

Ab Initio Holography

Sung-Sik Lee

McMaster University

Perimeter Institute

Can one find a **constructive proof** of the
AdS/CFT correspondence, which will
allows one to find holographic duals for
general QFTs?

What is behind AdS/CFT?

$$\text{RG} \approx \text{GR}$$

- Radial direction in the bulk = length scale of QFT
- Bulk variables : scale dependent coupling functions
- Radial evolution of the bulk fields correspond to the RG flow

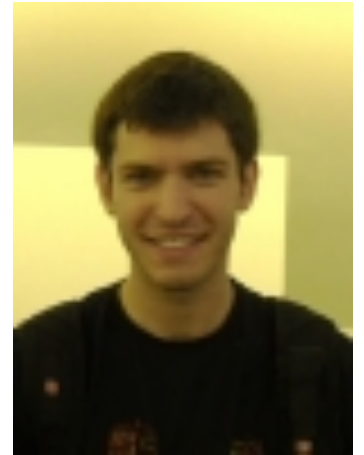
However, the connection between **RG** and **GR** is incomplete

RG	GR
RG flow is classical : Given initial condition, coupling functions are deterministic without uncertainty	Bulk variables have quantum fluctuations

One has to promote RG to **quantum RG**

Outline

- Construct a holographic dual for a concrete lattice model [$U(N)$ vector mode] via quantum renormalization group
- Characterize different phases from the emergent geometries in the bulk



Based on arXiv : 1503.06474

Peter Lunts, S. Bhattacharjee, J. Miller, E. Schnetter, Y.B. Kim, SL

Related references

General Connection between holography and RG

- E. T. Akhmedov, Phys. Lett. B 442 (1998) 152
- J. de Boer, E. Verlinde and H. Verlinde, J. High Energy Phys. 08, 003 (2000)
- S. R. Das and A. Jevicki, Phys. Rev. D 68 (2003) 044011.
- R. Gopakumar, Phys. Rev. D 70 (2004) 025009; *ibid.* 70 (2004) 025010.
- I. Heemskerk, J. Penedones, J. Polchinski and J. Sully, J. High Energy Phys. 10 (2009) 079.
- I. Heemskerk and J. Polchinski, arXiv:1010.1264
- T. Faulkner, H. Liu and M. Rangamani, arXiv:1010.4036.

Holographic duals for vector models

- S. R. Das and A. Jevicki, Phys. Rev. D 68 (2003) 044011.
- R. Koch, A. Jevicki, K. Jin and J. P. Rodrigues, arXiv:1008.0633.
- M. Douglas, L. Mazzucato, and S. Razamat, Phys. Rev. D 83 (2011) 071701.
- R. Leigh, O. Parrikar, A. Weiss, arXiv:1402.1430
- E. Mintun and J. Polchinski, arXiv:1411.3151

Vector model

$$\mathcal{S} = \int d^D x \left[|\nabla \vec{\phi}|^2 + m^2 |\vec{\phi}|^2 + \frac{\lambda}{N} (|\vec{\phi}|^2)^2 \right]$$

- Exact critical exponents in the large N limit
- But, still no exact Wilsonian RG
- Believed to be dual to Vasiliev's higher-spin gauge theory [Sezgin-Sundell, Polyakov-Klebanov, Giombi-Yin, ..]

Lattice regularization

U(N) vector model for N complex bosons

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} (\phi_i^* \cdot \phi_j)$$

i,j : 3-dimensional Euclidean lattice

$$\phi_i = (\phi_i^1, \phi_i^2, \dots, \phi_i^N)$$

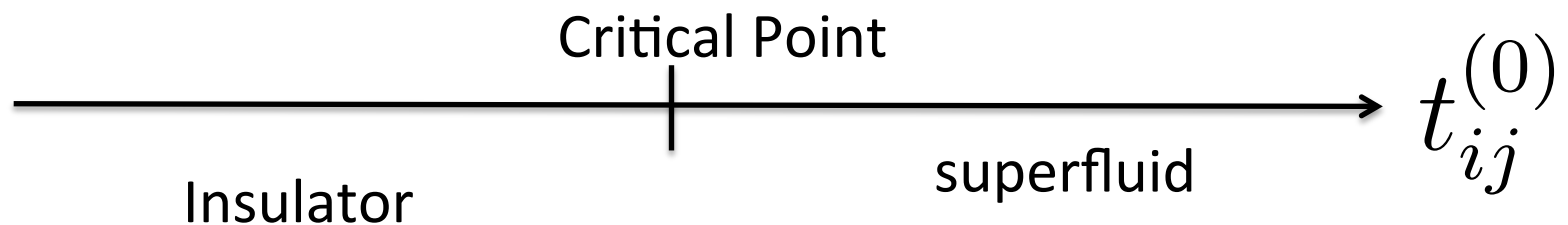
Two phases and one critical point

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} (\phi_i^* \cdot \phi_j)$$

On-site

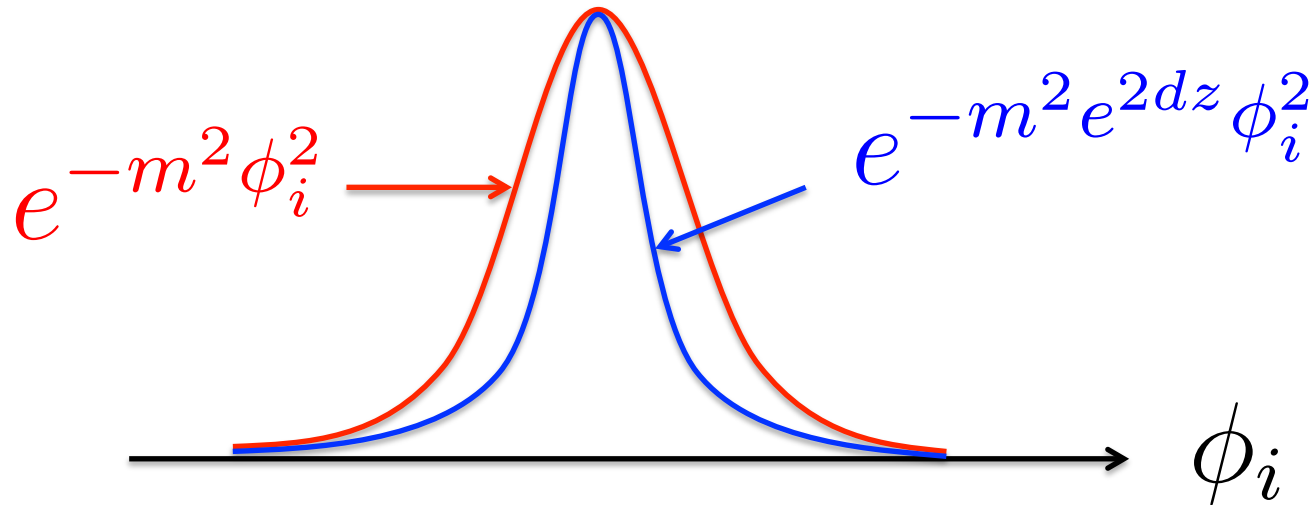
Hopping

- The on-site term describes the insulating fixed point
- Hopping term is treated as deformation to the fixed point
- If the hopping is large, it flows to superfluid
- Although the quadratic term is enough to describe the Insulating fixed point, quartic term is needed to describe the phase transition



Coarse Graining in Real Space

$$\begin{aligned} Z &= \int D\phi \, e^{-S[\phi; m^2, t_{ij}, \lambda]} \\ &= \int D\phi \, e^{-S[\phi; m^2 e^{2dz}, t_{ij} + \delta t_{ij}, \lambda + \delta \lambda] - \delta S} \end{aligned}$$



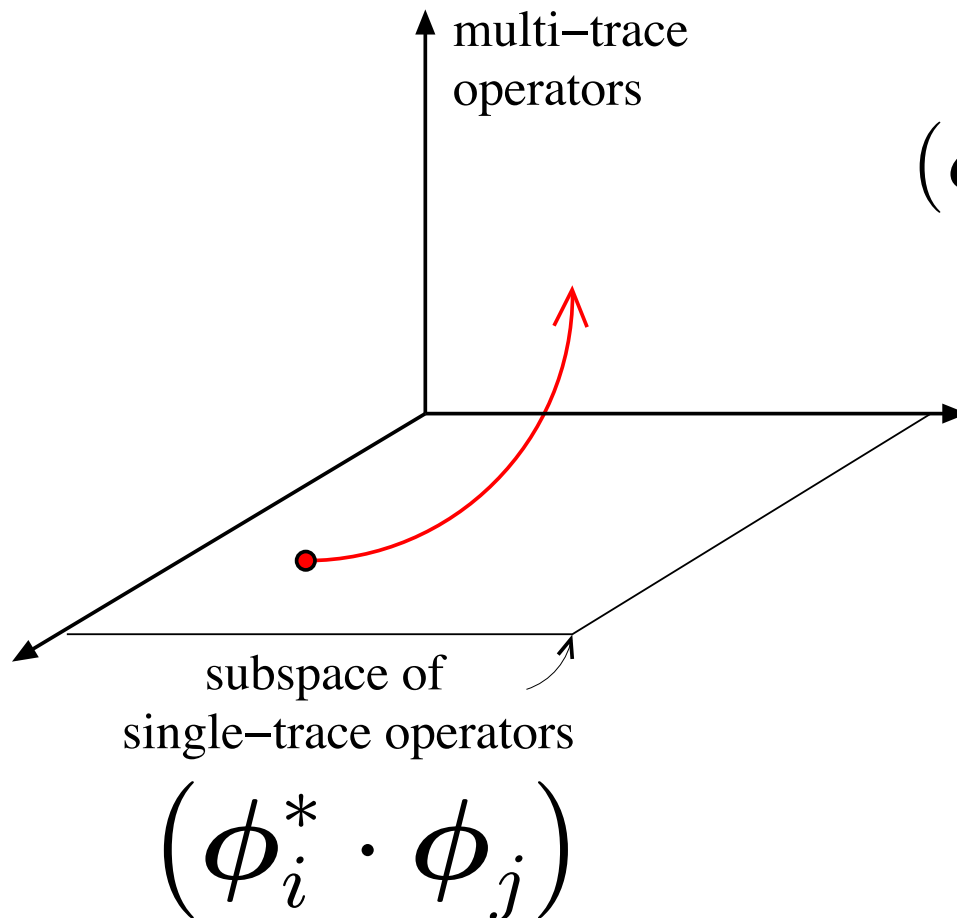
Renormalized Action

$$\begin{aligned}
 \tilde{\mathcal{S}}_1 = & 2Ndz \left\{ -\frac{1}{m^2} \sum_i t_{ii}^{(0)} \right\} \\
 & + 2dz \left\{ \frac{2\lambda \left(1 + \frac{1}{N}\right)}{m^2} \sum_i (\phi_i^* \cdot \phi_i) - \frac{4\lambda^2}{m^2 N^2} \sum_i (\phi_i^* \cdot \phi_i)^3 \right\} \\
 & + 2dz \left\{ \frac{2\lambda}{m^2 N} \sum_{ij} t_{ij}^{(0)} (\phi_i^* \cdot \phi_j) \{ (\phi_i^* \cdot \phi_i) + (\phi_j^* \cdot \phi_j) \} \right\} \\
 & + 2dz \left\{ -\frac{1}{m^2} \sum_{ijk} t_{ik}^{(0)} t_{kj}^{(0)} (\phi_i^* \cdot \phi_j) - 2\frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 + \sum_{ij} t_{ij}^{(0)} (\phi_i^* \cdot \phi_j) \right\} \\
 & - \sum_{ij} t_{ij}^{(0)} (\phi_i^* \cdot \phi_j) + \boxed{m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2}
 \end{aligned}$$

deformation

On-site (reference action)

Conventional RG



$$(\phi_i^* \cdot \phi_i)^n ,$$

$$(\phi_i^* \cdot \phi_i)^n (\phi_i^* \cdot \phi_j) , \dots$$

- Multi-trace deformations are generated

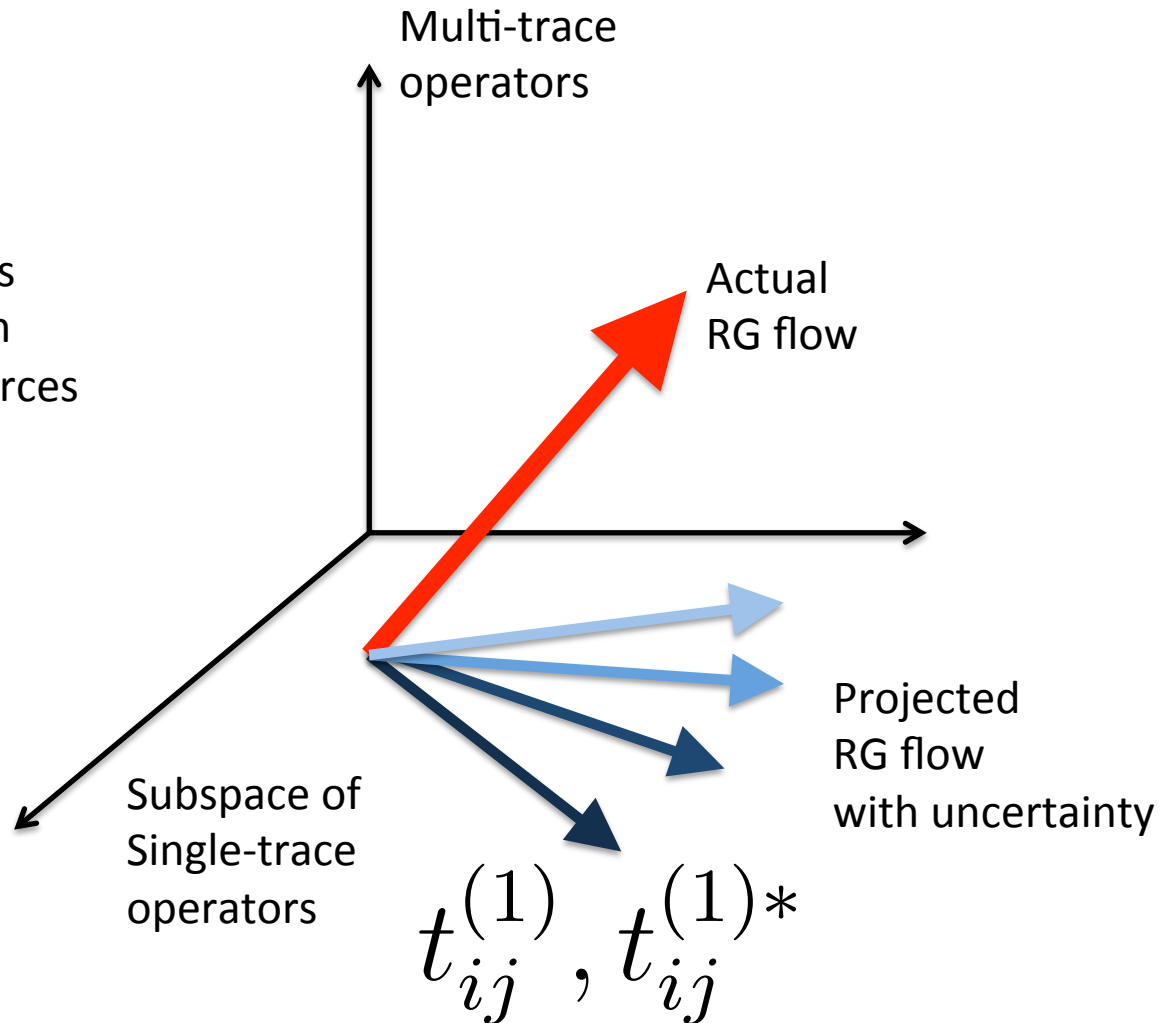
Multi-trace operator =
Single-trace operator
with dynamical sources

$$\mathcal{Z} = \int \mathcal{D}\phi \mathcal{D}\phi^* \boxed{\mathcal{D}t_{ij}^{(1)} \mathcal{D}t_{ij}^{*(1)}} e^{-\mathcal{S}_2},$$

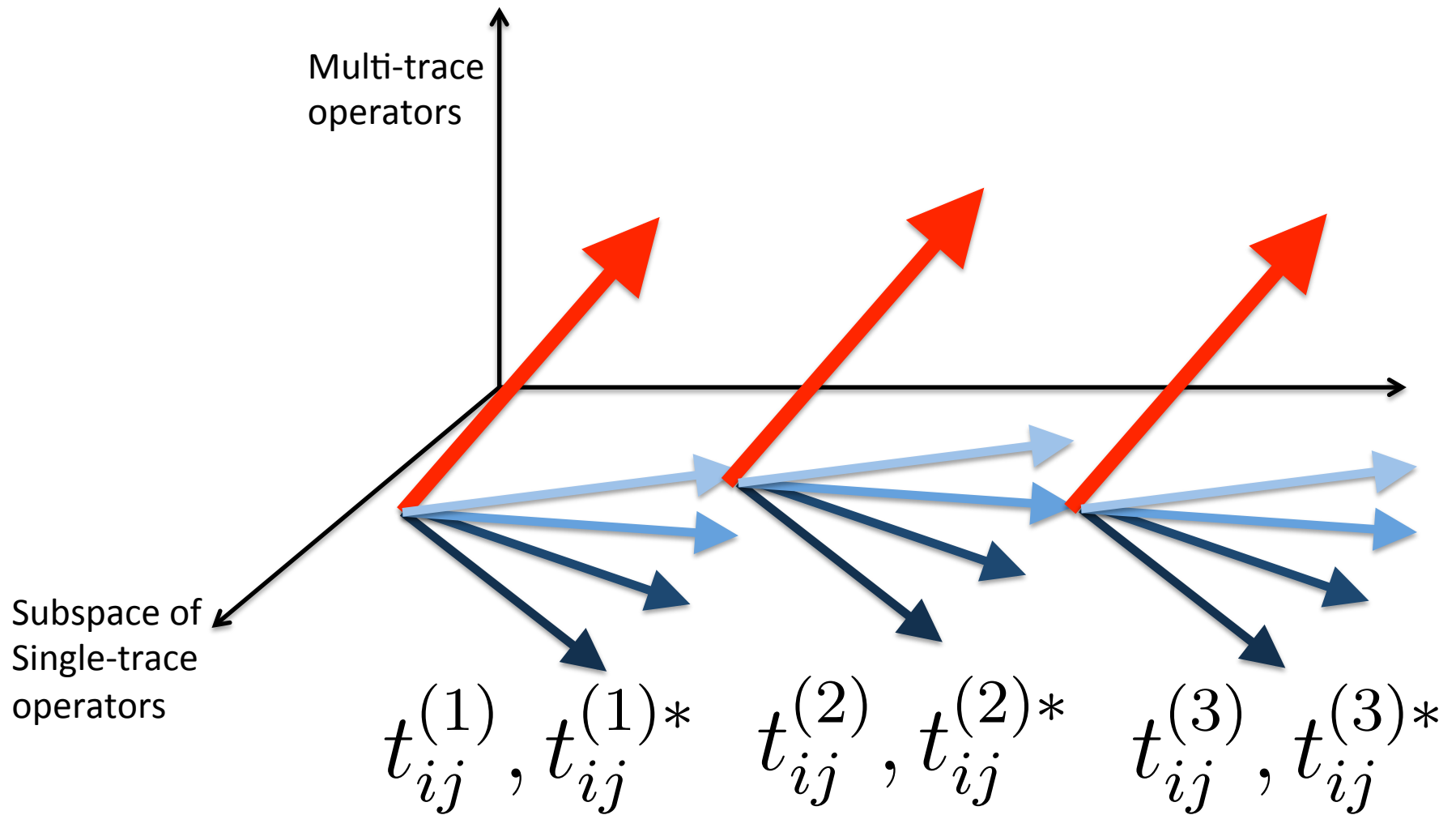
$$\begin{aligned} \mathcal{S}_2 = & N \left\{ - \sum_{ij} \left[(t_{ij}^{(0)} - t_{ij}^{(1)}) t_{ij}^{*(1)} \right] \right\} \\ & + 2N\alpha dz \left\{ -\frac{1}{m^2} \sum_i t_{ii}^{(0)} + \frac{2\lambda \left(1 + \frac{1}{N}\right)}{m^2} \sum_i t_{ii}^{*(1)} - \frac{4\lambda^2}{m^2} \sum_i \left(t_{ii}^{*(1)}\right)^3 \right\} \\ & + 2N\alpha dz \left\{ \frac{2\lambda}{m^2} \sum_{ij} t_{ij}^{(0)} t_{ij}^{*(1)} \left[t_{ii}^{*(1)} + t_{jj}^{*(1)} \right] - \frac{1}{m^2} \sum_{ijk} t_{ik}^{(0)} t_{kj}^{(0)} t_{ij}^{*(1)} - 2\lambda \sum_i \left(t_{ii}^{*(1)}\right)^2 + \sum_{ij} t_{ij}^{(0)} t_{ij}^{*(1)} \right\} \\ & \boxed{- \sum_{ij} t_{ij}^{(1)} (\phi_i^* \cdot \phi_j) + m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2} \end{aligned}$$

Projection creates uncertainty

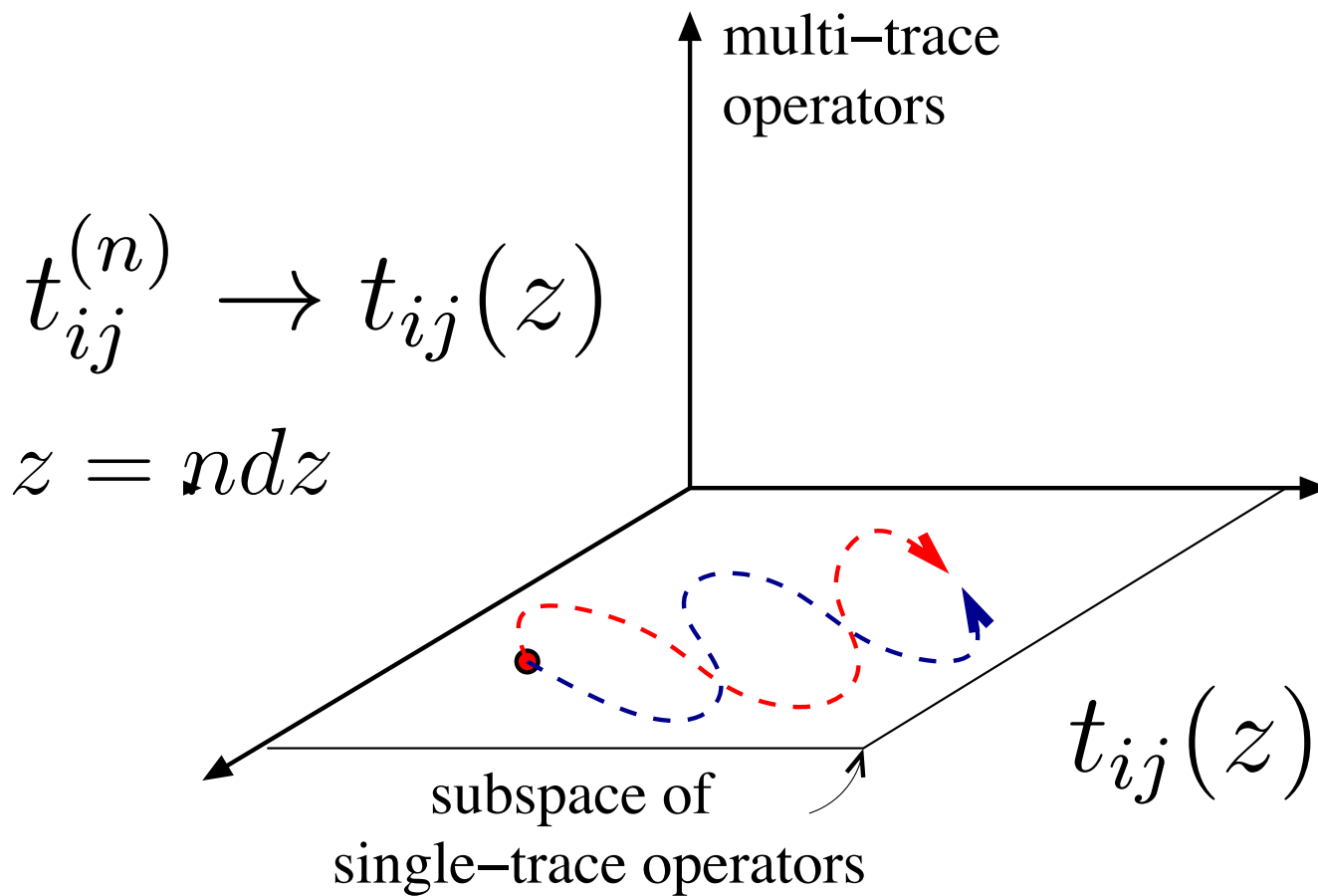
- Theory with multi-trace operators can be mapped into a theory with single-trace operators whose sources are dynamical



Quantum RG



Sum over all RG paths in the space of hoppings

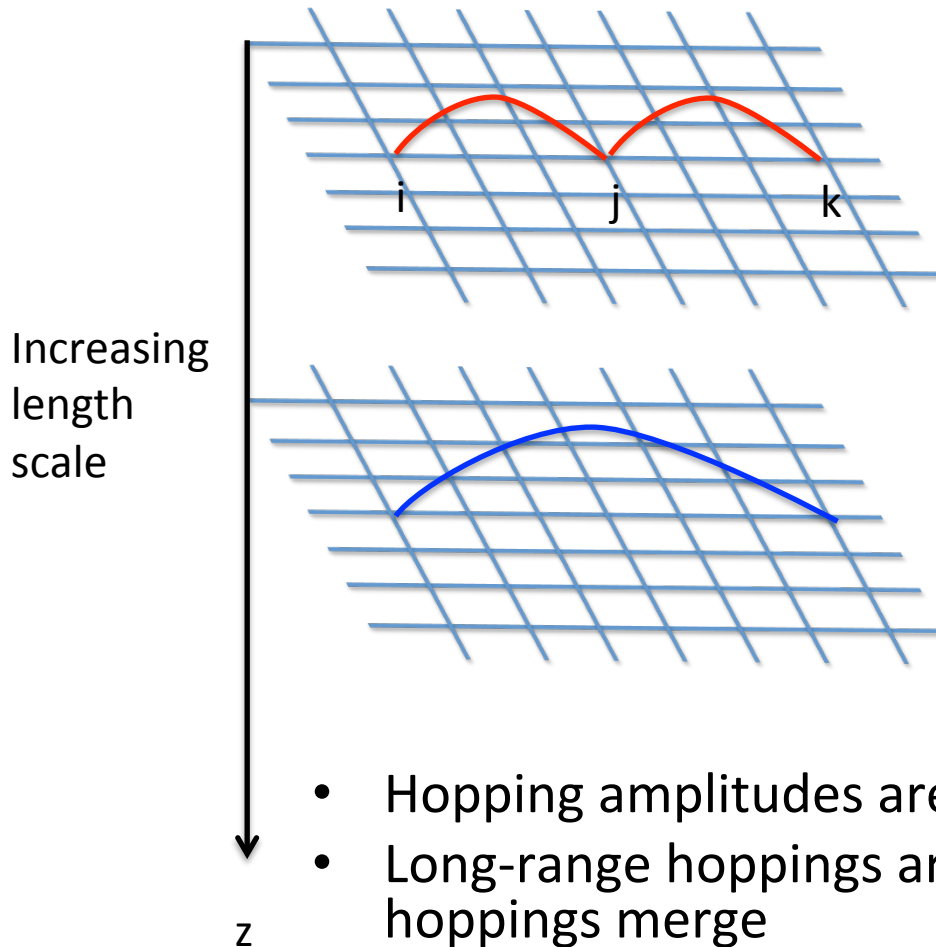


Weight for each RG path : Bulk Action

$$Z = \int Dt_{ij}(z)Dt_{ij}^*(z)e^{-NS_{bulk}[t,t^*]} \Big|_{t_{ij}(0)=t_{ij}^{(0)}}$$

$$\begin{aligned} \mathcal{S}_{Bulk} = & \int_0^\infty dz \left\{ \sum_{ij} t_{ij}^*(z) \partial_z t_{ij}(z) \right. \\ & + \sum_i \left[-\frac{2}{m^2} t_{ii}(z) + \frac{4\lambda \left(1 + \frac{1}{N}\right)}{m^2} t_{ii}^*(z) - 4\lambda (t_{ii}^*(z))^2 - \frac{8\lambda^2}{m^2} (t_{ii}^*(z))^3 \right] \\ & \left. + \sum_{ij} \left[2t_{ij}(z) t_{ij}^*(z) + \frac{4\lambda}{m^2} t_{ij}(z) t_{ij}^*(z) (t_{ii}^*(z) + t_{jj}^*(z)) \right] - \frac{2}{m^2} \sum_{ijk} [t_{ik}(z) t_{kj}(z) t_{ij}^*(z)] \right\} \end{aligned}$$

Bulk Geometry is quantum and dynamical



$$t_{ik}^\dagger t_{ij} t_{jk}$$

- Hopping amplitudes are promoted to quantum operators
- Long-range hoppings are generated as short-range hoppings merge
- Hopping fields determine the connectivity (geometry) of the bulk

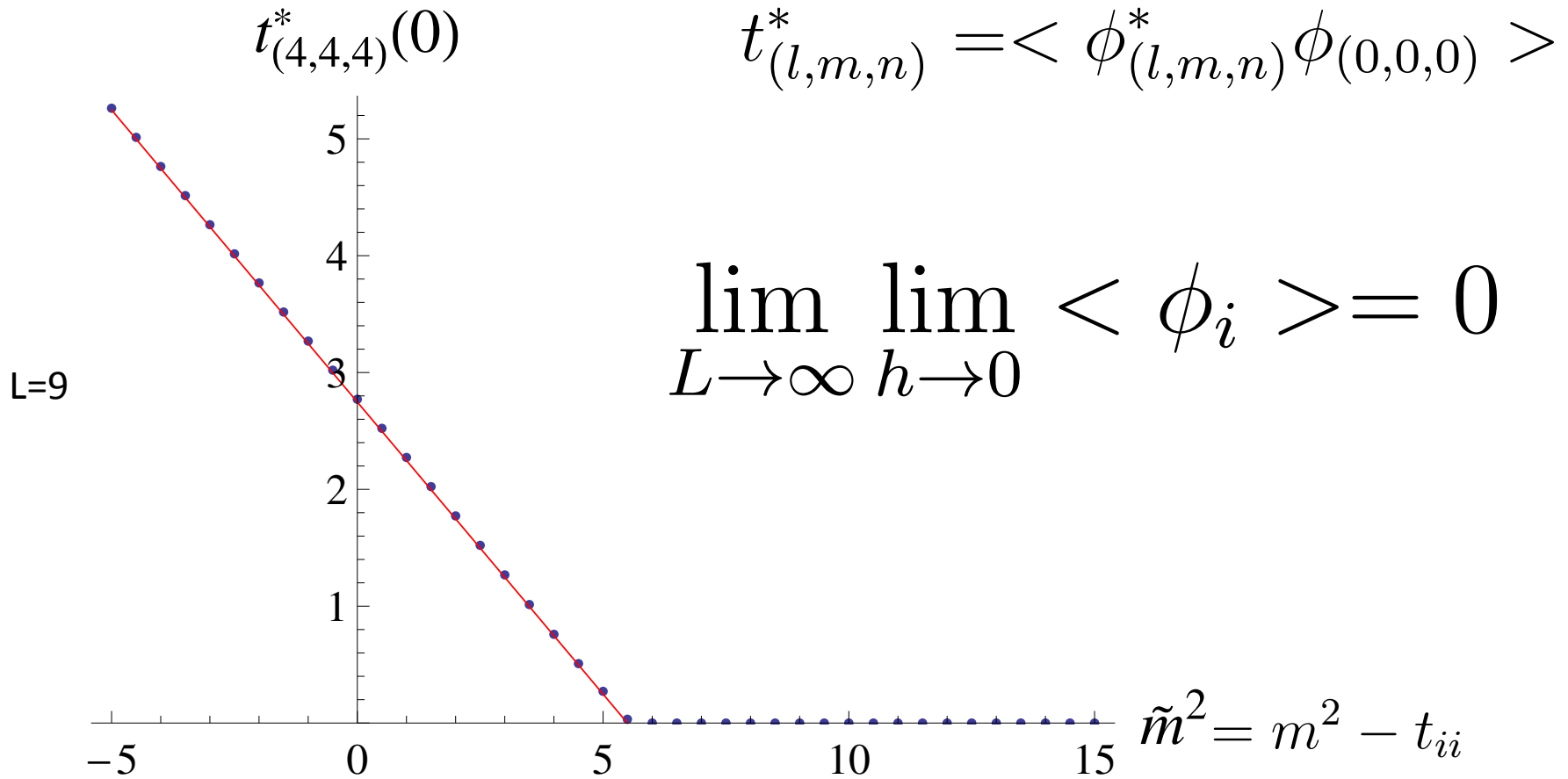
Saddle point approximation

$$\begin{aligned}\partial_z t_{ij} &= -2 \left\{ \frac{2\lambda \delta_{ij}}{m^2} - \delta_{ij} \left[4\lambda + \frac{12\lambda^2}{m^2} t_{ii}^* \right] t_{ii}^* + \frac{2\lambda \delta_{ij}}{m^2} \sum_k (t_{ik} t_{ik}^* + t_{ki} t_{ki}^*) \right. \\ &\quad \left. + \left[1 + \frac{2\lambda}{m^2} (t_{ii}^* + t_{jj}^*) \right] t_{ij} - \frac{1}{m^2} \sum_k t_{ik} t_{kj} \right\} \\ \partial_z t_{ij}^* &= 2 \left\{ -\frac{\delta_{ij}}{m^2} + \left[1 + \frac{2\lambda}{m^2} (t_{ii}^* + t_{jj}^*) \right] t_{ij}^* - \frac{1}{m^2} \sum_k (t_{ik}^* t_{jk} + t_{ki} t_{kj}^*) \right\}\end{aligned}$$

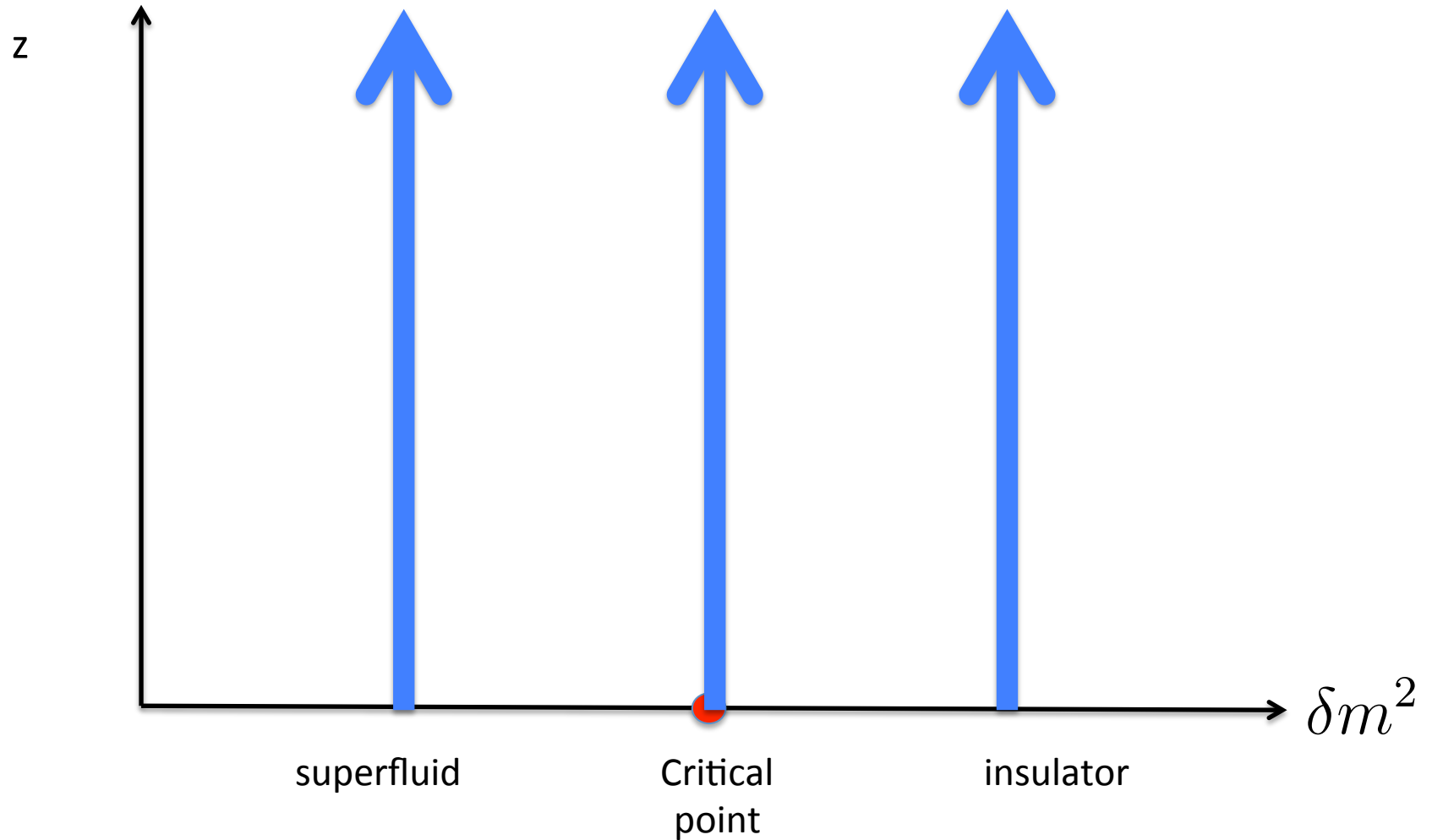
- In the large N limit, semi-classical RG path dominates the partition function
- At the saddle point t_{ij} , t_{ij}^* take independent values

$$t_{ij}^* = \frac{1}{N} \langle \phi_i^* \phi_j \rangle$$

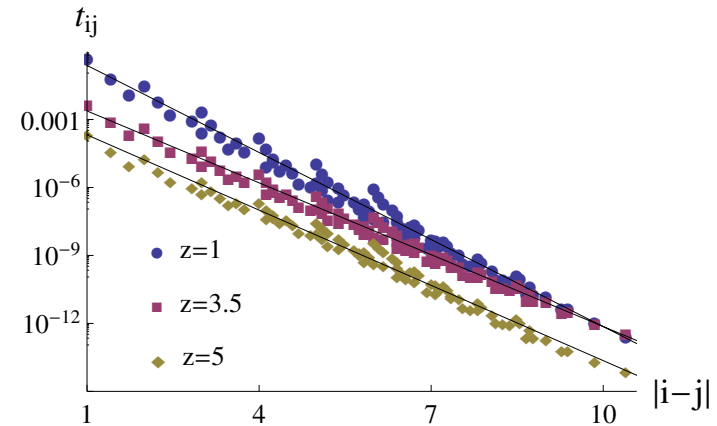
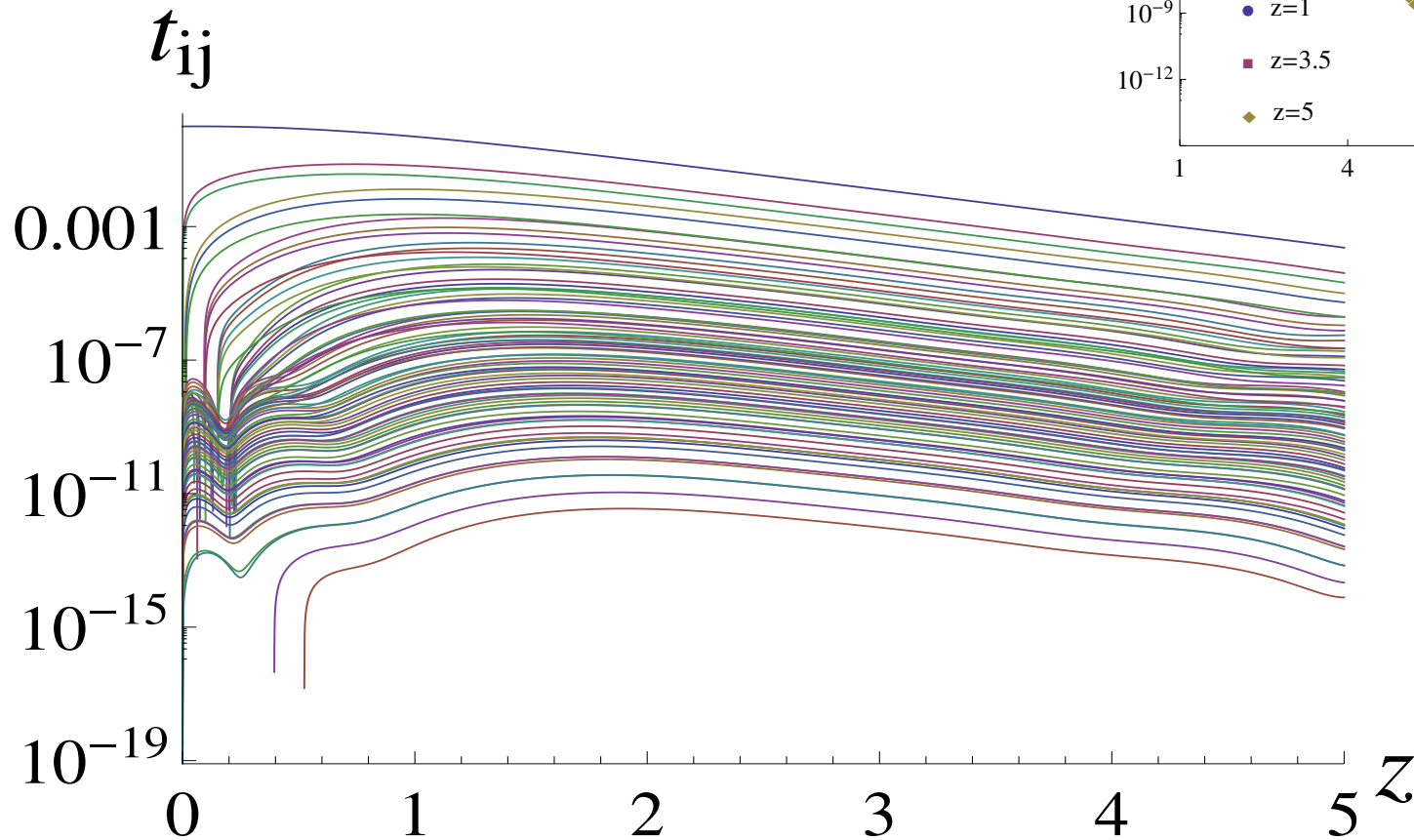
Field theory observable at the UV boundary



Bulk fields

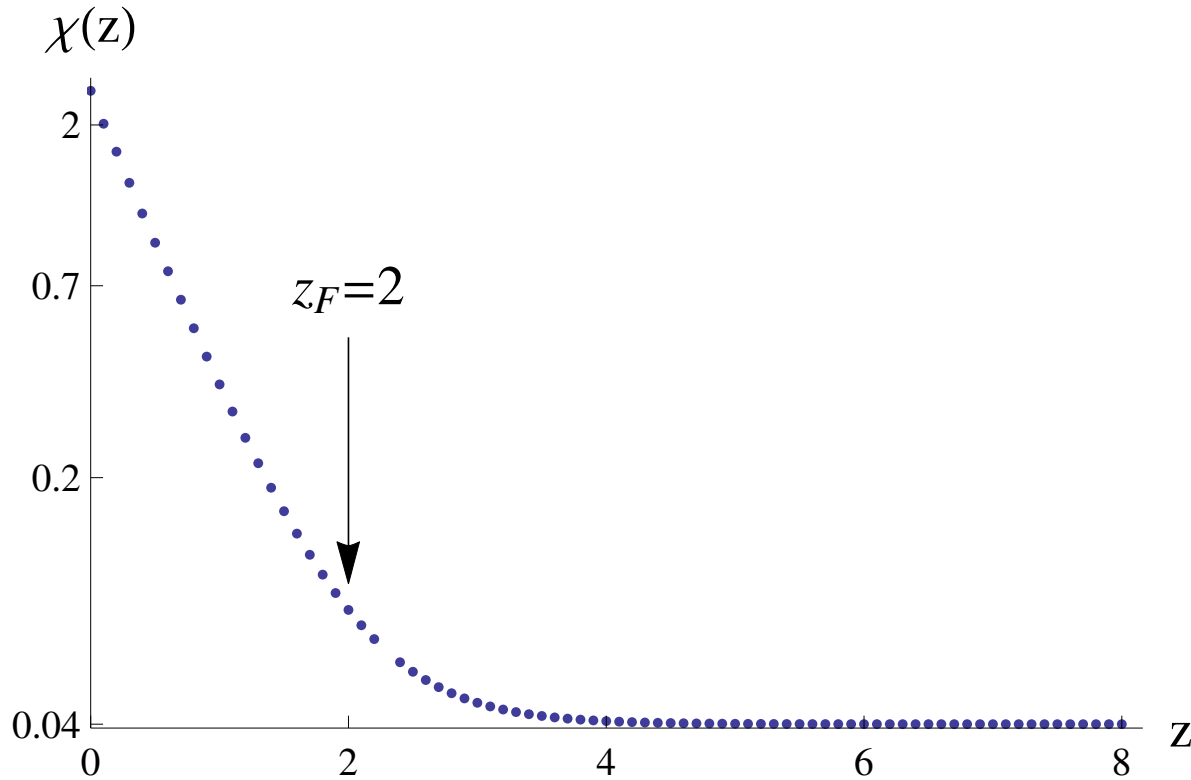


Insulating Phase



- $t_{ij}(z)$ decay exponentially both in $|i-j|$ and z

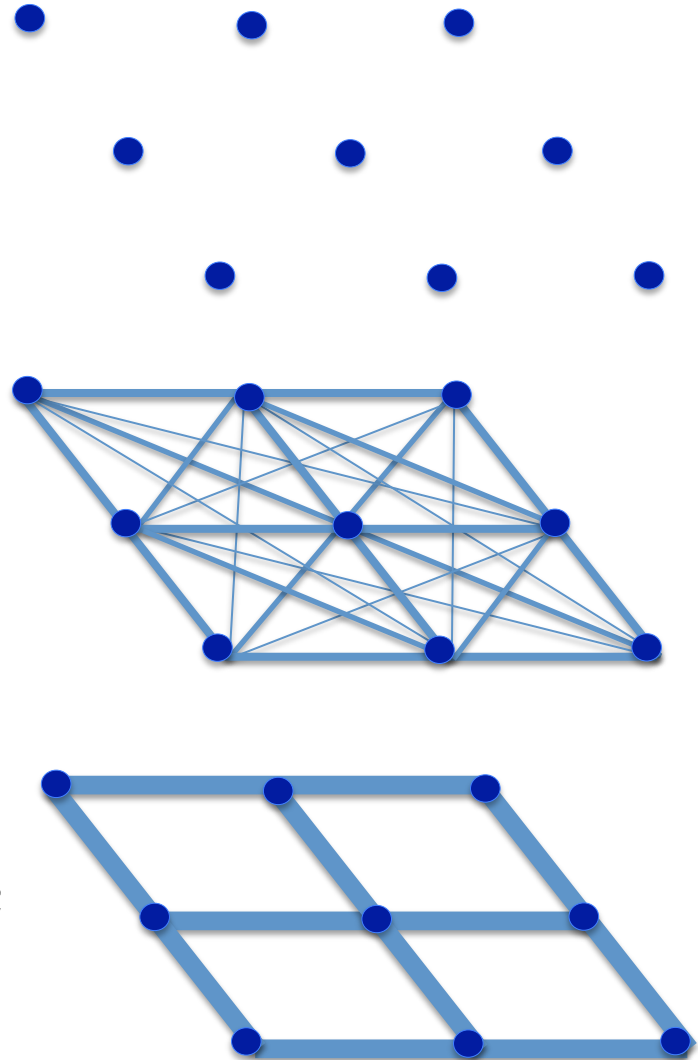
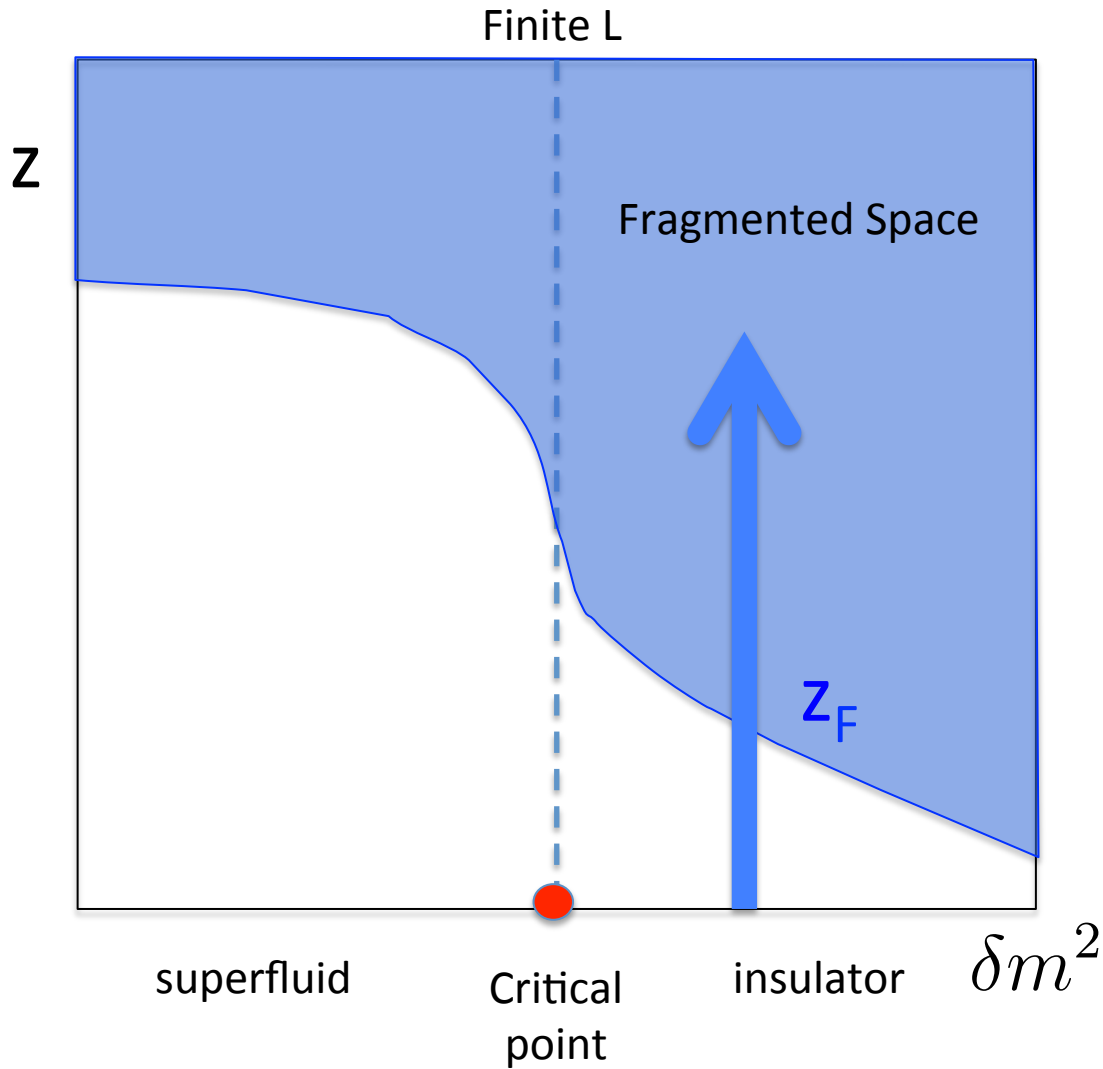
Fragmentation



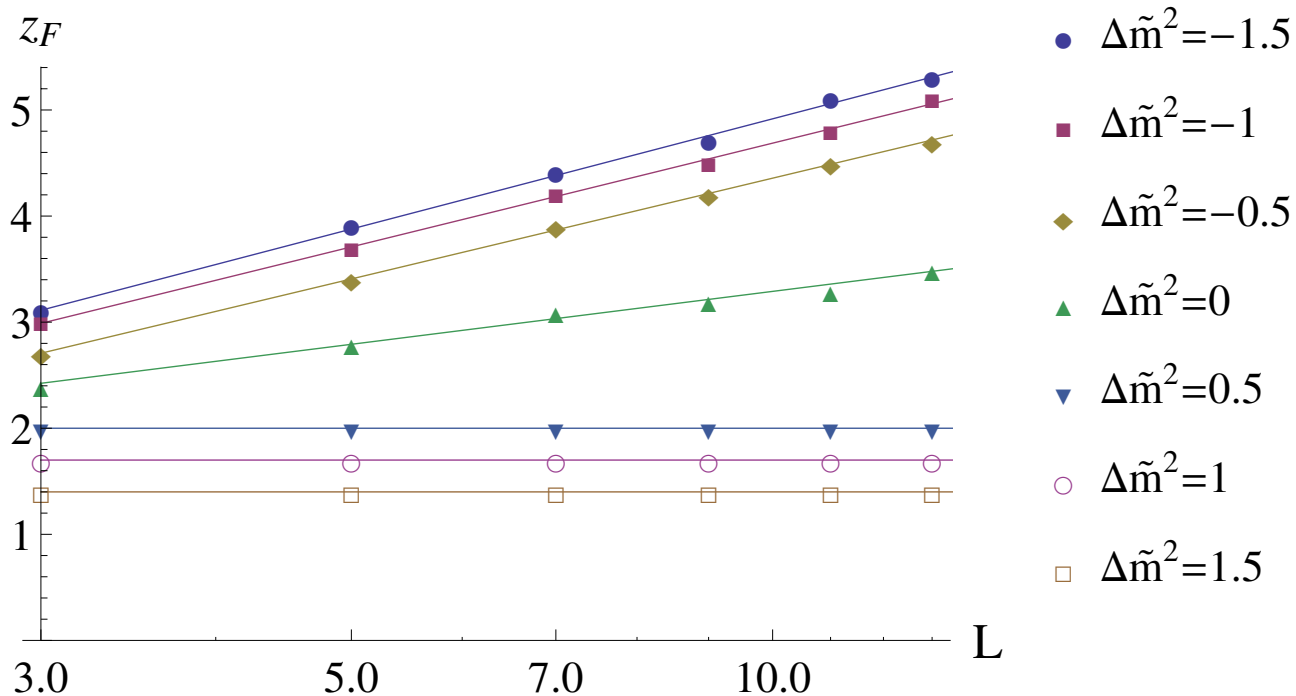
$$\chi = \sum_j t_{ij}^*$$

- For $z > z_F$, correlation length of $t_{ij}^*(z) \sim e^{-|i-j|/\xi}$ becomes much less than the lattice spacing (not a sharp transition)

Fragmentation in insulating phase

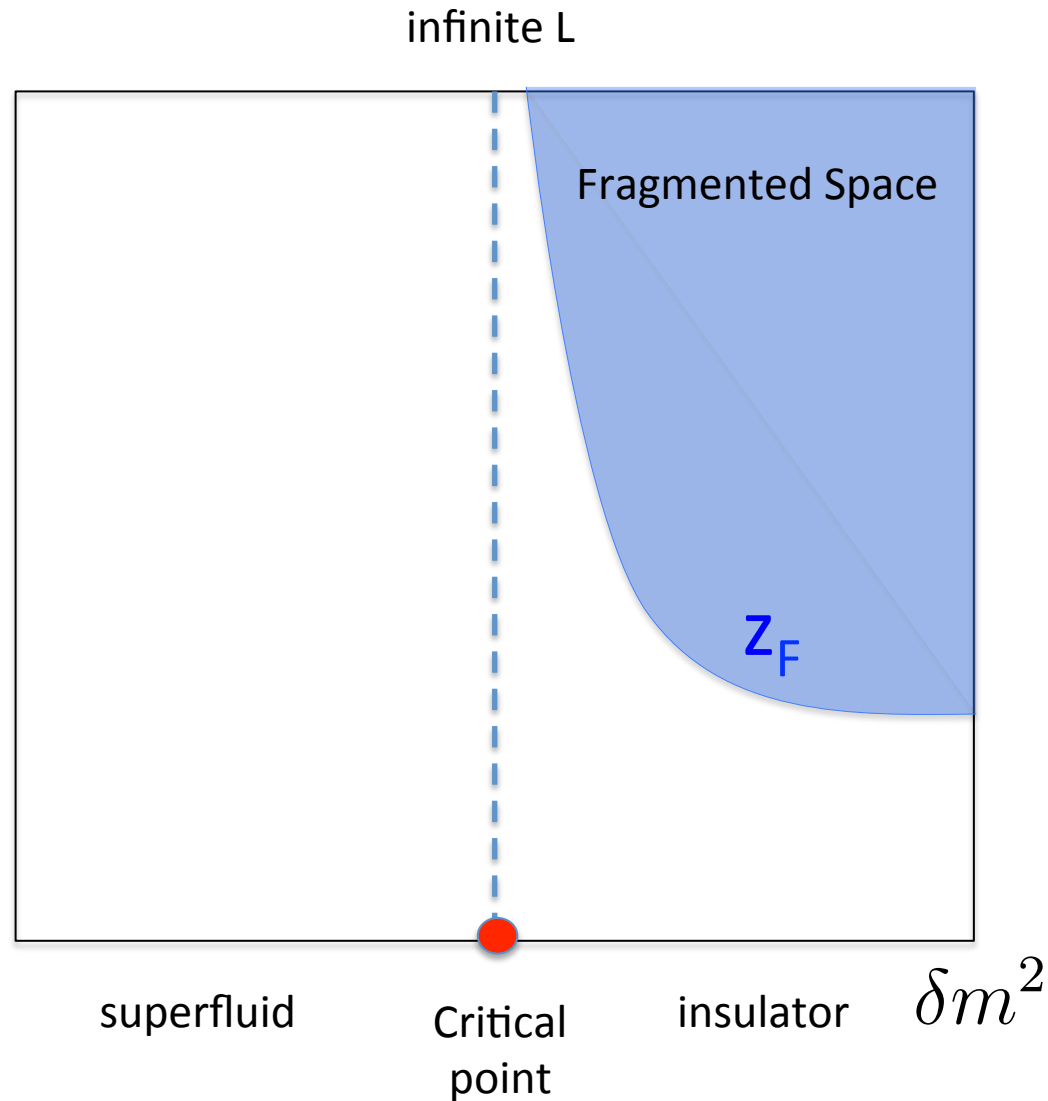


Fragmentation in superfluid phase is a finite size effect

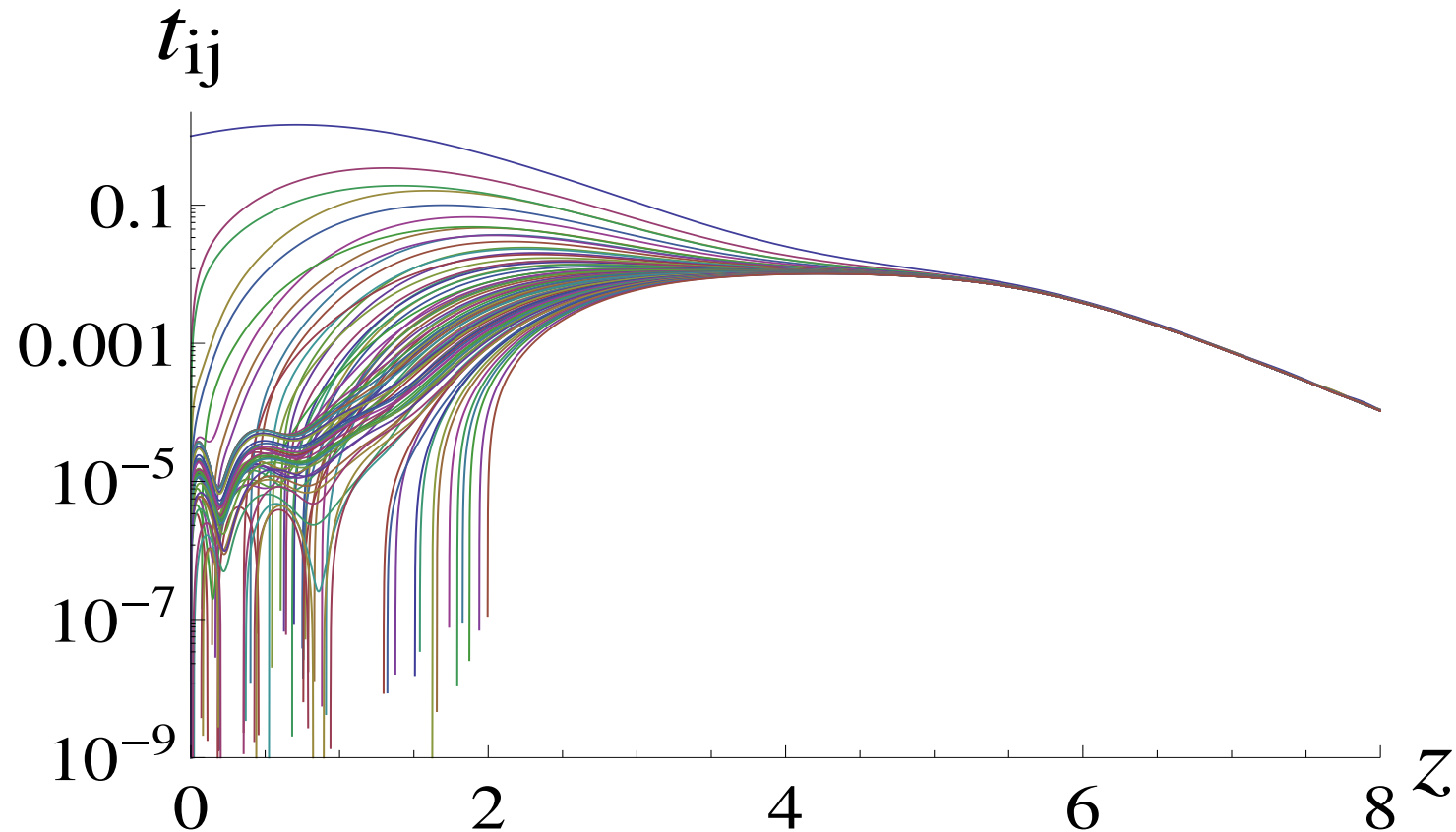


- In the insulating phase, z_F is independent of system size
- In the superfluid phase and at the critical point, z_F diverges in the thermodynamic limit

Fragmentation in the insulating phase

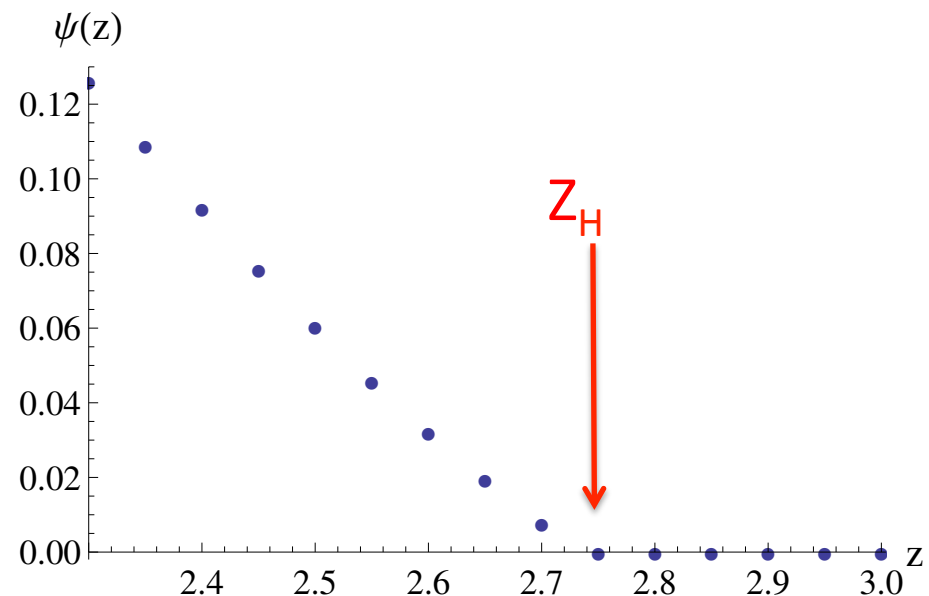
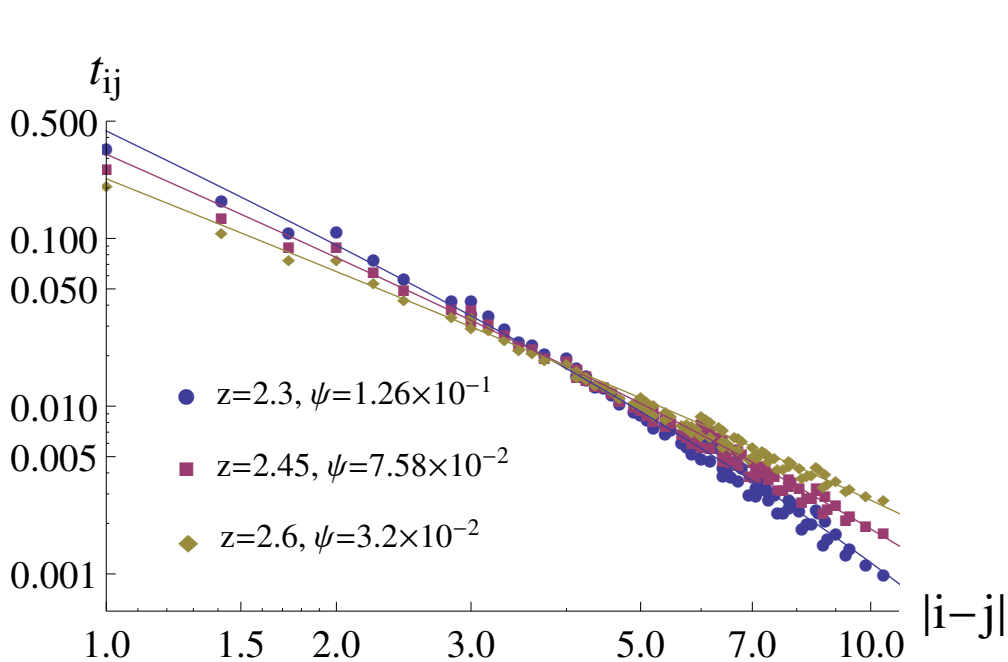


Superfluid Phase

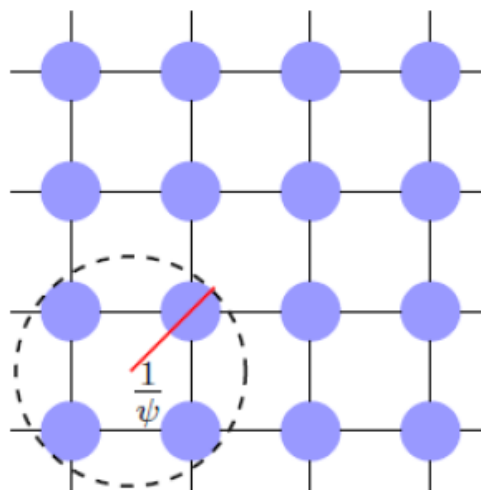


- In the superfluid phase, locality is lost in the bulk

Horizon in the superfluid phase



$$t_{ij}(z) \sim \frac{e^{-\psi(z)|i-j|}}{|i-j|^\kappa}$$



Range of hopping
diverges at horizon :
Volume shrinks to
zero

Why called horizon?

At finite temperature

$$\beta = \frac{1}{T}$$



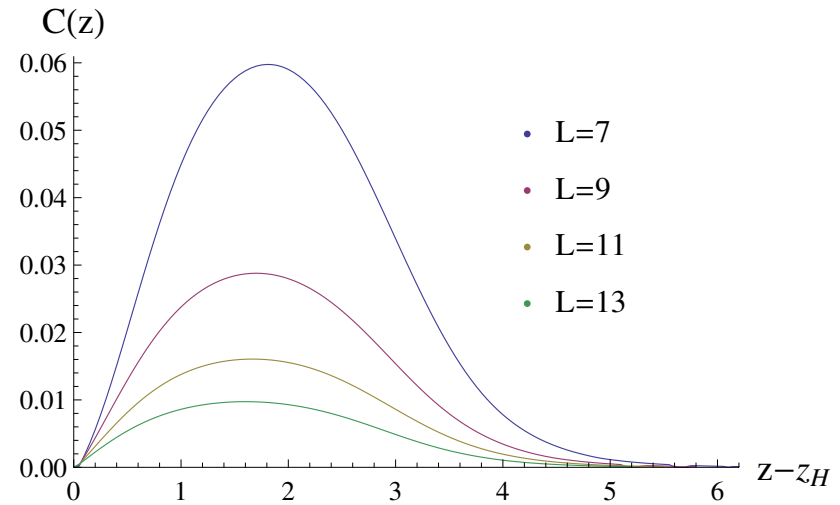
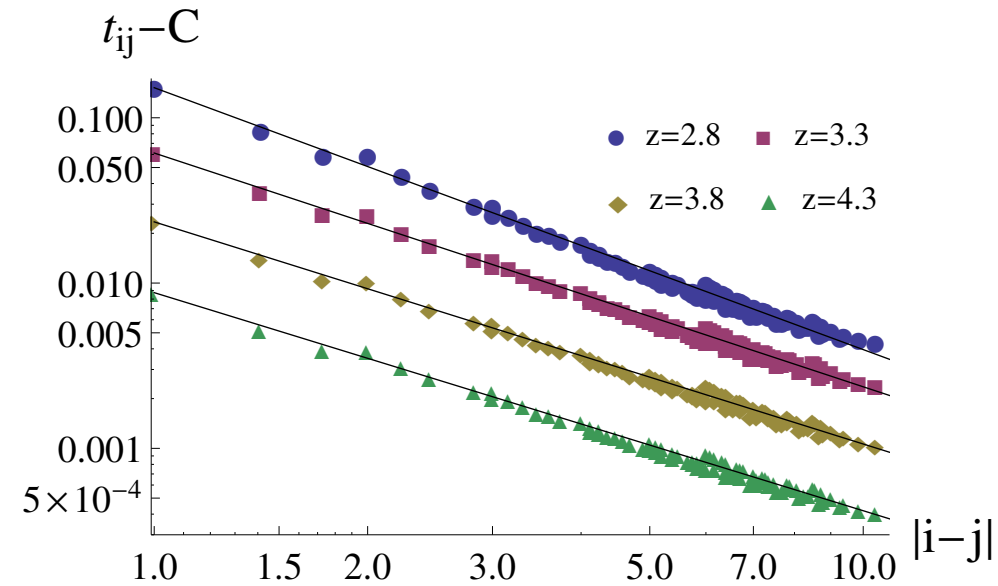
z_H

Horizon :
Loss of locality
in the direction of
the thermal circle



z

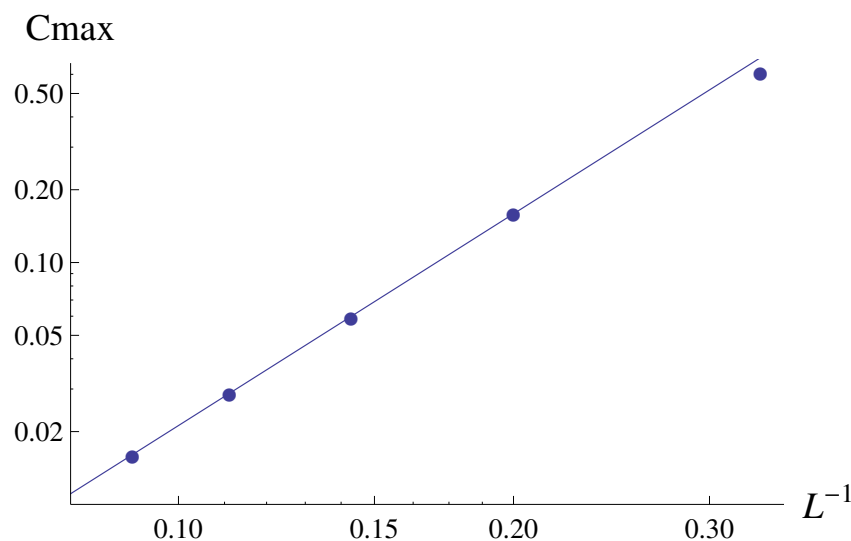
Beyond horizon



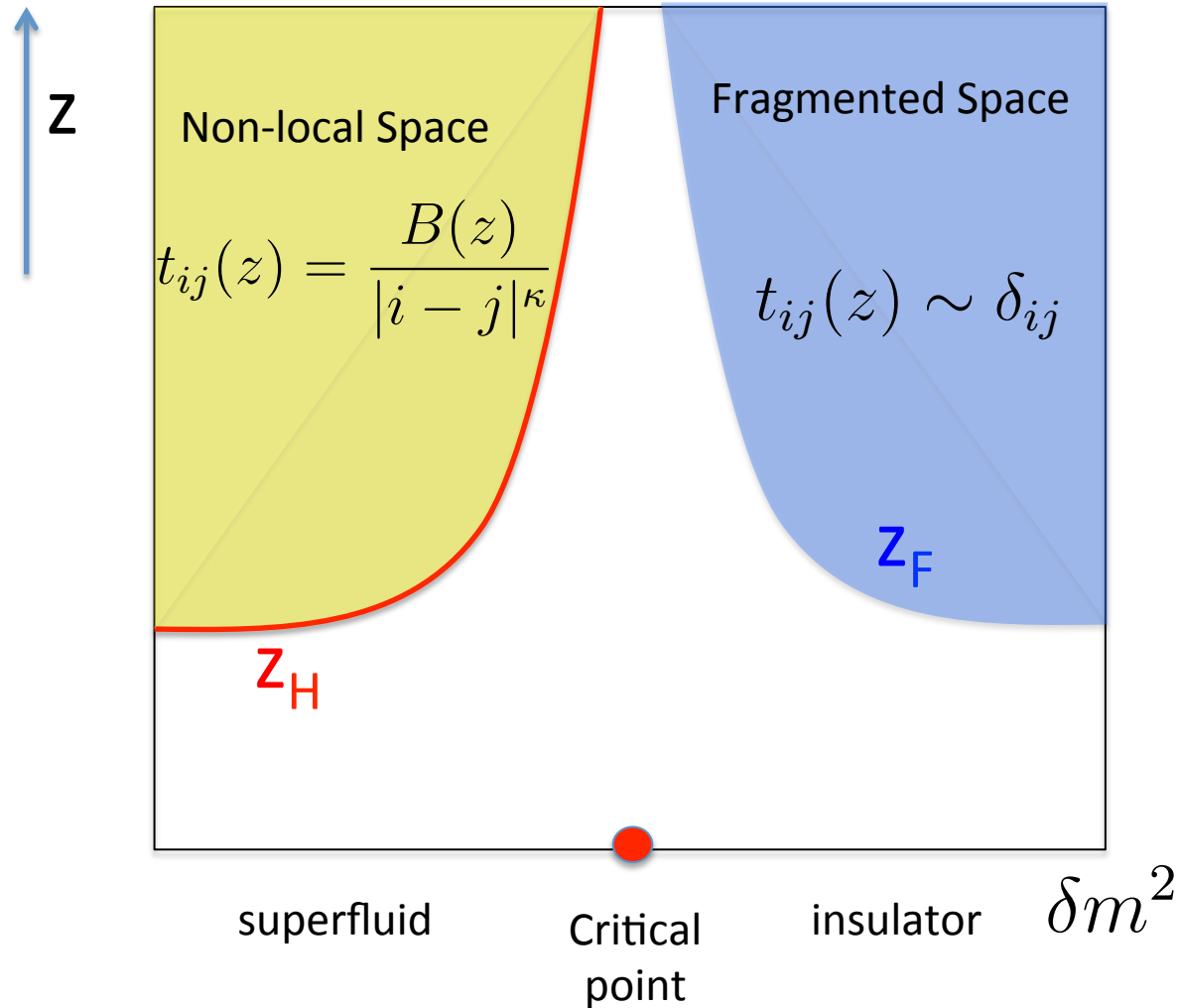
$$z \geq z_H \quad t_{ij}(z) = C(z) + \frac{B(z)}{|i-j|^\kappa}$$

Beyond horizon in the thermodynamic limit

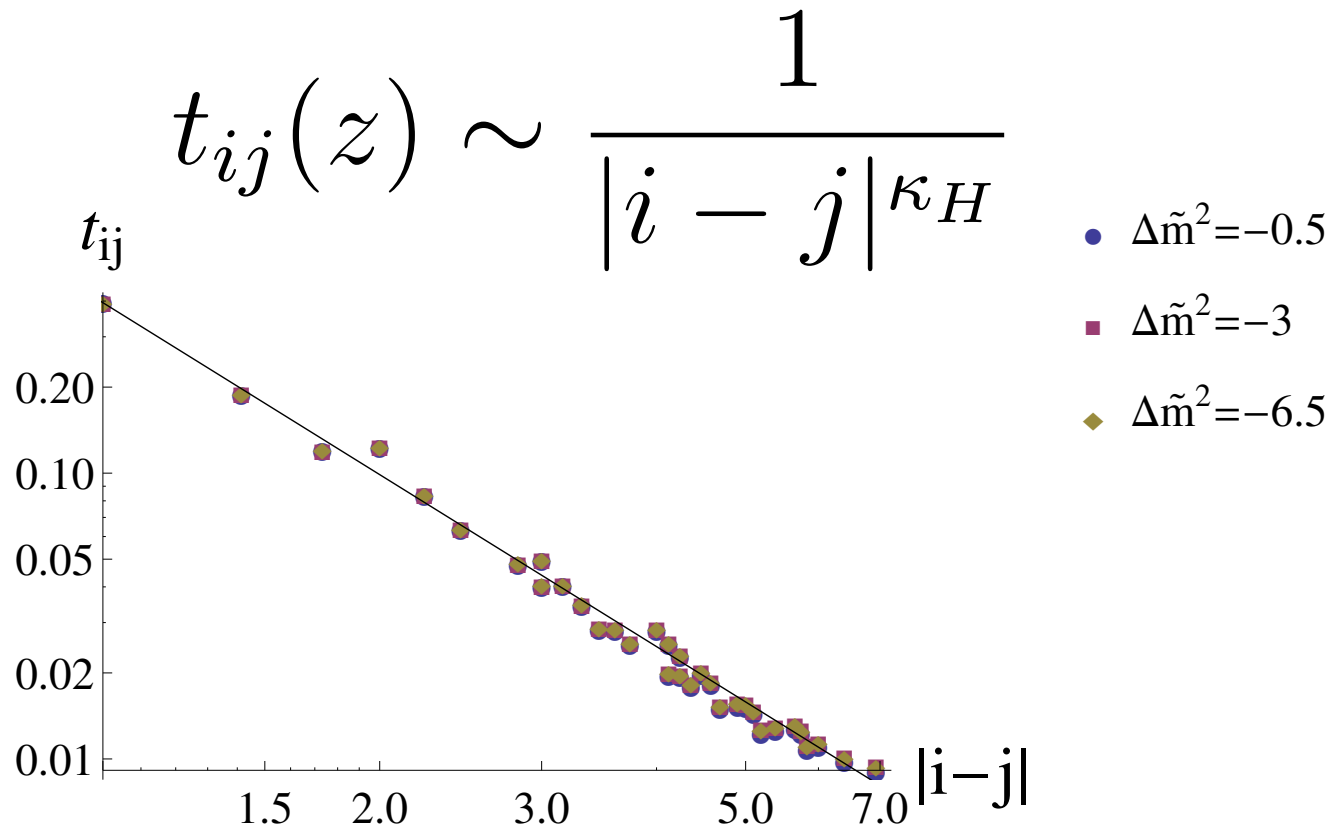
$$\lim_{L \rightarrow \infty} t_{ij}(z) = \underset{0}{\cancel{C(z)}} + \frac{B(z)}{|i - j|^\kappa}$$



Holographic phase diagram

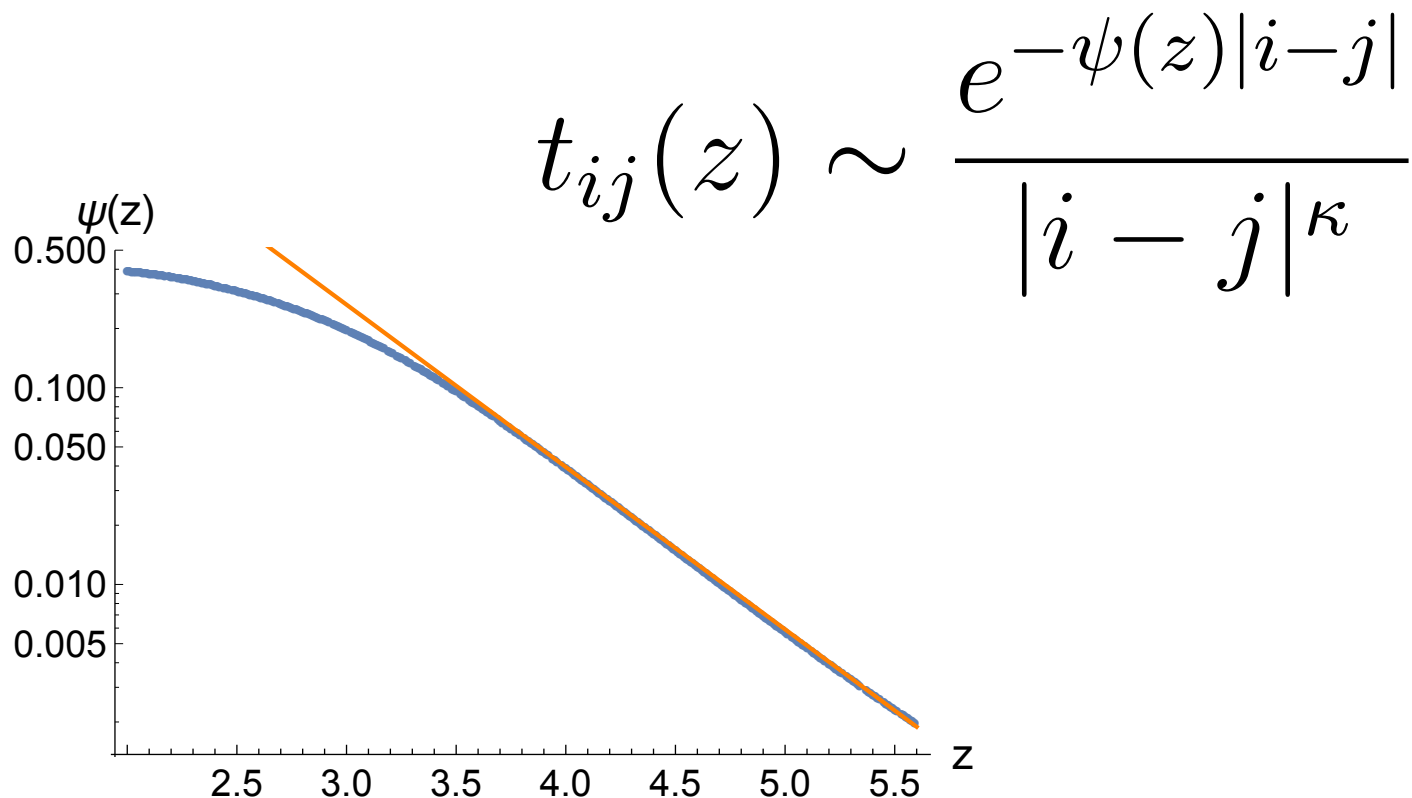


Universal power-law at the horizon



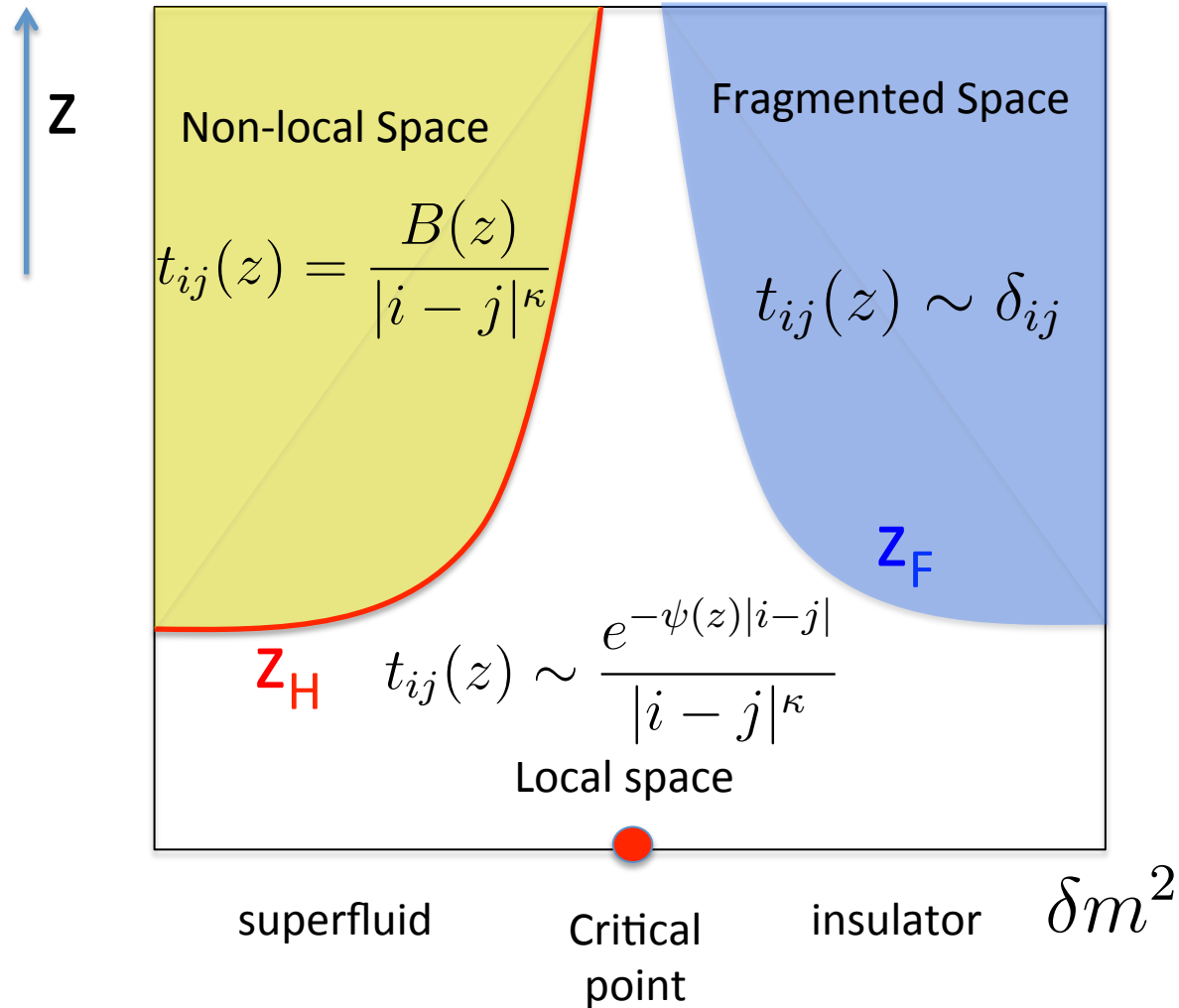
- At the horizon, the hopping fields decay with a universal exponent independent of δm^2

Critical Point

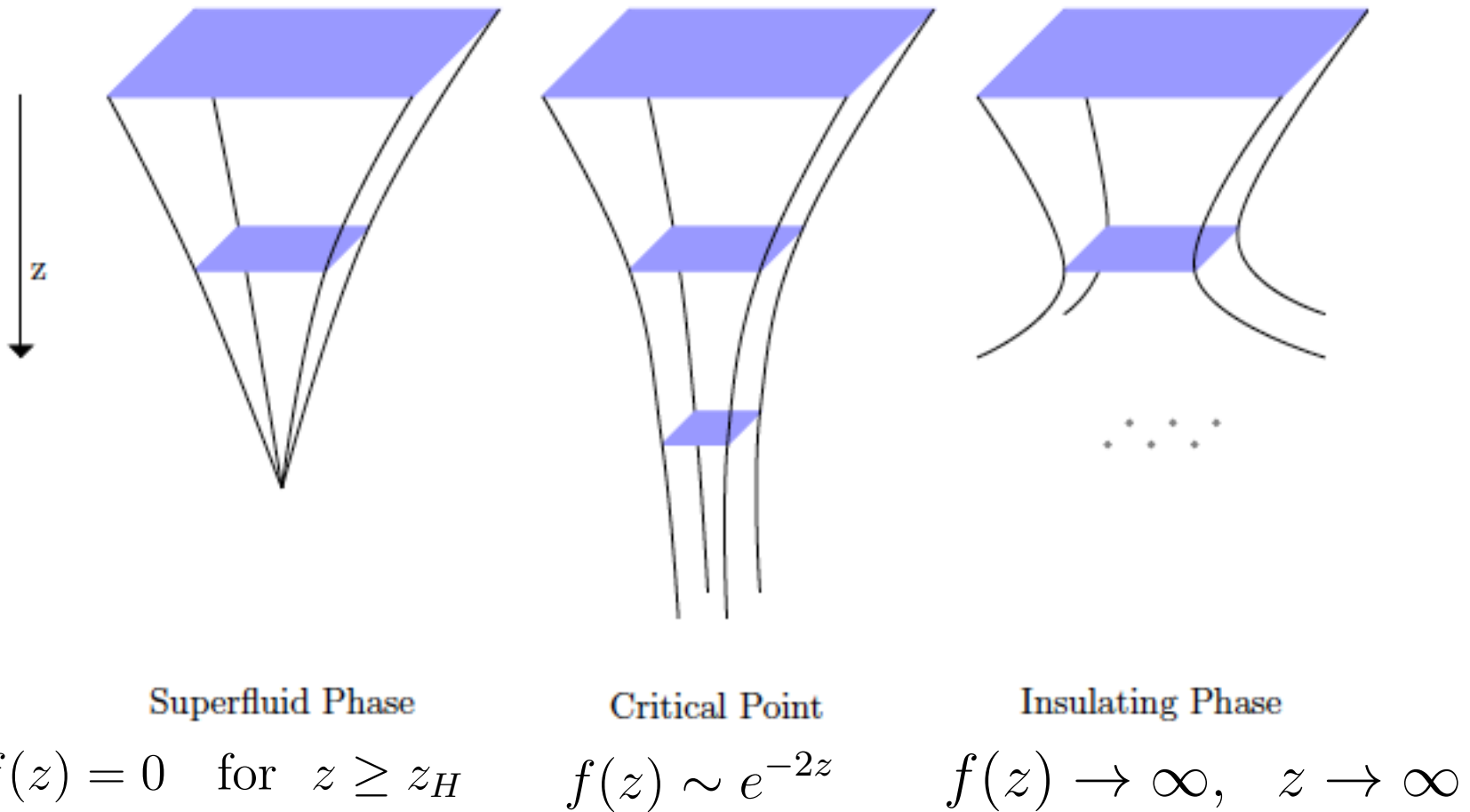


- At the critical point, the length scale associated with the hopping diverges exponentially in z

Holographic phase diagram



Locality as order parameter



$$ds^2 = dz^2 + f(z)dx_i^2$$

Summary

- Holographic solution of $U(N)$ vector model via quantum RG
- (Non-)Locality serves as an order parameter
 - Insulator : Fragmented geometry
 - Superfluid : Non-locality geometry behind horizon
 - Critical point : Local geometry

Open problem

- Analytic/continuum solution
- Connection to the Vasiliev theory
- Application to Fermi surface

Gauge symmetry in the bulk

$$\mathcal{S}_{Bulk} = \int_0^\infty dz \left\{ \sum_{ij} t_{ij}^*(z) \partial_z t_{ij}(z) \right. \quad \text{in the radial gauge} \\ \left. + \sum_i \left[-\frac{2}{m^2} t_{ii}(z) + \frac{4\lambda \left(1 + \frac{1}{N}\right)}{m^2} t_{ii}^*(z) - 4\lambda (t_{ii}^*(z))^2 - \frac{8\lambda^2}{m^2} (t_{ii}^*(z))^3 \right] \right. \\ \left. + \sum_{ij} \left[2t_{ij}(z) t_{ij}^*(z) + \frac{4\lambda}{m^2} t_{ij}(z) t_{ij}^*(z) (t_{ii}^*(z) + t_{jj}^*(z)) \right] - \frac{2}{m^2} \sum_{ijk} [t_{ik}(z) t_{kj}(z) t_{ij}^*(z)] \right\}$$

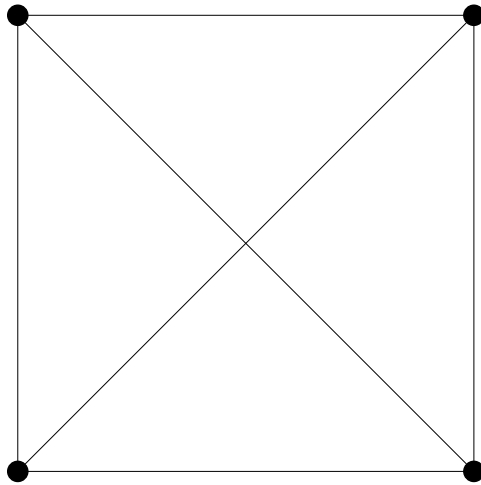
Higher spin symmetry : $t_{ij}(z) \rightarrow U_{ii'}^\dagger t_{i'j'}(z) U_{j'j}$
($\lambda=0$)

[R. Leigh, O. Parrikar, A. Weiss, arXiv:1402.1430]

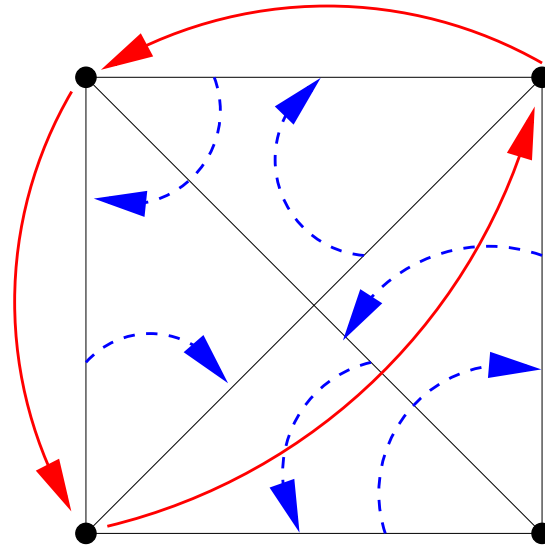
Only discrete diffeomorphism survives : $U_{ij} = \delta_{ij} + N_i$
for $\lambda \neq 0$

Origin of D-dim discrete diffeomorphism inv.

$$\mathcal{S}_0 = m^2 \sum_i (\phi_i^* \cdot \phi_i) + \frac{\lambda}{N} \sum_i (\phi_i^* \cdot \phi_i)^2 - \sum_{ij} t_{ij}^{(0)} (\phi_i^* \cdot \phi_j)$$



(a)



(b)

Permutation (event symmetry) in D-dimensional network