Entanglement and Correlation of Quantum Fields in an Expanding Universe

Yasusada Nambu (Nagoya University, Japan)

International Workshop on Strings, Black Holes and Quantum Information @ Tohoku Univ

2015/9/9

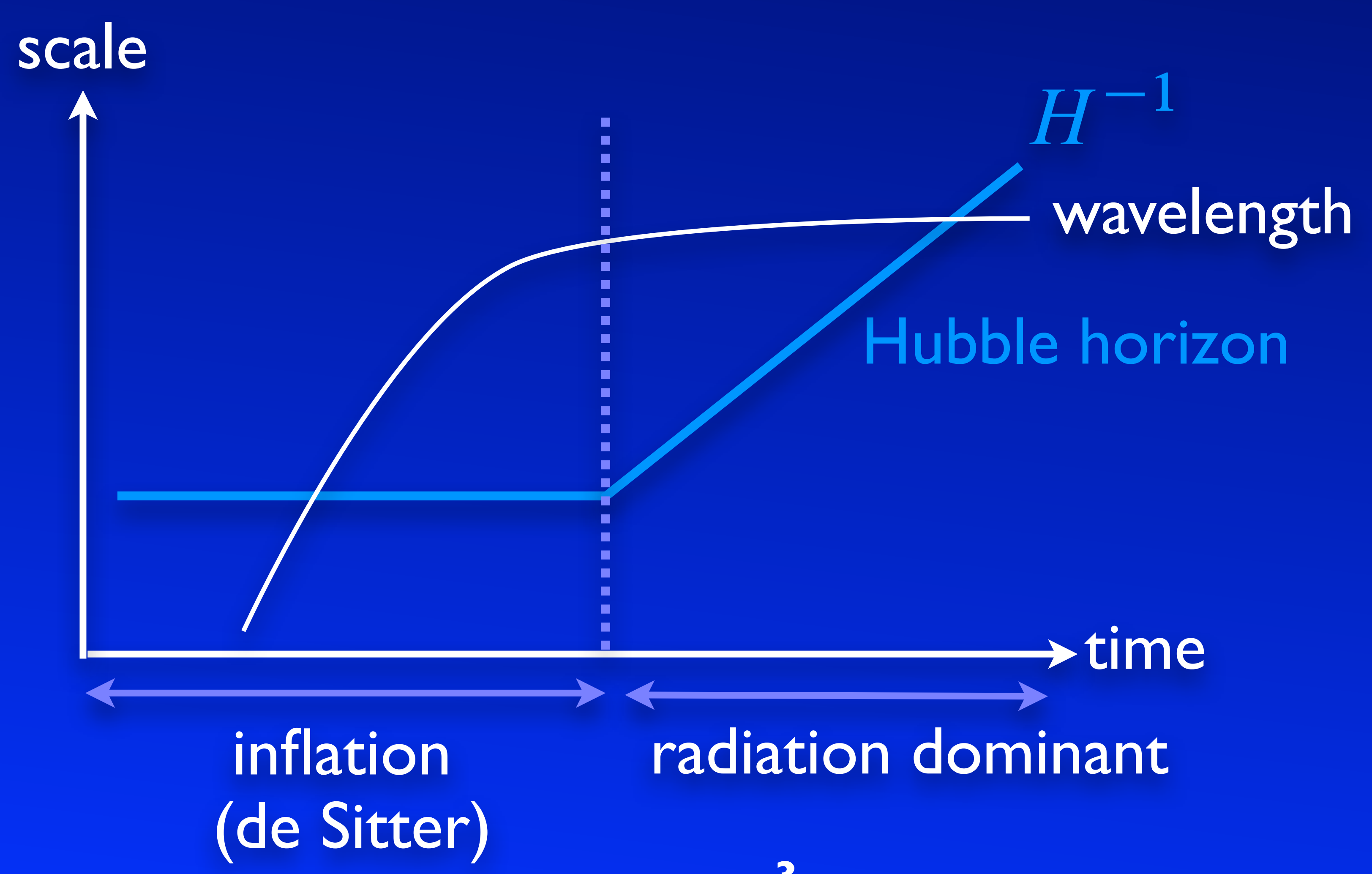
Contents

- Introduction
- Bipartite entanglement
- 1-dim lattice model of quantum field in a FRW universe
- Gaussian quantum discord
- Entanglement in k-space and quantum estimation in a FRW universe

Introduction

- Inflation provides us a mechanism to generate primordial quantum fluctuations that lead to the large scale structures in our present universe.

- As the present universe is classical, initial quantum fluctuations must lose its quantum nature in course of its evolution (quantum to classical transition).



- What are conditions for quantum fluctuations become classical?

loss of wave property (freeze out)

loss of quantum superposition (decoherence)

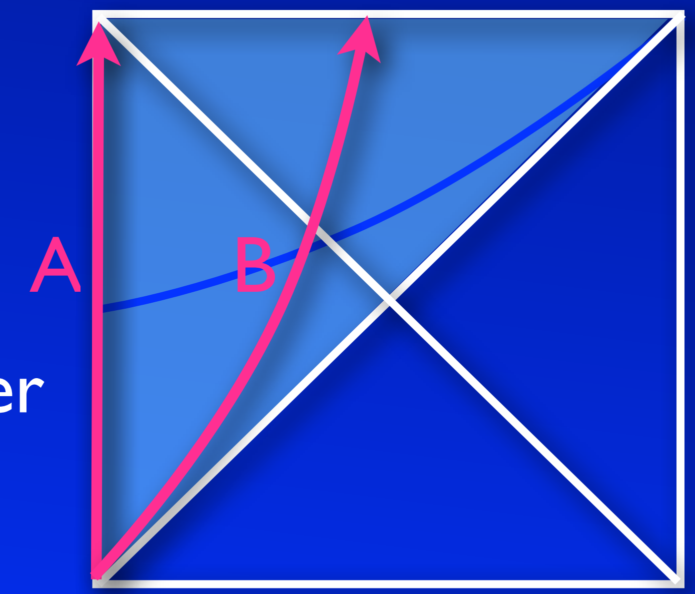
loss of quantum correlation (dis-entanglement)

Entanglement is purely quantum mechanical non-local correlation

> We want to understand the meaning of classicalization in terms of entanglement of quantum field

Evolution of spatial entanglement in an expanding universe

two comoving observer
in deSitter spacetime



Entanglement (two party)

bipartite entanglement

pure state

- A, B are separable

$$|A, B\rangle = |A\rangle|B\rangle$$

- A, B are entangled

$$|A, B\rangle = |a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle + \cdots$$

⇒ correlation peculiar to quantum mechanics

mixed state

- A, B are separable

$$\Leftrightarrow \quad \hat{\rho}_{AB} = \sum_j w_j \hat{\rho}_A^j \otimes \hat{\rho}_B^j, \quad \sum_j w_j = 1, \quad w_j \geq 0$$

- If the state cannot represented as this form, A, B are entangled

Separability: necessary and sufficient condition

(R.Simon 2000, L.Duan *et al.* 2000)

1 X 1 Gaussian state

A: $(\hat{q}_A, \hat{p}_A)$ B: $(\hat{q}_B, \hat{p}_B)$

$$\hat{\xi}_i = (\hat{q}_A, \hat{p}_A, \hat{q}_B, \hat{p}_B) \qquad [\hat{\xi}_j, \hat{\xi}_k] = i\,\Omega_{jk}$$

$$\boldsymbol{\Omega} = \begin{pmatrix} \boldsymbol{J} & 0 \\ 0 & \boldsymbol{J} \end{pmatrix} \qquad \boldsymbol{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

covariance matrix $V_{jk} = \frac{1}{2}\langle \hat{\xi}_j \hat{\xi}_k + \hat{\xi}_k \hat{\xi}_j \rangle$ $\qquad \langle \hat{A} \rangle = \mathrm{Tr}[\hat{\rho}\hat{A}]$

- positivity $\quad V + \frac{i}{2}\boldsymbol{\Omega} \geq 0 \qquad$ for arbitrary $\hat{A}$ $\quad \langle \hat{A}\hat{A}^\dagger \rangle \geq 0$

  $\hat{\rho} = \hat{\rho}^\dagger$

- partial transpose $\quad p_B \longrightarrow -p_B \qquad V \longrightarrow \tilde{V}$

A,B are separable $\Longleftrightarrow$ $\tilde{V} + \frac{i}{2}\boldsymbol{\Omega} \geq 0$ $\qquad$ 2X2, 2X3, 1XN

for M X N system

A,B is separable $\Longrightarrow$ $\tilde{V} + \frac{i}{2}\boldsymbol{\Omega} \geq 0$

Symplectic eigenvalue

symplectic transformation

$$S V S^T = \mathrm{diag}(\nu_+, \nu_+, \nu_-, \nu_-)$$

$$\nu_+ \geq \nu_- > 0$$

$$S \in \mathrm{Sp}(4, R)$$

$$S \Omega S^T = \Omega$$

- positivity $\nu_- \geq \frac{1}{2}$ uncertainty relation
- separability $\tilde{\nu}_- \geq \frac{1}{2}$

If these conditions are satisfied, A, B are separable (no entanglement)

Logarithmic negativity

$$E_N = -\min\left[\log_2(2\tilde{\nu}_-), 0\right]$$

$E_N > 0$ entangled

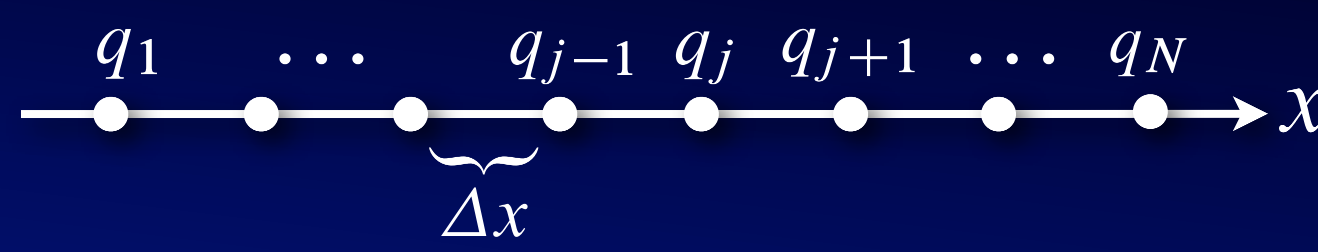
$E_N = 0$ separable

Entanglement of Quantum Field in a FRW Universe

YN, PRD80(2009)124031

- 1-dim lattice model (periodic BC)
- massless scalar

$$\Box\phi = 0$$



EOM for $q = a\phi$

$$q'' - \frac{a''}{a}q - \nabla^2 q = 0$$

scale factor $a(\eta)$

conformal time $\eta = \int \frac{dt}{a}$

discretize space

$$q_j'' - \frac{a''}{a}q_j + 2q_j - \alpha(q_{j+1} + q_{j-1}) = 0 \qquad \alpha = 1 - \frac{1}{2}(m\Delta x)^2$$

- quantization

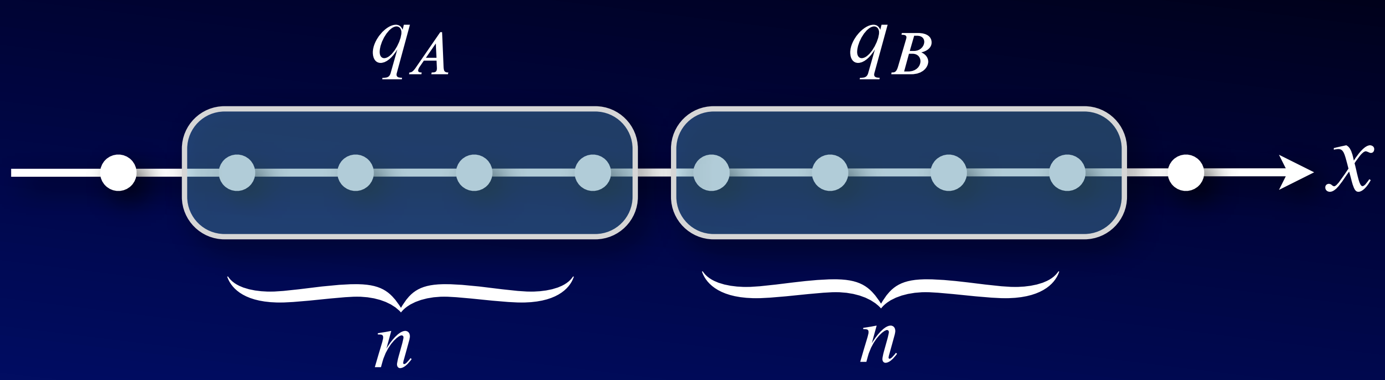
$$\hat{q}_j = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(f_k \hat{a}_k + f_k^* \hat{a}_{N-k}^\dagger \right) e^{i\theta_k j} \qquad \theta_k = \frac{2\pi k}{N}$$

$$f_k'' + \left(\omega_k^2 - \frac{a''}{a} \right) f_k = 0 \qquad \omega_k^2 = 2(1 - \alpha \cos\theta_k)$$

block variables

$$q_A = \frac{1}{\sqrt{n}} \sum_{j \in A} q_j$$

$$q_B = \frac{1}{\sqrt{n}} \sum_{j \in B} q_j$$



covariance matrix

$$V = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{C} \\ \boldsymbol{C} & \boldsymbol{A} \end{pmatrix} \qquad \boldsymbol{A} = \begin{pmatrix} a_1 & a_3 \\ a_3 & a_2 \end{pmatrix} \qquad \boldsymbol{C} = \begin{pmatrix} c_1 & c_3 \\ c_3 & c_2 \end{pmatrix}$$

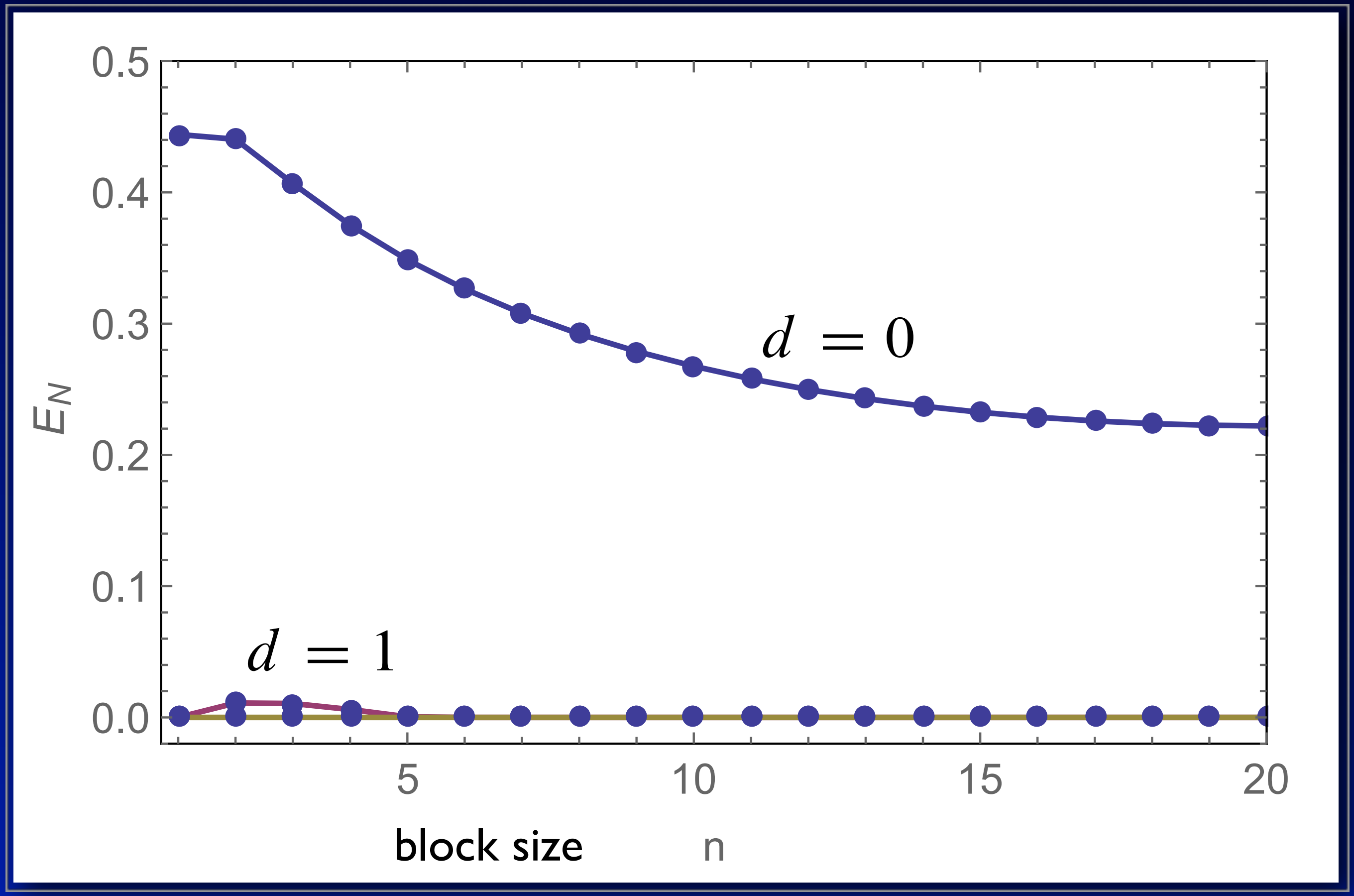
$$a_1 = \langle \hat{q}_A^2 \rangle \qquad a_2 = \langle \hat{p}_A^2 \rangle \qquad a_3 = \frac{1}{2} \langle \hat{q}_A \hat{p}_A + \hat{p}_A \hat{q}_A \rangle$$

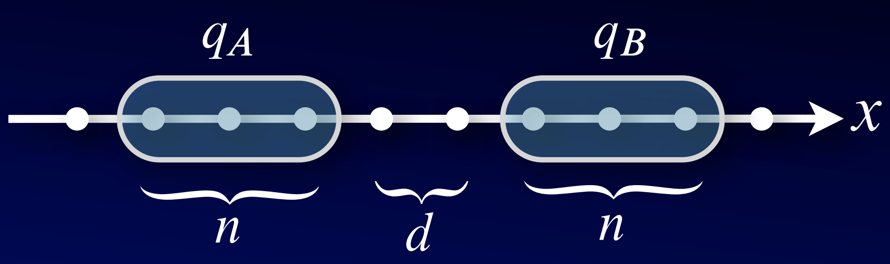
$$c_1 = \frac{1}{2} \langle \hat{q}_A \hat{q}_B + \hat{q}_B \hat{q}_A \rangle \qquad c_2 = \frac{1}{2} \langle \hat{p}_A \hat{p}_B + \hat{p}_B \hat{p}_A \rangle$$

$$c_3 = \frac{1}{2} \langle \hat{q}_A \hat{p}_B + \hat{p}_B \hat{q}_A \rangle$$

- these variables are time dependent N=100
- numerically calculate symplectic eigenvalue
- obtain logarithmic negativity and judge separability

Minkowski vacuum

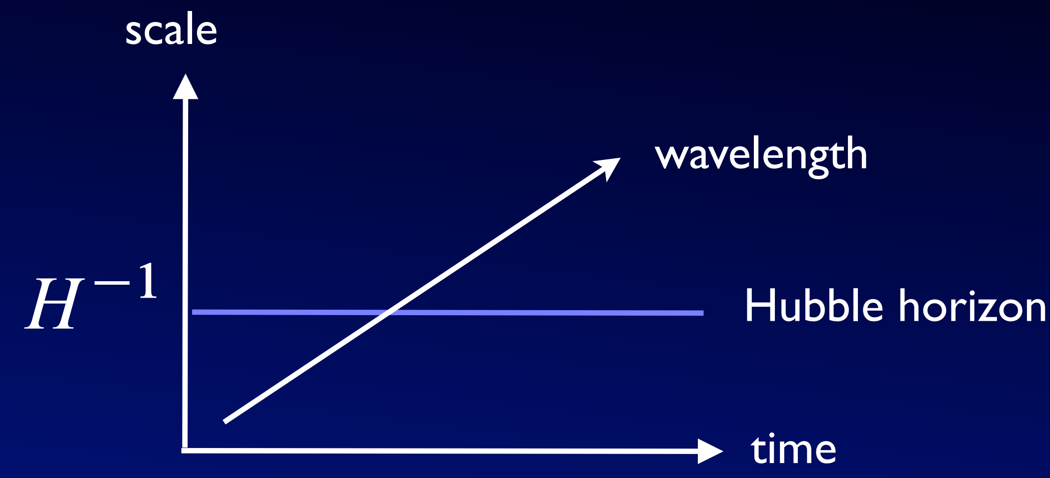


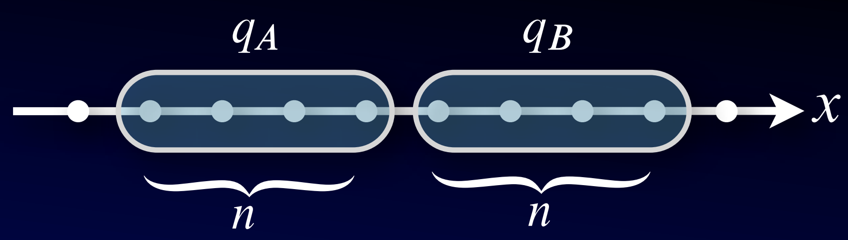


- Minkowski vacuum is entangled
- E_N<>0 for d=0, E_N=0 for d>1
- Value of entanglement depends on the definition of spatial regions
- Negativity is constant in time

De Sitter (Bunch-Davies vacuum)

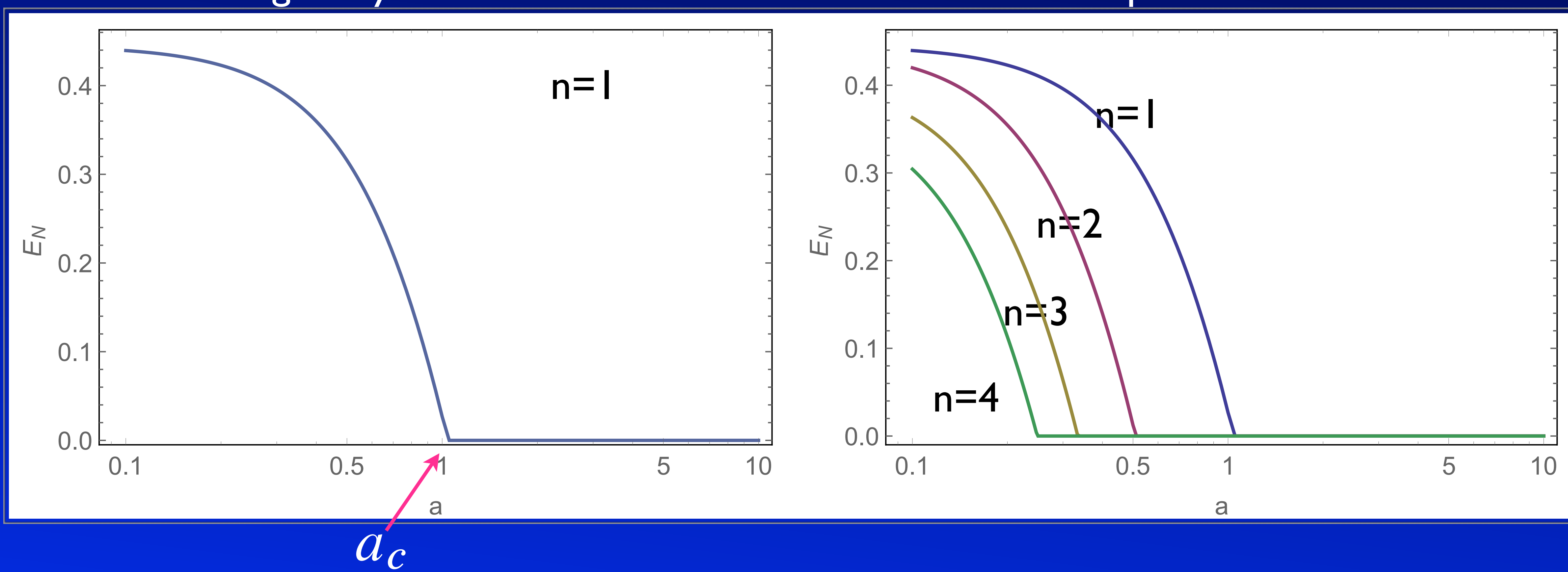
$$a(t) = e^{Ht}$$





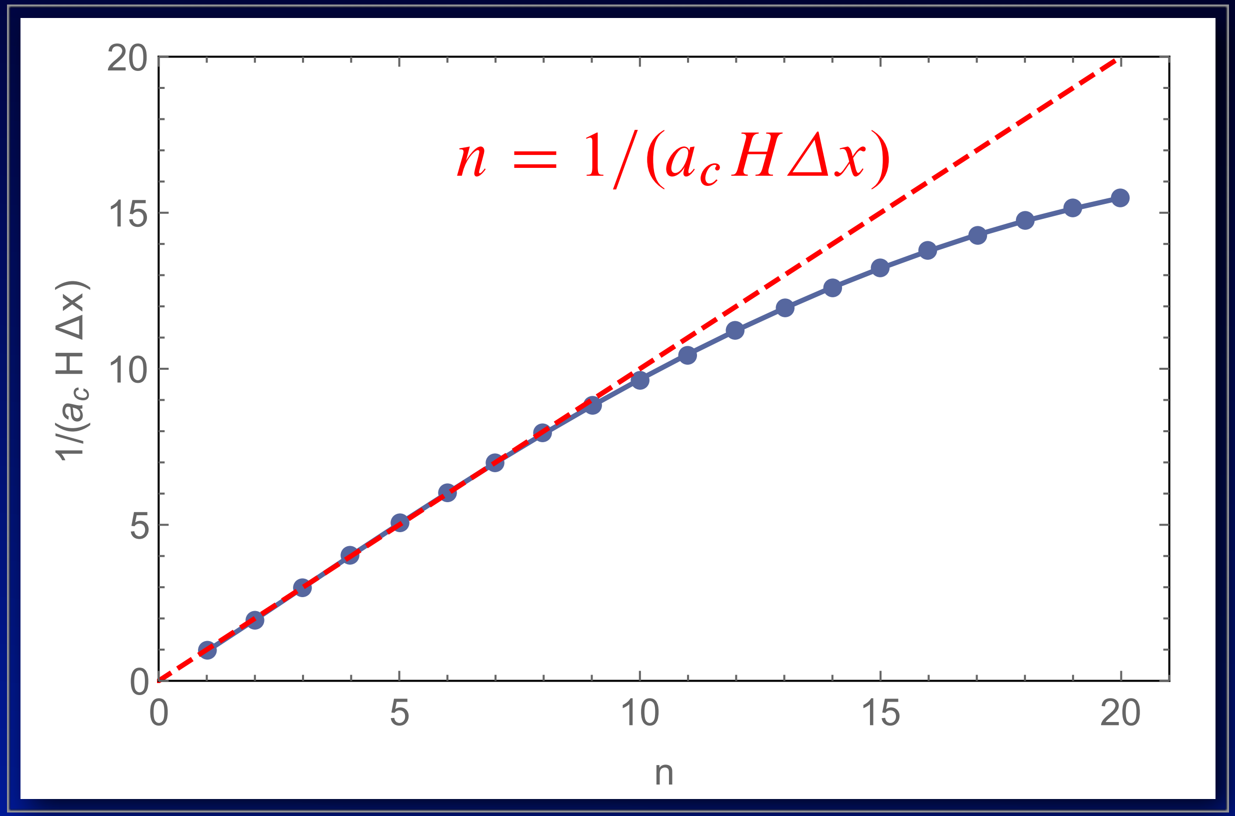
Evolution of negativity

block size dependence



- Initial entangled state evolves to separable state
- Separable time η_c depends on size of block

block size and separable time



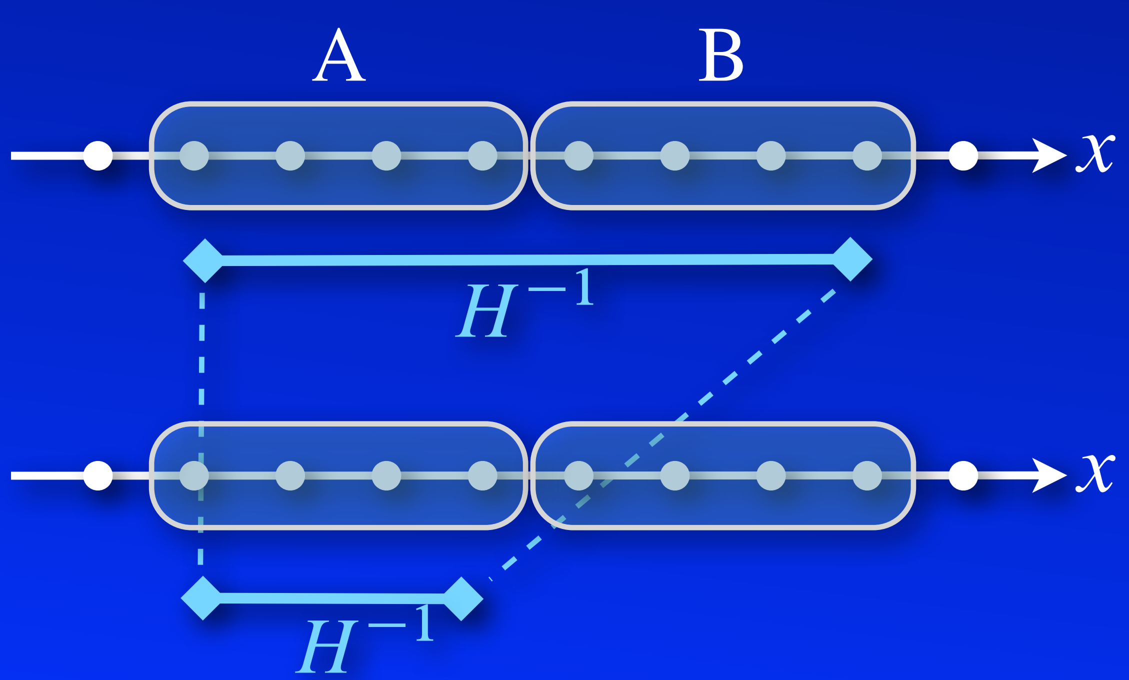
separable time η_c

$$n\Delta x = -\eta_c = \frac{1}{a_c H}$$

$$\therefore \quad a_c \times n\Delta x = H^{-1}$$

block size — Hubble length

When the block size is equal to the Hubble horizon scale, entanglement between blocks is lost.



initial entangled state
(initial vacuum state)

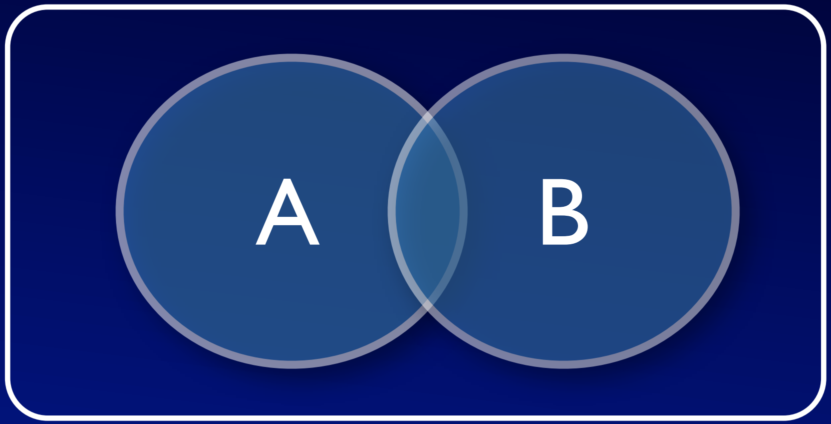
↓ de Sitter expansion

separable state

- quantum correlation is lost
- generation of "classical" fluctuation
- quantumness?

Information and Correlation of Scalar Field

Correlation of a bipartite system



Mutual information lack of information ⇔ entropy

$$I(A:B) = S(A) + S(A) - S(AB)$$

$$S(X) = -\sum_x p_x \log p_x$$ Shannon entropy for a classical variable

$$S(X) = -\mathrm{tr}(\rho_X \log \rho_X)$$ von Neumann entropy for a quantum state

For classical variables, by Bayes' rule

$$I(A:B) = S(A) - S(A|B) \equiv J(A:B)$$ mutual information in terms of conditional entropy

conditional entropy $S(A|B) = \sum_b p_b S(A|b)$

For quantum case, using a POVM measurement on B,

$$J(A|B) = S(A) - \sum_b p_b S(\rho_{A|b})$$

measurement op. of b

$$\{\Pi_b\}, \quad \sum_b \Pi_b = \mathbb{1}$$

$$p_b = \mathrm{tr}(\Pi_b \rho_{AB} \Pi_b)$$

and in general,

$$I(A:B) \neq J(A|B)$$

Maximal correlation obtained via measurement is defined by

$$J(A|B) = S(A) - \min_{\{\Pi_b\}} \sum_b p_b S(\rho_{A|b})$$

Difference between I and J quantifies 'quantumness' of correlation:

$$D(A|B) \equiv I(A:B) - J(A|B)$$

quantum discord

$$I = J + D$$

total correlation, classical correlation, quantum discord

Henderson and Vedral 2001
J. Phys. A 34, 6899

Ollivier and Zurek 2002
PRL 88, 01790

We can judge "quantumness" of quantum fluctuation using quantum discord.

Gaussian Quantum Discord

Adesso & Datta 2010
Giorda & Paris 2010

For 2-mode Gaussian state with a covariance matrix

$$V = \begin{pmatrix} A & C \\ C & B \end{pmatrix}$$

We consider a Gaussian measurement   coherent state POVM

$$\Pi_B(\eta) = D_B(\eta)\rho_M D_B^\dagger(\eta), \quad \pi^{-1}\int d^2\eta\,\Pi_B(\eta) = \mathbb{1}$$

$$D_B(\eta) = e^{\eta b^\dagger - \eta^* b}$$

$$b = \sqrt{\frac{\omega}{2}}x_B + \frac{i}{\sqrt{2\omega}}p_B$$

State of A after measurement of B is

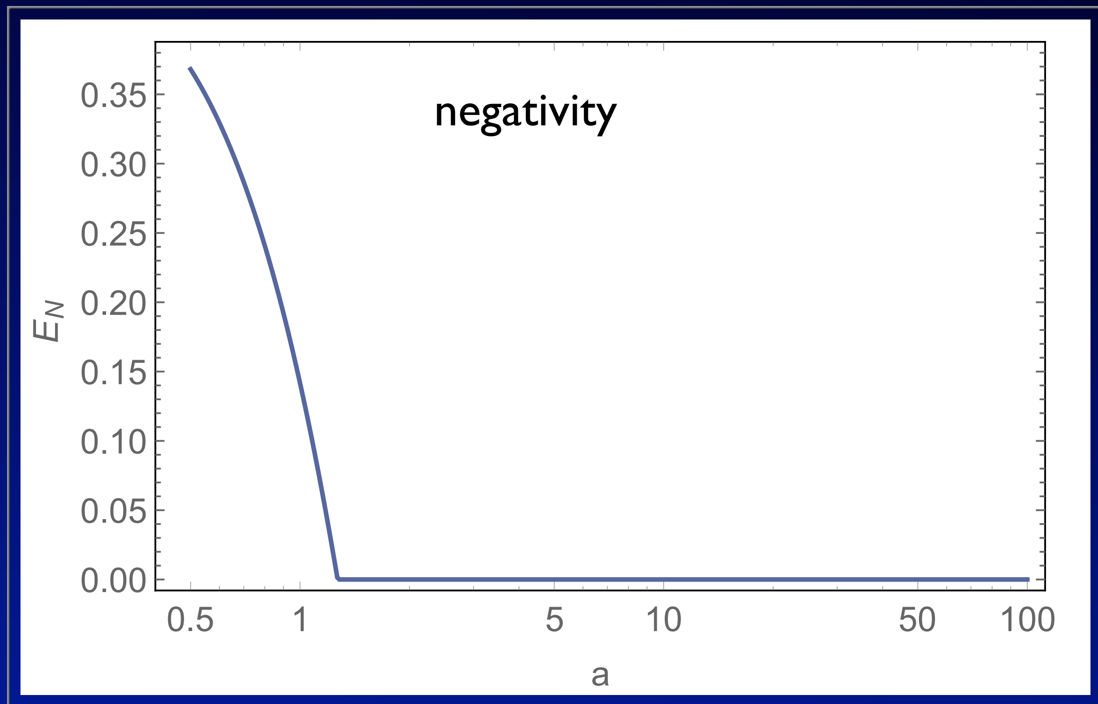
$$V_A' = A - C(B + V_M)^{-1}C^T$$

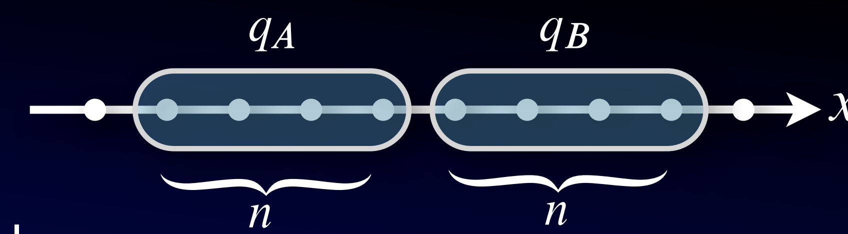
General form of Gaussian discord:

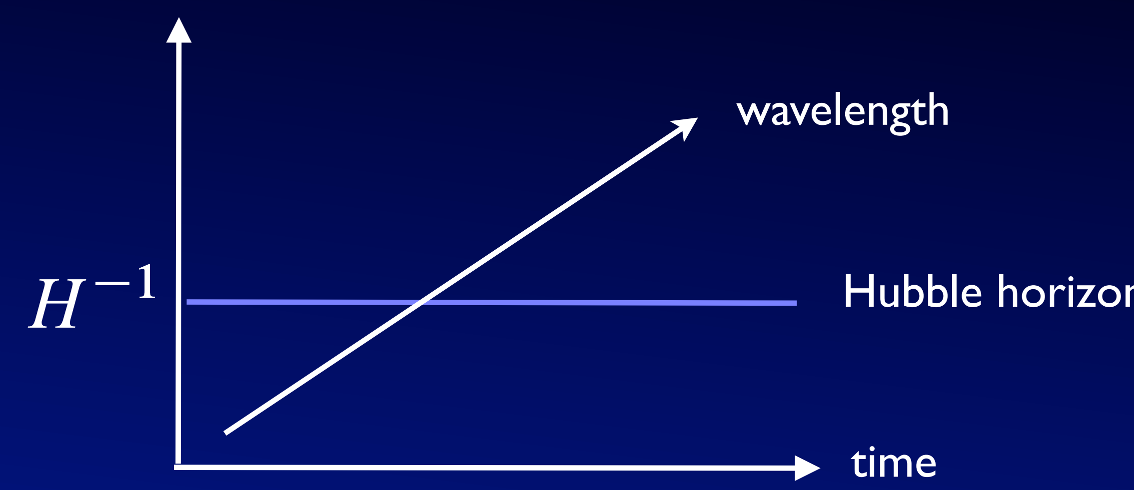
$$D = f(\sqrt{B}) - f(\nu_-) - f(\nu_+) + \max_{V_M} f(\sqrt{V_A'})$$

$$f(x) = \left(x + \frac{1}{2}\right)\log\left(x + \frac{1}{2}\right) - \left(x - \frac{1}{2}\right)\log\left(x - \frac{1}{2}\right)$$

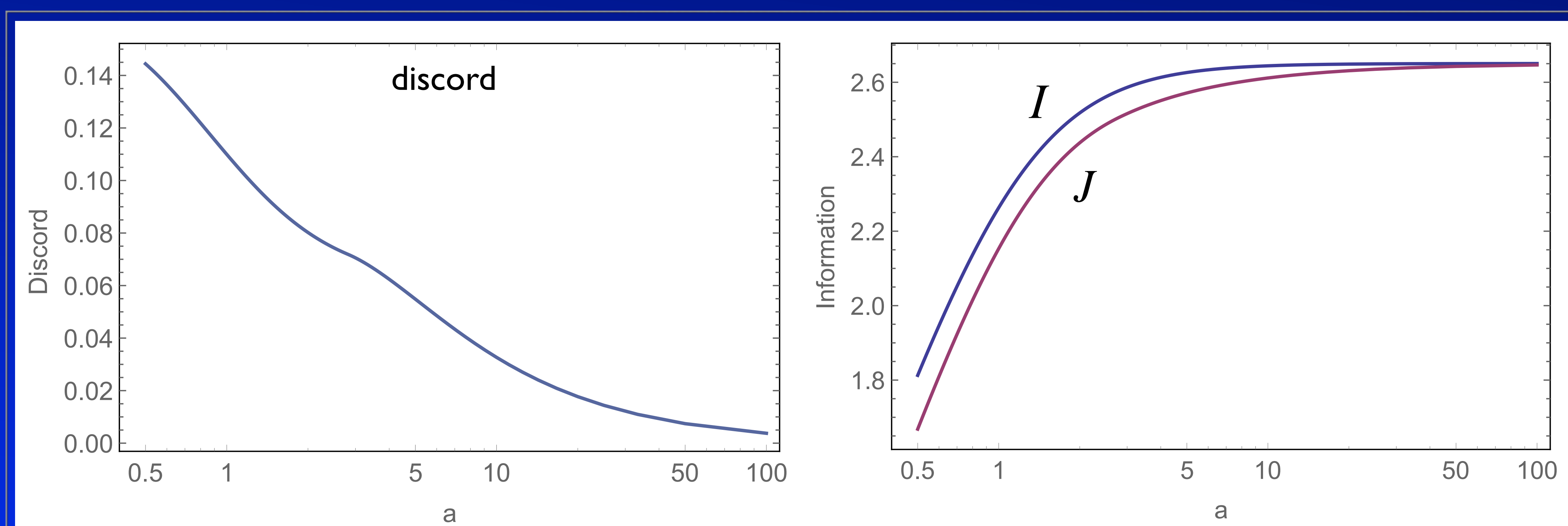
De Sitter (Bunch-Davies vacuum)







$$D = I - J$$

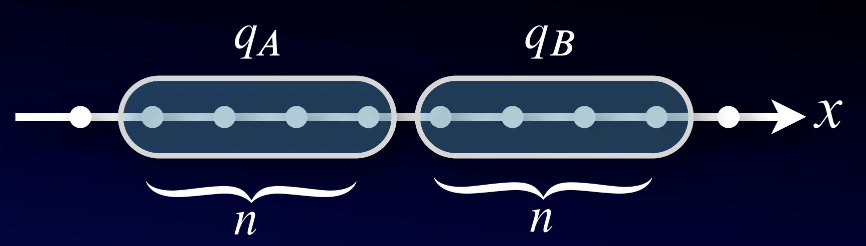


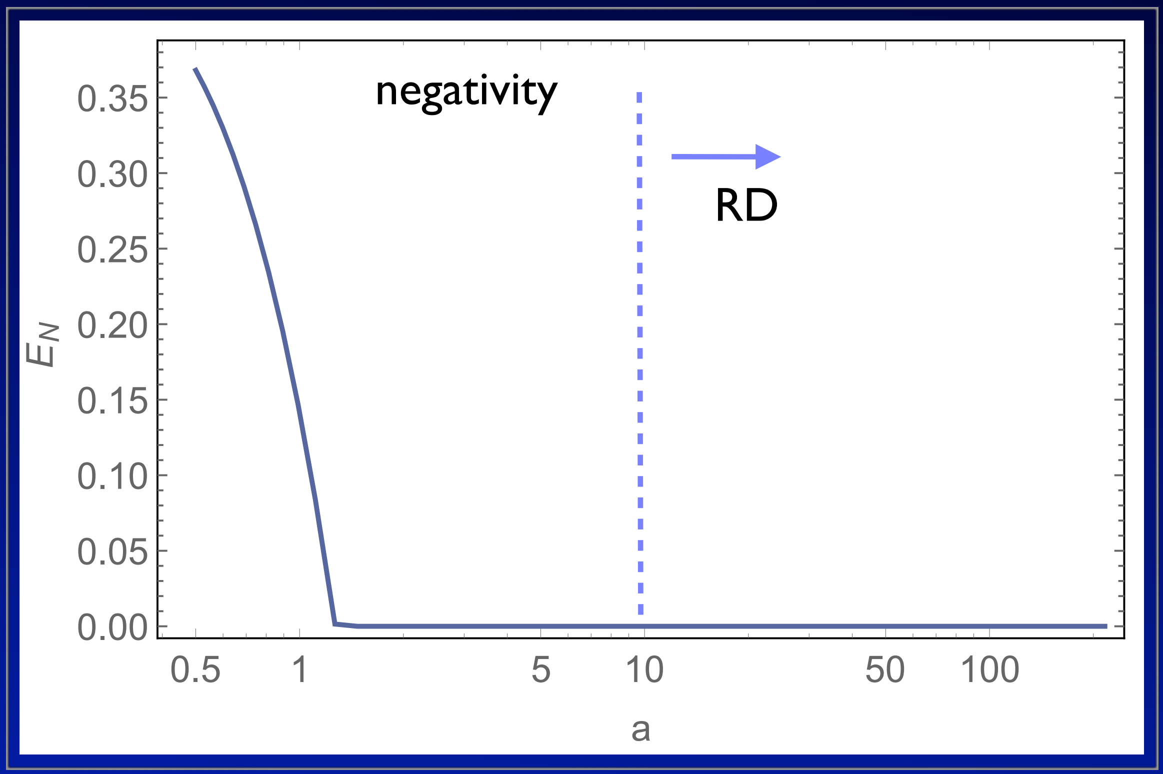
Asymptotically approaches to zero discord state

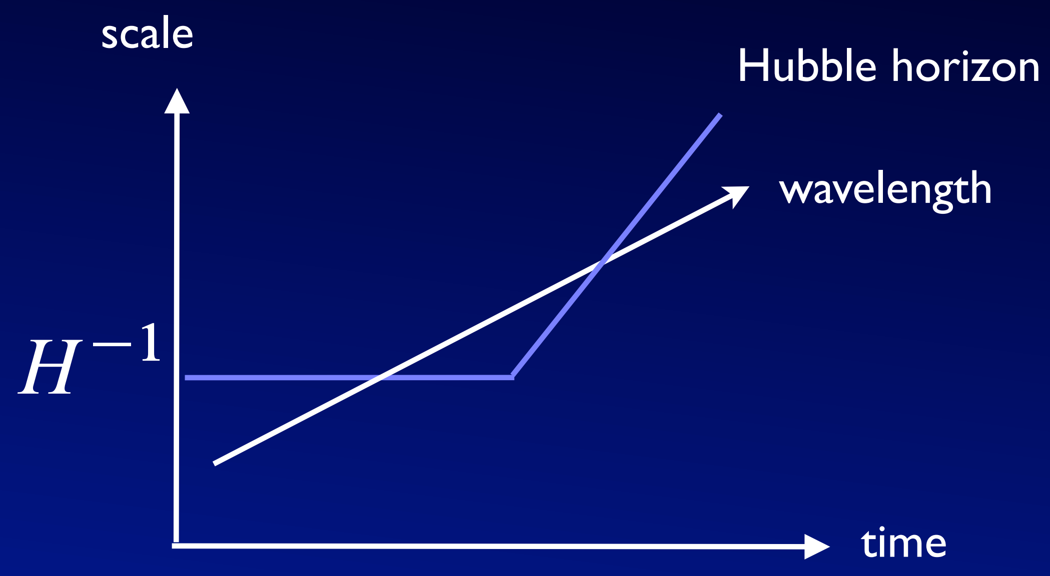
→ appearance of classical correlation

De Sitter → radiation dominant

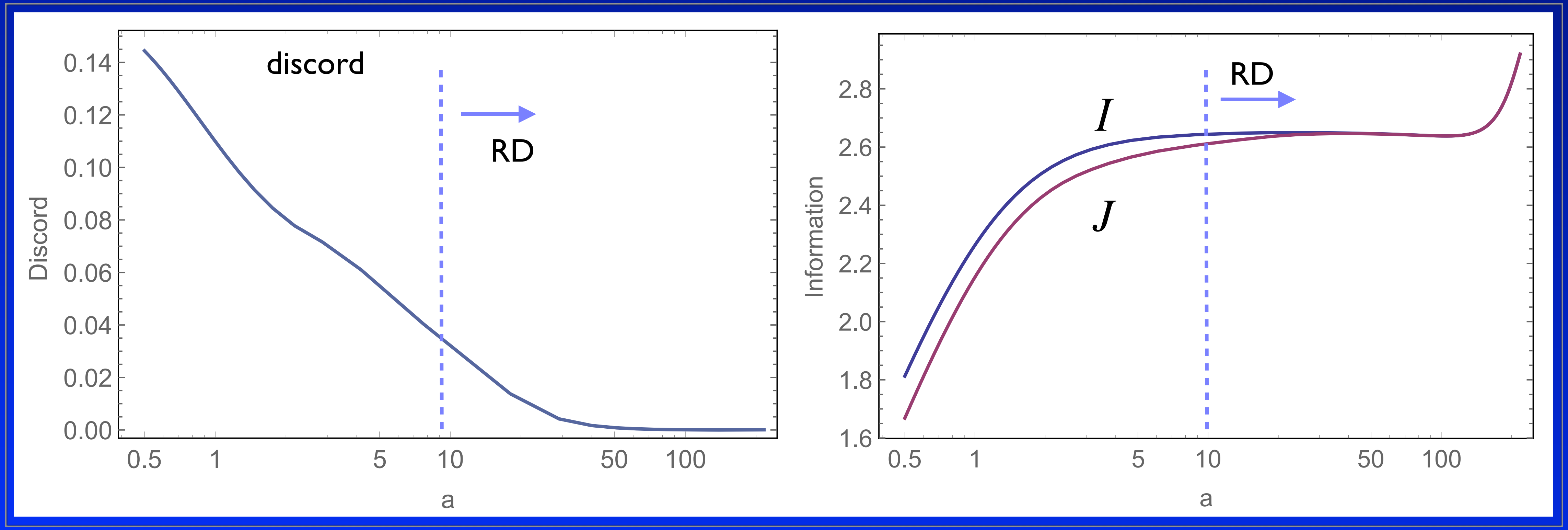
$a \propto e^{Ht}$ $\qquad$ $a \propto (t + t_0)^{1/2}$



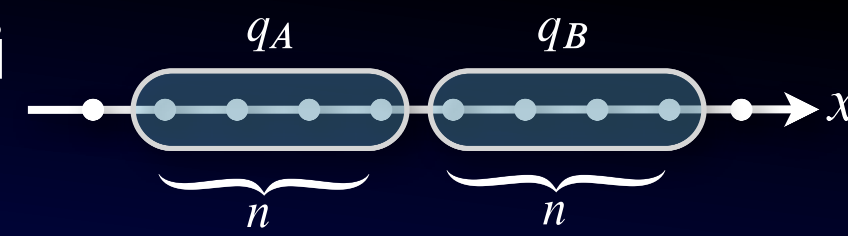




zero discord state in radiation dominant stage



Minkowski → cosmic expansion → Minkowski

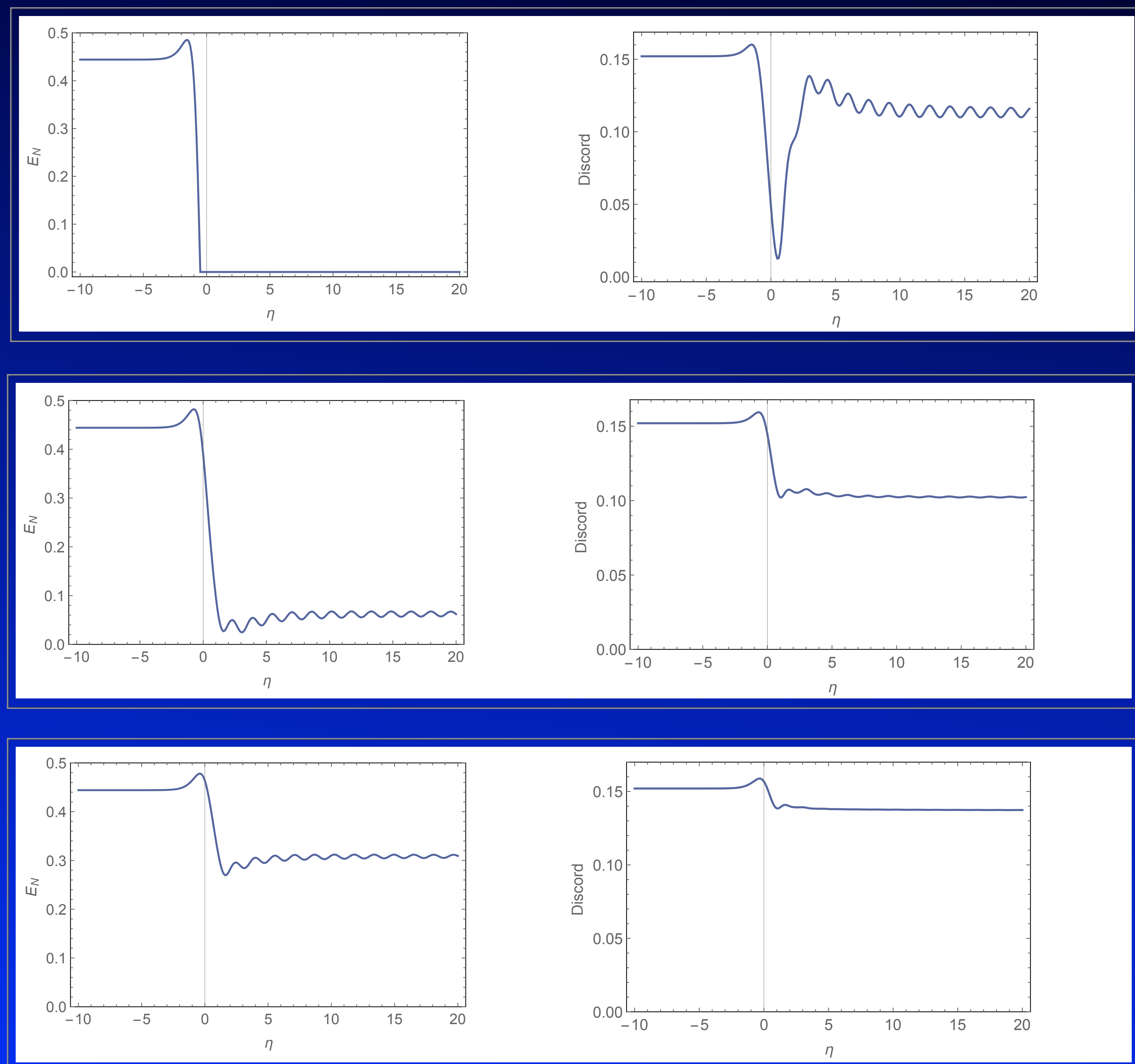


negativity | discord

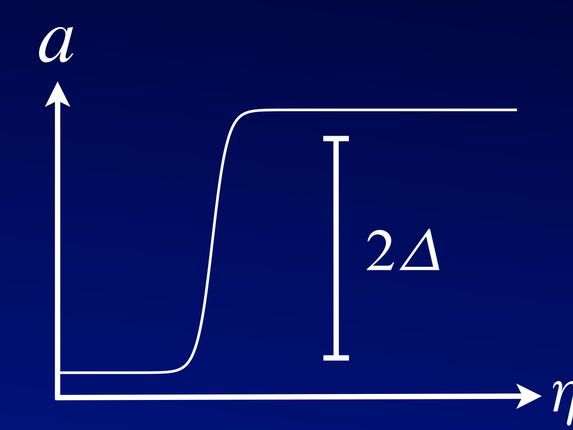
Δ=5

Δ=1

Δ=0.5



$$a(\eta) = 1 + \Delta(1 + \tanh(\eta))$$



large change of negativity

↕

small value of final discord

Evolution of entanglement and quantum correlation in De Sitter → radiation dominant universe (lattice model)

- Quantum fluctuations generated in de Sitter phase lose spatial entanglement when their wavelength exceed the Hubble horizon.

- Entanglement between adjacent spatial regions remains zero after the universe enters the phase of decelerated expansion.

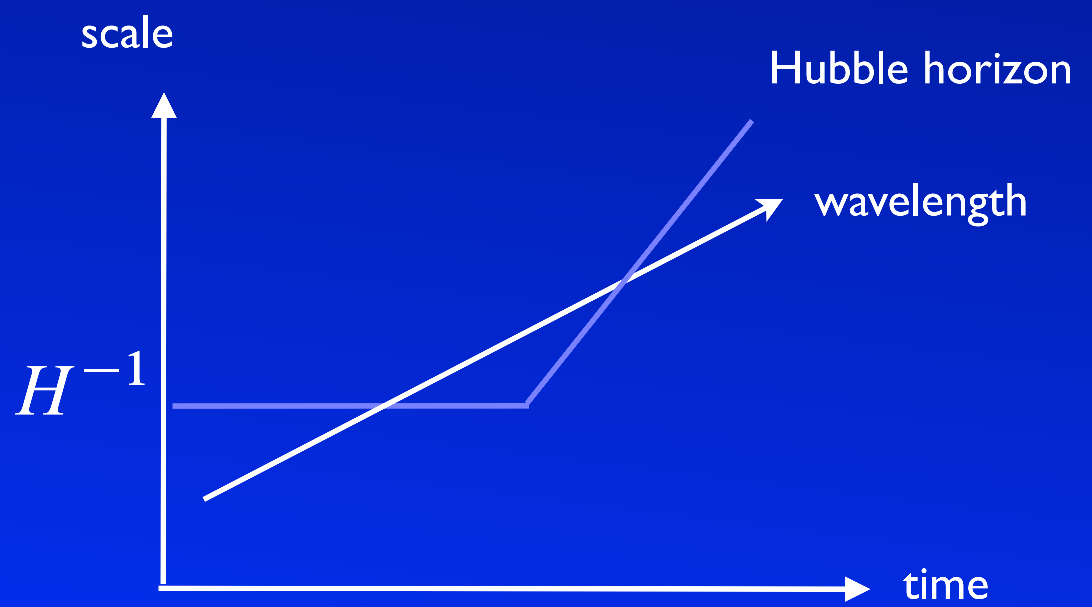
- Quantum discord has non-zero value even after the entanglement becomes zero.

- Asymptotically, quantum discord approaches zero and the zero discord state is attained.

zero discord state

⇕

total correlation=classical correlation



Quantum Estimation in Cosmology

- Estimation of model parameters

  expansion law, mass of fields, coupling of fields,......

- Classical theory: measurement result (probability)

  Fisher information

- Quantum theory: measurement result (probability) depends on POVM

  optimize about possible POVM

  quantum Fisher information

- We want to understand relation between Fisher information and entanglement in cosmological situations

Fisher information

$P(\xi|\theta)$: probability to obtain measurement result ξ with respect to POVM $\{\Pi_\xi\}$

θ : a parameter to be estimated

Fisher information

$$\mathcal{F}_\xi(\theta) = \sum_\xi P(\xi|\theta)\left(\frac{\partial \log P(\xi|\theta)}{\partial\theta}\right)^2$$

Unbiased error for θ satisfies (Cramer-Rao inequality)

$$(\Delta\theta)^2 \geq \frac{1}{\mathcal{F}_\xi(\theta)}$$

Larger Fisher information reduces the lower bound of error

Quantum Fisher information

$$\mathcal{F}_Q(\theta) = \max_{\{\Pi_\xi\}} \mathcal{F}_\xi(\theta)$$

defined by optimization wrt all POVM

This quantity can be represented by symmetric logarithm derivative:

$$\mathcal{F}_Q(\theta) = \mathrm{tr}(\rho\,\mathcal{L}_\theta^2), \quad \partial_\theta \rho \equiv \frac{1}{2}\{\rho, \mathcal{L}_\theta\}$$

For a state with form

$$\rho = \sum_n \lambda_n |\psi_n\rangle\langle\psi_n|$$

$$\mathcal{F}_Q(\theta) = \sum_m \frac{(\partial_\theta \lambda_m)^2}{\lambda_m} + 2\sum_{m\neq n} \frac{(\lambda_m - \lambda_n)^2}{\lambda_m + \lambda_n} |\langle\psi_m|\partial_\theta \psi_n\rangle|^2$$

Massive scalar field in a FRW universe

$$L = \int d^3x \sqrt{-g} \left(-\frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi + \frac{m^2}{2} \phi^2 \right)$$

$$\varphi = a\, \phi$$

$$\varphi_{\boldsymbol{k}} = \frac{1}{\sqrt{2\omega_k}} \left(b_{\boldsymbol{k}} + b_{-\boldsymbol{k}}{}^\dagger \right) \qquad p_{\boldsymbol{k}} = i \sqrt{\frac{\omega_k}{2}} \left(b_{\boldsymbol{k}}{}^\dagger - b_{-\boldsymbol{k}} \right)$$

$$\omega_k^2 = k^2 + m^2 a^2$$

Hamiltonian

$$H = \int d^3k \left[\frac{\omega_k}{2} \left(b_{\boldsymbol{k}}{}^\dagger b_{\boldsymbol{k}} + b_{\boldsymbol{k}} b_{\boldsymbol{k}}{}^\dagger \right) + i \frac{a'}{a} \left(b_{\boldsymbol{k}}{}^\dagger b_{-\boldsymbol{k}}{}^\dagger - b_{\boldsymbol{k}} b_{-\boldsymbol{k}} \right) \right]$$

generates entanglement between k,-k

$$\begin{pmatrix} b_{\boldsymbol{k}}(\eta) \\ b_{-\boldsymbol{k}}^\dagger(\eta) \end{pmatrix} = \begin{pmatrix} u_k & v_k \\ v_k^* & u_k^* \end{pmatrix} \begin{pmatrix} b_{\boldsymbol{k}}(\eta_0) \\ b_{-\boldsymbol{k}}^\dagger(\eta_0) \end{pmatrix}$$

$$u_k(\eta_0) = 1, \quad v_k(\eta_0) = 0$$

Bogoluibov coefficients

Initial "vacuum" state evolves to many particle state:

$$|0,\eta\rangle_S = \frac{1}{|u_k|^{1/2}} \exp\left[\frac{v_k}{2u_k^*}\left(a_{\boldsymbol{k}}{}^{\dagger} a_{-\boldsymbol{k}}{}^{\dagger}\right)\right]|0,\eta_0\rangle = \frac{1}{|u_k|}\sum_{n=0}^{\infty}\left(\frac{v_k}{u_k^*}\right)^n |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$$

2 mode squeezed state: entangled

Reduced density matrix

$$\rho = \sum_{n=0}^{\infty} \lambda_n |n_{\boldsymbol{k}}\rangle\langle n_{\boldsymbol{k}}|, \quad \lambda_n = \frac{\gamma^n}{|u_k|^2}, \quad \gamma \equiv \left|\frac{v_k}{u_k}\right|^2$$

Entanglement entropy

$$S_E = -\sum_{n=0}^{\infty} \lambda_n \log \lambda_n$$

Quantum Fisher information

$$\mathcal{F}_Q = \sum_{n=0}^{\infty} \lambda_n (\partial_\theta \log \lambda_n)^2$$

previous works

model universe with expansion law

$$a(\eta) = 1 + \epsilon(1 + \tanh \rho\eta)$$

- (k,-k) entanglement of a bosonic and a fermionic field

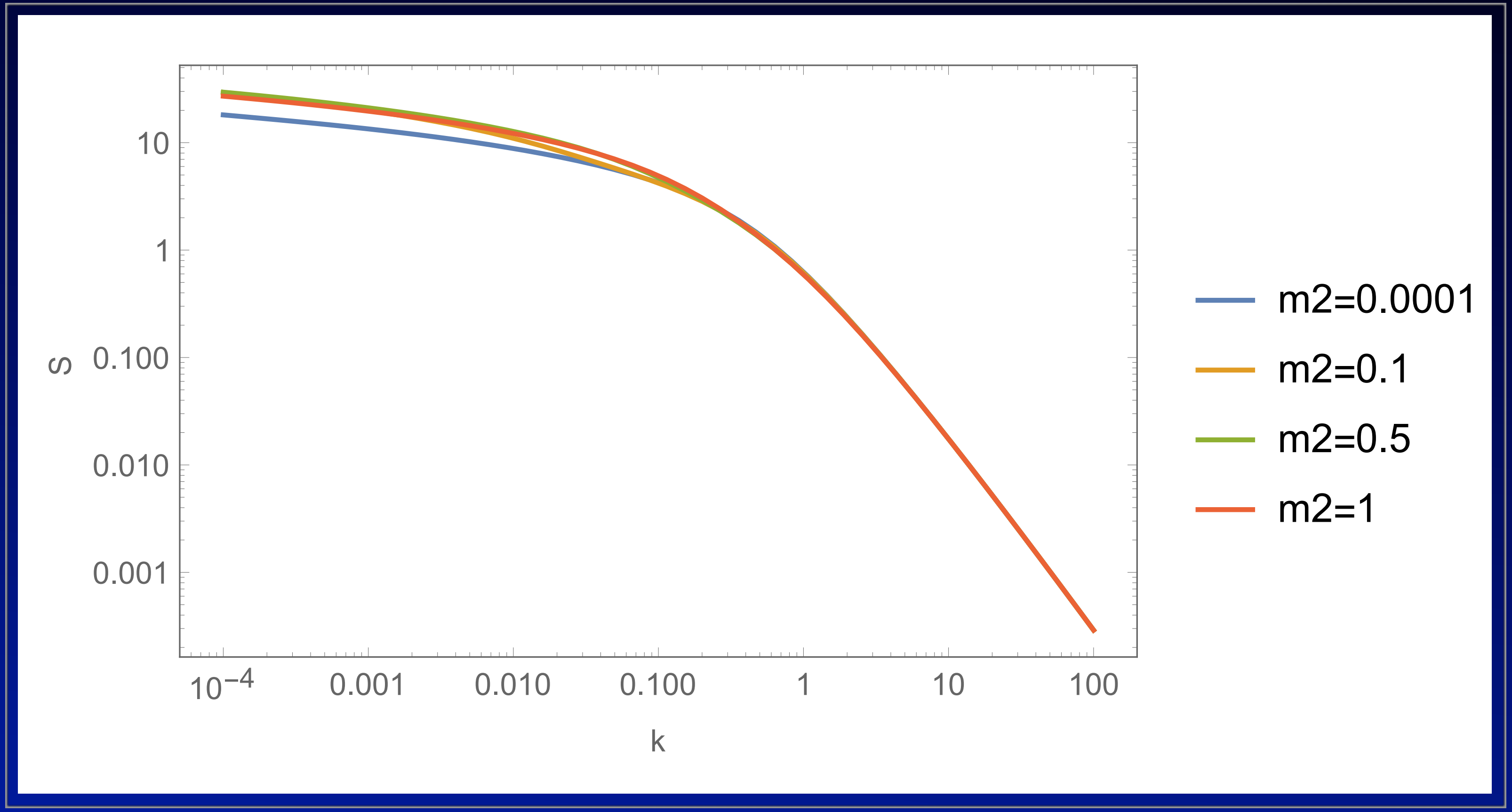
  I. Fuentes et al. PRD82, 045030(2010)

- estimation of expansion law using a fermionic field

  J. Wang et al. Nucl. Phys. B892(2015)390

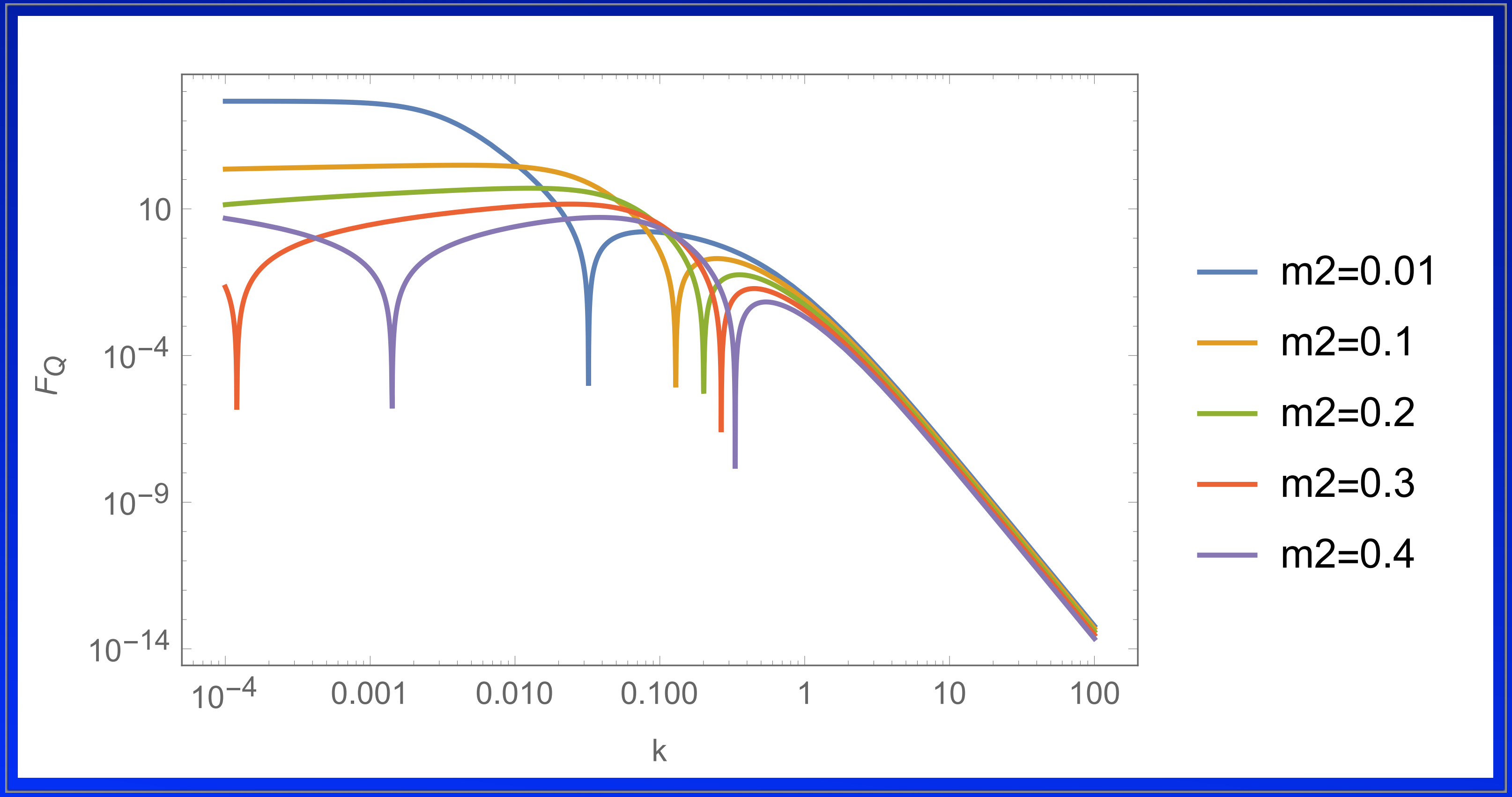
We consider a massive scalar field in de Sitter universe.

Entanglement entropy



no specific feature due to mass

Quantum Fisher information: estimation parameter is mass



oscillation due to mass appears in super horizon scale

QFI is sensitive to behavior of entanglement

physical meaning?

Summary

- Classicality (quantumness) of quantum field in a FRW universe
- Entanglement and quantum discord
- We confirm the quantum to classical transition of the scalar field using 1-dim lattice model.

The considering system approaches the zero discord state
Cause of this behavior? : particle creation due to cosmic expansion

increase of k-space entanglement ⟺ decrease of x-space entanglement
particle creation

- What is the relation between (k,-k)-entanglement and x-space entanglement?

- Entanglement, classical correlation, strength of energy(density) fluctuation

古典化の条件　（量子論の期待値を再現する確率分布の存在条件）

任意の F に対して次の関係を満たす分布関数$\mathcal{P}$が存在

$$\langle F(\hat{q}_A, \hat{p}_A, \hat{q}_B, \hat{p}_B)\rangle = \int d^2q d^2p\, \mathcal{P}(q_A, p_A, q_B, p_B) F(q_A, p_A, q_B, p_B)$$

$$\int d^2q d^2p\, \mathcal{P} = 1,\ \mathcal{P} > 0$$

- 1 自由度 X 1 自由度 Gaussian stateに対しては

系がseparable ⟺ $$\hat{\rho}_{AB} = \int d^2\alpha d^2\beta\ P(\alpha, \beta)\, |\alpha, \beta\rangle\langle\alpha, \beta|$$

(R.Simon 2000, L.Duan *et al.* 2000)

$P \geq 0$　　$|\alpha, \beta\rangle = |\alpha\rangle|\beta\rangle$ A, Bに対する coherent state

P-function　$$\langle : F(\hat{\boldsymbol{q}}, \hat{\boldsymbol{p}}) : \rangle = \int d^2q d^2p\ P(\boldsymbol{q}, \boldsymbol{p}) F(\boldsymbol{q}, \boldsymbol{p})$$

$$P(\xi) = \frac{1}{4\pi^2}\sqrt{\det \boldsymbol{P}}\exp\left(-\frac{1}{2}\xi^T \boldsymbol{P}\xi\right) \qquad \boldsymbol{P} = \boldsymbol{S}^T\left(V_{II} - \frac{\boldsymbol{I}}{2}\right)^{-1}\boldsymbol{S}$$

$$\boldsymbol{S} \in \mathrm{Sp}(2,R) \otimes \mathrm{Sp}(2,R)$$

standard form

$$V_{II} = \boldsymbol{S}V\boldsymbol{S}^T = \begin{pmatrix} ar & & cr & \\ & a/r & & c'/r \\ cr & & ar & \\ & c'/r & & a/r \end{pmatrix} \qquad r = \sqrt{\frac{a-|c'|}{a-|c|}}$$

$$V_{II} - \frac{\boldsymbol{I}}{2} \geq 0 \quad \Longleftrightarrow \quad \tilde{\nu}_- \geq \frac{1}{2}$$

P-funcの存在条件 separability

Wigner function

$$W(\boldsymbol{q},\boldsymbol{p}) = [\det V]^{-1/2} \exp\left(-\frac{1}{2}\boldsymbol{\xi}^T V^{-1}\boldsymbol{\xi}\right) \qquad \boldsymbol{\xi} = (q_A, p_A, q_B, p_B)^T$$

任意の関数$F(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}})$ に対して

$$\langle\{F(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}})\}_{\rm sym}\rangle = \int d^2q d^2p\, W(\boldsymbol{q},\boldsymbol{p})F(\boldsymbol{q},\boldsymbol{p})$$

Wigner func. : $V > 0$ なら存在

$$\langle :F(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}}):\rangle = \int d^2q d^2p\, P(\boldsymbol{q},\boldsymbol{p})F(\boldsymbol{q},\boldsymbol{p})$$

P-func. : separableなら存在

separableの条件下で古典化の条件は

$$\langle\{F(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}})\}_{\rm sym}\rangle \approx \langle :F(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}}):\rangle \approx \langle F(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}})\rangle$$

$\hat{q},\hat{p}$ の非可換性が無視できる

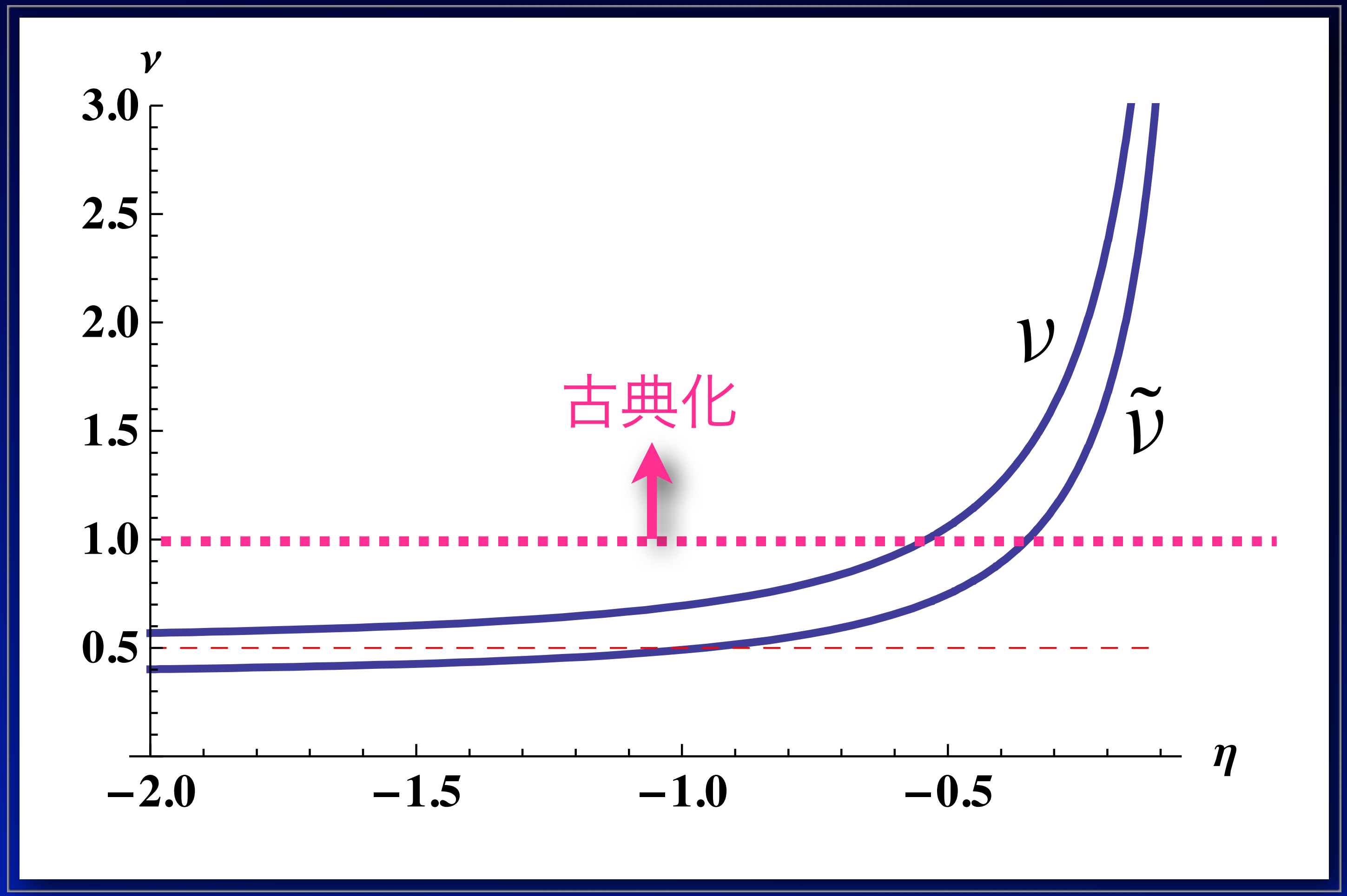
⇔

$$P(\boldsymbol{q},\boldsymbol{p}) \approx W(\boldsymbol{q},\boldsymbol{p})$$

$\nu, \tilde{\nu} \gg 1$ (nambu, 2008)

$$\Rightarrow \quad \langle F(\hat{\boldsymbol{q}},\hat{\boldsymbol{p}})\rangle \approx \int d^2q d^2p\, W(\boldsymbol{q},\boldsymbol{p})F(\boldsymbol{q},\boldsymbol{p})$$

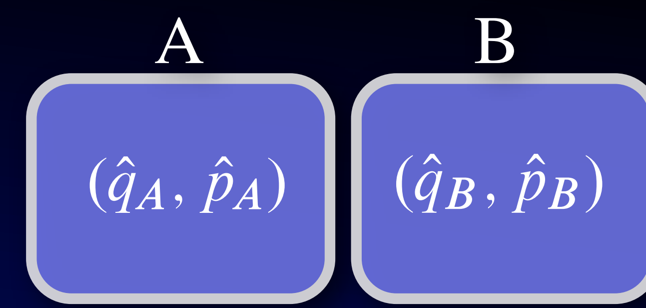
lattice model



symplectic eigenvalueの時間発展

- 漸近的には古典化条件が成立

- 古典分布関数の構造



$$W \approx W_1(\varphi_A, p_A) W_1(\varphi_B, p_B) \times \exp\left[\frac{c}{2a^2}(\varphi_A - \varphi_B)^2\right] \exp\left[-\frac{c'}{2a^2}(p_A + p_B)^2\right]$$

$\nu, \tilde{\nu} \gg 1 \ (c/a, c'/a \ll 1)$ でほぼ一定

量子相関の名残り

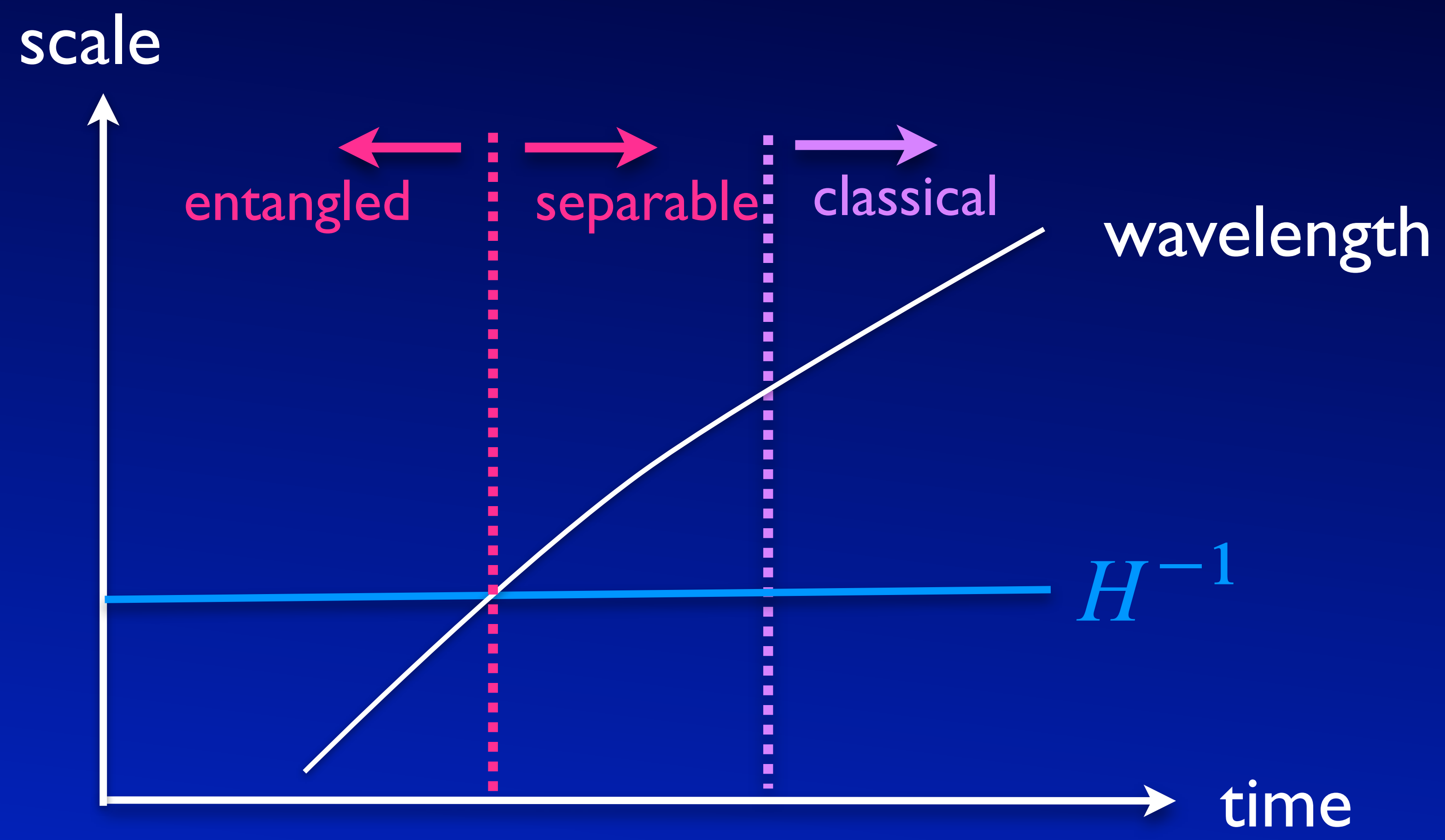
$\nu, \tilde{\nu} \gg 1$ ならば

$$W \approx W_1(\varphi_A, p_A) W_1(\varphi_B, p_B)$$

⇒ A, Bの変数は独立な確率変数として扱える

Summary

2体entanglementに基づく古典化に到る流れ



- 領域の大きさがhorizon scaleを超すと領域間はseparable

  ➡ horizonが量子相関の有無を決定

- separableになってからone Hubble time程度で“古典化”

  ➡ 相関関数を再現する古典分布関数の出現

今後の課題

- massの効果 (Compton wavelength)
- 空間次元
- 具体的なinflation modelでの評価
- entanglementとgeometryの関係
- N-party entanglement