An introductory course on
Real and Complex Microlocal Analysis
Sendai April 2016

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I will discuss some links between D-modules on a complex manifold and sheaves on a real manifold, from a microlocal point of view.

Lecture 1

D-modules [Kas03]: characteristic variety, direct and inverse images, holonomic systems.
Sheaves [KS90]: microsupport, direct and inverse images, constructible sheaves.
The de Rham functor: from D-modules to sheaves.

Lecture 2

Microlocalization for D-modules [SKK73, KS12]: microdifferential modules and DQ-modules.
Microlocalization for sheaves: the functor μhom and open problems [Tam15].

Lecture 3

The Riemann-Hilbert correspondence (regular case) after M. Kashiwara.
Subanalytic topology and the sheaf of temperate holomorphic functions, operations [KS01].
A modern proof of the regular R-H correspondence [KS16].

References

[Kas03] Masaki Kashiwara, *D-modules and Microlocal Calculus*, Translations of Mathematical Monographs, vol. 217, American Math. Soc., 2003.

[KS90] Masaki Kashiwara and Pierre Schapira, *Sheaves on manifolds*, Grundlehren der Mathematischen Wissenschaften, vol. 292, Springer-Verlag, Berlin, 1990.

[KS12] _____ , *Deformation quantization modules*, Astérisque, vol. 345, Soc. Math. France, 2012.

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[KS16] _____ , *Regular and irregular holonomic D-modules*, Lecture Note Series, London Math Society, 2016.

[SKK73] Mikio Sato, Takahiro Kawai, and Masaki Kashiwara, *Microfunctions and pseudo-differential equations*, Hyperfunctions and pseudo-differential equations (Proc. Conf., Katata, 1971; dedicated to the memory of André Martineau), Springer, Berlin, 1973, pp. 265–529. Lecture Notes in Math., Vol. 287.

[Tam15] Dmitry Tamarkin, *Microlocal category* (2015), available at `arXiv:1511.08961`.