Tohoku Forum for Creativity
April 20, 2015

Physics Utrecht
ΕΜΜΕΦ

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The role of Black Holes and Conformal Symmetry in Quantum Gravity

Why are *symmetries* so important for physics?

- Translation symmetry, in space and in time
- Rotation symmetry
- Galilei transformations ($\longrightarrow$ Lorentz transformations)
- local and global particle symmetries ...

what about scale transformations ?

$$x^{\pm} \;=\; \tfrac{1}{\sqrt{2}}(x^3 \pm x^0)$$

Lorentz transformations:

$$\begin{aligned} x^+ &\;\to\; \lambda\, x^+ \\ x^- &\;\to\; \lambda^{-1} x^- \end{aligned}$$

Scale transformations (holds for *electro magnetism*):

$$\begin{aligned} x^+ &\;\to\; \lambda\, x^+ \\ x^- &\;\to\; \lambda\, x^- \end{aligned}$$

Combine: basic transformation

$$\begin{aligned} x^+ &\;\to\; \lambda\, x^+ \\ x^- &\;\to\; x^- \end{aligned}$$

local scale transformation = (local) conformal transformation:

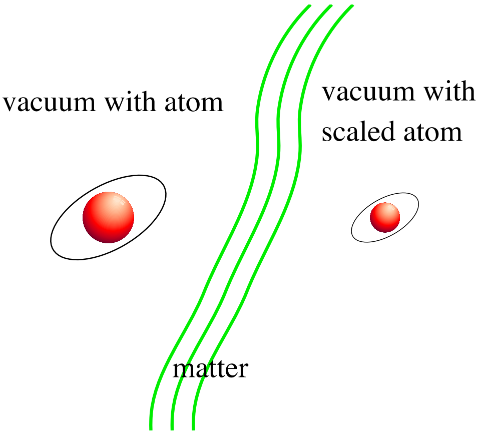
$$g_{\mu\nu} \;\rightarrow\; \Omega^2(\vec{x}, t)\, g_{\mu\nu}$$

Real world (beyond electro magnetism) is not invariant under global or local scale transformations.

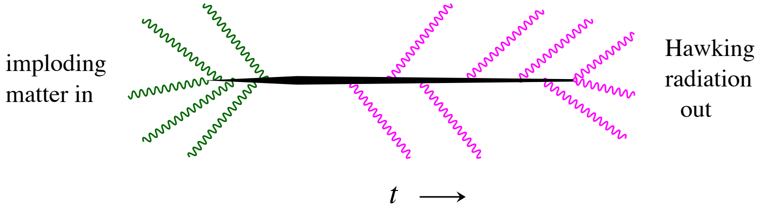
$$\text{Symmtry breaking may be} \begin{cases} \text{explicitly broken} \\ \text{or} \\ \text{spontaneously broken} \end{cases}$$

In latter case: the *vacuum state* breaks the symmetry.

As in BEH mechanism


vacuum with atom
vacuum with
scaled atom
matter

Black holes can be ‘produced’ and can decay

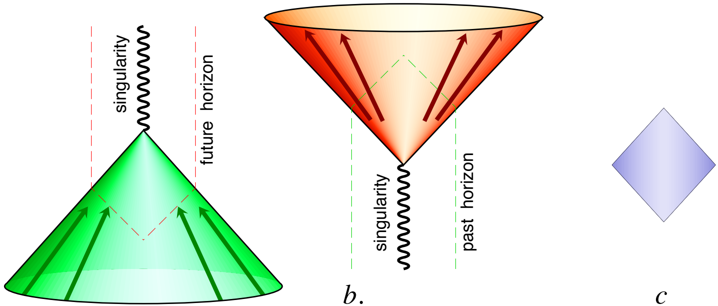


All known laws of physics are *CPT* invariant. Why not also black holes?

Black hole complementarity: an outside observer of a black hole should see the same states Nature can be in as an observer falling in

Hawking radiation normally emerges with a thermal Maxwell distribution.

Ingoing matter may be in any form, such as a *collapsing shell*



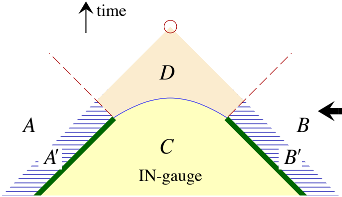
$a.$ $b.$ c

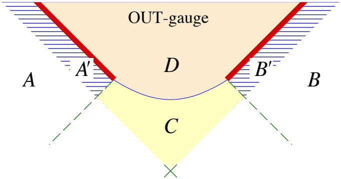
[With low probability] Hawking particles can also form a shell:
CPT can be preserved . . .

Conjecture: Not only does an outside observer have access to the same info as the inside observer, they also see the same

causal order: *They see the same light cones.*

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = 0. \text{ Therefore: } g_{\mu\nu}^{\mathrm{in}} = \Omega(\vec{x}, t) g_{\mu\nu}^{\mathrm{out}}$$





Conformal *in* - gauge conformal *out* - gauge

Usual description of black hole metric:

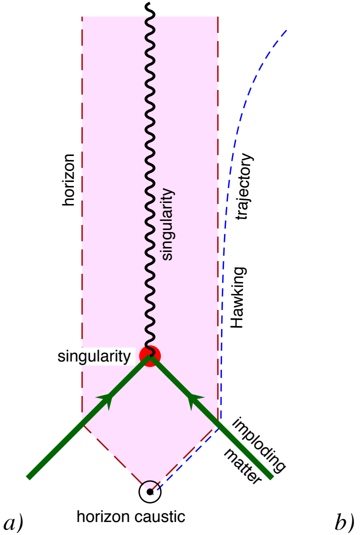
$$\mathrm{d}s^2 = \Big(-(1-2M/r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-2M/r} + r^2\mathrm{d}\Omega^2\Big)\ ,$$

With conformal symmetry, $r \to M(\tilde{t})\, r$,

$$\mathrm{d}s^2 = M^2(\,\tilde{t}\,)\Big(-(1-2/r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-2/r} + r^2\mathrm{d}\Omega^2\Big)$$

$\tilde{t} \ = \ t^{\mathrm{retarded}}$?

No singularity ! no horizon !



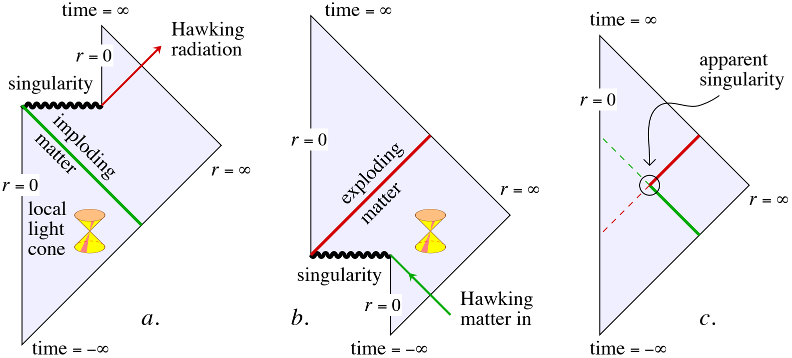
No
singularity

No
horizon !

Under conformal transformation:

$$T_{\mu\nu} \to T_{\mu\nu} + \Omega^{-2}(4\partial_\mu\Omega\partial_\nu\Omega - g_{\mu\nu}(\partial\Omega)^2) - \tfrac{1}{2}\Omega^{-1}(4D_\mu\partial_\nu\Omega - g_{\mu\nu}D^2\Omega) \ .$$

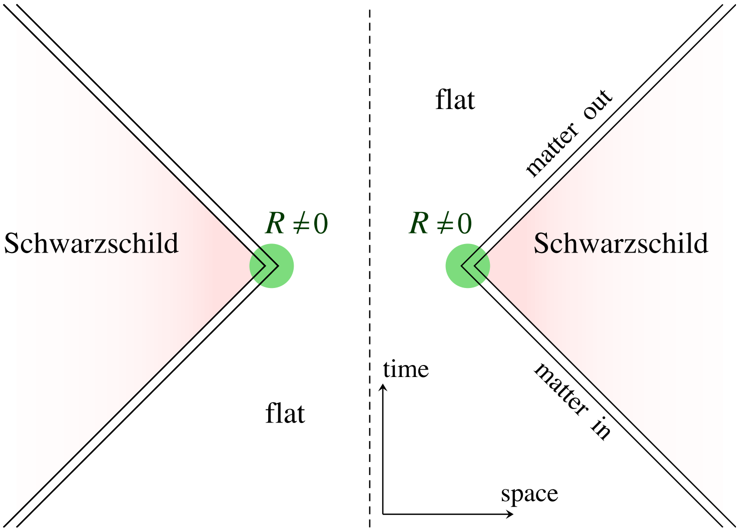
Use any one of the 10 components of $T_{\mu\nu}$ to *fix conformal gauge* :



$T_{--} = 0$,

$T_{++} = 0$

past: $T_{--} = 0$
future: $T_{++} = 0$


flat
matter out
Schwarzschild
$R \neq 0$
$R \neq 0$
Schwarzschild
time
flat
matter in
space

Conformal symmetry is present in *standard canonical gravity coupled to* (non-conformal) *matter*

$$S^{\,\mathrm{total}} = \int \mathrm{d}^4x\sqrt{-g}\,\left(\frac{1}{16\pi G_N}(R-2\Lambda)+\mathcal{L}^{\mathrm{matter}}\right)$$

Write $g_{\mu\nu}(x) \overset{\mathrm{def}}{=} \omega^2(x)\,\hat{g}_{\mu\nu}(x)$ *All are dynamical variables.*

Obvious invariance: $\hat{g}_{\mu\nu} \rightarrow \Omega^2(x)\hat{g}_{\mu\nu}\ , \quad \omega \rightarrow \Omega^{-1}\omega .$

$$S = \int \mathrm{d}^nx\sqrt{-\hat{g}}\left(\frac{1}{16\pi G_N}\big(\omega^2\hat{R} - 2\,\omega^4\Lambda + 6\,\hat{g}^{\,\mu\nu}\partial_\mu\omega\,\partial_\nu\omega\big)+\mathcal{L}^{\mathrm{matter}}\right)$$

$$
\begin{aligned}
&\mathcal{L}^{\mathrm{matter}}(A_\mu, \varphi,\, \psi, \bar{\psi},\, \hat{g}_{\mu\nu}, \omega) \;= \\
&-\tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \bar{\psi}\, \hat{\gamma}^\mu \hat{D}_\mu\, \psi - \tfrac{1}{2} \hat{g}^{\mu\nu} D_\mu \varphi\, D_\nu \varphi - \tfrac{1}{2} m^2_\varphi \omega^2 \varphi^2 \\
&-\tfrac{1}{12} \hat{R} \varphi^2 - V_4(\varphi) - \omega V_3(\varphi) \\
&-\bar{\psi}(y_i \varphi_i + i y^5_i \gamma^5 \varphi_i + m_d \omega)\psi
\end{aligned}
$$

Then write ω as $\tilde{\kappa}\chi$, where $\tilde{\kappa} = \sqrt{\frac{4\pi G}{3}}$, so that

$$
\mathcal{L}^{\mathrm{grav}} = \sqrt{-\hat{g}} \left(\tfrac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + \tfrac{1}{12} \hat{R} \chi^2 \right)
$$

$$
\begin{aligned}
\mathcal{L}^{\mathrm{total}} \;=\; & -\tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \bar{\psi}\, \hat{\gamma}^\mu \hat{D}_\mu\, \psi - \tfrac{1}{2}(D\varphi)^2 + \tfrac{1}{2}(\partial\chi)^2 \\
&-\tfrac{1}{2}(\tilde{\kappa} m_\varphi)^2 \chi^2 \varphi^2 + \tfrac{1}{12} \hat{R}(-\varphi^2 + \chi^2) - V_4(\varphi) - \tilde{\kappa}\chi V_3(\varphi) \\
&-\bar{\psi}(y_i \varphi_i + i y^5_i \gamma^5 \varphi_i + \tilde{\kappa} m_d \chi)\psi \quad - \tilde{\Lambda}\chi^4
\end{aligned}
$$

$$\mathcal{L}^{\text{total}} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \bar{\psi}\,\hat{\gamma}^\mu \hat{D}_\mu \psi - \frac{1}{2}(D\varphi)^2 + \frac{1}{2}(\partial\chi)^2$$

$$-\frac{1}{2}(\tilde{\kappa} m_\varphi)^2 \chi^2 \varphi^2 + \frac{1}{12}\hat{R}(-\varphi^2 + \chi^2) - V_4(\varphi) - \tilde{\kappa}\chi V_3(\varphi)$$

$$-\bar{\psi}(y_i \varphi_i + i y_i^5 \gamma^5 \varphi_i + \tilde{\kappa} m_d \chi)\psi \quad - \tilde{\Lambda}\chi^4 \qquad \tilde{\Lambda} = \frac{1}{6}\tilde{\kappa}^2 \Lambda$$

$$
\begin{aligned}
\mathcal{L}^{\rm total} \;=\; & -\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} - \bar\psi\,\hat\gamma^\mu\hat D_\mu\,\psi - \tfrac12(D\varphi)^2 + \tfrac12(\partial\chi)^2 \\
& -\tfrac12(\tilde\kappa m_\varphi)^2\chi^2\varphi^2 + \tfrac{1}{12}\hat R(-\varphi^2+\chi^2) - V_4(\varphi) - \tilde\kappa\chi V_3(\varphi) \\
& -\bar\psi(y_i\varphi_i + iy^5_i\gamma^5\varphi_i + \tilde\kappa m_d\chi)\psi \quad - \tilde\Lambda\chi^4 \qquad \tilde\Lambda = \tfrac16\tilde\kappa^2\Lambda
\end{aligned}
$$

Write: $\chi = i\eta$, *vacuum expectation value :*

$$\langle\,\chi\,\rangle = 1/\tilde\kappa\ , \quad \langle\,\eta\,\rangle = -i/\tilde\kappa$$

Conformal invariance is trivial, but it is *spontaneously broken* by the *vacuum exp. values*.

For 2 reasons this is nevertheless important:

– Different observers do not agree about “vacuum”

are the Hawking particles real?

– Conformal anomalies

Our theory has *only* conformally invariant terms:

$g_{\rm gauge}$, $\lambda_{\rm scalars}$, y_i, y_i^5, and also $\tilde\kappa m_\varphi$, $\tilde\kappa m_d$,
and even $\tilde\Lambda$ (cosm.const)

But $\mathcal{L}$ does not have a kinetic term for $\hat g_{\mu\nu}$
($\mathcal{L}^{\rm EH}$ went entirely into kinetic term for χ)

Classically, equation $\partial S/\partial\hat g_{\mu\nu} = 0$ leads to
$T^\eta_{\mu\nu} + T^{\rm matter}_{\mu\nu} = 0$, $\qquad T^\chi_{\mu\nu} - T^{\rm matter}_{\mu\nu} = 0$

which is Einstein's equ.: $T^\chi_{\mu\nu} = \frac{1}{8\pi G}\, G_{\mu\nu}$. In short:

- Take a background conformal metric $\hat g_{\mu\nu}$.
- Solve Eqs. for all matter fields and the field χ
- Find all $T_{\mu\nu}$. Check that they cancel. If not:
- Adjust $\hat g_{\mu\nu}$ and try again.

In this theory, there seem to be no horizons and no singularities.

Black holes are replaced by locally finite field configurations, all with complete local conformal symmetry.

Black hole entropy should follow automatically, but this could not be checked.

Theory resembles the B.E.H. theory for electroweak theory :

- Possibility of a *"renormalizable" gauge condition* :
- Choose ω such that the *total activity* (like # vertices of Feynman diagrams) is limited to a maximum in all points x.

Conformal anomalies: *nothing special should happen*
if ω (or χ or η) tend to 0.

Therefore, conformal transformations should not affect any of the conformal coupling costants.

Therefore, *all* β *functions should vanish*.
(We are **at** a fixed point)

This leads to equations that fix *all constants of nature* (!)

If only we knew the algebra of all matter fields, the values of all physical constants (including masses and cosmological constant) could be calculated.

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THE END