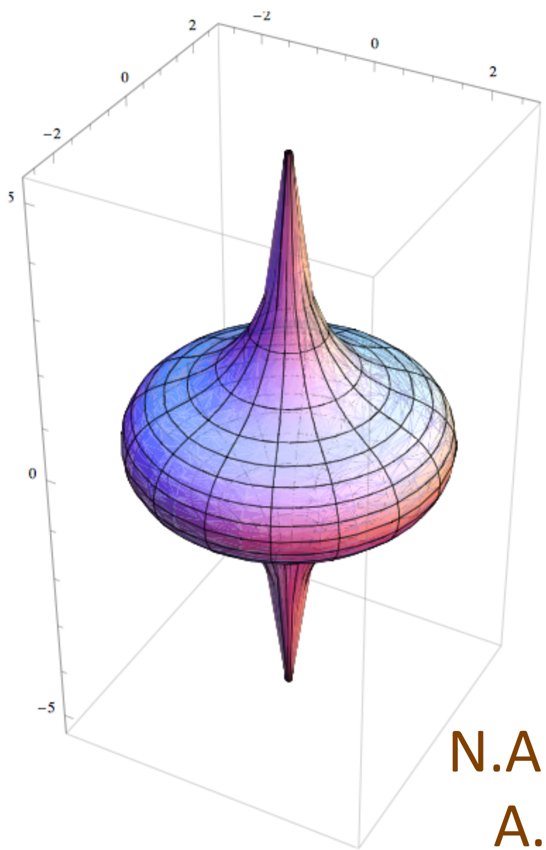
Super-Entropic Black Holes

Robert Mann



R. Hennigar, D. Kubiznak, N. Muskoe

R. Hennigar, D. Kubiznak, R.B. Mann
Phys Rev Lett **115** (2015) 031101 arXiv: 1411.4309
R. Hennigar, D. Kubiznak, R.B. Mann, N. Muskoe,
JHEP **1506** (2015) 096 arXiv: 1504.07529

N.Altimirano, W. Brenna, B. Dolan, T. Delsate
A. Frassino, S. Gunasekaran, D. Kastor, A. Kostouki, A. Rajagopal, Z. Sherkatgnad,
F. Simovic, J. Traschen

Review of Black Hole Chemistry

Thermodynamics | Gravity

Enthalpy $H \leftrightarrow M$ Mass

Temperature $T \leftrightarrow \frac{\hbar\kappa}{2\pi}$ Surface gravity

Entropy $S \leftrightarrow \frac{A}{4\hbar}$ Horizon Area

$$dH = TdS + VdP + \cdots \leftrightarrow dM = \frac{\kappa}{8\pi}dA + VdP + \cdots$$

First Law | First Law

$$H = E + PV + \cdots \leftrightarrow M = E - \rho V$$

Mass
= Total Energy
- Vacuum
Contribution
(infinite)

The Chemistry of AdS Black Holes

Include gauge charges:

First Law

$$\delta M = T_h \delta S_h + \sum_i (\Omega_h^i - \Omega_\infty^i)\delta J^i + \Phi_h \delta Q + V_h \delta P$$

Smarr Relation

$$\frac{D-3}{D-2} M = T_h S_h + \sum_i (\Omega_h^i - \Omega_\infty^i) J^i + \frac{D-3}{D-2}\Phi_h Q - \frac{2}{D-2} P V_h$$

Even for D=2

Kubiznak/Frassino/Mann/Mureika (to appear)

Thermodynamic Potential:
Gibbs Free Energy

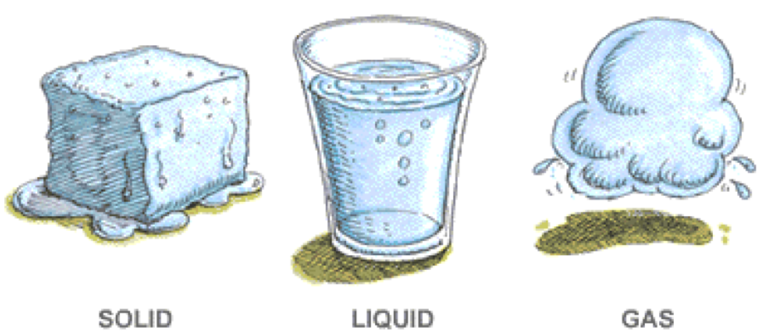
$$G = M - TS = G(T, P, J_i, Q)$$

- Equilibrium: Global minimum of Gibbs Free Energy
- Local Stability: Positivity of the Specific Heat

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_{P, J_i, Q} > 0$$

New Results from Black Hole Chemisty

- Hawking Page Transition
  - solid/liquid phase transition with infinite coexistence line
- Black Holes as Van der Waals Fluids
  - Complete correspondence between intrinsic and extrinsic variables
- Reentrant Phase Transitions
  - Change from one phase to another and back again as one parameter (eg. temperature) monotonically changes
- Black Hole Triple Points ←→ Solid/Liquid/Gas



Hawking-Page Transition

S.W. Hawking & D.N. Page
Comm Math. Phys. 87 (1983) 577

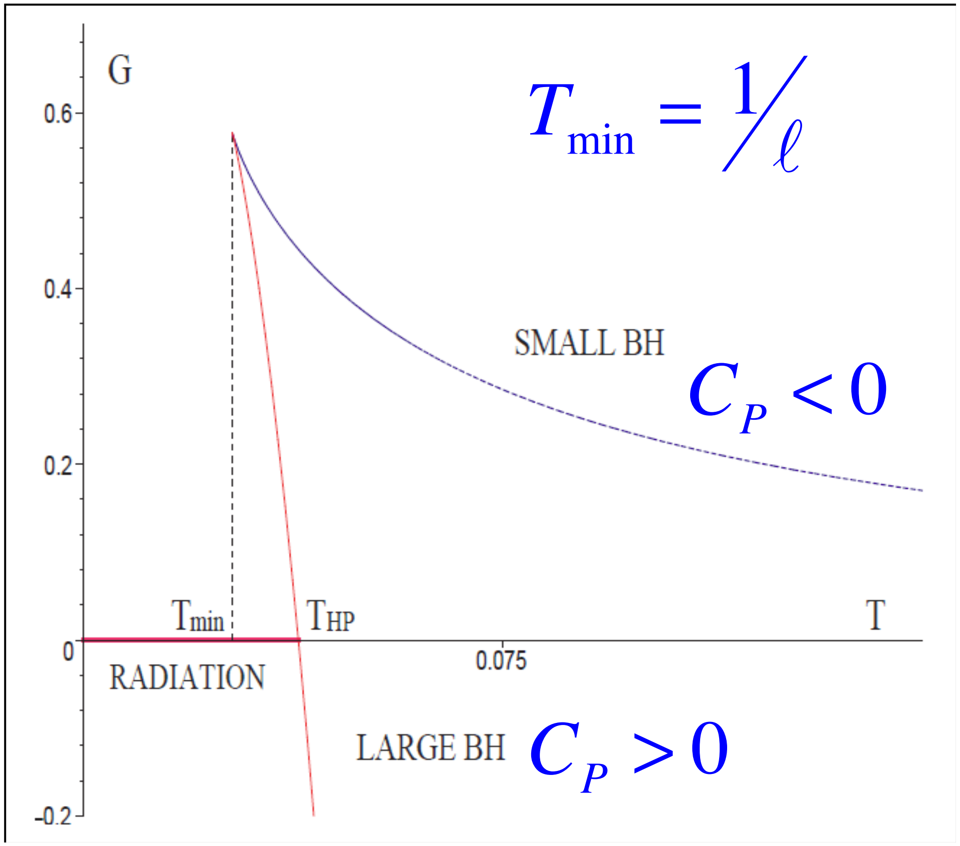
D-dim'l Schwarzschild-AdS Black hole

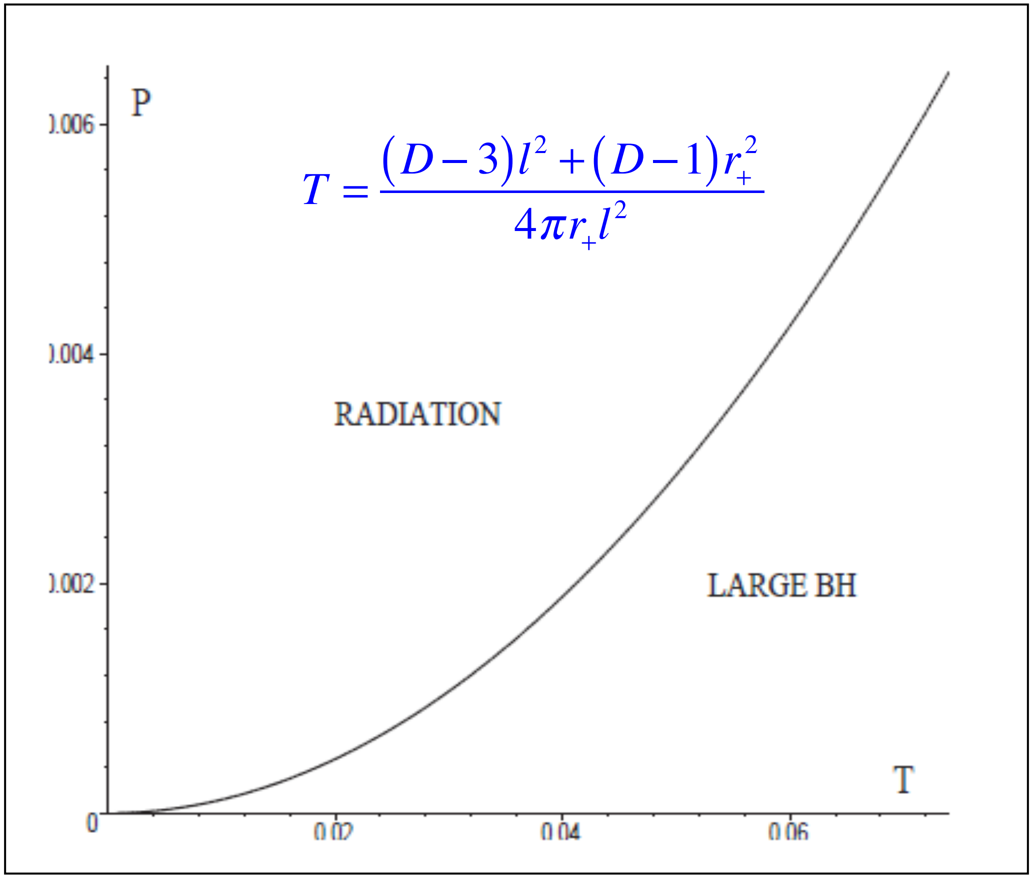
$$ds^2 = -Vdt^2 + \frac{dr^2}{V} + r^2 d\Omega^2_{k,D-2} \qquad V = k - \frac{\tilde{M}}{r^{D-3}} + \frac{r^2}{\ell^2} \qquad k = \begin{cases} 1 & \text{spherical} \\ 0 & \text{planar} \\ -1 & \text{hyperbolic} \end{cases}$$

• AF black holes evaporate by Hawking radiation
• AdS is like a confining box
→ static black holes in thermal equilibrium

$$T < T_{\min} \Rightarrow \text{ gas of particles}$$

1st order transition
between gas of particles and large black holes at Tc





$$T = \frac{(D-3)l^2 + (D-1)r_+^2}{4\pi r_+ l^2}$$

Phase transition in dual CFT (quark-gluon plasma)

Witten (1998)

Fluid interpretation:
solid/liquid PT
(infinite coexistence line)

D. Kubiznak/RBM
arXiv [1404.2126]

Equation of state

$$Pv = T - \frac{k}{2\pi v}$$

depends on the horizon topology

$$v = 2l_P^2 r_+ = 2\left(\frac{3V}{4\pi}\right)^{1/3} = 6\frac{V}{N}$$

$$N = \frac{A}{l_P^2}$$

Planar black holes
←→ ideal gas

Van der Waals Fluids

$$Pv^3 - (kT + bP)v^2 + a(v - b) = 0$$

Critical Point

$$\frac{\partial P}{\partial v} = 0, \quad \frac{\partial^2 P}{\partial v} = 0$$

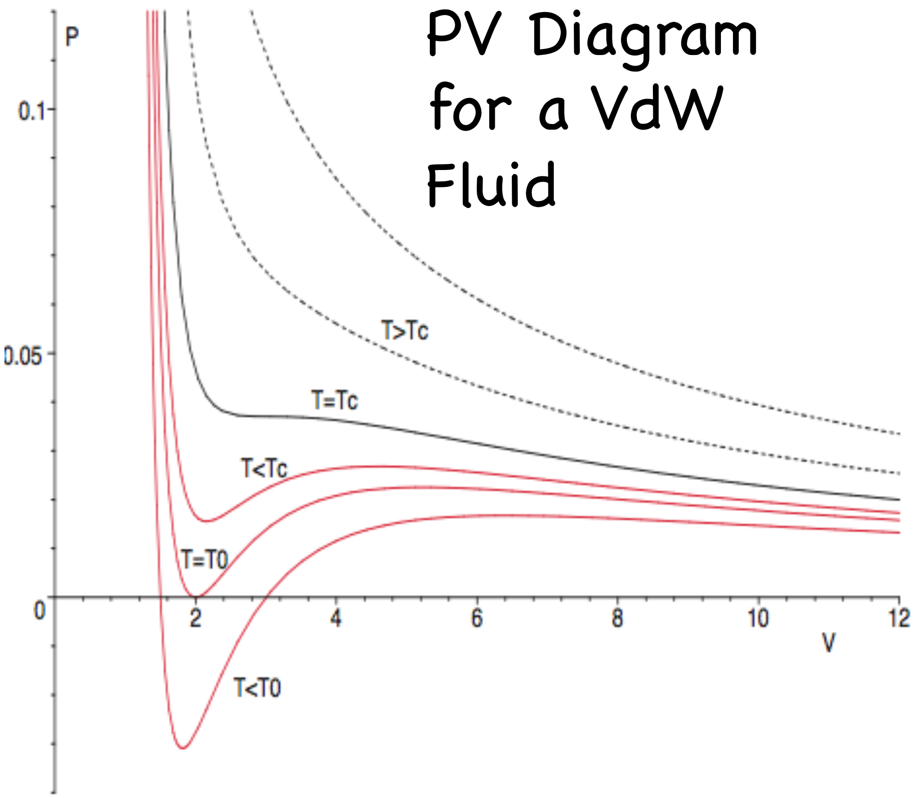
$$kT_c = \frac{8a}{27b}, \quad v_c = 3b, \quad P_c = \frac{a}{27b^2}$$

$$p = \frac{P}{P_c}, \quad v = \frac{v}{v_c}, \quad \tau = \frac{T}{T_c}$$

$$8\tau = (3v - 1)\left(p + \frac{3}{v^2}\right)$$

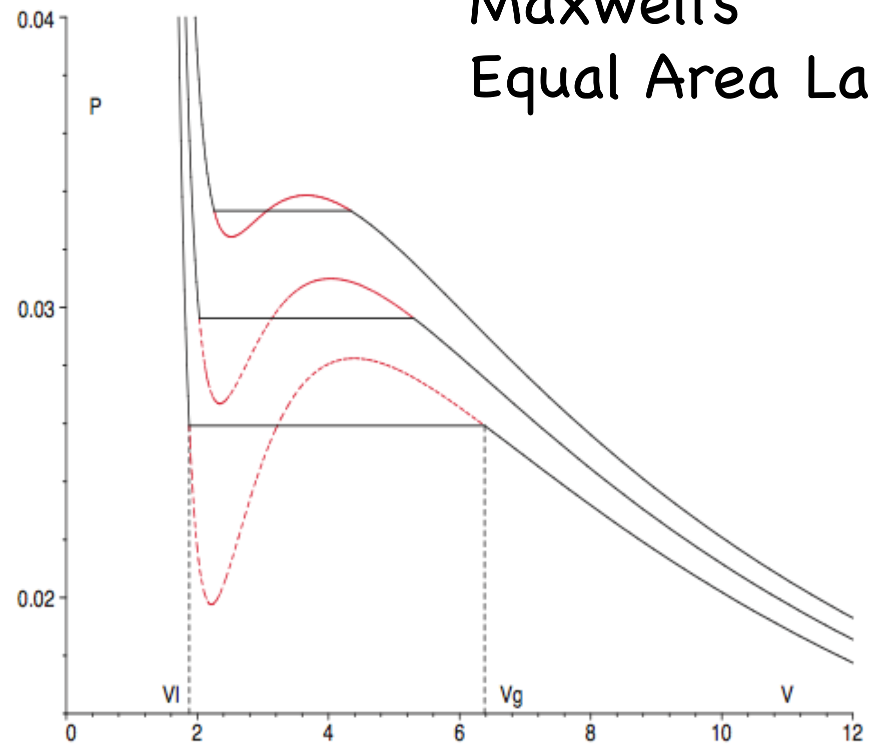
law of corresponding states

PV Diagram for a VdW Fluid



$$\frac{P_c v_c}{kT_c} = \frac{3}{8}$$

Maxwell's Equal Area Law



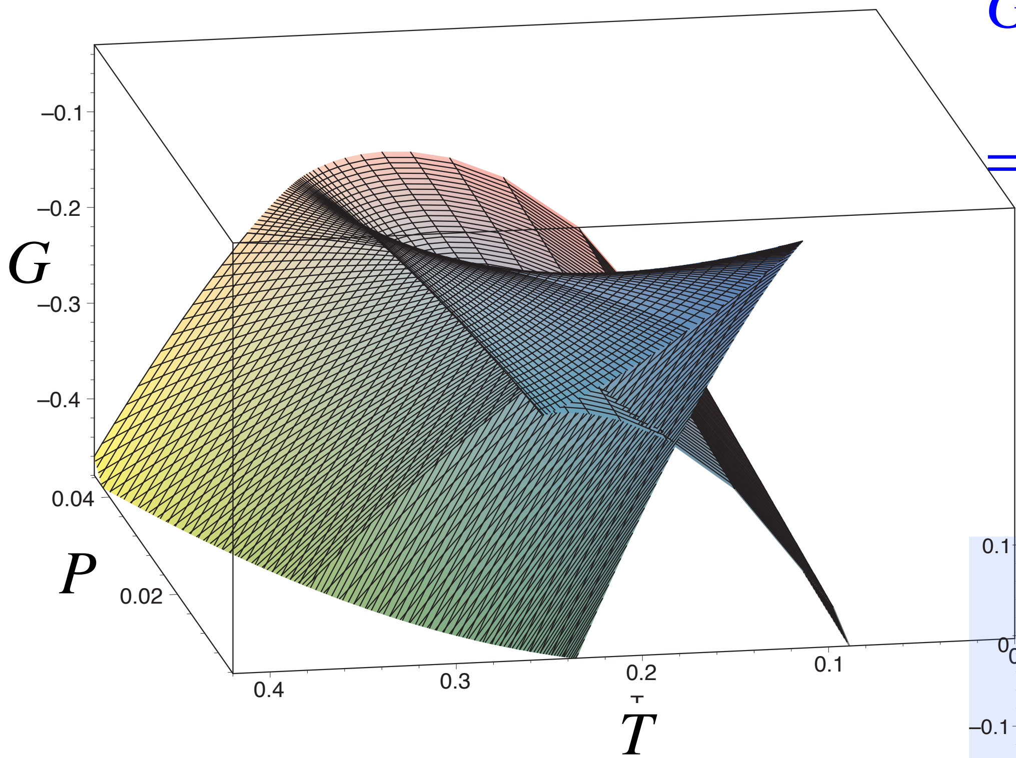
$$\oint v dP = 0$$

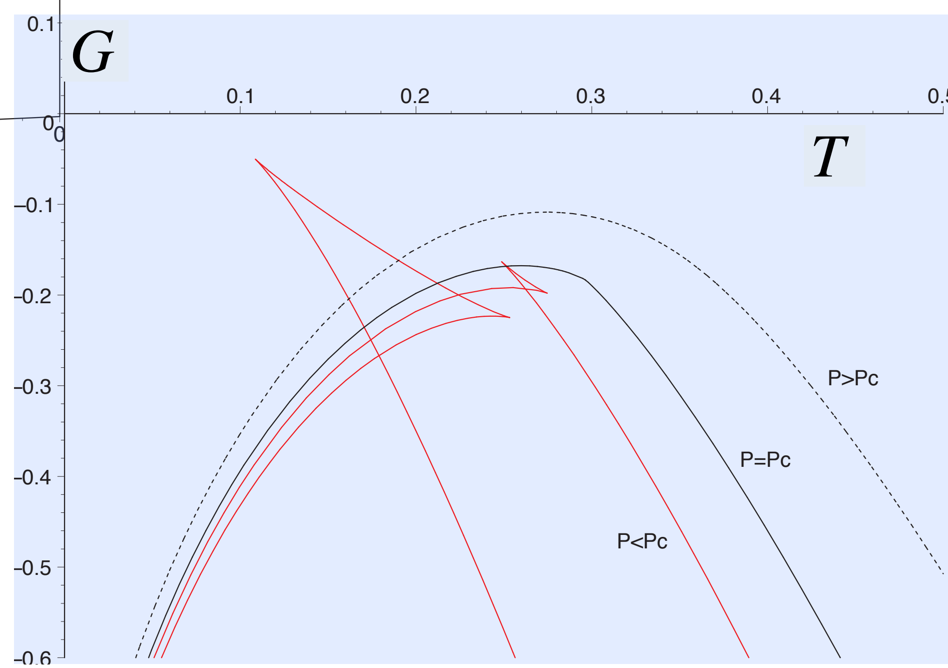
Gibbs Free Energy

$$G = G(T, P)$$

$$= -kT\left(1+\ln\left[\frac{(v-b)T^{3/2}}{\Phi}\right]\right)-\frac{a}{v}+Pv$$

characteristic of the gas





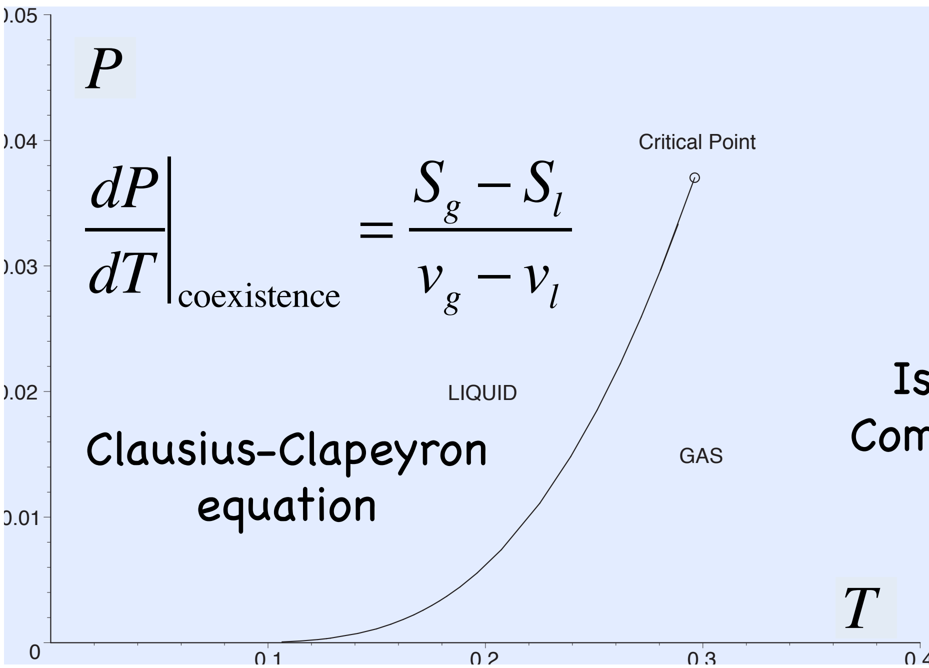
First Law

$$dG = -SdT + v\, dP$$

Critical Exponents

$p = \frac{P}{P_c}, \quad v = \frac{v}{v_c}, \quad \tau = \frac{T}{T_c}$

Line of Coexistence



$$\left.\frac{dP}{dT}\right|_{\text{coexistence}} = \frac{S_g - S_l}{v_g - v_l}$$

Clausius-Clapeyron equation

Specific Heat $$C_v = T\left.\frac{\partial S}{\partial T}\right|_v \propto |\tau - 1|^{-\alpha}$$

Order Parameter $$\eta = v_g - v_l \propto |\tau - 1|^{\beta}$$

Isothermal Compressibility $$\kappa_T = -\frac{1}{v}\left.\frac{\partial v}{\partial P}\right|_T \propto |\tau - 1|^{-\gamma}$$

Critical Isotherm $$|P - P_c| \propto |v - v_c|^{\delta}$$

For a VdW Fluid $\alpha = 0 \quad \beta = \frac{1}{2} \quad \gamma = 1 \quad \delta = 3$

Charged AdS Black Holes as Van der Waals Fluids

Kubiznak/Mann
JHEP 1207
(2012) 033

$$ds^2 = -Vdt^2 + \frac{dr^2}{V} + r^2 d\Omega_2^2 \qquad F = dA \quad \begin{cases} V = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{r^2}{l^2} \\ A = -\frac{Q}{r}dt \end{cases}$$

Temperature $T = \frac{1}{\beta} = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right)$

Entropy $S = \frac{A}{4} = \pi r_+^2$ Pressure $P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi}\frac{1}{l^2}$

Potential $\Phi = \frac{Q}{r_+}$ Volume $V = \frac{4}{3}\pi r_+^3$

$$M = 2(TS - PV) + \Phi Q$$

Smarr Relation

$$dM = TdS + VdP + \Phi dQ$$

First Law

Equation of State

$$r_+ = \left(\frac{3V}{4\pi}\right)^{1/3}$$

$$T = \frac{1}{4\pi r_+}\left(1 + \frac{3r_+^2}{l^2} - \frac{Q^2}{r_+^2}\right)$$

$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi}\frac{1}{l^2}$$

$$P = \frac{T}{2r_+} - \frac{1}{8\pi r_+^2} + \frac{Q^2}{8\pi r_+^4}$$

Physical Equation of State

Thermodynamic Volume

$$\text{Press} = \frac{\hbar c}{l_P^2}P \quad \text{Temp} = \frac{\hbar c}{k}T$$

$$\text{Press} = \frac{k\text{Temp}}{2l_P^2 r_+} + \ldots$$

$$v = 2l_P^2 r_+$$

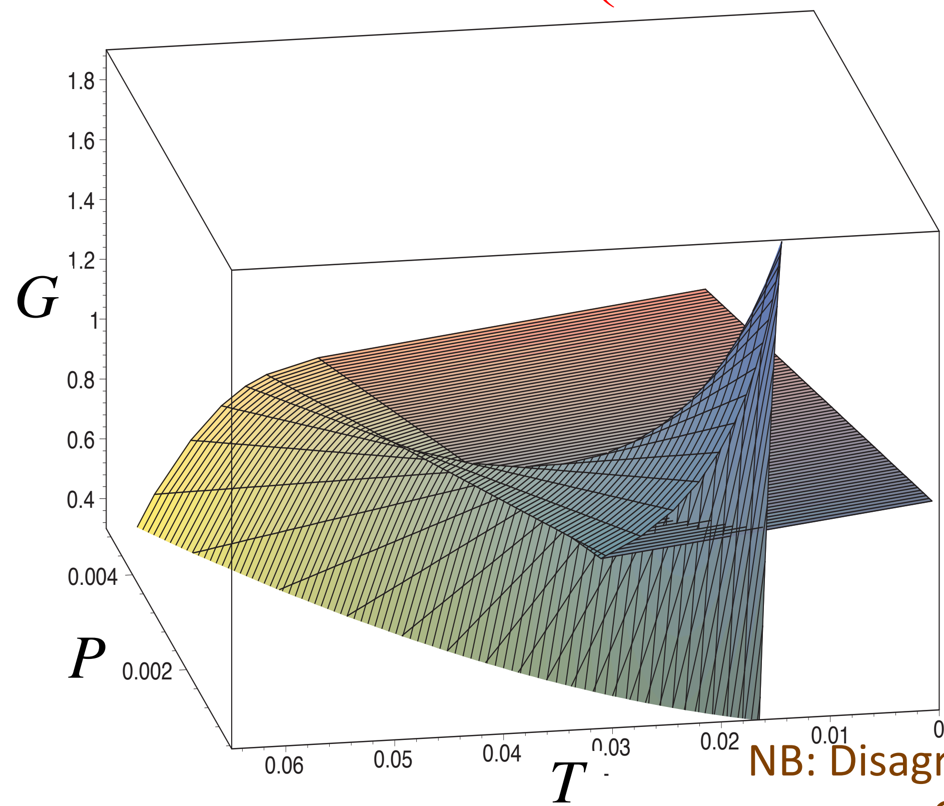
Van der Waal's Equation

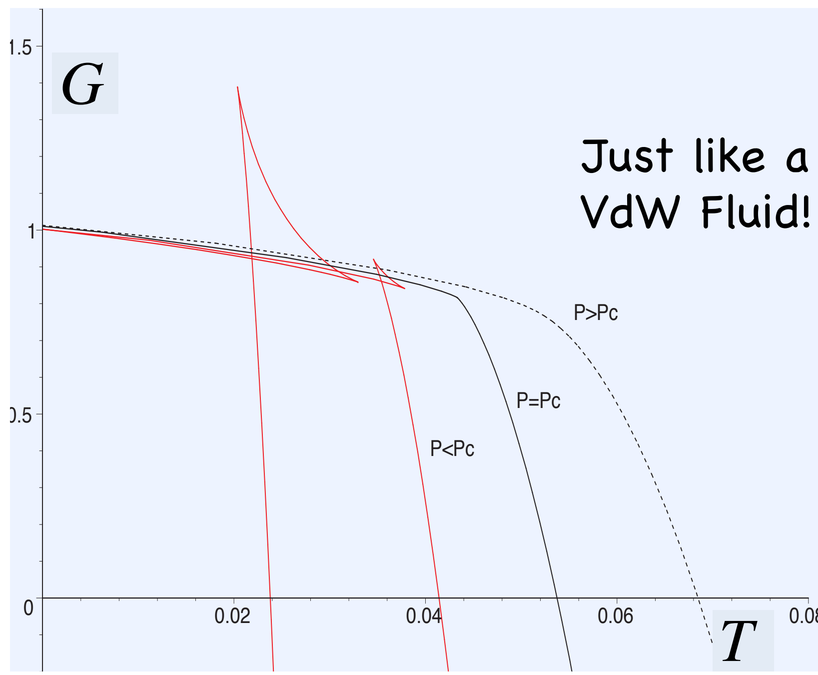
$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}$$

Gibbs Free Energy of AdS RN BH

$$I = -\frac{1}{16\pi}\int_M \sqrt{-g}\left(R - F^2 + \frac{6}{l^2}\right) - \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{h}K - \frac{1}{4\pi}\underbrace{\int_{\partial M} d^3x\sqrt{h}n_a F^{ab}A_b}_{\text{Fixed Charge}} + I_c$$

$$G = G(T,P) = \frac{1}{4}\left(r_+ - \frac{8\pi}{3}Pr_+^3 + \frac{3Q^2}{r_+}\right)$$





Just like a VdW Fluid!

NB: Disagree with
Chamblin et.al. PRDD60 (1999) 064018; 104026

Critical Behaviour

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4}$$

$$\frac{\partial P}{\partial v} = 0 \qquad \frac{\partial^2 P}{\partial v} = 0$$

$$T_c = \frac{\sqrt{6}}{18\pi Q}$$

$$v_c = 2\sqrt{6}Q$$

$$P_c = \frac{1}{96\pi Q^2}$$

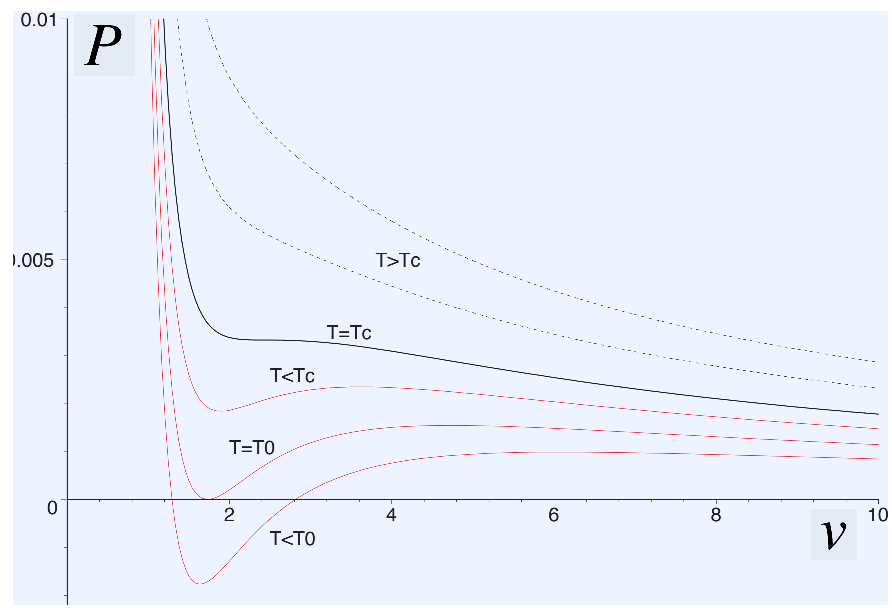
$$\frac{P_c v_c}{kT_c} = \frac{3}{8}$$

Just like a VdW Fluid!

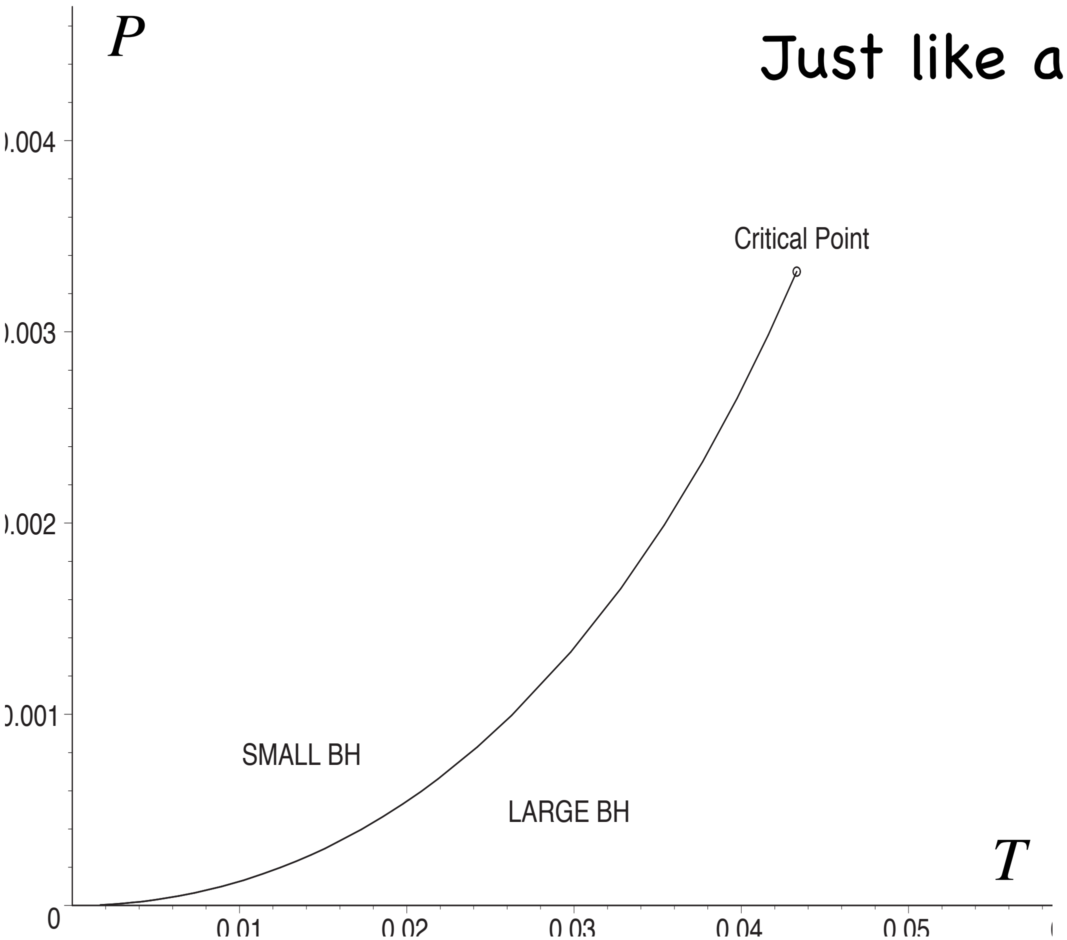
$$p = \frac{P}{P_c}, \quad \nu = \frac{v}{v_c}, \quad \tau = \frac{T}{T_c}$$

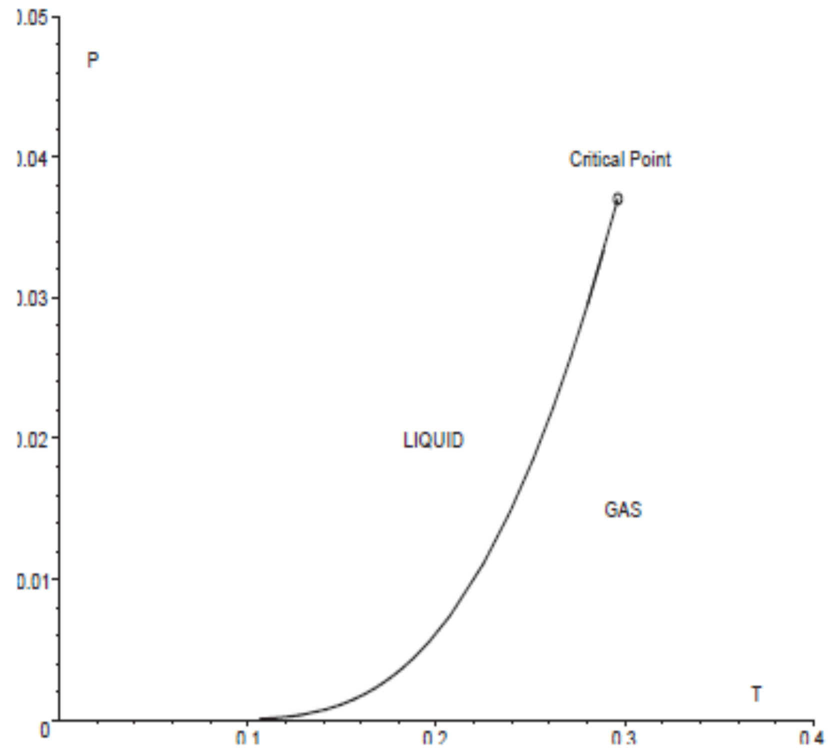
$$8\tau = 3\nu\left(p + \frac{2}{\nu^2}\right) - \frac{1}{\nu^3}$$

law of corresponding states



Just like a VdW Fluid!





$$\alpha = 0 \quad \beta = \frac{1}{2} \quad \gamma = 1 \quad \delta = 3$$

govern volume, compressibility, specific heat, and pressure near the critical point

Higher Dimensional VdW Black Holes

Gunasekaran/Kubiznak/Mann
JHEP 1211 (2012) 110

$$H = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j}^{N} W_{ij}$$

d dimensions, N particles

$$a = -\frac{1}{2}\int W(|\vec{r}|)d\mathbf{r}$$

$$Z = \int d\mathbf{r}_1 \ldots d\mathbf{r}_N d\mathbf{p}_1 \ldots d\mathbf{p}_N e^{-\beta H} = \left(\frac{2\pi m}{\beta}\right)^{\frac{N(d-1)}{2}} V^N \left(1 + \frac{\beta N^2 a}{V}\right) + \cdots$$

$$F = -kT \lg Z = -\frac{d-1}{2} kTN \lg(2\pi mkT) - kTN \lg V - \frac{N^2 a}{V} + O(a^2)$$

$$P = -\frac{\partial F}{\partial V}$$

$$\left(P + \frac{a}{v^2}\right)(v - b) = kT$$

VdW in higher dimensions is the same!

Charged D-Dim'l AdS Black Holes

$$ds^2 = -Vdt^2 + \frac{dr^2}{V} + r^2 d\Omega_{D-2}^2 \qquad F = dA \quad \begin{cases} V = 1 - \dfrac{m}{r^{D-3}} + \dfrac{q^2}{r^{2(D-3)}} + \dfrac{r^2}{l^2} \\ A = -\sqrt{\dfrac{D-2}{2(D-3)}} \dfrac{q}{r^{D-3}} dt \end{cases}$$

$$M = \frac{D-2}{16\pi} \omega_{D-2} m$$

Temperature $T = \frac{D-3}{4\pi r_+}\left(1 - \frac{q^2}{r_+^{2(D-3)}} + \frac{D-1}{D-3}\frac{r_+^2}{l^2}\right)$

$$Q = \frac{\sqrt{2(D-2)(D-3)}}{8\pi} \omega_{D-2}\, q$$

Entropy $S = \frac{A_{D-2}}{4} = \frac{\omega_{D-2} r_+^{D-2}}{4}$ Pressure $P = -\frac{\Lambda}{8\pi} = \frac{(D-1)(D-2)}{16\pi l^2}$

Potential $\Phi = \sqrt{\frac{D-2}{2(D-3)}} \frac{q}{r_+^{D-3}}$ Volume $V = \frac{\omega_{D-2} r_+^{D-1}}{D-1}$ $\omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma[(D-1)/2]}$

$$M = \frac{D-2}{D-3} TS - \frac{2}{D-3} VP + \Phi Q \qquad dM = TdS + VdP + \Phi dQ$$

Smarr Relation First Law

Equation of State

$$r_+ = \left(\frac{(D-1)V}{4\pi}\right)^{1/(D-1)}$$

$$T = \frac{D-3}{4\pi r_+}\left(1 - \frac{q^2}{r_+^{2(D-3)}} + \frac{D-1}{D-3}\frac{r_+^2}{l^2}\right)$$

$$P = \frac{(D-1)(D-2)}{16\pi l^2}$$

$$P = \frac{T(D-2)}{4r_+} - \frac{(D-3)(D-2)}{16\pi r_+^2} + \frac{q^2(D-3)(D-2)}{16\pi r_+^{2(D-2)}}$$

Physical Equation of State

Thermodynamic Volume

$$\text{Press} = \frac{\hbar c P}{l_P^{d-2}} \quad \text{Temp} = \frac{\hbar c T}{k} \Rightarrow \text{Press} = \frac{k(D-2)\text{Temp}}{4 l_P^{d-2} r_+} + \ldots \Rightarrow v = \frac{2 l_P^{D-2} r_+}{D-2}$$

Van der Waal's Equation

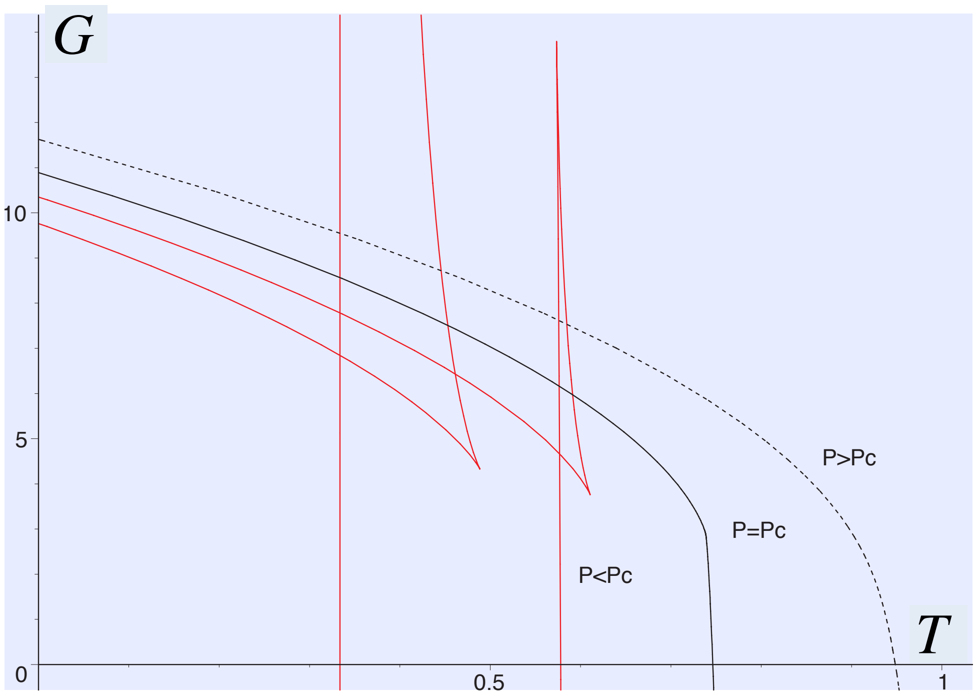
$$P = \frac{T}{v} - \frac{D-3}{(D-2)\pi v^2} + \frac{(D-3)q^2}{4\pi v^{2(D-2)}\kappa^{2D-5}}$$

$$r_+ = \kappa v = \frac{D-2}{2} v$$

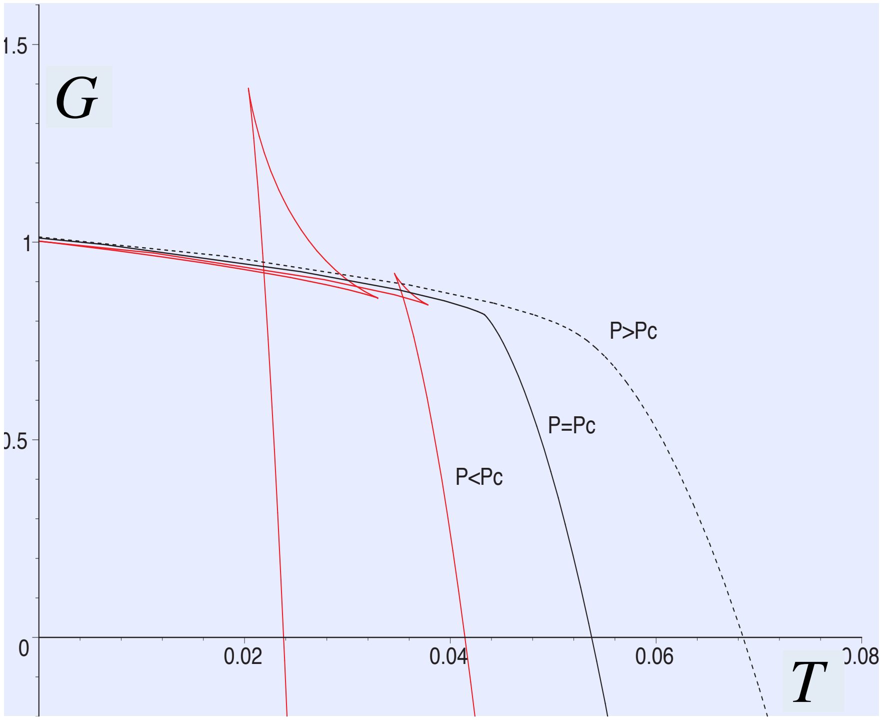
Gibbs Free Energy of D-Dim AdS RN BH

$$G = G(T,P) = \frac{\omega_{D-2}}{16\pi}\left(r_+^{D-3} - \frac{16\pi P r_+^{D-1}}{(D-1)(D-2)} + \frac{(2D-5)q^2}{r_+^{D-3}} \right)$$

10 Dimensions



4 Dimensions



Critical Behaviour

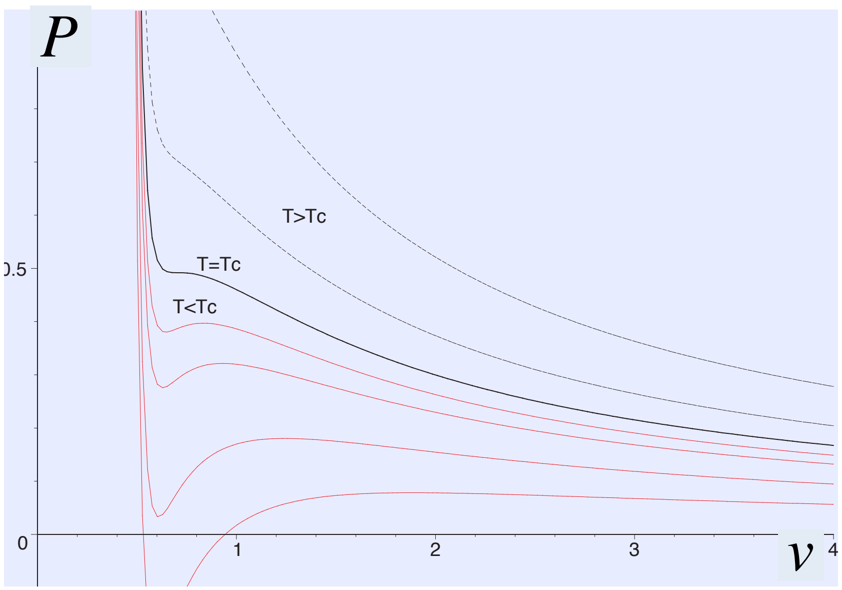
$$P = \frac{T}{v} - \frac{D-3}{(D-2)\pi v^2} + \frac{(D-3)q^2}{4\pi v^{2(D-2)}\kappa^{2D-5}}$$

$$\frac{\partial P}{\partial v} = 0 \quad \frac{\partial^2 P}{\partial v} = 0$$

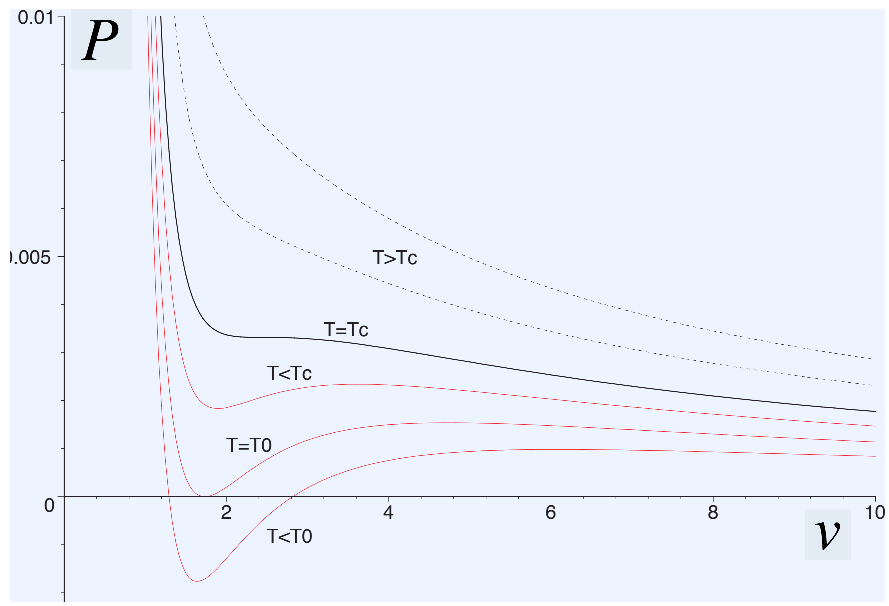
$$\frac{P_c v_c}{kT_c} = \frac{2D-5}{4D-8}$$

$$4(D-2)\tau = (2D-5)v\left(p + \frac{D-2}{(D-3)v^2}\right) - \frac{1}{(D-3)v^3}$$

law of corresponding states



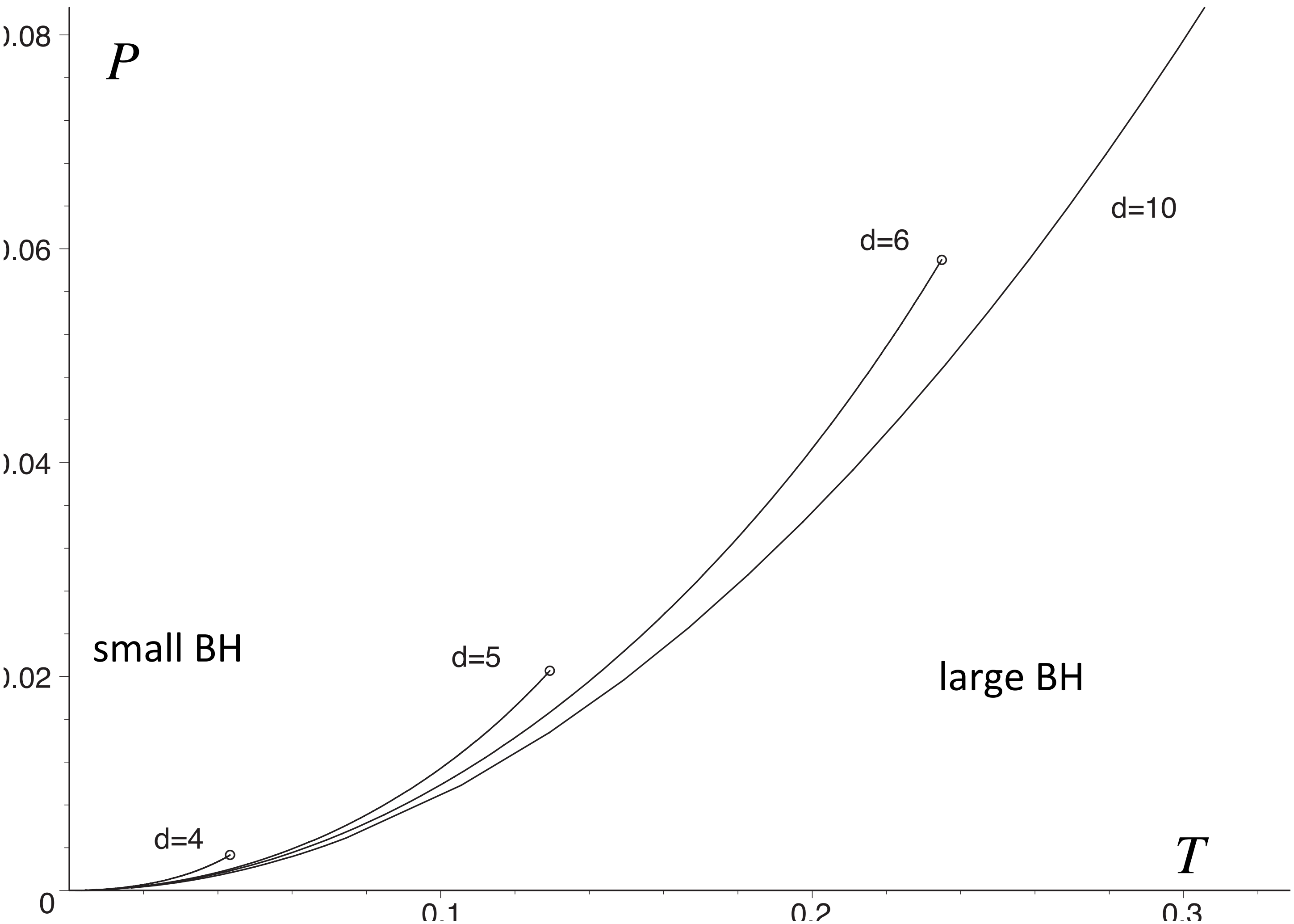
10 Dimensions



4 Dimensions

$$\alpha = 0 \quad \beta = \frac{1}{2} \quad \gamma = 1 \quad \delta = 3$$

Just like a
4d- VdW
Fluid!



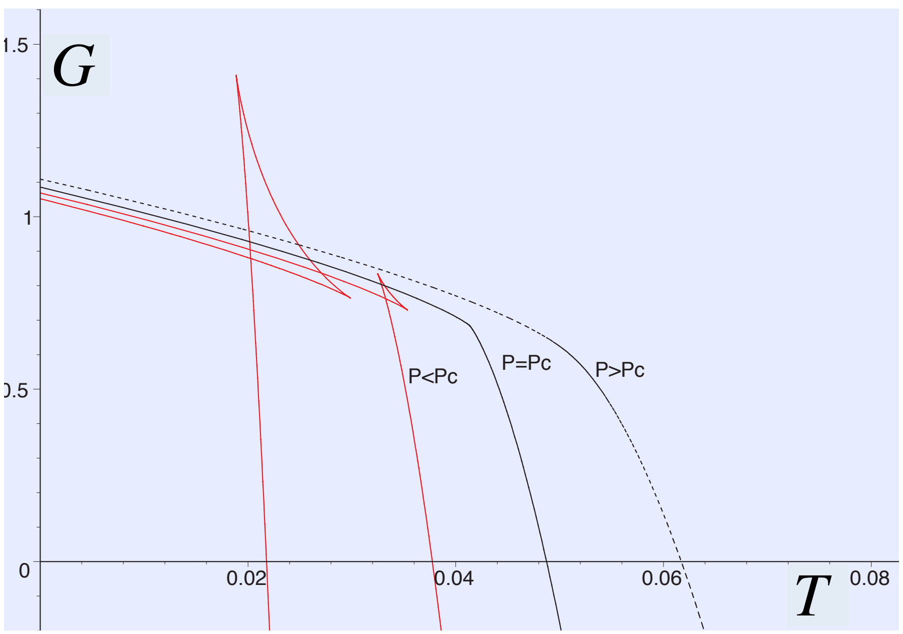
Kerr-Newman Black Holes

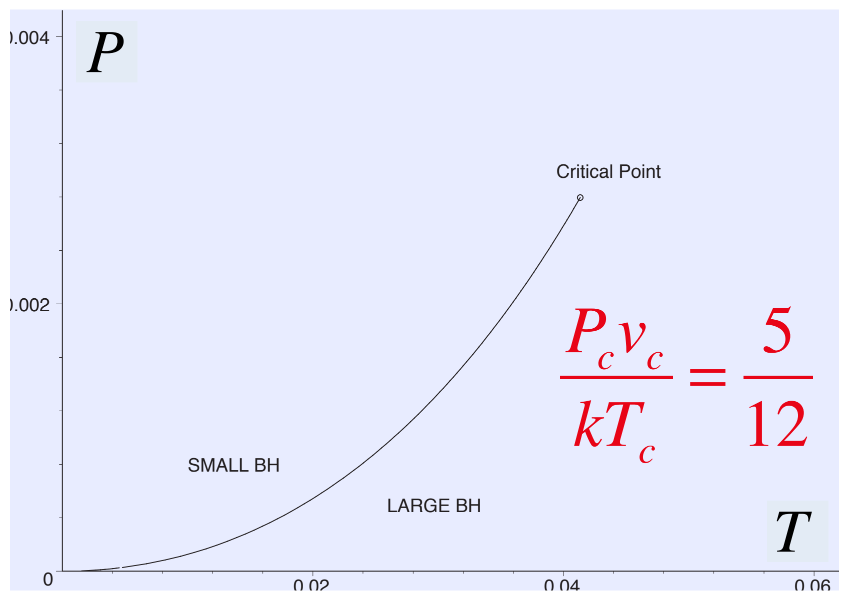
N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann
Galaxies 2 (2014) 89

$$ds^2 = -\frac{\Delta}{\rho^2}\left[dt - \frac{a\sin^2\theta}{\Xi}d\varphi\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\Sigma}d\theta^2 + \frac{\Sigma\sin^2\theta}{\rho^2}\left[a\,dt - \frac{r^2+a^2}{\Xi}d\varphi\right]^2$$

$$\Delta = (r^2+a^2)\left(1+\frac{r^2}{l^2}\right) - 2mr + q^2 \qquad \Sigma = 1 - \frac{a^2}{l^2}\cos^2\theta \qquad \Xi = 1 - \frac{a^2}{l^2} \qquad \rho^2 = r^2 + a^2\cos^2\theta$$

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2Q^2}{\pi v^4} + \frac{48J^2}{\pi v^6} - \frac{96Q^2(24Q^2+5v^2+6\pi T v^3)J^2}{\pi v^6(8Q^2+v^2+\pi T v^3)^2} + O\left(J^4\right)$$



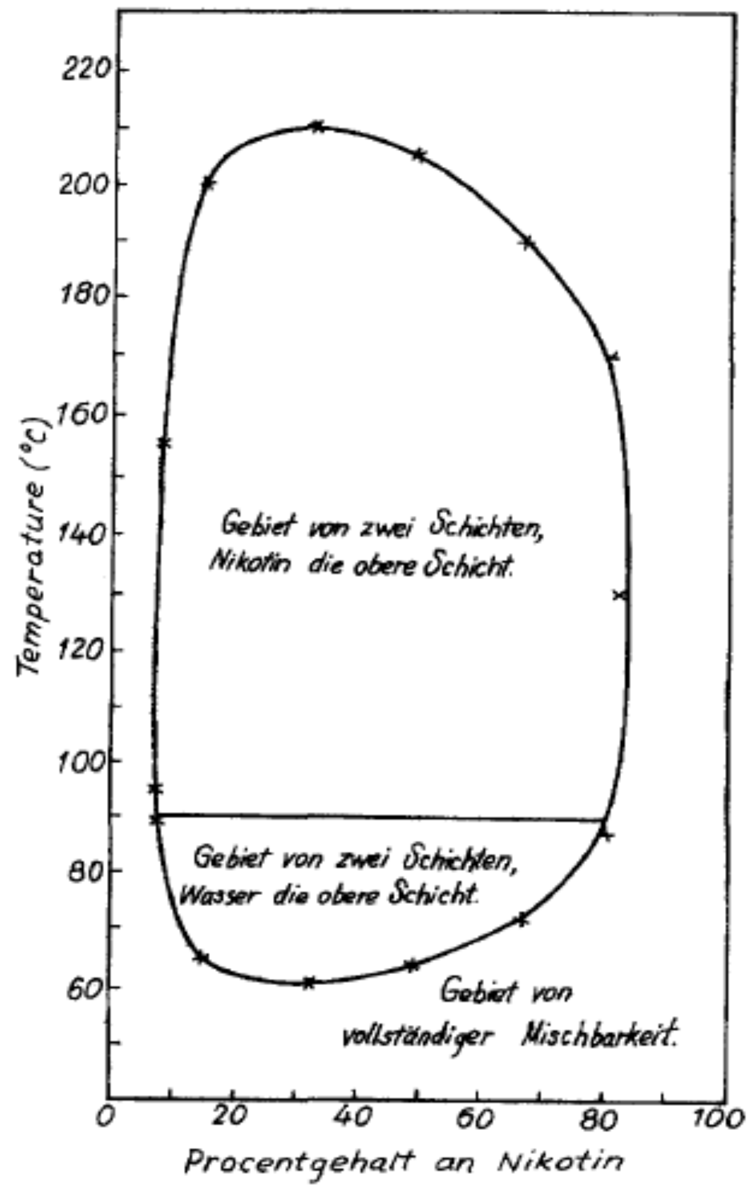


$$\frac{P_c v_c}{kT_c} = \frac{5}{12}$$

Reentrant Phase Transitions

RPT: If a **monotonic** variation of any thermodynamic quantity that results in two (or more) phase transitions such that the **final state is macroscopically similar** to the initial state.

First observed in nicotine/water



C. Hudson Z. Phys. Chem. 47 (1904) 113.

- multicomponent fluid systems
- gels
- ferroelectrics
- liquid crystals
- binary gases

T. Narayanan and A. Kumar Physics Reports 249 (1994) 135

Single-Rotation Black Holes

$$ds^2 = -\frac{\Delta}{\rho^2}\left[dt - \frac{a\sin^2\theta d\varphi}{\Xi}\right]^2 + \frac{\Sigma\sin^2\theta}{\rho^2}\left[adt - \frac{\left(r^2 + a^2\right)d\varphi}{\Xi}\right]^2$$

$$+\frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\Sigma}d\theta^2 + r^2\cos^2\theta d\Omega_{D-2}$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\Sigma = 1 - \frac{a^2}{l^2}\cos^2\theta \quad \Xi = 1 - \frac{a^2}{l^2}$$

$$\Delta = (r^2 + a^2)\left(1 + \frac{r^2}{l^2}\right) - 2mr^{5-D}$$

$$M = \frac{\omega_{D-2}}{4\pi}\frac{m}{\Xi^2}\left(1 + \frac{(D-4)\Xi}{2}\right) \quad J = \frac{\omega_{D-2}}{4\pi}\frac{ma}{\Xi^2} \quad \Omega_H = \frac{a}{l^2}\frac{r_+^2 + l^2}{r_+^2 + a^2},$$

$$T = \frac{1}{2\pi}[r_+(\frac{r_+^2}{l^2} + 1)\left(\frac{1}{a^2 + r_+^2} + \frac{D-3}{2r_+^2}\right) - \frac{1}{r_+}] \quad S = \frac{\omega_{D-2}}{4}\frac{(a^2 + r_+^2)r_+^{d-4}}{\Xi} = \frac{A}{4}$$

$$\frac{D-3}{D-2}M = TS + \Omega_H J - \frac{2VP}{D-2} \qquad dM = TdS + VdP + \Omega_H dJ$$

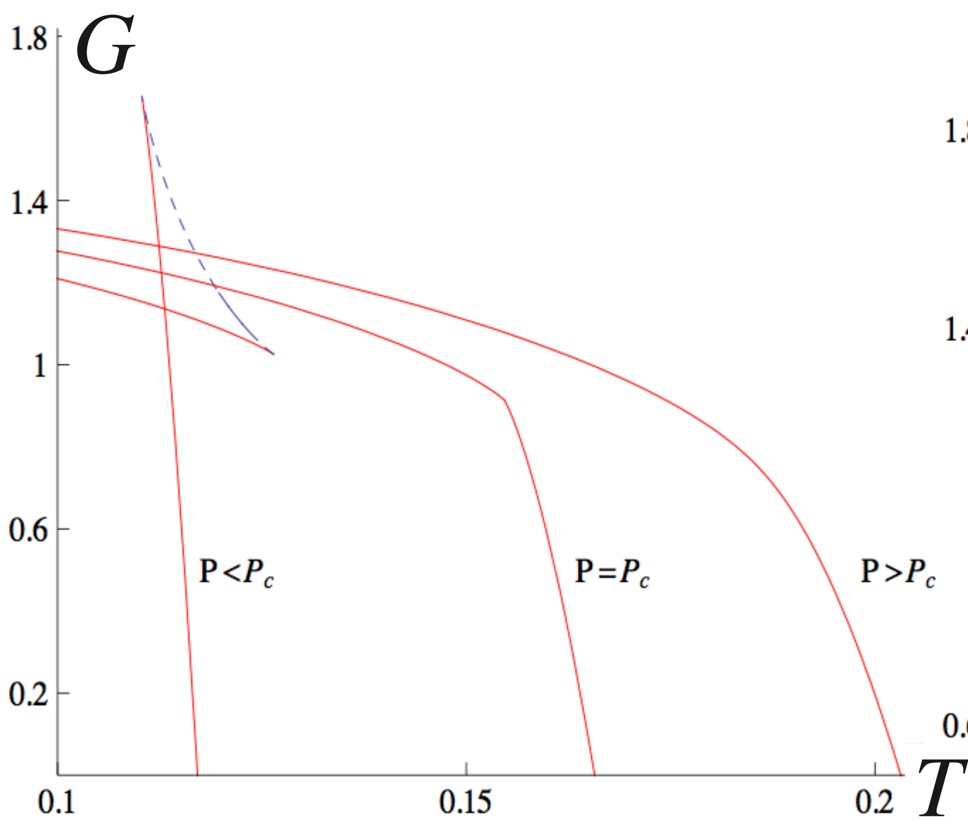
Smarr Relation

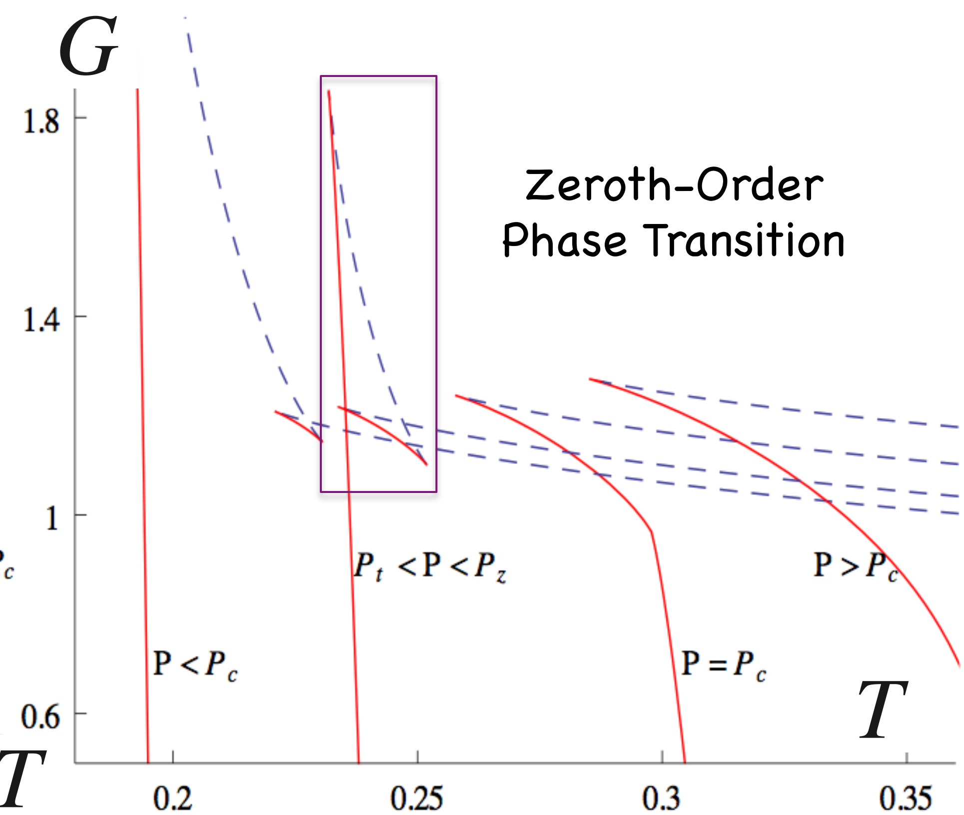
First Law

Dimensional Dependence of G

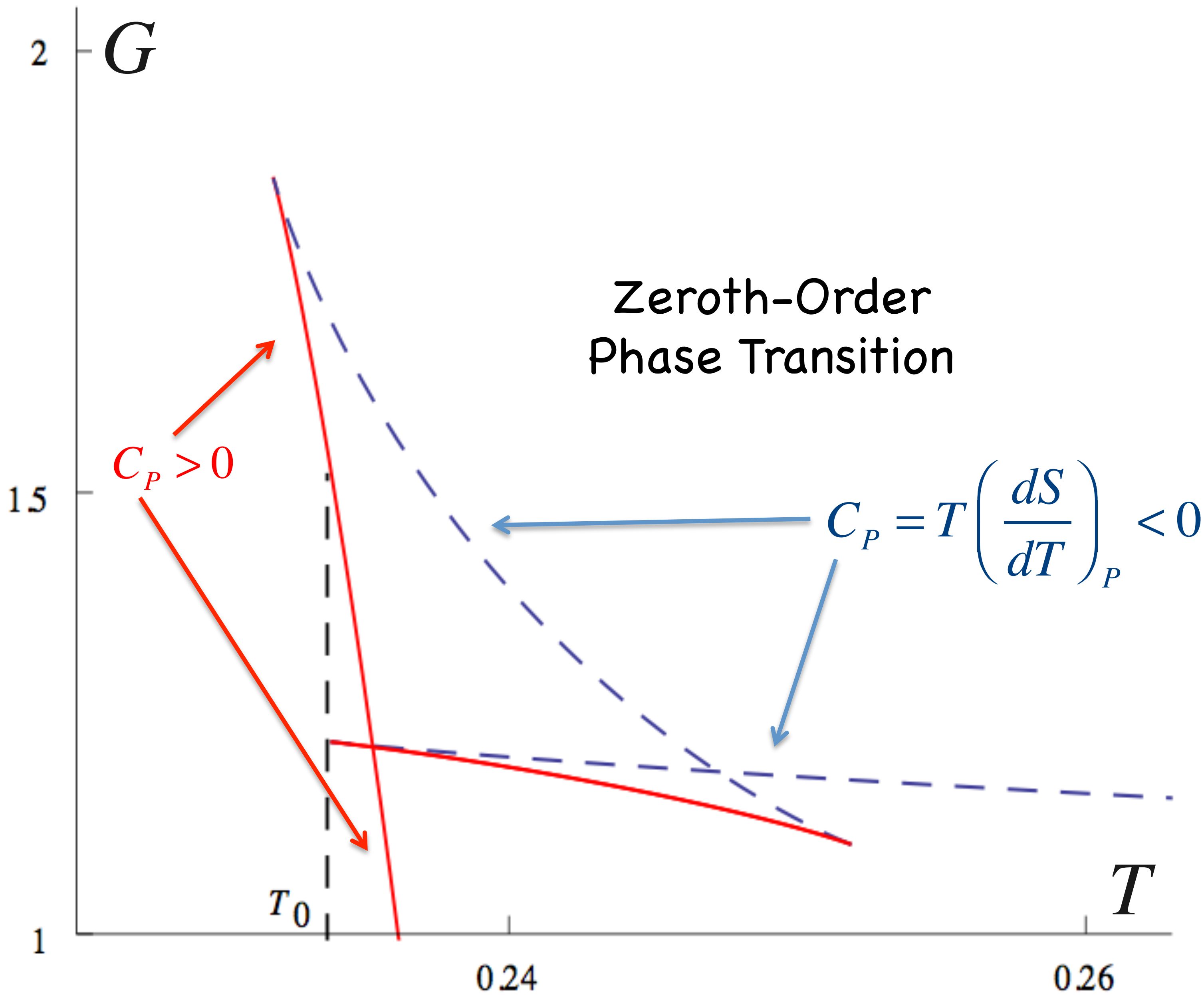
$D = 5$

$D > 5$

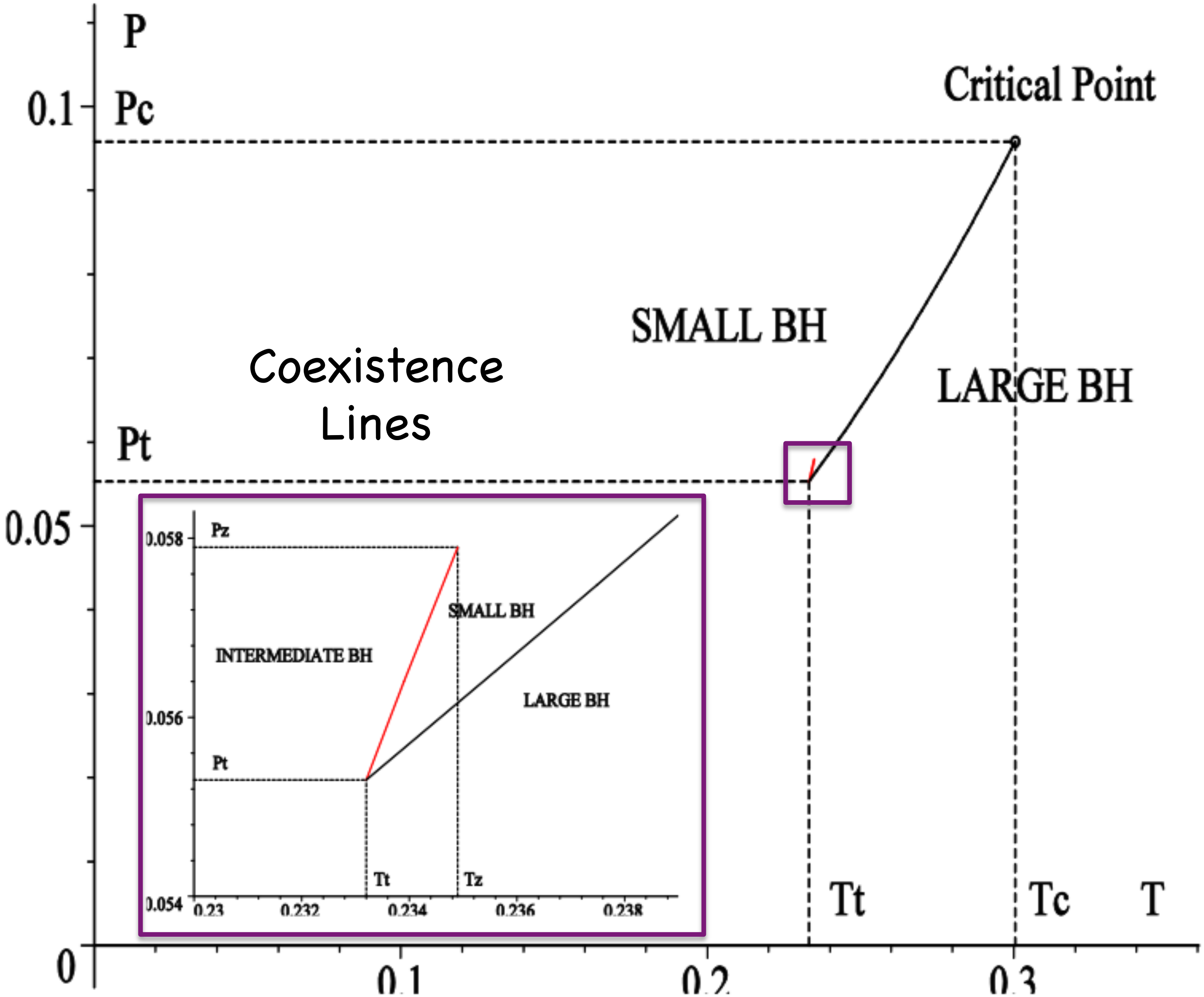




Reentrant Phase Transitions in D>5



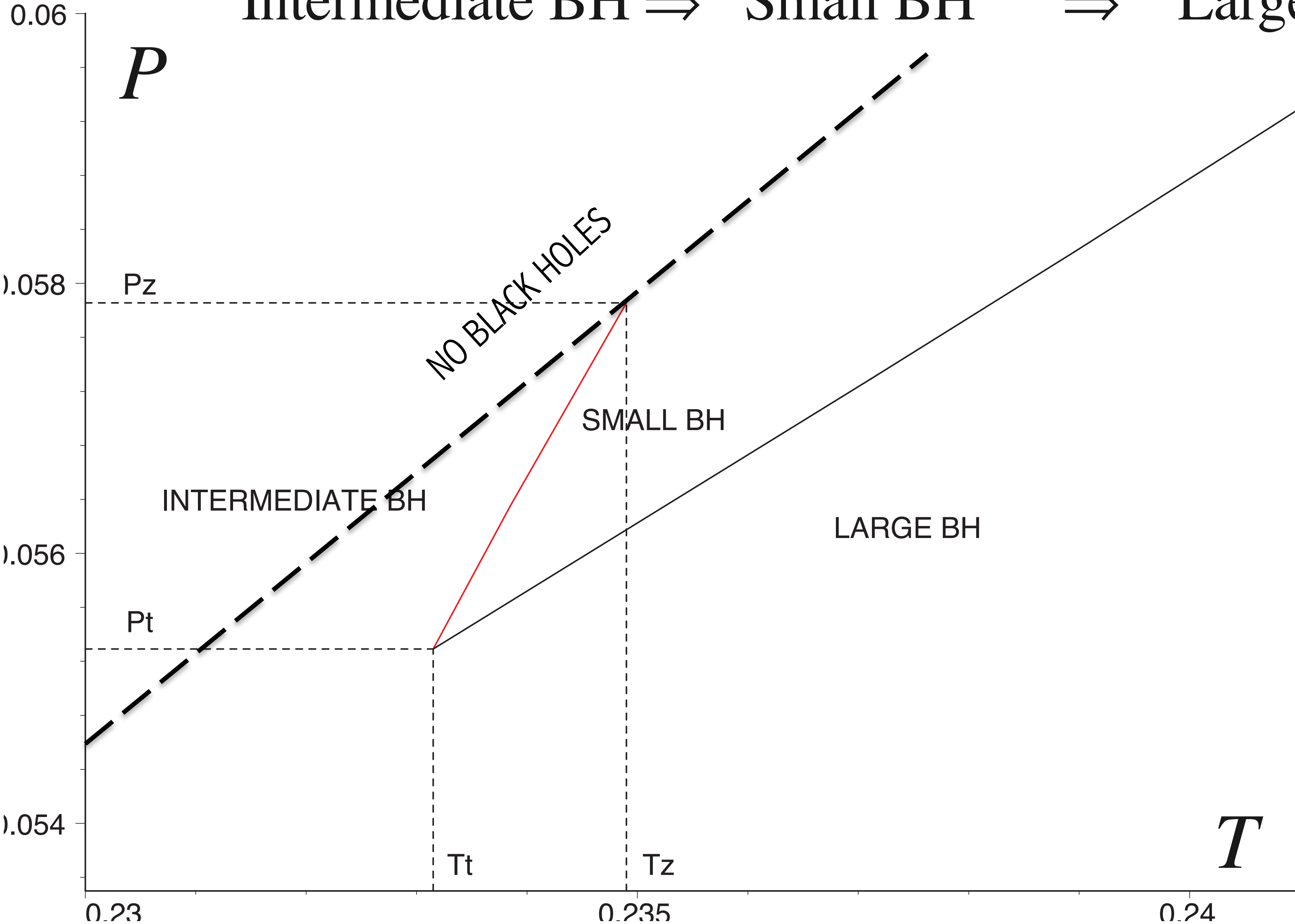
Reentrant Phase Transitions in D>5



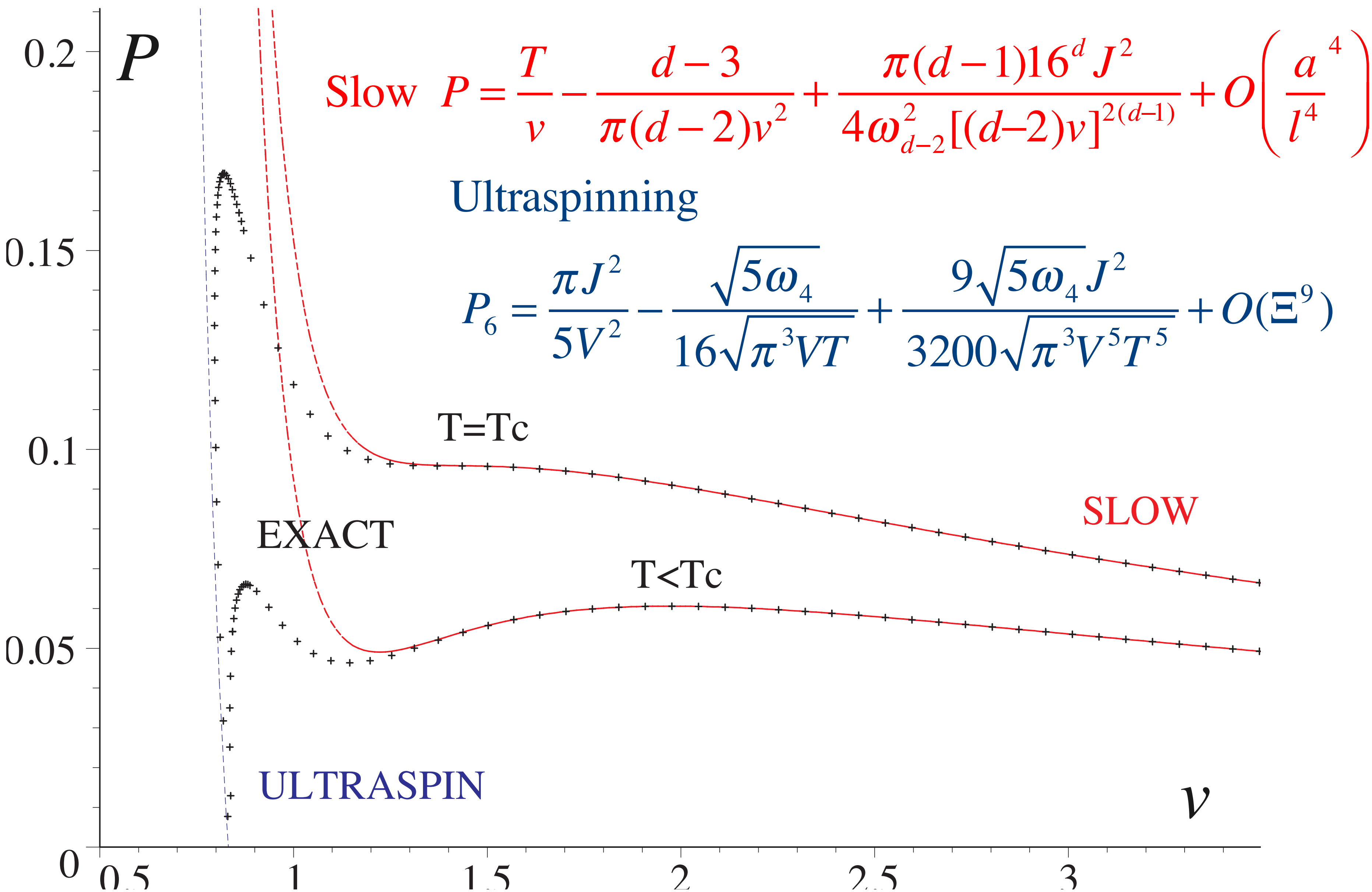
Low T | **Medium T** | **High T**

mixed ⇒ water/nicotine ⇒ mixed

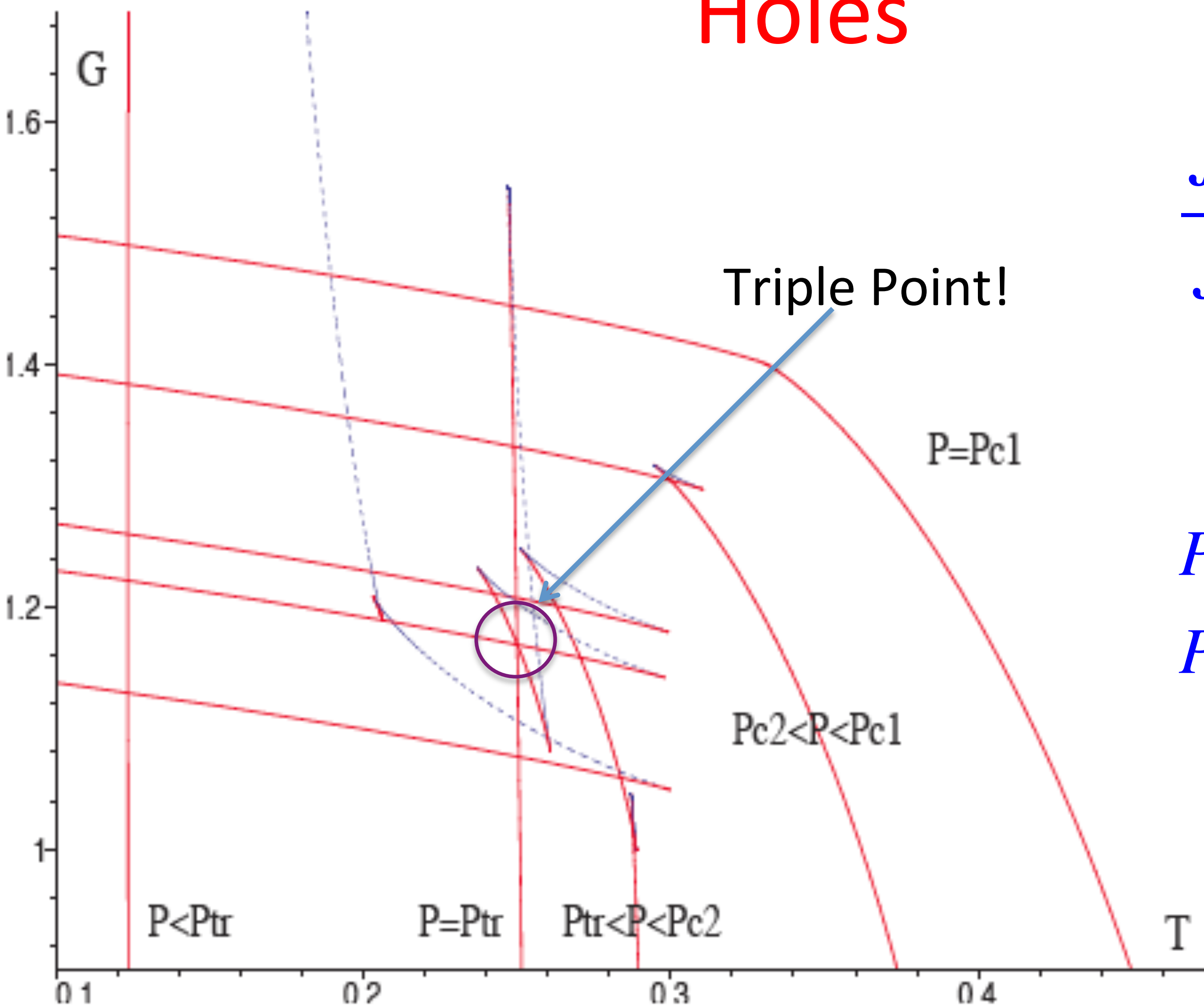
Intermediate BH ⇒ Small BH ⇒ Large BH



Slow and Ultraspinning Limits



Triple Points in Multiply Rotating Black Holes



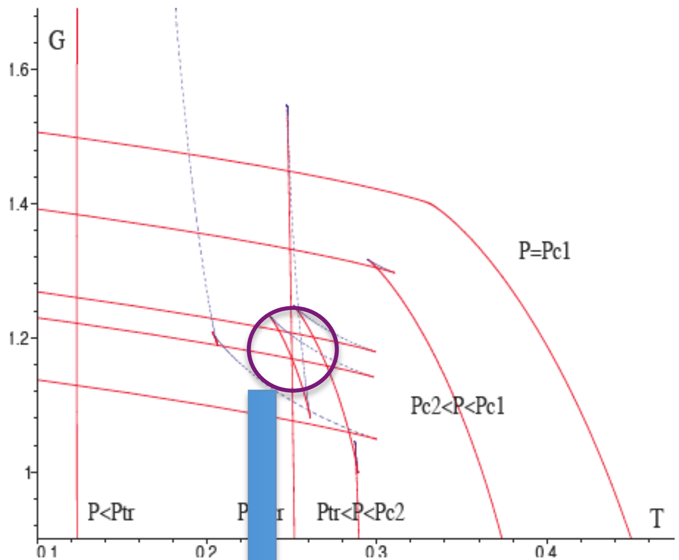
$$\frac{J_2}{J_1} = 0.05$$

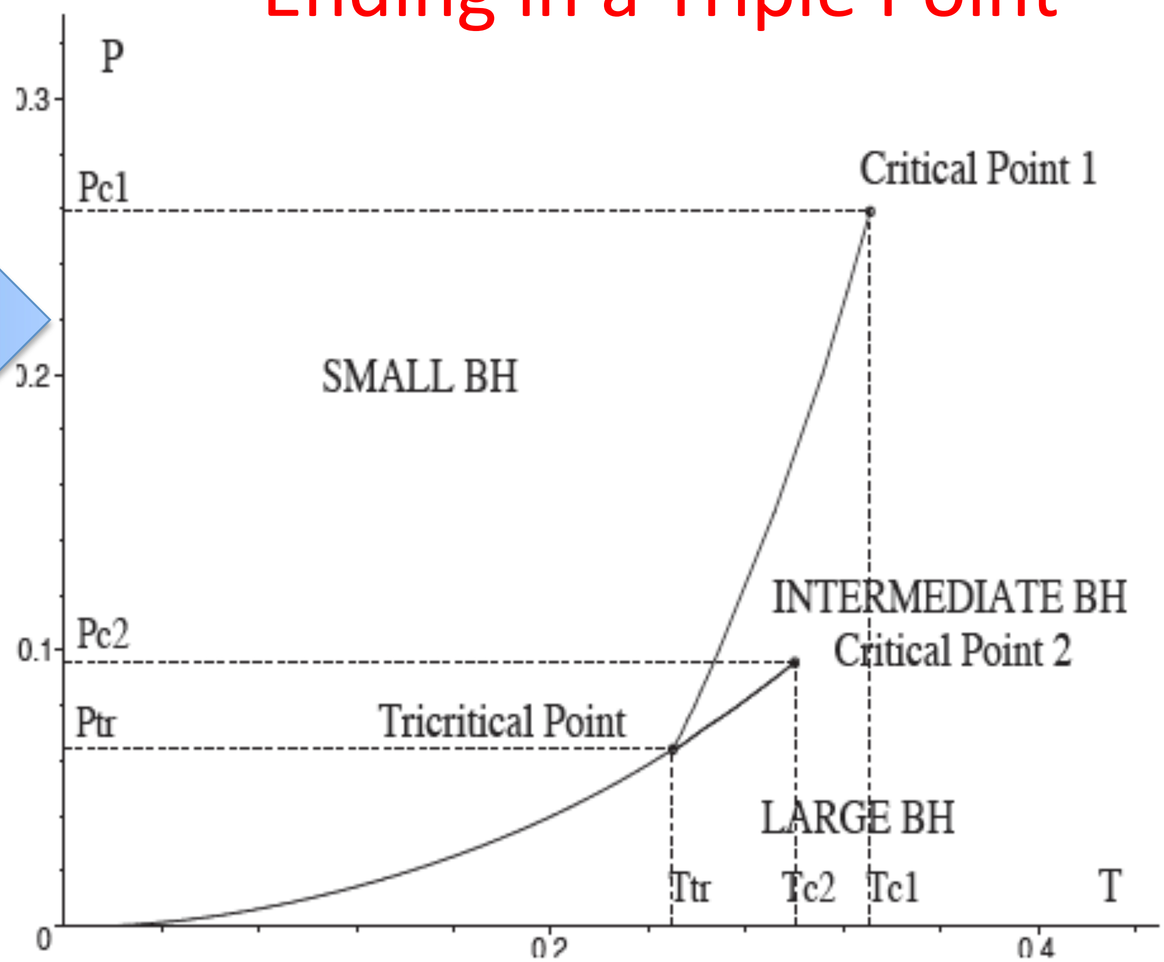
$$P_{c1} = 0.259$$

$$P_{c1} = 0.0956$$

N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann
CQG 31 (2104) 042001 arXiV:1308.2672

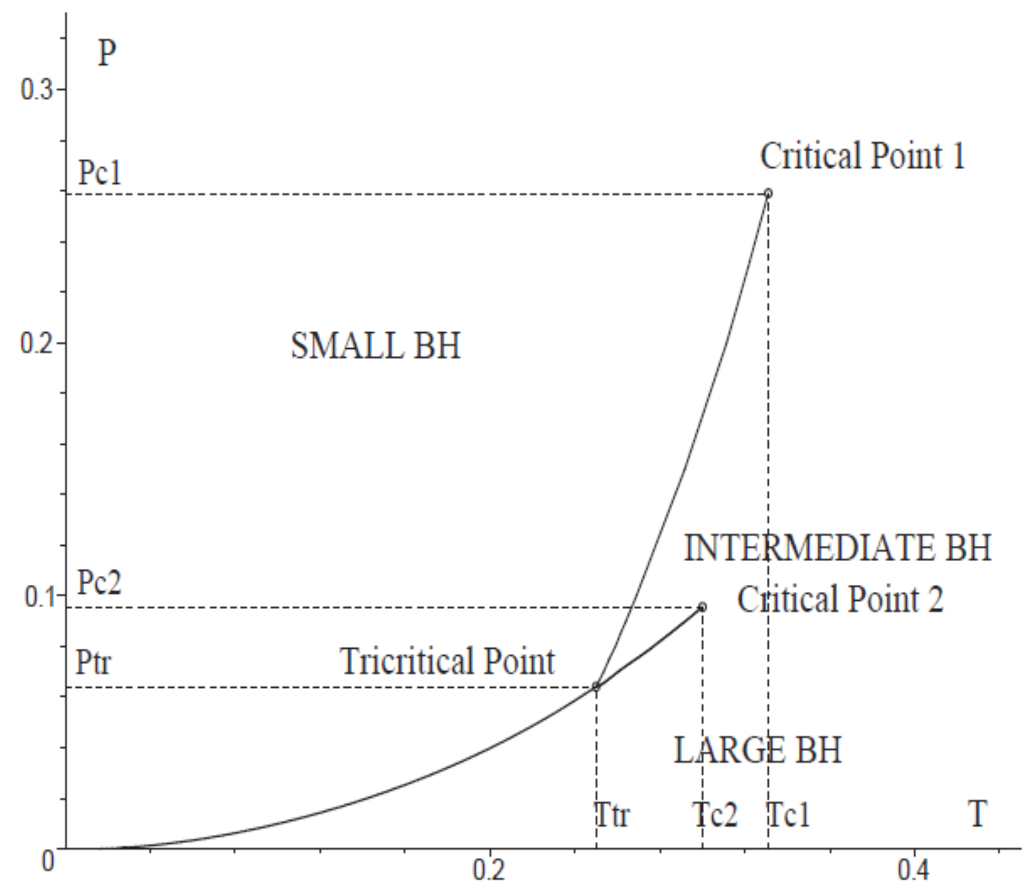
Reentrant Phase transition Ending in a Triple Point

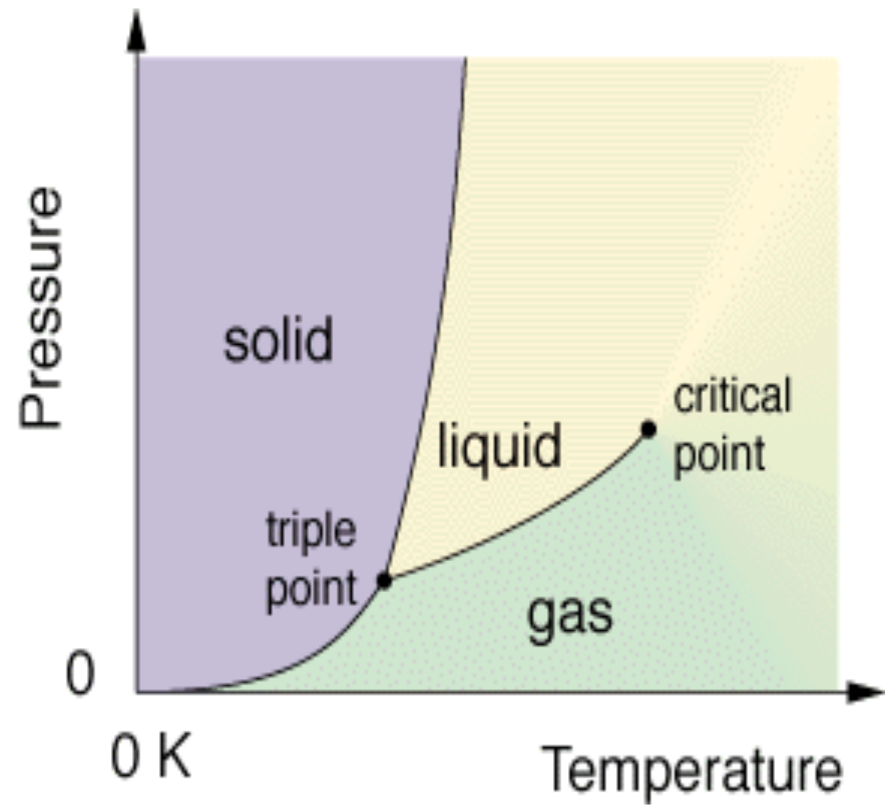




$$\frac{J_2}{J_1} = 0.05$$

The Black Hole Triple Point





N. Altimirano, D. Kubiznak, Z. Sherkatgnad, R.B. Mann CQG 31 (2014) 042001

Newer Results from Black Hole Chemistry

- Lovelock Gravity
  - Multiple re-entrant phase transitions

A. Frassino, D.Kubiznak, R.B. Mann, F. Simovic JHEP **1409** (2014) 080

- Thermodynamic Singularities
  - Isotherms cross at a particular value of ν
  - Global minimum of Gibbs free energy still defined
- Isolated critical points
  - Some black holes do not have standard critical exponents

B. Dolan, A. Kostouki, D.Kubiznak, R.B. Mann, CQG **31** (2014) 242001

- Quasitopological black holes
  - Exhibit similar phenomena but in lower dimensions

W.G. Brenna, R. Hennigar, R.B. Mann, JHEP (to appear)

- Van der Waals black holes
  - Black holes with exotic matter yield exact VdW equation

A. Rajagopal, D. Kubinzank, R.B. Mann, PL **B737** (2014) 277

T. Delsate, RBM JHEP **1502** (2015) 070

Isolated Critical Points

B. Dolan, A. Kostouki,
D.Kubiznak,
R.B. Mann,
CQG 31 (2014) 242001

$$\mathcal{L} = \frac{1}{16\pi G_N} \sum_{k=0}^{K} \hat{\alpha}_{(k)} \mathcal{L}^{(k)} = \frac{2^{-k}}{16\pi G_N} \sum_{k=0}^{K} \hat{\alpha}_{(k)} \delta^{a_1 b_1 \ldots a_k b_k}_{c_1 d_1 \ldots c_k d_k} R_{a_1 b_1}{}^{c_1 d_1} \ldots R_{a_k b_k}{}^{c_k d_k}$$

$$\alpha_k = \alpha A^{K-k} \binom{K}{k}$$

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_\kappa^2$$

$$A^{-1} = \sqrt[K-1]{K\alpha}$$

$$f = \kappa + r^2 A \left[1 - \left(\frac{m_0 r^{1-d} - \alpha_0}{\alpha A^K} + 1 \right)^{1/K} \right]$$

$$B \equiv \frac{\kappa}{r_+^2} + A$$

Equation of State

$$P = \frac{(d-1)(d-2)\alpha}{16\pi G_N} \left[B^{K-1} \left(\frac{2K(2\pi r_+ T + \kappa)}{(d-1) r_+^2} - B \right) + A^K \right]$$

$\kappa = -1$

$K = \text{odd}$

Isolated Critical Point where two 1st order Phase Transitions merge

$$\kappa = -1 \qquad \frac{\partial^k P}{\partial r_+^k} = 0$$
$$K = \text{odd} \qquad k = 1, \ldots, K-1$$

$$r_c = \frac{1}{\sqrt{A}} \qquad T_c = \frac{1}{2\pi r_c}$$

$$P_c = \frac{(d-1)(d-2)\alpha}{16\pi G_N} A^K$$

$$\frac{P}{P_c} = 1 + K\frac{2^K}{d-1}\omega^{K-1}\tau + \frac{(K-d+1)2^K}{d-1}\omega^K + \ldots$$

$$\tilde{\alpha} = 0, \;\; \tilde{\beta} = 1, \;\; \tilde{\gamma} = K-1, \; \tilde{\delta} = K$$

Non-Standard Critical Exponents

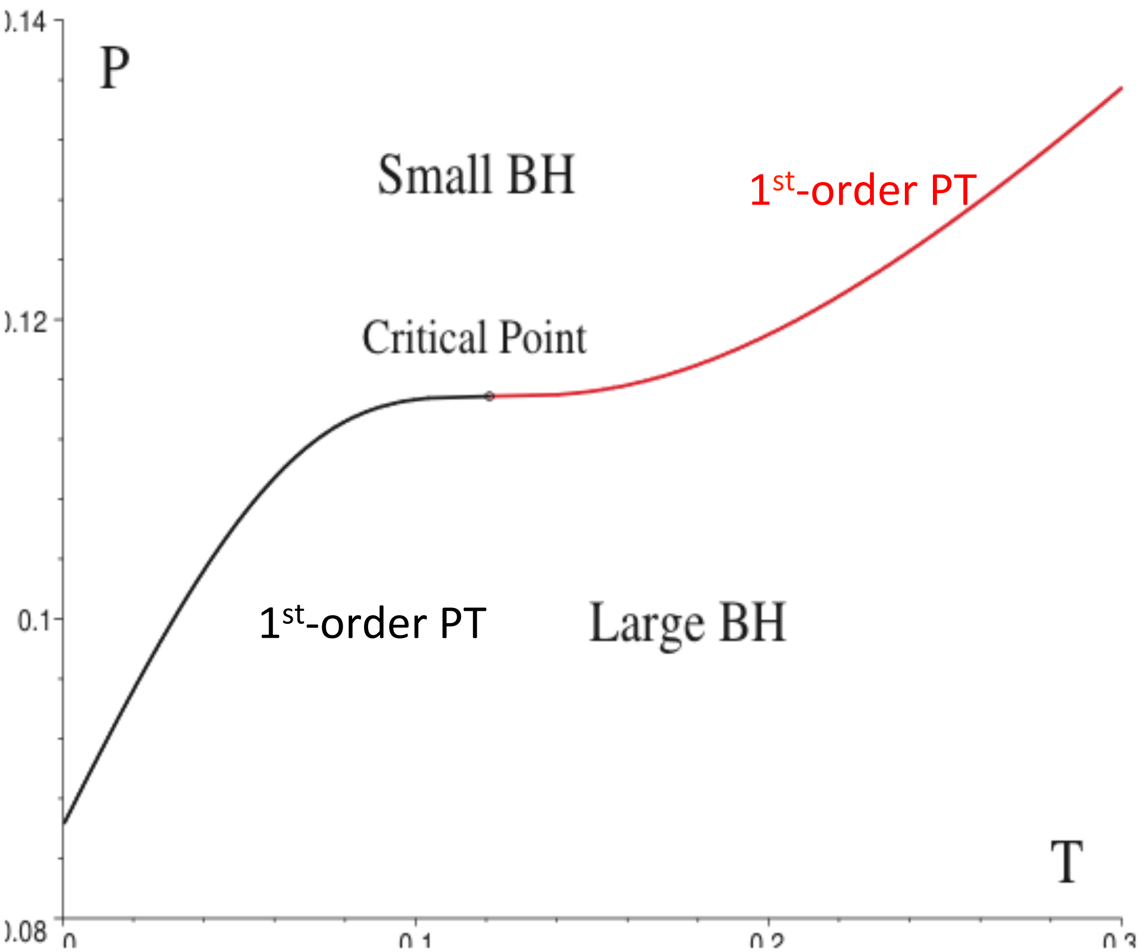
$$\frac{p}{p_c} = 1 + A\frac{t}{t_c} + B\frac{t}{t_c}\frac{v-v_c}{v_c} + C\left(\frac{v-v_c}{v_c}\right)^3 + \ldots$$

Most black holes ←→Mean field thy
→standard critical exponents

$$\frac{p}{p_c} = 1 + B\frac{t}{t_c}\left(\frac{v-v_c}{v_c}\right)^2 + C\left(\frac{v-v_c}{v_c}\right)^3 + .$$

Topological black holes ←→scaling rel'ns violated
→non-standard critical exponents

$$\tilde{\alpha}=0,\ \tilde{\beta}=1,\ \tilde{\gamma}=K-1,\ \tilde{\delta}=K$$



$$\tilde{\gamma}=\tilde{\beta}(\tilde{\delta}-1)$$

Widom Relation ✓

$$\tilde{\alpha}+2\tilde{\beta}+\tilde{\gamma}\geq 2$$

Rushbrooke inequality ✓

Ehrenfest relations ✓

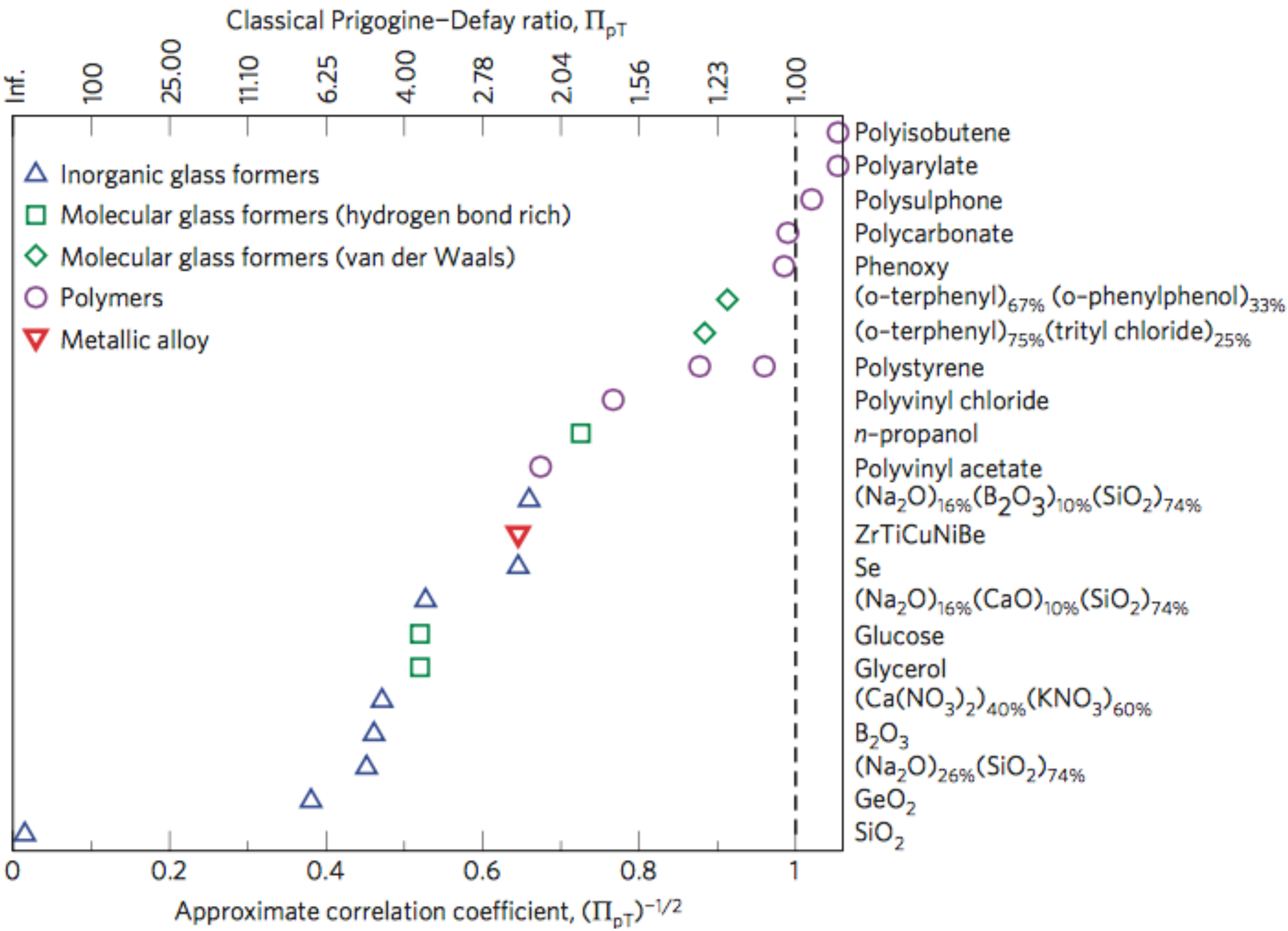
$$\Pi=1/K$$

Prigogine-De Fay ratio

Suggests a liquid-glass type of phase transition

Similar phenomena present for Quasitopological Black Holes

W.G. Brenna, R. Hennigar, R.B. Mann, JHEP (to appear)



Classical Prigogine−Defay ratio, Π_{pT}
Inf.
100
25.00
11.10
6.25
4.00
2.78
2.04
1.56
1.23
1.00
Inorganic glass formers
Molecular glass formers (hydrogen bond rich)
Molecular glass formers (van der Waals)
Polymers
Metallic alloy
Polyisobutene
Polyarylate
Polysulphone
Polycarbonate
Phenoxy
(o-terphenyl)$_{67\%}$ (o-phenylphenol)$_{33\%}$
(o-terphenyl)$_{75\%}$(trityl chloride)$_{25\%}$
Polystyrene
Polyvinyl chloride
n-propanol
Polyvinyl acetate
$(Na_2O)_{16\%}(B_2O_3)_{10\%}(SiO_2)_{74\%}$
ZrTiCuNiBe
Se
$(Na_2O)_{16\%}(CaO)_{10\%}(SiO_2)_{74\%}$
Glucose
Glycerol
$(Ca(NO_3)_2)_{40\%}(KNO_3)_{60\%}$
B_2O_3
$(Na_2O)_{26\%}(SiO_2)_{74\%}$
GeO_2
SiO_2
0
0.2
0.4
0.6
0.8
1
Approximate correlation coefficient, $(\Pi_{pT})^{-1/2}$

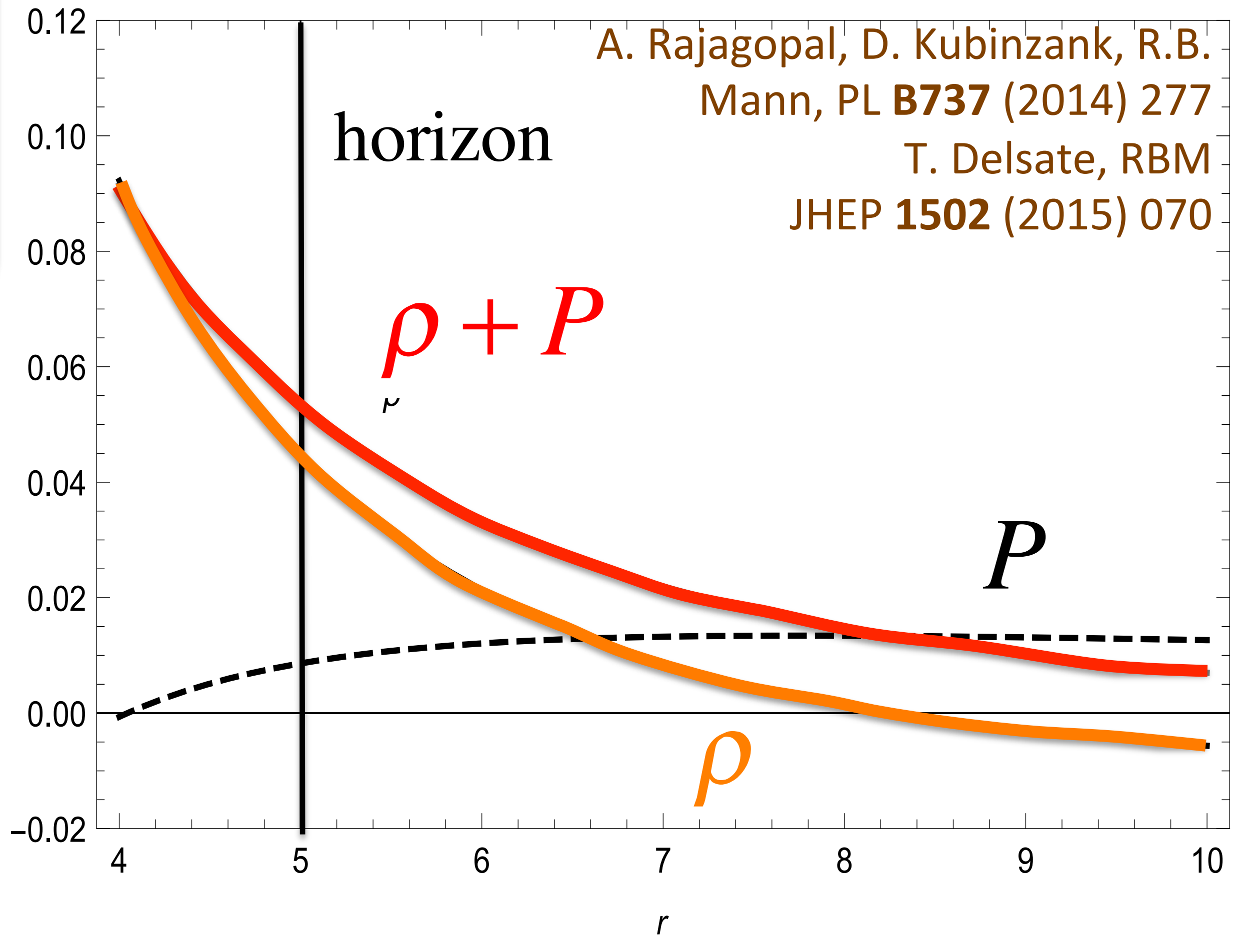
Van der Waals Black Holes

$$T = \left(P + \frac{a}{v^2}\right)(v - b)$$

Q: What Black Hole yields the exact Van der Waals equation?

A: A spherical black Hole inside a distribution of exotic aniostropic matter

But it is always possible to satisfy the energy conditions within a region of the horizon



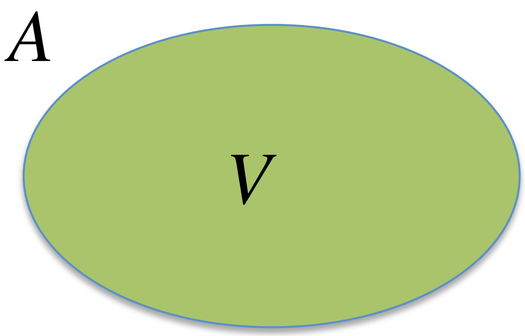
A. Rajagopal, D. Kubinzank, R.B. Mann, PL **B737** (2014) 277

T. Delsate, RBM JHEP **1502** (2015) 070

Reverse Isoperimetric Inequality

Q: What is the smallest area that encloses a given Euclidean volume V?

A: A spherical surface



$$\omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma\left(\frac{D-1}{2}\right)}$$

$$R = \left(\frac{(D-1)V}{\omega_{D-2}}\right)^{1/D-1}\left(\frac{\omega_{D-2}}{A}\right)^{1/D-2} \le 1$$

Cvetic/Gibbons/Kubiznak/Pope
PRD84 (2011) 024037

Conjecture: All black holes obey the Reverse Isoperimetric Inequality $R \geq 1$

Implication: For a given thermodynamic volume, the entropy of a black hole is maximized by the Schwarzschild-AdS solution

Kerr-(A)dS Black Hole

$$A_h = \frac{\omega_{D-2}}{r_h^{1-\epsilon}} \prod_i \frac{r_h^2 + a_i^2}{\Xi_i}$$

$$V_h = \frac{r_h A_h}{D-1} + \frac{8\pi}{(D-1)(D-2)} \sum_i a_i J_i = \frac{r_h A_h}{D-1}\left[1 + \frac{1 \pm g^2 r_h^2}{(D-2) r_h^2} \sum_i \frac{a_i^2}{\Xi_i}\right]$$

$$R^{D-1} = r_h \left[1 + \frac{z}{D-2}\right]\left[\frac{1}{r_h^{1-\epsilon}} \prod_i \frac{(r_h^2 + a_i^2)}{\Xi_i}\right]^{\frac{1}{2-D}} = \left[1 + \frac{z}{D-2}\right]\left[\prod_i \frac{(r_h^2 + a_i^2)}{r_h^2 \Xi_i}\right]^{\frac{1}{2-D}}$$

$$\geq \left[1 + \frac{z}{D-2}\right]\left[\frac{2}{D-1}\left(\sum_i \frac{1}{\Xi_i} + \sum_i \frac{a_i^2}{r_h^2 \Xi_i}\right)\right]^{\frac{D-1}{4-2D}} = \left[1 + \frac{z}{D-2}\right]\left[1 + \frac{2z}{D-1}\right]^{\frac{D-1}{4-2D}} \equiv F(z)$$

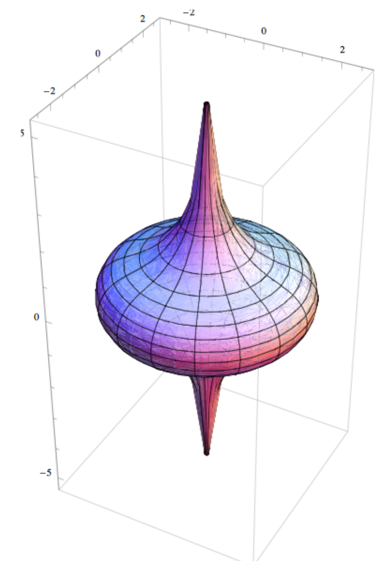
$$F(0) = 1 \qquad \frac{dF(z)}{dz} > 0 \quad \Rightarrow \quad F(z) \geq 1$$

$$\Rightarrow \quad R \geq 1$$

Always?

What is a Super-Entropic Black Hole?

- New ultraspinning limit to the class of Kerr black hole metrics
- Non-compact horizons with finite area
- Asymptotically AdS, but with boundary rotating at the speed of light
- First counterexamples to the Reverse Isoperimetric Inequality → Super-entropic!



Super-Entropic Black Holes

Kerr-Newman AdS Black Hole

R. Hennigar, D. Kubiznak, R.B. Mann
Phys Rev Lett **115** (2015) 031101

$$ds^2 = -\frac{\Delta_a}{\Sigma_a}\left[dt - \frac{a\sin^2\theta}{\Xi}d\phi\right]^2 + \frac{\Sigma_a}{\Delta_a}dr^2 + \frac{\Sigma_a}{S}d\theta^2 + \frac{S\sin^2\theta}{\Sigma_a}\left[adt - \frac{r^2+a^2}{\Xi}d\phi\right]^2$$

$$A = -\frac{qr}{\Sigma_a}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)$$

$$\Sigma_a = r^2 + a^2\cos^2\theta, \quad \Xi = 1 - \frac{a^2}{l^2},$$

$$S = 1 - \frac{a^2}{l^2}\cos^2\theta \qquad \Delta_a = (r^2+a^2)(1+\frac{r^2}{l^2}) - 2mr + q^2$$

1) $\psi = \phi/\Xi$ 2) $a \to l$

3) Compactify $\psi \sim \psi + \mu$

$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - l\sin^2\theta d\psi\right]^2 + \frac{\Sigma}{\Delta}dr^2 + \frac{\Sigma}{\sin^2\theta}d\theta^2 + \frac{\sin^4\theta}{\Sigma}\left[ldt - (r^2+l^2)d\psi\right]^2$$

$$A - \frac{qr}{\Sigma}\left(dt - l\sin^2\theta d\psi\right)$$

$$\Sigma = r^2 + l^2\cos^2\theta$$

$$\Delta = (l + \frac{r^2}{l})^2 - 2mr + q^2$$

D. Klemm PRD 89 (2014) 048007

$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - l\sin^2\theta d\psi\right]^2 + \frac{\Sigma}{\Delta}dr^2 + \frac{\Sigma}{\sin^2\theta}d\theta^2 + \frac{\sin^4\theta}{\Sigma}\left[ldt - (r^2 + l^2)d\psi\right]^2$$

$$M = \frac{\mu m}{2\pi}, \quad J = Ml, \quad \Omega = \frac{l}{r_+^2 + l^2} \qquad A - \frac{qr}{\Sigma}\left(dt - l\sin^2\theta d\psi\right)$$

$$T = \frac{1}{4\pi r_+}\left(3\frac{r_+^2}{l^2} - 1 - \frac{q^2}{l^2 + r_+^2}\right)$$

$$S = \frac{\mu}{2}(l^2 + r_+^2) \quad \Phi = \frac{qr_+}{r_+^2 + l^2} \quad Q = \frac{\mu q}{2\pi}$$

chemical potential

$$dM = TdS + VdP + \Omega dJ + \Phi dQ + Kd\mu$$

$$V = \frac{r_+ A}{3} = \frac{2}{3}\mu r_+\left(r_+^2 + l^2\right) \quad K = \frac{(l^2 - r_+^2)[(r_+^2 + l^2)^2 + q^2 l^2]}{8\pi l^2 r_+(r_+^2 + l^2)}$$

Basic Properties

1) Both non-extremal and extremal cases exist

$$m \geq m_0 \equiv 2r_0\left(\frac{r_0^2}{l^2}+1\right) \qquad r_0^2 \equiv \frac{l^2}{3}\left[-1+\left(4+\frac{3q^2}{l^2}\right)^{\frac{1}{2}}\right]$$

2) No Closed Time-like curves

$$g_{\psi\psi} = \frac{l^4 \sin^4\theta}{l^2\cos^2\theta + r^2}\left(2mr - q^2\right) > 0 \text{ outside horizo}$$

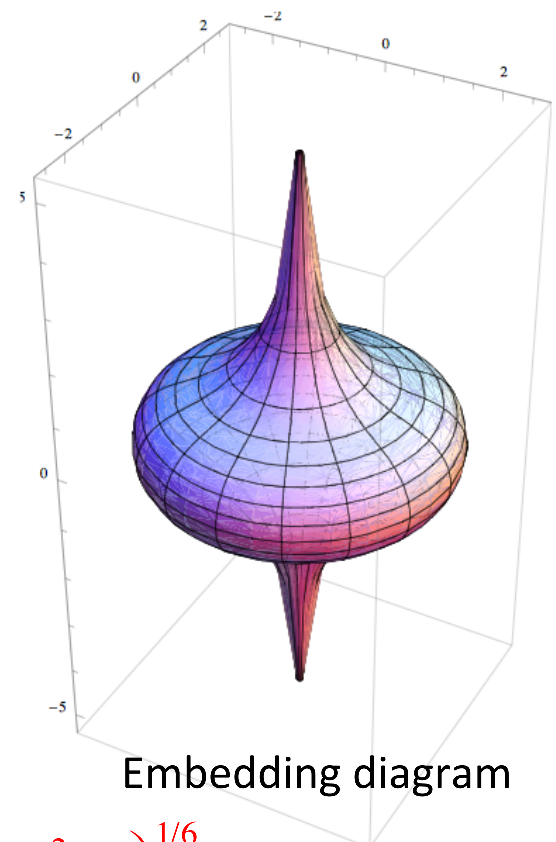
3) Horizon is non-compact

$$\kappa = l(1-\cos\theta)$$

$$ds_h^2 = (r_+^2 + l^2)\left[\frac{d\kappa^2}{4\kappa^2} + \frac{4\kappa^2}{l^2}d\psi^2\right]$$

4) Ergosphere

$$\Delta - l^2 \sin^4\theta \leq 0$$



Embedding diagram

5) Violate the Reverse Isoperimetric Inequality

$$R = \left(\frac{(D-1)V}{\omega_{D-2}}\right)^{1/D-1}\left(\frac{\omega_{D-2}}{A}\right)^{1/D-2} = \left(\frac{r_+ A}{2\mu}\right)^{1/3}\left(\frac{2\mu}{A}\right)^{1/2} = \left(\frac{r_+^2}{r_+^2+l^2}\right)^{1/6} < 1$$

Super-entropic!

Non-Compact Horizon?

Killing-Yano form

$$b=(l^2\cos^2\theta-r^2)dt-l(l^2\cos^2\theta-r^2\sin^2\theta)d\psi$$

Separate Hamilton-Jacobi equation

$$\frac{\partial \mathbf{S}}{\partial \lambda}+g^{ab}\frac{\partial \mathbf{S}}{\partial x^a}\frac{\partial \mathbf{S}}{\partial x^b}=0 \qquad \partial_a \mathbf{S}=u_a$$

Carter Constant

$$C=-\sigma r^2-\Delta R'^2$$
$$+\frac{1}{\Delta}\left[-(r^2+l^2)E+lh\right]^2$$
$$=\sin^2\theta\Lambda'^2+\sigma l^2\cos^2\theta$$
$$+\frac{1}{\sin^4\theta}\left[h-l\sin^2\theta E\right]^2$$

$$u_t=-E \quad u_\psi=h \quad \sigma_r=\pm \quad \sigma_\theta=\pm$$

$$\dot t=\frac{E(2mr-q^2)l^2}{\Sigma\Delta}+\frac{lh(\Delta-\sin^2\theta(r^2+l^2))}{\Sigma\Delta\sin^2\theta}$$

$$\dot\psi=\frac{h(\Delta-\sin^4\theta l^2)}{\Sigma\Delta\sin^4\theta}-\frac{lE(\Delta-\sin^2\theta(r^2+l^2))}{\Sigma\Delta\sin^2\theta}$$

$$\dot r=\frac{\sigma_r}{\Sigma}\sqrt{\left[lh-(r^2+l^2)E\right]^2-\Delta C-\sigma\Delta r^2}$$

$$\dot\theta=\frac{\sigma_\theta\sin\theta}{\Sigma}\sqrt{C-\frac{1}{\sin^4\theta}\left[h-l\sin^2\theta E\right]^2-\sigma l^2\cos^2\theta}$$

Near $\theta = 0, \pi$ $\qquad \dot{\theta} = -\frac{\sin\theta}{\Sigma}\sqrt{C - l^2 E^2} \qquad \dot{\psi} \neq 0$ Ingoing Null Geodesic

$\Rightarrow \quad \dot{\theta}/\theta \approx -b^2 = -\sqrt{C_* - l^2 E^2}/r_*^2 = \text{constant} \quad \Rightarrow \quad \theta \to e^{-b^2 \tau}$

Can show $\Rightarrow$ $t = -\frac{1}{k}\int \frac{d\theta}{\sin\theta} = -\frac{1}{k}\ln(\tan\frac{\theta}{2}) + \text{const.}$

Null Geodesics eternally spiral toward the poles

- Polar axes are not part of the spacetime
- Horizon has the topology of a sphere with 2 punctures
- Qualitatively new kind of black hole with finite area and non-compact horizon
- Super-entropic: for a given volume their entropy is *more* than that of Schwarzschild-AdS

Asymptotic Structure

$$ds^2 = -\frac{\Delta_a}{\Sigma_a}\left[dt - \frac{a\sin^2\theta}{\Xi}d\phi\right]^2 + \frac{\Sigma_a}{\Delta_a}dr^2 + \frac{\Sigma_a}{S}d\theta^2 + \frac{S\sin^2\theta}{\Sigma_a}\left[adt - \frac{r^2+a^2}{\Xi}d\phi\right]^2$$

$$A = -\frac{qr}{\Sigma_a}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)$$

1) $\psi = \phi + (1-x)\frac{at}{l^2}$ 2) $\psi \to \frac{\psi}{\Xi}$

3) $a \to l$ 4) Compactify $\psi \sim \psi + \mu$

$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - l\sin^2\theta d\psi\right]^2 + \frac{\Sigma}{\Delta}dr^2 + \frac{\Sigma}{\sin^2\theta}d\theta^2 + \frac{\sin^4\theta}{\Sigma}\left[ldt - (r^2+l^2)d\psi\right]^2$$

$$A - \frac{qr}{\Sigma}\left(dt - l\sin^2\theta d\psi\right)$$

$$ds^2_{\text{bdry}} = -dt^2 - 2l\sin^2\theta dt d\psi + \frac{l^2}{\sin^2\theta}d\theta^2$$

- Obtain same metric as before: all choices of rotating frame yield same limit
- Conformal boundary is AdS_3 : azimuthal coordinate becomes null

Higher-Dimensional Super-Entropic Black Holes

R. Hennigar, D. Kubiznak, N. Muskoe, R.B.M. JHEP **1506** (2015) 096

Begin with singly-spinning Kerr

$$ds^2 = -\frac{\Delta_a}{\rho_a^2}\left[dt - \frac{a}{\Xi}\sin^2\theta d\phi\right]^2 + \frac{\rho_a^2}{\Delta_a}dr^2 + \frac{\rho_a^2}{\Sigma_a}d\theta^2 + \frac{\Sigma_a \sin^2\theta}{\rho^2}\left[a dt - \frac{r^2+a^2}{\Xi}d\phi\right]^2$$

$$+ r^2\cos^2\theta d\Omega_{d-4}^2$$

$$\Delta_a = (r^2+a^2)(1+\frac{r^2}{l^2}) - 2mr^{5-d}$$

$$\rho_a^2 = r^2 + a^2\cos^2\theta \quad \Xi = 1 - \frac{a^2}{l^2} \quad \Sigma_a = 1 - \frac{a^2}{l^2}\cos^2\theta$$

1) $\psi = \phi / \Xi$ 2) $a \to l$

3) Compactify $\psi \sim \psi + \mu$

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - l\sin^2\theta d\psi)^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\sin^2\theta}d\theta^2$$

$$+\frac{\sin^4\theta}{\rho^2}[l dt - (r^2+l^2)d\psi]^2 + r^2\cos^2\theta d\Omega_{d-4}^2$$

$$\Delta = (l + \frac{r^2}{l})^2 - 2mr^{5-d}$$

$$\rho^2 = r^2 + l^2\cos^2\theta$$

Singly-Spinning d-dimensional SBH

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - l\sin^2\theta d\psi)^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\sin^2\theta}d\theta^2$$
$$+\frac{\sin^4\theta}{\rho^2}[ldt - (r^2+l^2)d\psi]^2 + r^2\cos^2\theta d\Omega_{d-4}^2$$

Same basic properties as the d=4 case

$$M = \frac{\omega_{d-2}}{8\pi}(d-2)m \qquad J = \frac{2}{d-2}Ml$$

$$\Omega = \frac{l}{r_+^2 + l^2} \qquad T = \frac{1}{4\pi r_+ l^2}[(d-5)l^2 + r_+^2(d-1)]$$

$$S = \frac{\omega_{d-2}}{4}(l^2 + r_+^2)r_+^{d-4} = \frac{A}{4} \qquad V = \frac{r_+ A}{d-1}$$

$$\mathcal{R} = \left(\frac{r_+ A}{\omega_{d-2}}\right)^{\frac{1}{d-1}}\left(\frac{\omega_{d-2}}{A}\right)^{\frac{1}{d-2}} = \left(\frac{r_+^2}{l^2 + r_+^2}\right)^{\frac{1}{(d-1)(d-2)}} < 1$$

Super-entropic!

SBH's in Minimal Gauged Supergravity

Begin with the general doubly-rotating charged solution

$$ds^2 = d\gamma^2 - \frac{2qv\omega}{\Sigma} + \frac{f\omega^2}{\Sigma^2} + \frac{\Sigma dr^2}{\Delta} + \frac{\Sigma d\theta^2}{S} \qquad A = \frac{\sqrt{3}q\omega}{\Sigma}$$

Chong/Cvetic/Lu/Pope, PRL **95** (2005) 161301

$$d\gamma^2 = -\frac{S\rho^2 dt^2}{\Xi_a \Xi_b l^2} + \frac{r^2+a^2}{\Xi_a}\sin^2\theta d\phi^2 + \frac{r^2+b^2}{\Xi_b}\cos^2\theta d\psi^2$$

$$v = b\sin^2\theta d\phi + a\cos^2\theta d\psi$$

$$\omega = \frac{Sdt}{\Xi_a\Xi_b} - a\sin^2\theta\frac{d\phi}{\Xi_a} - b\cos^2\theta\frac{d\psi}{\Xi_b}$$

$$S = \Xi_a\cos^2\theta + \Xi_b\sin^2\theta$$

$$\Delta = \frac{(r^2+a^2)(r^2+b^2)\rho^2/l^2 + q^2 + 2abq}{r^2} - 2m$$

$$\Sigma = r^2 + a^2\cos^2\theta + b^2\sin^2\theta, \quad \rho^2 = r^2 + l^2$$

$$\Xi_a = 1 - \frac{a^2}{l^2}, \quad \Xi_b = 1 - \frac{b^2}{l^2} \quad f = 2m\Sigma - q^2 + \frac{2abq}{l^2}\Sigma$$

1) $\phi = \phi_R + \frac{a}{l^2}t$

$\psi = \psi_R + \frac{b}{l^2}t$

2) $\varphi = \phi_R / \Xi_a$

3) $a \to l$

4) Compactify

$\varphi \sim \varphi + \mu$

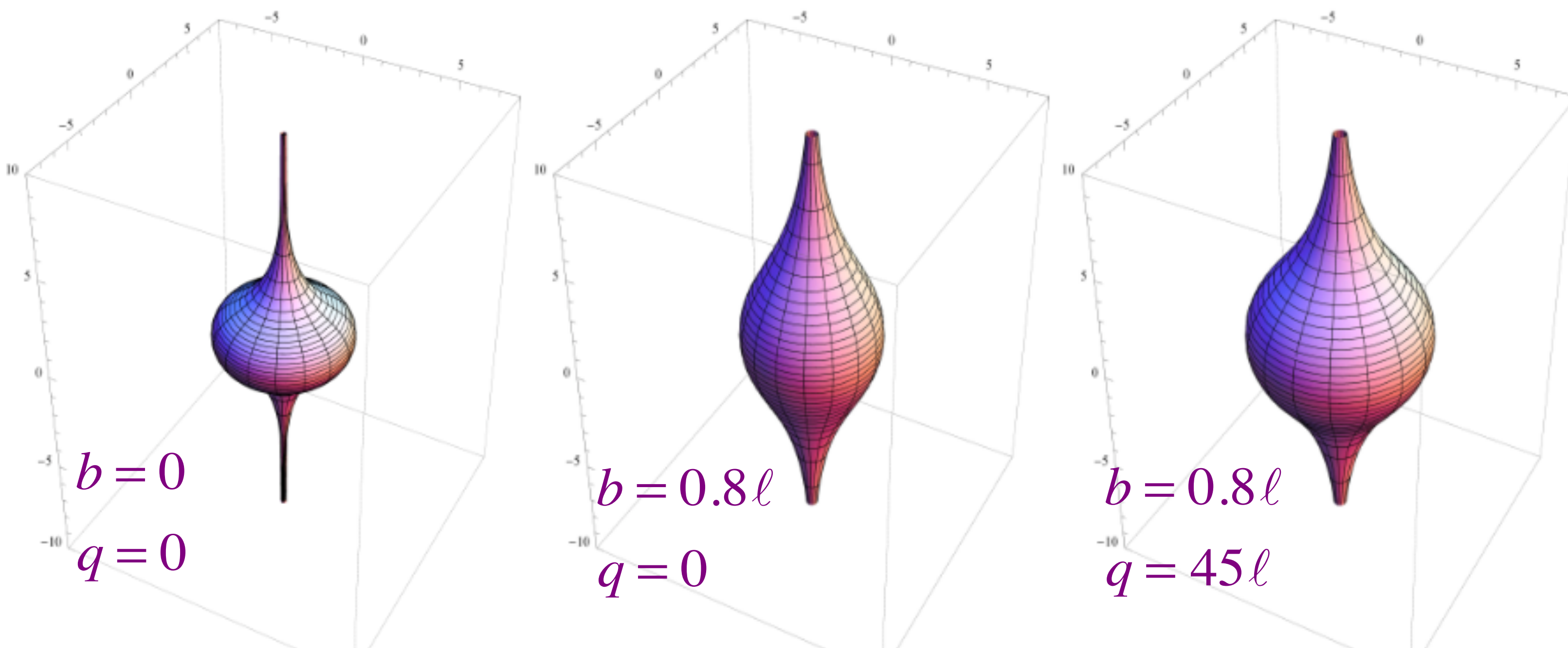
Doubly-Spinning Charged 5-dimensional SUGRA SBH

$$ds^2 = d\gamma_s^2 - \frac{2qv_s\omega_s}{\Sigma} + \frac{f\omega_s^2}{\Sigma^2} + \frac{\Sigma dr^2}{\Delta} + \frac{\Sigma d\theta^2}{\Xi_b \sin^2\theta} \qquad A = \frac{\sqrt{3}q\omega_s}{\Sigma}$$

$$\Delta = \frac{\rho^4(r^2+b^2)/l^2 + q^2 + 2lbq}{r^2} - 2m \qquad \Xi_b = 1 - \frac{b^2}{l^2}$$

$$f = 2m\Sigma - q^2 + \frac{2bq}{l}\Sigma \qquad \Sigma = r^2 + l^2\cos^2\theta + b^2\sin^2\theta$$

Note: Only 1 super-entropic limit!



Thermodynamics of the SUGRA SBH

$$M=\frac{\mu}{8}\frac{(m+bq/l)(2+\Xi_b)}{\Xi_b^2}\qquad \Phi=\frac{\sqrt{3}qr_+^2}{(b^2+r_+^2)\rho_+^2+blq}\qquad Q=\frac{\mu\sqrt{3}q}{8\Xi_b}$$

$$J_\varphi=\frac{\mu}{4}\frac{lm+bq}{\Xi_b}\qquad J_\psi=\frac{\mu}{8}\frac{2bm+q(b^2+l^2)/l}{\Xi_b^2}$$

$$\Omega_\varphi=\frac{l(b^2+r_+^2)+bq}{\rho_+^2(b^2+r_+^2)+lbq},\qquad \Omega_\psi=\frac{b\rho_+^4/l^2+ql}{\rho_+^2(b^2+r_+^2)+lbq}$$

$$T=\frac{r_+^4\left[2+(2r_+^2+b^2)/l^2\right]-(bl+q)^2}{2\pi r_+\left[\rho_+^2(b^2+r_+^2)+lbq\right]}\qquad S=\frac{\mu\pi\left[(b^2+r_+^2)\rho_+^2+blq\right]}{4r_+\Xi_b}=\frac{A}{4}$$

TD Volume

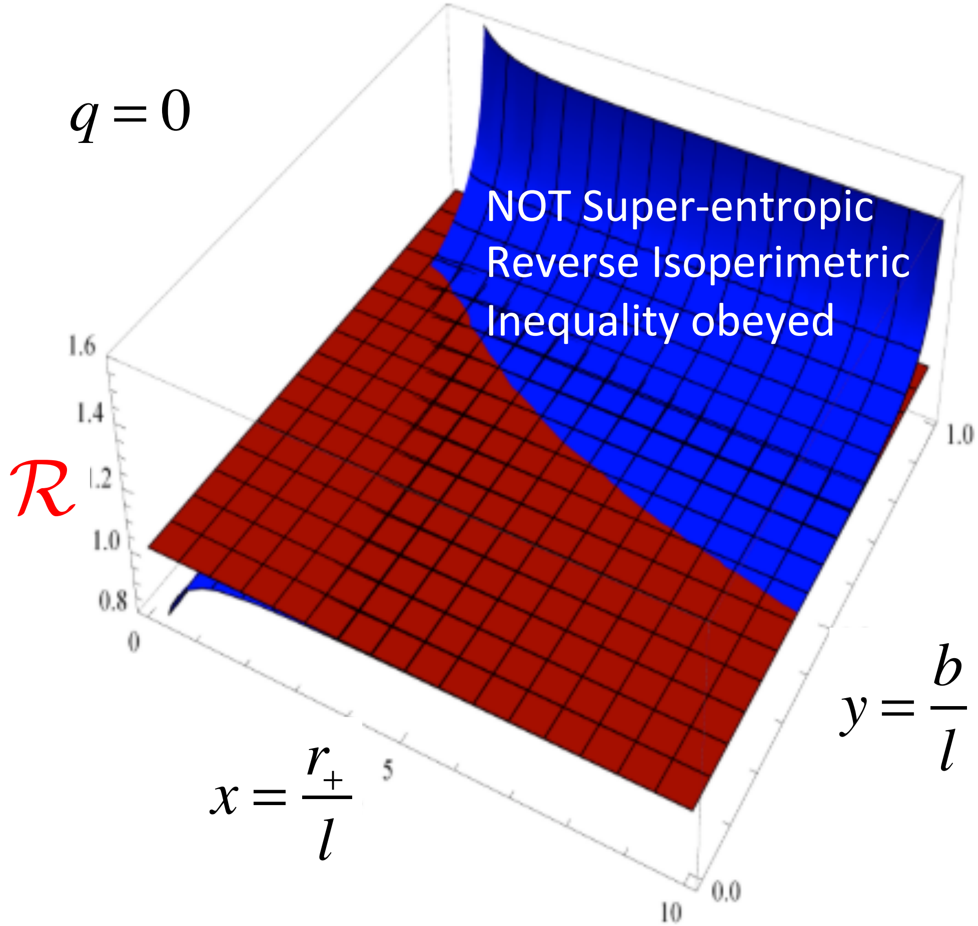
$$V=\frac{\mu\pi}{12r_+^2l^2\Xi_b^2}([(b^2+3r_+^2)l^2-2b^2r_+^2]\rho_+^2(b^2+r_+^2)$$

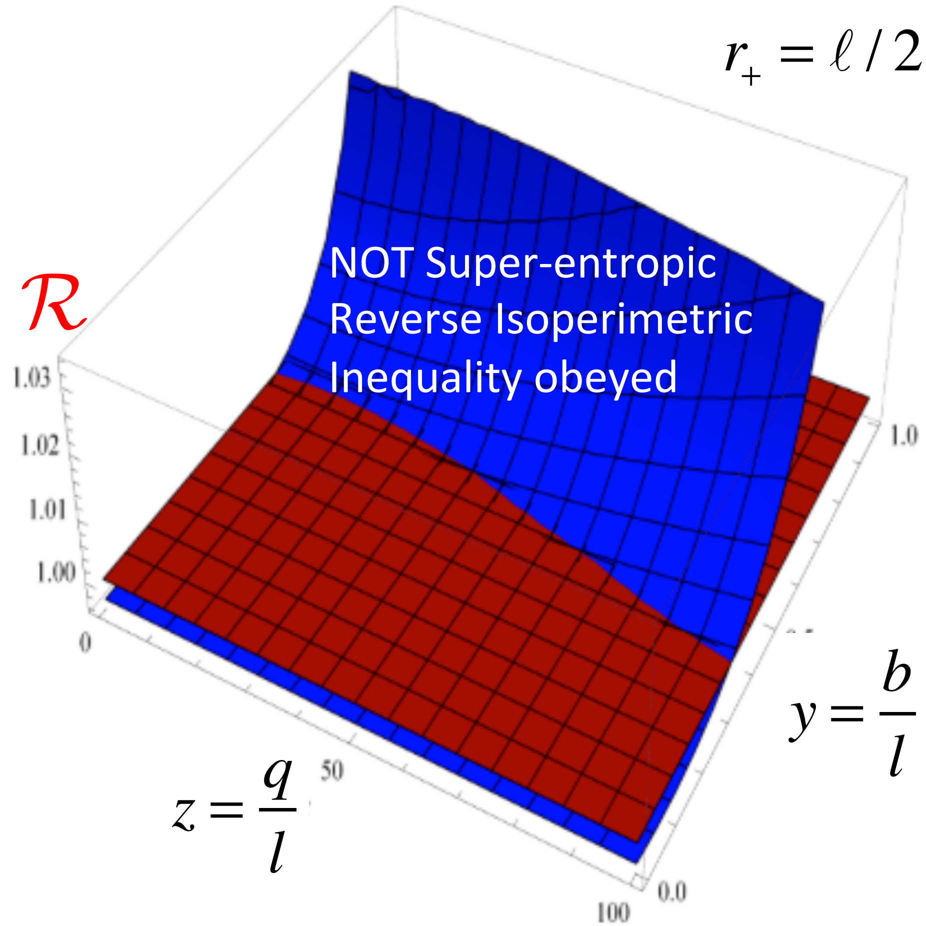
$$+qbl\left[(2b^2+3r_+^2)l^2+lbq-b^2r_+^2\right]$$

Super-entropic?

Sometimes Super-entropic SUGRA Black Holes

$$\mathcal{R}^{12}_{q=0} = \left(\frac{1}{27}\right)\frac{(3x^2+y^2-2x^2y^2)^3}{x^2(1-y^2)^2(x^2+1)(x^2+y^2)}$$





Larger orthogonal spin removes the effect

Multispinning Super-Entropic Black Holes

Metric for Multiply rotating Black Hole

$$ds^2 = d\gamma^2 + \frac{2m}{U}\omega^2 + \frac{Udr^2}{F-2m} + d\Omega^2$$

$$d = 2N + 1 + \varepsilon$$

$$\sum_{i=1}^{N+\varepsilon} \mu_i^2 = 1$$

$$d\gamma^2 = -\frac{W\rho^2}{l^2}dt^2 + \sum_{i=1}^{N} \frac{r^2 + a_i^2}{\Xi_i}\mu_i^2 d\phi_i^2$$

$$\rho^2 = r^2 + l^2 \qquad F = \frac{r^{\varepsilon-2}\rho^2}{l^2}\prod_{i=1}^{N}(r^2 + a_i^2)$$

$$d\Omega^2 = \sum_{i=1}^{N+\varepsilon} \frac{r^2 + a_i^2}{\Xi_i}d\mu_i^2 - \frac{1}{W\rho^2}\left(\sum_{i=1}^{N+\varepsilon} \frac{r^2 + a_i^2}{\Xi_i}\mu_i d\mu_i\right)^2$$

$$\Xi_i = 1 - \frac{a_i^2}{l^2} \qquad W = \sum_{i=1}^{N+\varepsilon} \frac{\mu_i^2}{\Xi_i}$$

$$\omega = Wdt - \sum_{i=1}^{N} \frac{a_i \mu_i^2 d\phi_i}{\Xi_i}$$

$$U = r^{\varepsilon}\sum_{i=1}^{N+\varepsilon} \frac{\mu_i^2}{r^2 + a_i^2}\prod_{j}^{N}(r^2 + a_j^2)$$

1) $\phi_j = \phi_j + \frac{a_j}{l^2}t$ 2) $\varphi_j = \phi_j / \Xi_j$

3) $a_j \to l$ 4) Compactify $\varphi_j \sim \varphi_j + \mu$

$$ds^2 = d\gamma_s^2 + \frac{2m}{U}\omega_s^2 + \frac{U dr^2}{F - 2m} + d\Omega_s^2$$

$$\rho^2 = r^2 + l^2 \quad F = \frac{r^{\varepsilon-2}\rho^4}{l^2}\prod_{i\neq j}^{N}(r^2 + a_i^2) \quad \Xi_i = 1 - \frac{a_i^2}{l^2} \text{ for } i \neq j$$

$$d\Omega_s^2 = \sum_{i\neq j}\frac{r^2 + a_i^2}{\Xi_i}d\mu_i^2 - 2\frac{d\mu_j}{\mu_j}\left(\sum_{i\neq j}^{N+\varepsilon}\frac{r^2 + a_i^2}{\Xi_i}\mu_i d\mu_i\right) + \frac{d\mu_j^2}{\mu_j^2}(\rho^2\hat{W} + l^2\mu_j^2)$$

$$\hat{W} = \sum_{i\neq j}^{N+\varepsilon}\frac{\mu_i^2}{\Xi_i} \qquad U = r^{\varepsilon}\left(\mu_j^2 + \sum_{i\neq j}\frac{\mu_i^2\rho^2}{r^2 + a_i^2}\right)\prod_{k\neq j}^{N}(r^2 + a_k^2)$$

- Killing-Yano tensor exists → complete integrability of geodesic motion
- Non-compact horizon with finite area
- Sometimes super-entropic

$$V = \frac{r_+ A}{d-1} + \frac{8\pi}{(d-1)(d-2)}\sum_{i\neq j} a_i J_i$$

Super-entropic for small values of the rotation parameters

SBHs are a New Ultra-spinning Limit

Multiply rotating Black Hole

$$ds^2 = d\gamma^2 + \frac{2m}{U}\omega^2 + \frac{Udr^2}{F-2m} + d\Omega^2$$

$$t = \epsilon^2 \hat{t} \quad r = \epsilon^2 \hat{r} \quad \mu_j = \epsilon^{(d-1)/2}\sigma / l$$

$$\epsilon = \Xi_j^{1/(d-5)} \to 0 \quad \text{as} \quad a_j \to l$$

$$ds^2 = -d\hat{t}^2 + \frac{2\hat{m}}{\hat{U}}(d\hat{t} - \sum_{i\neq j}^{N} \hat{a}_i \mu_i^2 d\phi_i)^2$$

$$+ \frac{\hat{U}d\hat{r}^2}{\hat{F} - 2\hat{m}} + d\sigma^2 + \sigma^2 d\varphi^2$$

$$+ \sum_{i\neq j}^{N+\varepsilon} (\hat{r}^2 + \hat{a}_i^2) d\mu_i^2 + \sum_{i\neq j}^{N} (\hat{r}^2 + \hat{a}_i^2)\mu_i^2 d\phi_i^2$$

$$\sum_{i\neq j}^{N+\varepsilon} \mu_i^2 = 1$$

$$\hat{F} = \hat{r}^{\varepsilon-2} \prod_{i\neq j}^{N} (\hat{r}^2 + \hat{a}_i^2)$$

$$\hat{U} = \hat{r}^{\varepsilon} \left(\sum_{i\neq j}^{N+\varepsilon} \frac{\mu_i^2}{\hat{r}^2 + \hat{a}_i^2} \right) \prod_{k\neq j}^{N} (\hat{r}^2 + \hat{a}_k^2)$$

Standard ultra-spinning black brane limit

- Neither the same as nor compatible with the super-entropic limit

Summary

- A new class of ultra-spinning black holes has been obtained
- These black holes are super-entropic: they have more entropy than their thermodynamic volume naively permits
- Open questions
  - Stability of SBHs?
  - Compatibility with other limits of rotating BHs?
  - Phase transitions? New phenomena?
  - Meaning for AdS/CFT?