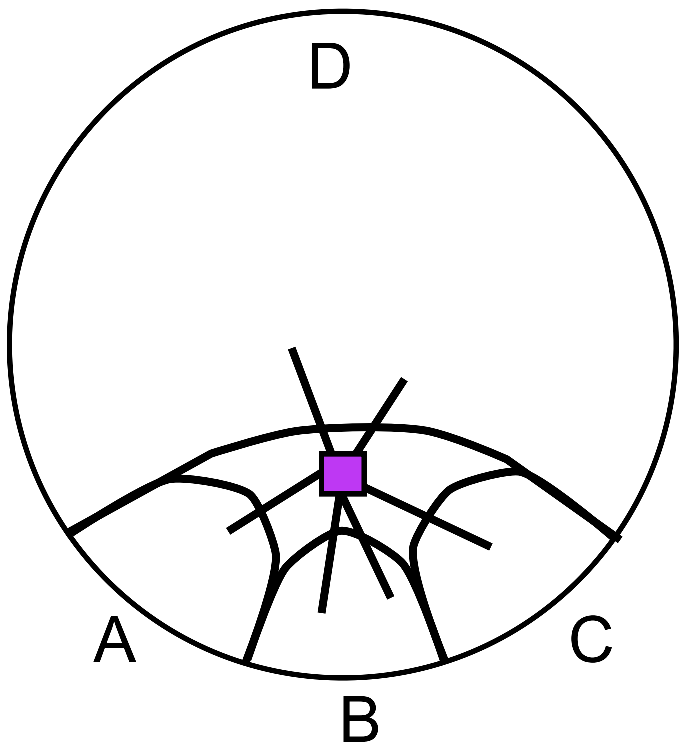


Residual regions in holographic state



Negativity of tripartite entanglement

- Perfect tensor (state) is the key for negative tripartite entanglement !



-- Split 2n-perfect state into four subsets A, B, C, D.

-- Assume 0< |A|, |B|, |C|, |D| < n+1

Then the tripartite entanglement is always negative !

$$I(A,B,C) = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$$

Summary of the talk

- A toy model of the AdS/CFT correspondence

  Causal wedge and entanglement wedge reconstruction

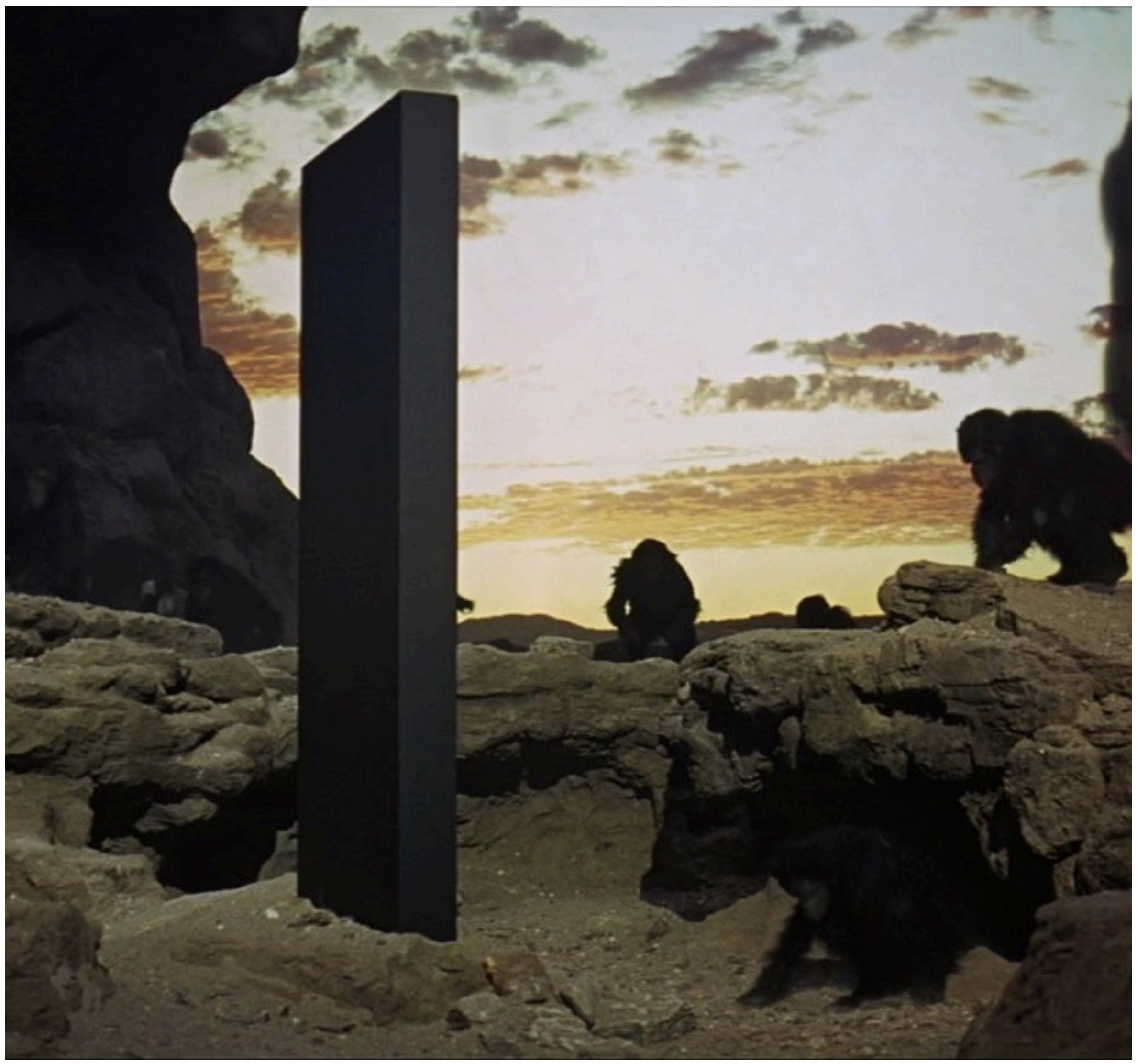
- The RT formula is exact for a single connected region

  Negativity of tripartite entanglement

- Toward the AdS/CFT limit ?

- Dynamics, fast scrambling ?

Von Neumann vs Peter Shor



Entanglement entropy (1964) vs Quantum code (1995)