TOHOKU FORUM for CREATIVITY

Physics Utrecht

EMMEΦ

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The Cellular Automaton Interpretation of Quantum Mechanics

First Principles

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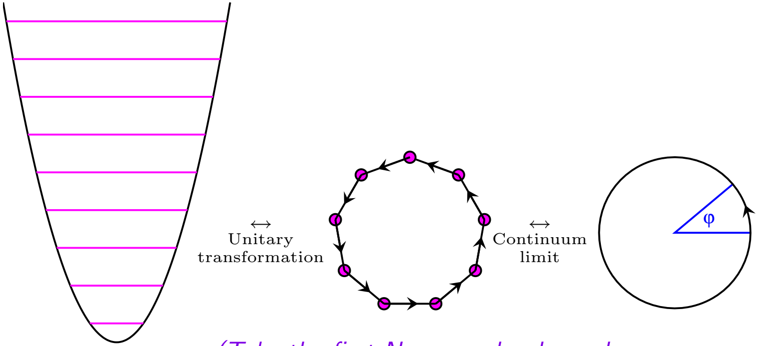
The harmonic oscillator.



(Take the first N energy levels, and make finite Fourier transformation)

Quantum Oscillator ↔ Classical periodic system

In the harmonic oscillator,

$x(t)$ and $p(t)$ are QM operators, or *observables*

$\varphi(t)$ is a very special type of observable operator called *beable*:

Beables are operators $\mathcal{B}_i(t)$ with the property that, at all $t,\ t'$:

$$[\mathcal{B}_i(t),\ \mathcal{B}_j(t')] = 0\ .$$

In many models, one can find such a set of beable operators.

example:

Massless, chiral, non interacting neutrinos are *deterministic:*

Second-quantised theory: $H = -i\,\psi^\dagger\,\sigma_i\partial_i\,\psi$

First quantised theory: $H = \sigma_i p_i$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{O}(t) \;=\; i[\mathcal{O}(t),\, H]$$

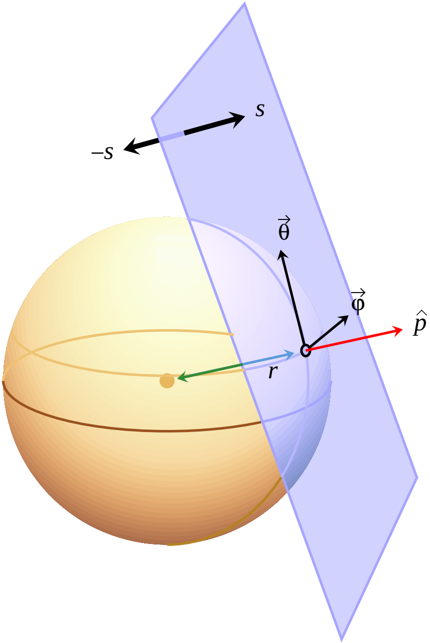
Beables $\{\mathcal{O}_i^{\mathrm{op}}\} \;=\; \{\hat{p},\, s,\, r\}:$

$$\hat{p} \equiv \pm\vec{p}/|p|\ , \quad s \equiv \hat{p}\cdot\vec{\sigma}\ , \quad r \equiv \tfrac{1}{2}(\hat{p}\cdot\vec{x} + \vec{x}\cdot\hat{p})\ .$$

$$|\hat{p}| = 1\,, \qquad s = \pm 1\,, \qquad \infty < r < \infty$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{p} = 0\,, \qquad \frac{\mathrm{d}}{\mathrm{d}t} = 0\,, \qquad \frac{\mathrm{d}}{\mathrm{d}t}r = s$$

These beables form a *complete set*



The neutrino sheet.
Beables: $\{\hat{p},\ s,\ r\}$

The eigenstates of these operators span the entire Hilbert space.

Introducing operators in this basis, one can reconstruct the usual operators $\vec{x}\,,\ \vec{p}\,,\ \sigma_i$

Interesting mathematical physics:

$$x_i = \hat{p}_i\,(r - \frac{i}{p_r}) + \varepsilon_{ijk}\,\hat{p}_j\,L_k^{\rm ont}/p_r + \frac{1}{2p_r}\Big(-\varphi_i\,s_1 + \theta_i\,s_2 + \frac{\hat{p}_3}{\sqrt{1-\hat{P}_3^2}}\,\varphi_i\,s_3\Big) \tag{1}$$

θ_i and φ_i are beables, functions of $\hat{q}$.
$L_k^{\rm ont}$ are generators of rotations of the sheet,
$s_3 = s$, s_1 and s_2 are spin flip operators.

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The Hamiltonian of the first-quantised theory has no ground state, but, just as in Dirac's theory, the second quantised theory *does* have a ground state

The (finite) Automaton

Consider a system that can be in a very large number, N, of different states, named (0), (1), (2), $\cdots$, $(N-1)$. Suppose it obeys a *deterministic* evolution law:
over a fundamental unit time step δt the states are permuted according to a given element P of the permutation group P_N.

Now consider the *representation* of this permutator in vector space:

$$P = \begin{pmatrix} 0 & 0 & 0 & \cdots & 1 & 0 \\ 1 & 0 & & & & \\ 0 & 1 & & & & \\ 0 & & & & & \\ \vdots & & & \ddots & & \end{pmatrix}$$

write as

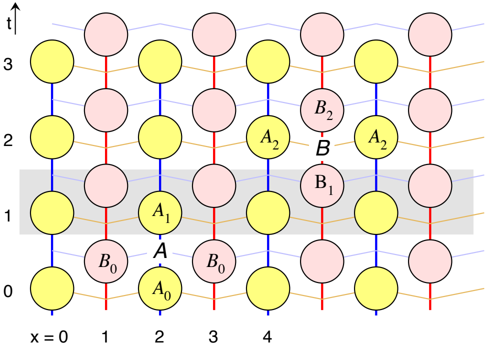
$$U(\delta t) = e^{-iA}$$

$$A = H\,\delta t$$

Possible solution (*except for ground state*):

$$H\delta t \;=\; i \log U \;=\; \pi + \sum_{n=1}^{\infty} \frac{1}{i n\,\delta t}\Big(U(n\,\delta t) - U(-n\,\delta t)\Big)\;.$$

The cellular automaton



$$U = e^{-iH} \;=\; e^{-iA}\, e^{-iB} \;; \quad A = \textstyle\sum_x A(x) \;, \quad B = \textstyle\sum_x B(x)$$

where $[A(x),\, A(x')] = 0$, $[B(x),\, B(x')] = 0$; $[A(x),\, B(x')] \neq 0$ only if x and x' are neighbors. Baker Campbell Hausdorff :

$$H = A + B - \tfrac{1}{2} i[A, B] - \tfrac{1}{12}([A, [A, B]] + [[A, B], B]) + \cdots .$$

CAI

The Cellular Automaton Interpretation of Quantum Mechanics

If the Hamiltonian of the world happens to be that of an automaton, we can identify observables called *Beables*.

beables $\mathcal{B}_i(t)$ are ordinary quantum operators that happen to obey $[\mathcal{B}_i(t), \mathcal{B}_j(t')] = 0$.

The eigenstates of $\mathcal{B}_i(t)$ at a given time t form a *basis*, called the *ontological* (ontic) *basis*.

In a given quantum theory, it's not known how to construct an ontic basis.

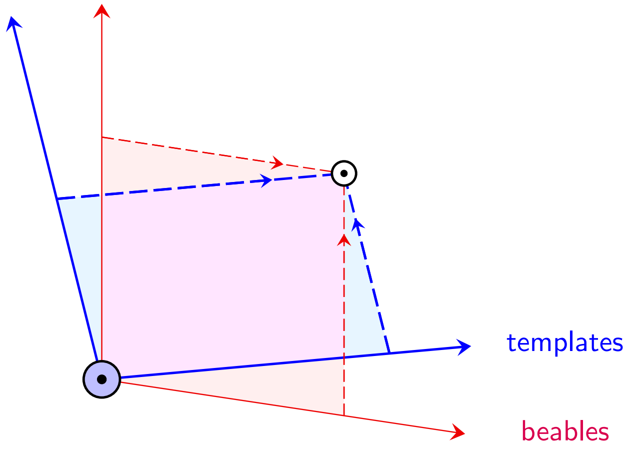
But one can come very close . . .

The CAI *assumes* that it exists. Its ontic states can be constructed from the ordinary quantum states.

If the beables can be constructed *more or less locally* from the known states, then we have a classical, "hidden variable theory".

The use of Templates

Hydrogen atom, plane waves of in- or out-particles, *etc.*



The states we normally use to do quantum mechanics are called *template states*. They form a basis of the kind normally used. This is a unitary transformation. Templates are quantum superpositions of ontic states and *vice versa*.

They all obey Schrödinger's equation!

In a quantum calculation, we may assume the intial state to be an ontic state, $|\psi\rangle_{\rm ont}$. This state will be some *superposition of template states* $|k\rangle_{\rm template}$:

$$|\psi\rangle_{\rm ont}=\sum_k \alpha_k|k\rangle_{\rm template} \tag{1}$$

In practice, we use some given template state of our choice. It will be related to the ontic states by

$$|k\rangle_{\rm template}=\sum_n \lambda_n|n\rangle_{\rm ont}\ , \tag{2}$$

where

$|\lambda_n|^2$ are the probabilities that we actually have ontic state $|n\rangle_{\rm ont}$.

Classical states

How are the *classical* states related to the *ontic* states?

Imagine a *planet*. The interior is very different from the local vacuum state. Vacuum state has *vacuum fluctuations*.

Take 1 mm^3 of matter inside the planet. Using statistics, looking at the ontic states, we may establish, with some probability, that the fluctuations are different from vacuum.

Combining the statistics of billions of small regions inside the planet, we can establish *with certainty* that there is a planet, by looking at the ontic state.

But what holds for a planet should then be true for all classical configurations, hence:

All classical states are ontological states!

Classical states do not superimpose.

Measurements

Paraphrase a simple "experiment":

First, make the initial state. We take a *template* for that (such as plane in-going waves). Remember:

$$|k\rangle_{\text{template}} = \sum_n \lambda_n |n\rangle_{\text{ont}} \ , \qquad (2)$$

Here, $P_n = |\lambda_n|^2$. λ_n are conserved in time.

Compute the final state, using Schrödinger equ. or Scattering matrix. The final state template is associated to some definite classical state. Compute

$$_{\text{template}}^{\text{classical}}\langle \ell | k \rangle_{\text{template}} = \sum_k \lambda_n \,^{\text{classical}}\langle \ell | n \rangle_{\text{ont}} \qquad (3)$$

<u>Ontic States evolve into Ontic States</u>, and the classical states are ontological $\rightarrow$ $\langle \ell | n \rangle_{\text{ont}} = \delta_{kn}$. Therefore:

$$\boxed{P_n = |\lambda_n|^2 \text{ are the Born probabilities.}}$$

The Born probabilities coincide with the probabilistic distributions reflecting the unknown details of the initial states.

And that's exactly how probabilities arise in an "ordinary" classical deterministic theory.

Ontological states form an orthonormal set: superpositions of ontological states are never ontological states themselves.
The universe is in an ontological state.

Collapse of the Wave function

When we use a template, we find the final state to be

$$\alpha_1|k_1\rangle + \alpha_2|k_2\rangle + \cdots$$

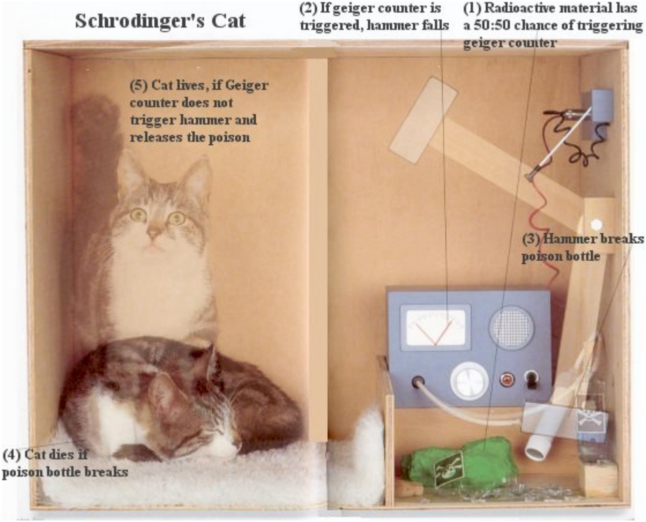
According to "Copenhagen", $P_1 = |\alpha_1|^2,\ P_2 = |\alpha_2|^2\,,\ \cdots$

Why is the final state only one of these states? Why are P_i probabilities?

The CAI gives the answer: $|k_1\rangle$ is a possible ontic final state,

and so is $k_2\rangle$, but $\alpha_1|k_1\rangle + \alpha_2|k_2\rangle$ is *not* an ontic state.
That's why it never occurs in the real world.

Schrödinger's cat is ontic when it is dead, also when it is alive, but *not* when it is in a superposition.



From: Mr. Kent's Chemistry Page

How do quantum operators arise in a CA (Cellular Automaton)?

The planetary system as prototype of *classical* dynamical models. Planets are in "states" $|\{\vec{x}_i, \vec{v}_i\}\rangle$, described by their positions $\vec{x}_i$ and velocities $\vec{v}_i$.

The evolution laws is "deterministic" by construction. But we can define operators just as in QM :

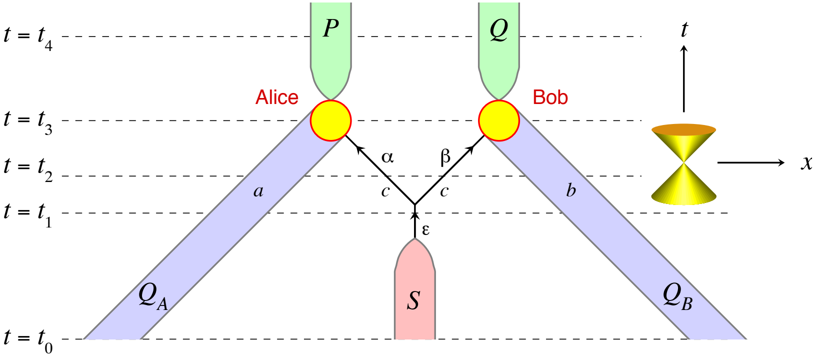
The **Earth - Mars exchange operator**, X_{EM} puts Mars where Earth is and Earth where Mars is, while also exchanging the velocity vectors. Leave the Moon and other planets where they are.

$$X_{EM}^2 = 1 \quad \rightarrow \quad X_{EM} = \pm 1.$$

Can we *measure* X_{EM}? Is it $+1$ or -1? How does X_{EM} evolve?

Counterfactual reality : you can't measure Earth's and Mars' positions *and* X_{EM} at the same time.

Why it is all wrong: Bell's theorem



In the Bell experiment, at $t = t_0$, one must demand that those degrees of freedom that later force Alice and Bob to make their decisions, and the source that emits two entangled particles,

need to have *3 - body correlations of the form*

(the Mousedropping Function)

But Alice and Bob have *free will*. How can their actions be correlated with what the decaying atom did, at time $t = t_2 \ll t_3$?

Answer: they don't have free will: *superdeterminism*.

The Mouse-dropping argument

" Your theory is absurd. Suppose Alice and Bob both
carry with them a cage, with in it a mouse.

" At $t = t_1$, an atom emits two entangled photons.

" At $t = t_2 \gg t_1$ both Alice and Bob count how many droppings
their mouse has produced.

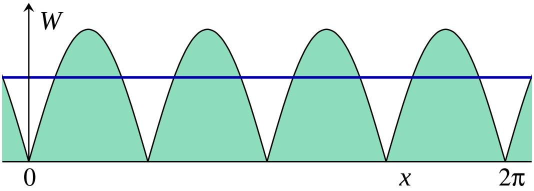
" At $t = t_3$, immediately after t_2, they set their polarisation filters
according to whether the number of droppings is *even* or *odd*.

" And now you tell me that the decaying atom already knew,
in andvance, how the bowels of these mice work?

" This is ridiculous! "

The mouse droppings function:

$$W(a,b,c) \;=\; \frac{1}{2\pi^2}\left|\sin(4c-2a-2b)\right| \;.$$



$$\begin{aligned} c &= \text{joint polarisations entangled particles} \\ a &= \text{filter polarisation chosen by Alice} \\ b &= \text{filter polarisation chosen by Bob} \\ x &= 2c-a-b \end{aligned}$$

What happened according to the CAI ?

We have the *ontology conservation law* :
Ontic states evolve into ontic states.

$$ {}^{\text{classical}}_{\text{template}}\langle \ell | k \rangle_{\text{template}} = \sum_k \lambda_n \, {}^{\text{classical}}\langle \ell | n \rangle_{\text{ont}} $$

If Alice makes an infinitesimal modification of her settings, the *classical* state will change $\rightarrow$ all ontic states will change:

$$ {}^{\text{classical}}_{\text{template}}\langle \ell + \delta\ell | k \rangle_{\text{template}} = \sum_k \lambda_m \, {}^{\text{classical}}\langle \ell + \delta\ell | m \rangle_{\text{ont}} $$

All Alice's ontological states $|m\rangle_{\text{ont}}$ are now different from all $|n\rangle_{\text{ont}}$ that she had before.

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So, both her past light-cone and her future light-cone are now entirely different. These light-cones do overlap with Bob's.
Does this affect Bob's world, and that of the decaying atom S?

If all ontological states had equal probabilities, the answer would be no. But one can easily imagine that some ontic states are more probable than others.
In that case, the *counterfactual* experiment $\ell \to \ell + \delta\ell$ would lead to drastically different probabilities. So it is easy to generate non-vanishing correlation functions that disobey Bell.
Ransom: all ontic states in the universe are associated with strong spacelike correlations. These correlations obey the ontology conservation law.
The photons c then automatically align in such a way that, after detection by Alice and Bob, they are still in an ontic state.

Conspiracy

Is this *conspiracy* ? Not if the ontological nature of a physical state is *conserved in time*. If, at late times, a photon is observed to be in a given polarization state, it has been in *exactly the same state* from the very moment it was emitted by the source (omniscient photons).

These are future-past correlations. The conspiracy argument now demands that the "ontological basis" be *unobservable!*

Non-observable hidden variables?

"Shut up and calculate!"

The anticlimax

The Cellular automaton can perfectly well be described by a quantum mechanical Hamiltonian.
However, we would like this Hamiltonian to reflect the fact that the automaton is *local* :

$$H = \sum_{\vec{x}} \mathcal{H}(\vec{x}) \ , \qquad [\mathcal{H}(\vec{x}), \mathcal{H}(\vec{x}\,')] = 0 \quad \text{if} \quad |\vec{x} - \vec{x}\,'| > \varepsilon \ . \tag{5}$$

But also: $$\langle\ H\ \rangle \geq 0 \ . \tag{6}$$

It is easy to find an H that obeys (5), and one that obeys (6), but *it is difficult* to find an Hamiltonian obeying both (5) and (6).

In *gravity*,

$\mathcal{H}(\vec{x})$ plays an important role as the *generator of a local time diffeomorphism*.

Also, $\mathcal{H}(\vec{x}) = T_{00}(\vec{x},\, t)$ is he (classical) source of space-time curvature — the gravitational field.

Could this mean that *quantum gravity* is essential for the understanding of locality in QM?

How does the *hierarchy problem* enter?
(Where doe those *large numbers* in physics come from?)

arXiv: 1204.4926
arXiv: 1205.4107
arXiv: 1207.3612
arXiv: 1405.1548.

THE END