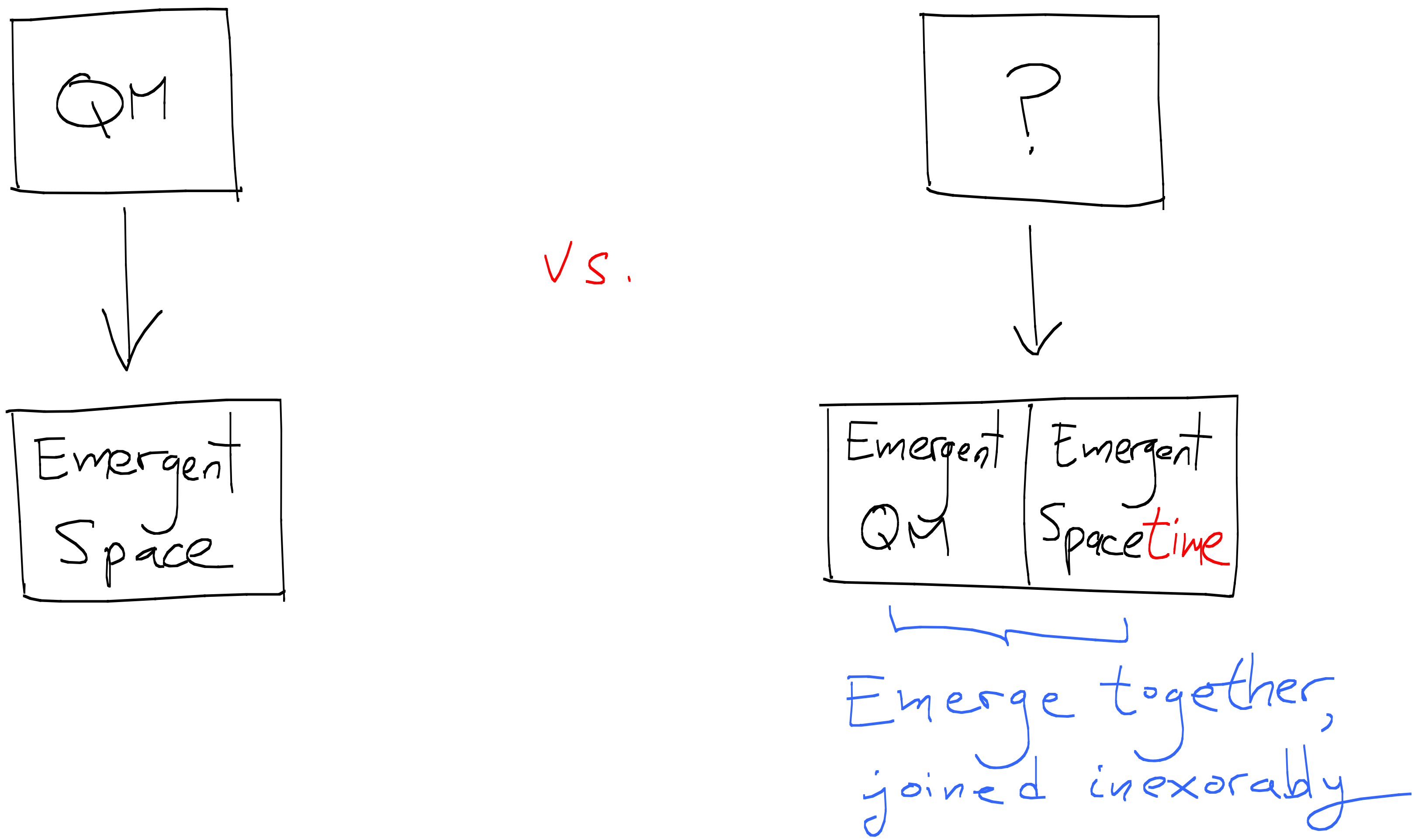
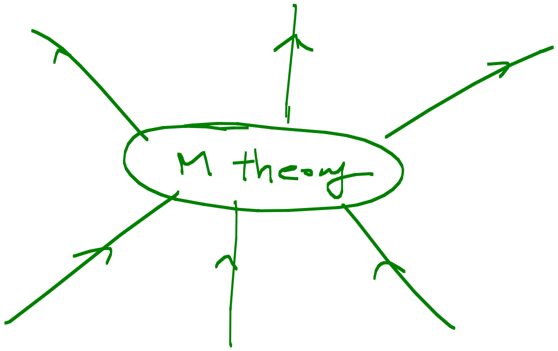
Spacetime, QM

+

Positive Geometry at Infinity


QM
Emergent Space
vs.
?
Emergent QM
Emergent Spacetime
Emerge together, joined inexorably

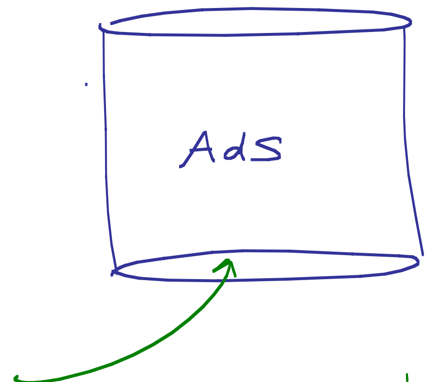
Big Tension



One Unified Theory!

Landscape of connected solutions. UNIFIED in FLAT SPACE

vs.

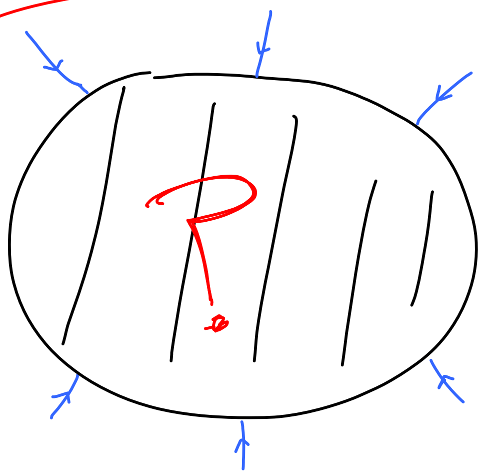


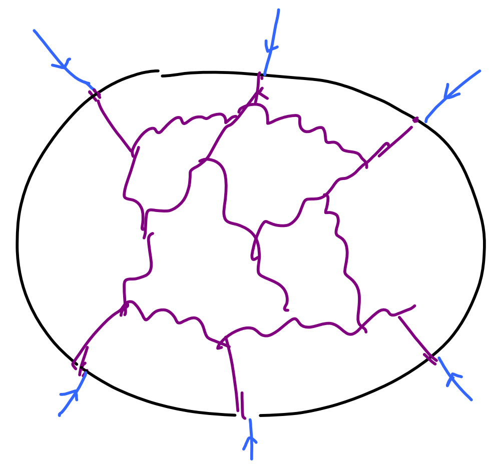
Any old Quantum Theory

Is a different theory in AdS!

LOOKING FOR MORE UNIVERSAL/UNIFIED "BULK" DESCRIPTION $\leftrightarrow$ STUDY FLAT SPACE S-MATRIX, WITHOUT SHACKLES OF MANIFEST QM, FOR A ROAD BACK INTO THE BULK

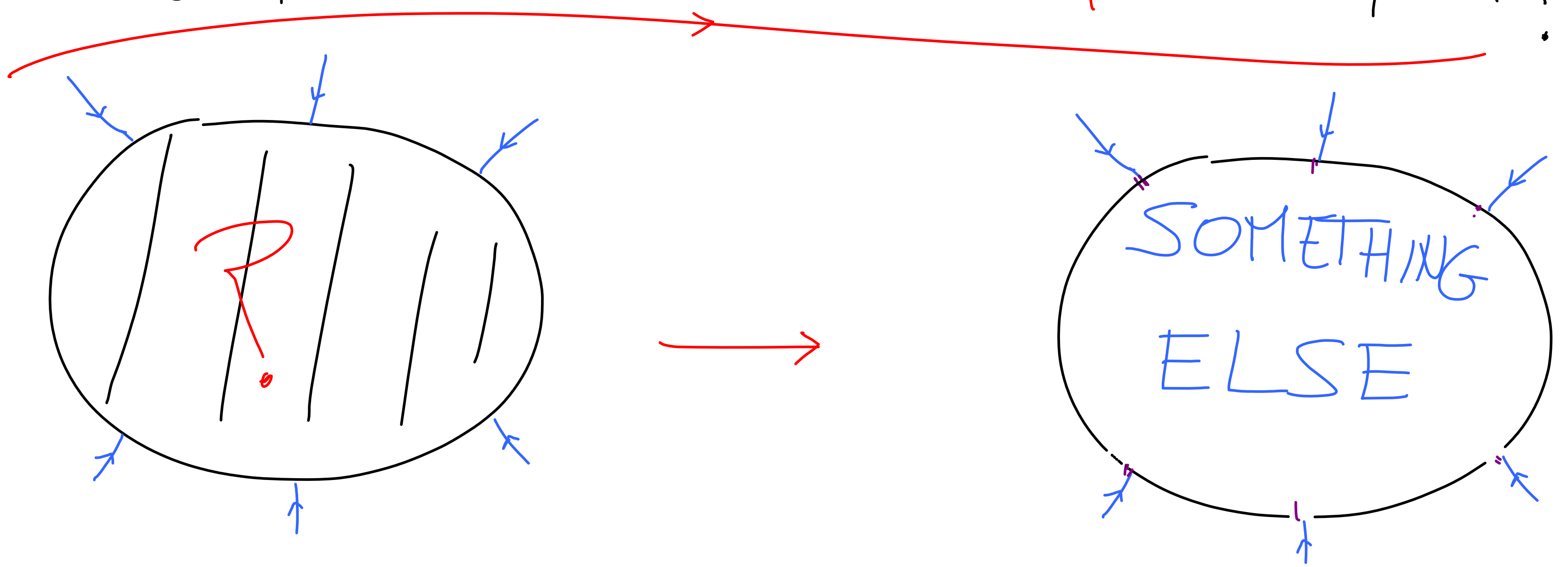
What is the Q to which A is the Answer?





Local, Unitary Evolution in Spacetime

What is the Q to which A is the Answer?



New Strategy: Look For

NEW PRINCIPLES

from which CAUSAL, UNITARY

evolution — local Spacetime Physics + QM,

emerge together.

The Canvas

* Physical momenta
* "Twistor" variables
* "Celestial Sphere"

⋮

* Spatial Future

Kin. Space
$\sim \partial$ Minkowski

(Amps)

(Ψ_{Univ})

Note Unlike
e.g ∂AdS:

NO USEFUL TIME
NO LOCALITY

WHAT IDEAS BREATHE PHYSICS-LIFE INTO THIS SPACE?

The Canvas

* Physical momenta
* "Twistor" variables
* "Celestial Sphere"

⋮

* Spatial Future

Kin. Space ~ ∂ Minkowski

(Amps)

(Ψ_{Univ})

Note Unlike e.g ∂AdS:

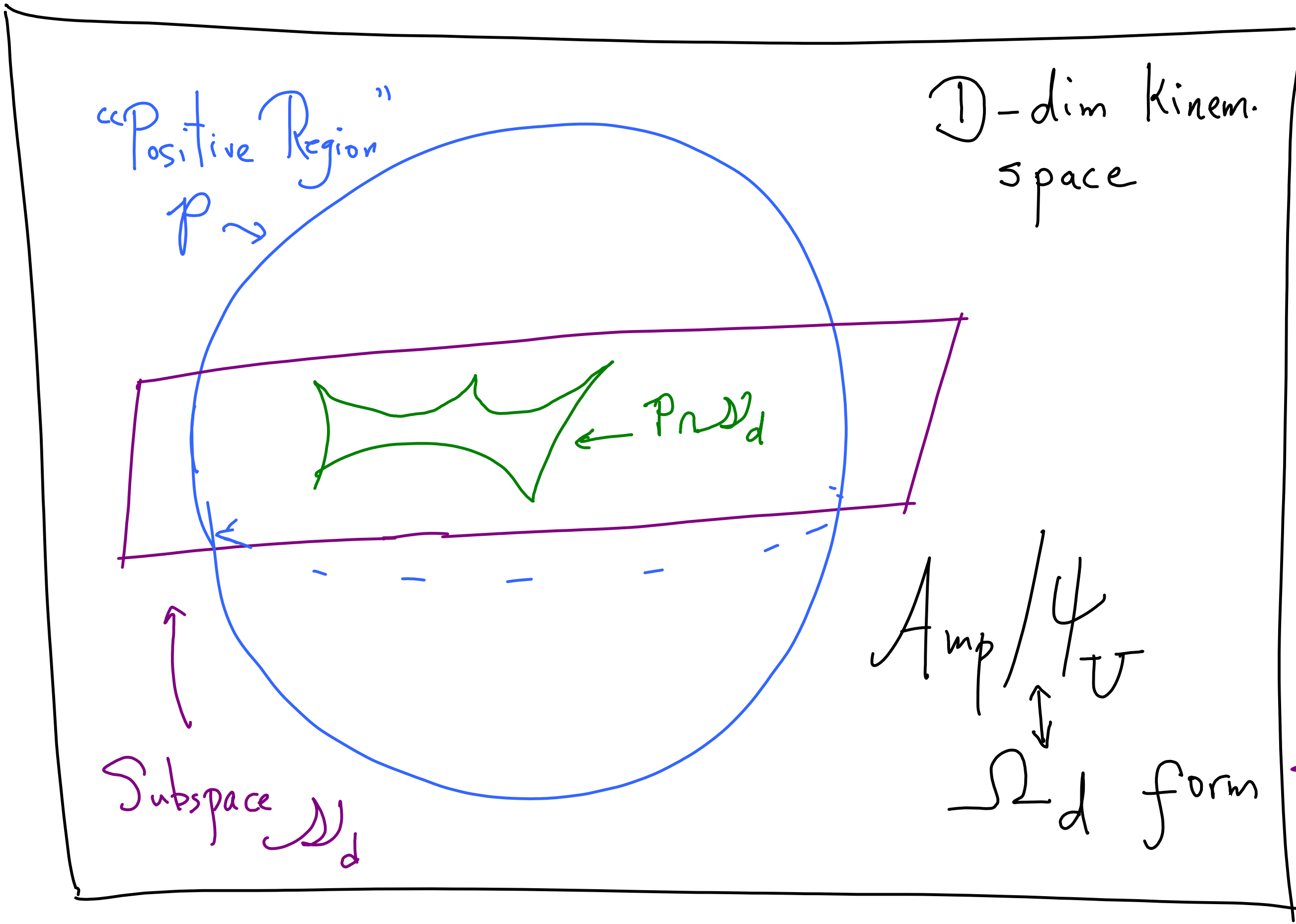
NO USEFUL TIME

NO LOCALITY

EMERGING PICTURE:

Combinatorics ⟷ "Positive Geometries" ⟷ Canonical Forms

General Picture

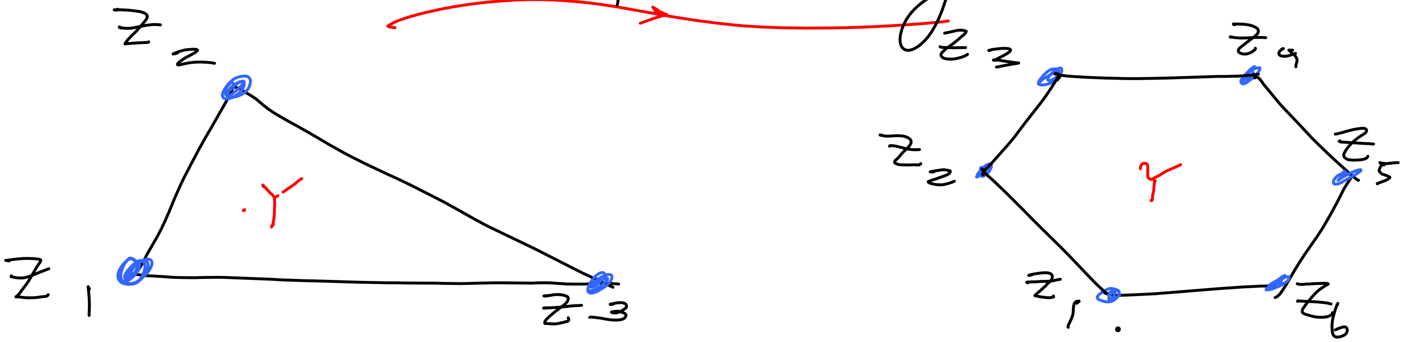


Ω_d fixed thusly:

$\mathcal{S}_d$ intersects $\mathcal{P}$ in a POSITIVE GEOMETRY

Ω_d Pulls Back to CANONICAL FORM

Positive Geometry



$$Y^I = c_a Z_a^I$$

$c_a \geq 0$

$\langle Z_{a_1} Z_{a_2} Z_{a_3} \rangle \geq 0$

$a_1 < a_2 < a_3$

+ Canonical Form $\Omega_{\mathcal{P}}[Y; Z_a]$: dlog. sing on all ∂'s of $\mathcal{P}$

(Tree) Amplituhedron

$$Y_\alpha^I = C_{\alpha a} Z_a^I$$

Region in $G[k, k+m]$

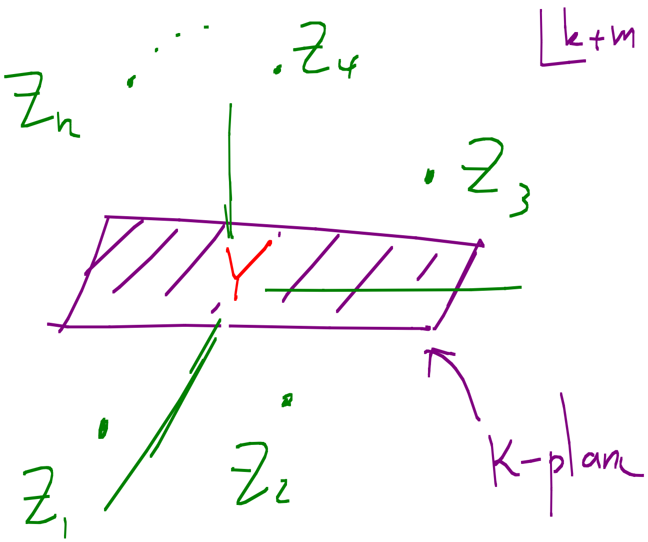
$(C_{a_1} \cdots C_{a_k}) \geq 0$
"Positive Grassmannian"

$\langle Z_{a_1} \cdots Z_{a_{k+m}} \rangle \geq 0$
"Positive" external data

Generalizes Polytopes into the Grassmannian

(I. Gelfand ~ '90s: "You Polytope people are doing trivial things. You should generalize polytopes into Grassmannians!")

Are You In or Out?



When is Y in the amplituhedron?

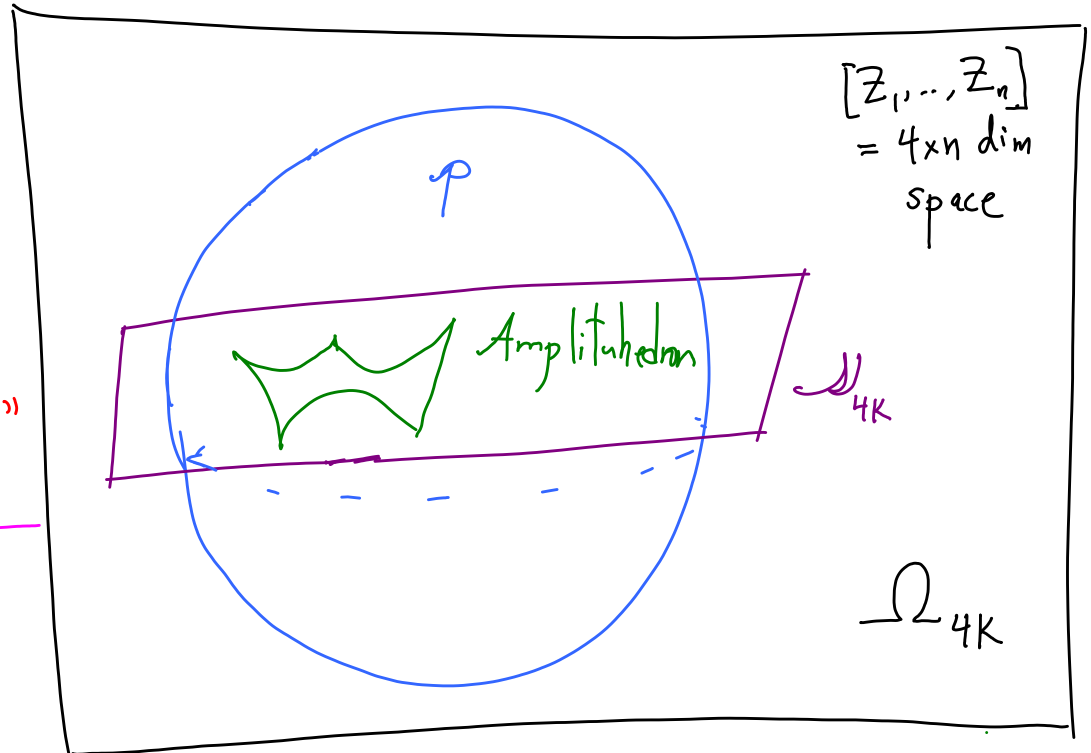
Project through $Y \to m$ dim. picture. What does this picture look like?

{Physics $m=4$: This is precisely conf. of "momentum twistors"}

$\mathcal{P}$: Config. of $\{Z_1, \ldots, Z_n\}$ has fixed "binary code"

$\Rightarrow$ physical poles > 0

$+$ maximal "winding #"



In full:

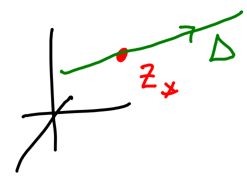
$\langle i\, i{+}1\, j\, j{+}1 \rangle > 0$

$\{\langle 1234 \rangle, \ldots, \langle 123n \rangle\}$ has k sgn flips

$\langle AB_\alpha\, i\, i{+}1 \rangle > 0$, $\langle AB_\alpha AB_\beta \rangle > 0$

$\{\langle AB_\alpha 12 \rangle, \ldots, \langle AB_\alpha 1n \rangle\}$ has $k+2$ sgn flips

$\mathcal{W}_{4K}$: Affine subspace



$$Z_a^I = Z_{*a}^I + y_\alpha^I \Delta_a^\alpha$$

$$\begin{pmatrix} Z_* \\ \hline \Delta \end{pmatrix} \cap G_+(4+K, n)$$

* The Physics of Locality + Unitarity — ST + QM — arise, joined at the hip, from this primitive combinatorics + geometry in kin. space @ infinity

* This mathematical structure describes real-world collisions of gluons — from cosmic rays to the LHC — in leading HE, tree approximation.

Planar $\mathcal{N}=4$ SYM Amps

ϕ^3 theory, Pions, Gluons...

$\Psi_{\mho}$ [φ^N theories]

Amplituhedra

(Generalized) Associahedra

Cosmological Polytopes

Many new developments in past 6 months, will be discussed in talks @ Amplitudes 2018 later this week

Planar $\mathcal{N}=4$ SYM Amps

Amplituhedra

ϕ^3 theory, Pions, Gluons...

(Generalized) Associahedra

$\Psi_{\mathcal{U}}$ [φ^N theories]

Cosmological Polytopes

Effective Field Theory

"EFThedron"
Hidden Positive Geometry
of Causality + Unitarity

Conformal Field Theory

"CFThedra"
Positive Geometry of Conformal Bootstrap.

The EFT-hedron

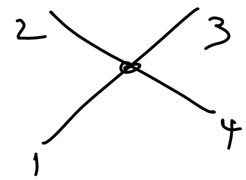
w/ Huang + Huang

* Universal Geometries controlling EFT

* In particular, infinitely many quantitative predictions about quantum gravity in the real world:

Locality [Causality], Unitarity $\Longrightarrow$ Positivity

e.g. $\mathcal{L} = -F_{\mu\nu}^2 + c\,F_{\mu\nu}^4 + \cdots$



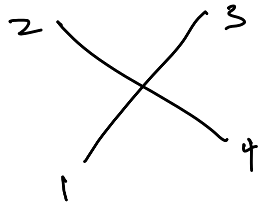
$A = c(s^2+t^2+u^2)$

$c > 0$

* Crucial for protecting consistency of Horizon thermodynamics!

* Related to weak gravity: $c<0 \Longleftrightarrow \delta\left(M/Q\right)\Big|_{\substack{\text{extr}\\ \text{BH}}} < 0$

Higher-Dim Ops in 4-particle Scattering

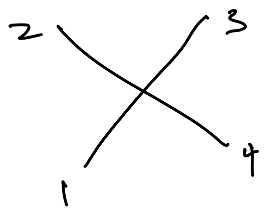


$$A^{\text{LE, contact}} = \left(\langle 12\rangle[34]\right)^{2s}\left[\sum a_{D,q}\, s^{D-q}\, t^{q}\right]$$

carries dim. $\rightarrow t_D$ $\qquad \hat{a}_{D,q} \leftarrow$ dimless, projective

$$\left(t_0, t_1, t_2, t_3 \ldots\right) \qquad \begin{pmatrix}\hat{a}_{0,0}\end{pmatrix}\begin{pmatrix}\hat{a}_{1,0}\\ \hat{a}_{1,1}\end{pmatrix}\begin{pmatrix}\hat{a}_{2,0}\\ \hat{a}_{2,1}\\ \hat{a}_{2,2}\end{pmatrix}\begin{pmatrix}\hat{a}_{3,0}\\ \hat{a}_{3,1}\\ \hat{a}_{3,2}\\ \hat{a}_{3,3}\end{pmatrix}\ldots$$

Higher-Dim Ops in 4-particle Scattering



$$A^{LE,\,\text{contact}} = \left(\langle 12\rangle[34]\right)^{2S}\left[\sum a_{D,q}\, s^{D-q}\, t^{q}\right]$$

carries dim. $\longrightarrow t_D$ $\quad \hat{a}_{D,q} \longleftarrow$ dimless, projective

$$\left(t_0, t_1, t_2, t_3 \ldots\right)$$

Satisfy ∞'ly many non-linear inequalities

$$\begin{pmatrix}\hat{a}_{0,0}\end{pmatrix}\begin{pmatrix}\hat{a}_{1,0}\\ \hat{a}_{1,1}\end{pmatrix}\begin{pmatrix}\hat{a}_{2,0}\\ \hat{a}_{2,1}\\ \hat{a}_{2,2}\end{pmatrix}\begin{pmatrix}\hat{a}_{3,0}\\ \hat{a}_{3,1}\\ \hat{a}_{3,2}\\ \hat{a}_{3,3}\end{pmatrix}\cdots$$

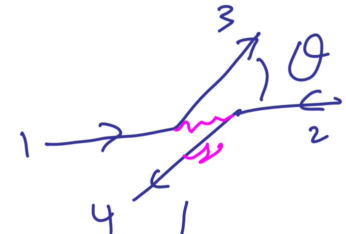
Must lie within Specific Polytopes

[Special cases of a single more abstract unified statement]

Causality + Unitarity

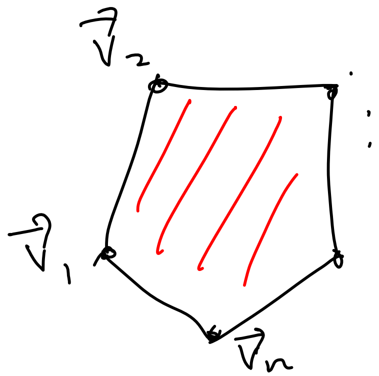
"subtraction terms"

$$-A(s,t) = a(t) + b(t)\, s + \sum_n \frac{R_n\left[\cos\theta = 1+\frac{2t}{m_n^2}\right]}{m_n^2 - s} + u\text{-chann.}$$



* $\boxed{m_n^2 > 0}$
* $R_n[\cos\theta] = \sum_{\ell} \boxed{P_{n,\ell} > 0}\, G_\ell[\cos\theta]$

Polytopes 101



$$\vec{A} = \frac{w_1 \vec{V}_1 + \cdots + w_n \vec{V}_n}{w_1 + \cdots + w_n}$$

$$\boxed{w_a > 0}$$

Better Projectively: $A^I = \begin{pmatrix} 1 \\ \vec{A} \end{pmatrix}, \; V_i^I = \begin{pmatrix} 1 \\ \vec{V}_i \end{pmatrix}$

$$A^I = w_1 V_1^I + \cdots + w_n V_n^I$$ "Convex Hull"

Check in A is inside? "Face" description $A^I W_I^{(a)} \geq 0$

Facet structure completely captured by $\langle V_{a_1} \cdots V_{a_D} \rangle \begin{cases} > 0 \\ < 0 \\ = 0 \end{cases}$

Very Special Class of Polytopes

$$\begin{array}{c} \\ \updownarrow D \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{pmatrix} V_1 & V_2 & \cdots & V_n \end{pmatrix} \end{array} \leftarrow \text{Ordering}$$

$$\langle V_{a_1} \cdots V_{a_D} \rangle > 0 \quad \text{for } a_1 < \cdots < a_D$$

Matrix: "Total Positivity", "Positive Grassmannian"

Vectors, vertices of "Cyclic Polytope" = "k=1 Amplituhedron"

Know all Facets!

$$\langle A\, V_i V_{i+1}\, V_j V_{j+1} \cdots V_k V_{k+1} \rangle \geq 0$$

Surprise: Hidden Total Positivity In

$$\frac{P_n}{m_n^2 - s}\,, \qquad G_\nu[\cos\theta]$$

Locality + Unitarity

Total Positivity of Propagators

$$-A(s) = \sum \frac{p_n}{m_n^2 - s} = \sum a_p s^p \Longrightarrow$$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \end{pmatrix} = \sum \frac{p_n}{m_n^2} \begin{pmatrix} 1 \\ m_n^{-2} \\ (m_n^{-2})^2 \\ \vdots \end{pmatrix} = \underset{x \geq 0}{\mathrm{Conv.}} \left\{ \begin{pmatrix} 1 \\ x \\ x^2 \\ \vdots \end{pmatrix}_s \right\}$$

"Moment Curve"

Now

$$\begin{vmatrix} 1 & \cdots & 1 \\ x_1 & & x_{D+1} \\ \vdots & & \vdots \\ x_1^D & & x_{D+1}^D \end{vmatrix} = \prod_{i<j} (x_j - x_i) > 0$$

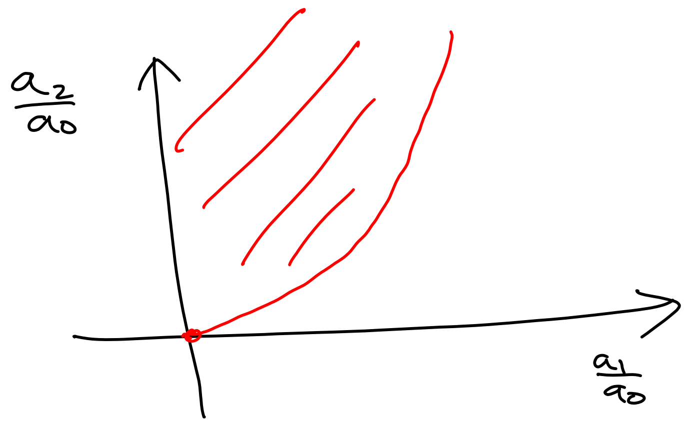
non-trivial $\longrightarrow$

$$\begin{vmatrix} \boxed{a_0} & a_1 & a_2 & \cdots \\ a_1 & a_2 & a_3 & \cdots \\ a_2 & a_3 & a_4 & \cdots \end{vmatrix}$$

<u>All</u> minors of "Hankel matrix" are Positive!

e.g. $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \underset{x \geq 0}{\mathrm{conv}} \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix}$. Trivially $a_{0,1,2} > 0$

[old known fact]

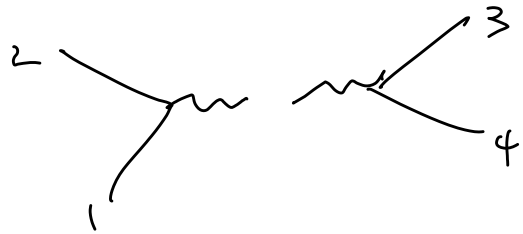
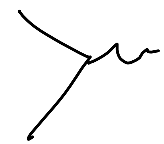
But 

$a_1 > 0$

$a_2 a_0 - a_1^2 > 0$

$$\begin{pmatrix} \boxed{\begin{matrix} \boxed{a_0} & a_1 \\ \boxed{a_1} & a_2 \end{matrix}} \end{pmatrix}$$

Hidden Positivity of Gegenbauers

$$G_{s}[\cos\theta] =$$  $$= \Big|$$  $$\Big|^2$$ when $\cos\theta \to 1$

Standard

But expand $G_{s}[1+y] = G_{s,q}\, y^{q} \;\ldots$

$$
\begin{array}{c|cccccc}
 & s=0 & 1 & 2 & 3 & 4 & \cdots \\
\hline
y^0 & 1 & 1 & 1 & 1 & 1 & \cdots \\
y^1 & 0 & 1 & 3 & 6 & 10 & \cdots \\
y^2 & 0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \cdots \\
y^3 & 0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{array}
$$

$$
\begin{array}{c}
\\ y^0 \\ y^1 \\ y^2 \\ y^3 \\ \vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc} s=0 & 1 & 2 & 3 & 4 & \cdots \end{array} \\
\begin{pmatrix}
\boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \cdots \\
0 & 1 & 3 & 6 & 10 & \cdots \\
0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \cdots \\
0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{pmatrix}
\end{array}
$$

Obvious

$$
\begin{array}{c c}
 & \begin{array}{cccccc} s=0 & 1 & 2 & 3 & 4 & \cdots \end{array} \\
\begin{array}{c} y^0 \\ y^1 \\ y^2 \\ y^3 \\ \vdots \end{array} &
\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & \cdots \\
0 & \boxed{1} & \boxed{3} & \boxed{6} & \boxed{10} & \cdots \\
0 & 0 & \boxed{\frac{3}{2}} & \boxed{\frac{15}{2}} & \boxed{\frac{45}{2}} & \cdots \\
0 & 0 & 0 & \boxed{\frac{5}{2}} & \boxed{\frac{35}{2}} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{pmatrix}
\end{array}
$$

Trivial, noticed by many (Martin '60s)

$$
\begin{array}{c|c}
 & \begin{array}{cccccc} s=0 & 1 & 2 & 3 & 4 & \cdots \end{array} \\
\begin{array}{c} y^0 \\ y^1 \\ y^2 \\ y^3 \\ \vdots \end{array} &
\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & \cdots \\
0 & 1 & 3 & 6 & 10 & \cdots \\
0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \cdots \\
0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{pmatrix}
\end{array}
$$

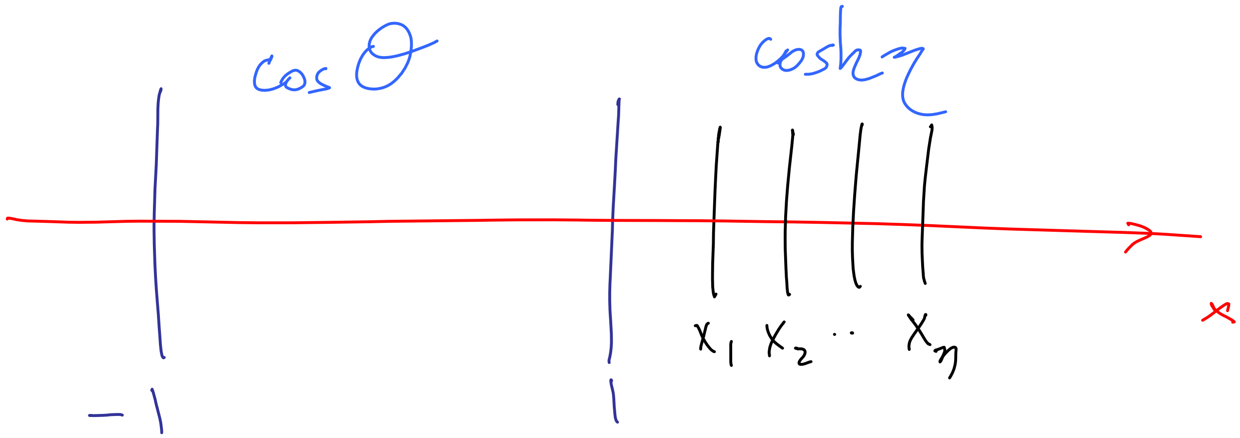
Totally Positive!

$$
\begin{array}{c}
\\
y^0 \\
y^1 \\
y^2 \\
y^3 \\
\vdots
\end{array}
\begin{array}{c}
\begin{array}{cccccc}
s=0 & 1 & 2 & 3 & 4 & \cdots
\end{array}
\\
\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & \cdots \\
0 & 1 & 3 & 6 & 10 & \cdots \\
0 & 0 & \frac{3}{2} & \frac{15}{2} & \frac{45}{2} & \cdots \\
0 & 0 & 0 & \frac{5}{2} & \frac{35}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots
\end{pmatrix}
\end{array}
$$

Top k rows: Positive Grassmannian

Cyclic Polytope

Positivity seen in (2,2) Signature

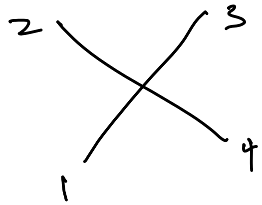


$$\det\begin{pmatrix} G_{\nu_1}(x_1) & G_{\nu_2}(x_1) & \cdots & G_{\nu_n}(x_1) \\ \vdots & \vdots & & \vdots \\ G_{\nu_1}(x_n) & G_{\nu_2}(x_n) & & G_{\nu_n}(x_n) \end{pmatrix} > 0$$

$x_1 < \cdots < x_n$

$\nu_1 < \cdots < \nu_n$

Higher-Dim Ops in 4-particle Scattering



$$A^{LE,\,\text{contact}} = \left(\langle 12\rangle[34]\right)^{2s}\Big[\sum a_{D,q}\, s^{D-q}\, t^{q}\Big]$$

carries dim. → t_D $\quad \hat{a}_{D,q}$ ← dimless, projective

$$\left(t_0, t_1, t_2, t_3 \ldots\right)$$

Satisfy ∞'ly many non-linear inequalities!

$$\begin{pmatrix}\hat{a}_{0,0}\end{pmatrix} \begin{pmatrix}\hat{a}_{1,0}\\ \hat{a}_{1,1}\end{pmatrix} \begin{pmatrix}\hat{a}_{2,0}\\ \hat{a}_{2,1}\\ \hat{a}_{2,2}\end{pmatrix} \begin{pmatrix}\hat{a}_{3,0}\\ \hat{a}_{3,1}\\ \hat{a}_{3,2}\\ \hat{a}_{3,3}\end{pmatrix} \cdots$$

Must lie within Specific Polytopes!

1 Pure photon

Consider the configuration (-, +, +, -) where we have

$$\langle 14\rangle^2[23]^2 \left(\sum_{i,j} g_{i,j} z^i t^j\right) \tag{1}$$

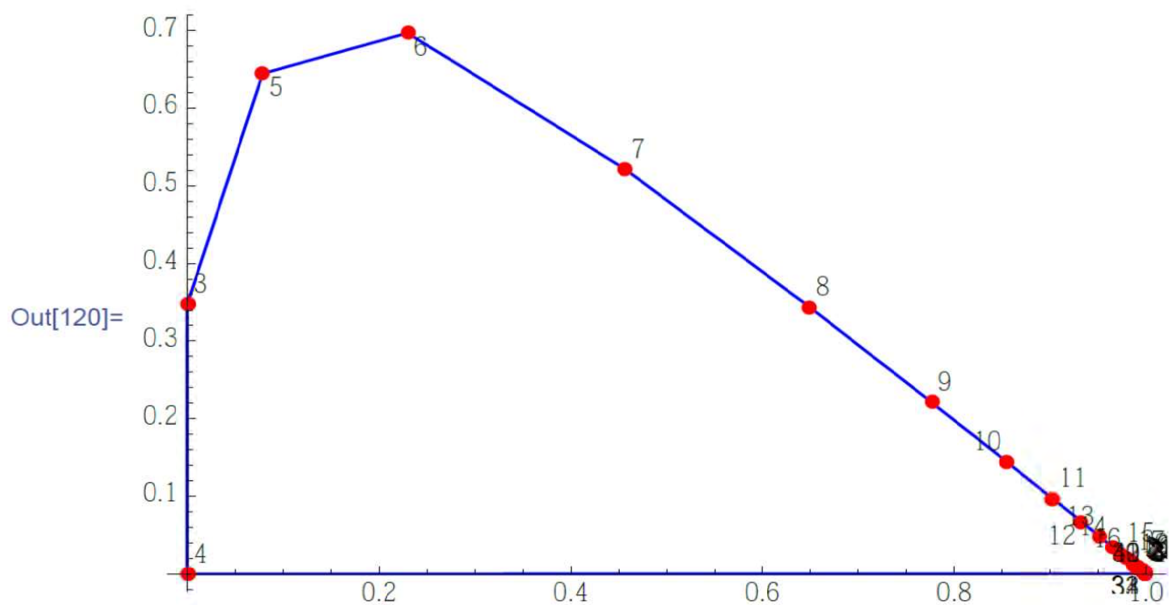
Note that due to the $\langle 14\rangle^2[23]^2 \sim t^2$ prefactor, we will not have any t-pole obstruction.

- (z^2, t^2): The space is one-dimensional, and the bound is simply

$$-\frac{7}{20} < \frac{g_{2,0}}{g_{0,2}}$$

- (z^4, z^2t^2, t^4): The critical spin is $s_c = 4$, spin-2 is inside the hull, i.e. not a vertex. The boundaries are:

$$\langle X, i, i+1\rangle > 0 \text{ for}, \quad i \geq 5, \langle X, 4, 3\rangle > 0, \quad \langle X, 3, 5\rangle > 0 \tag{2}$$



(3)

This plot was made by plotting the vectors

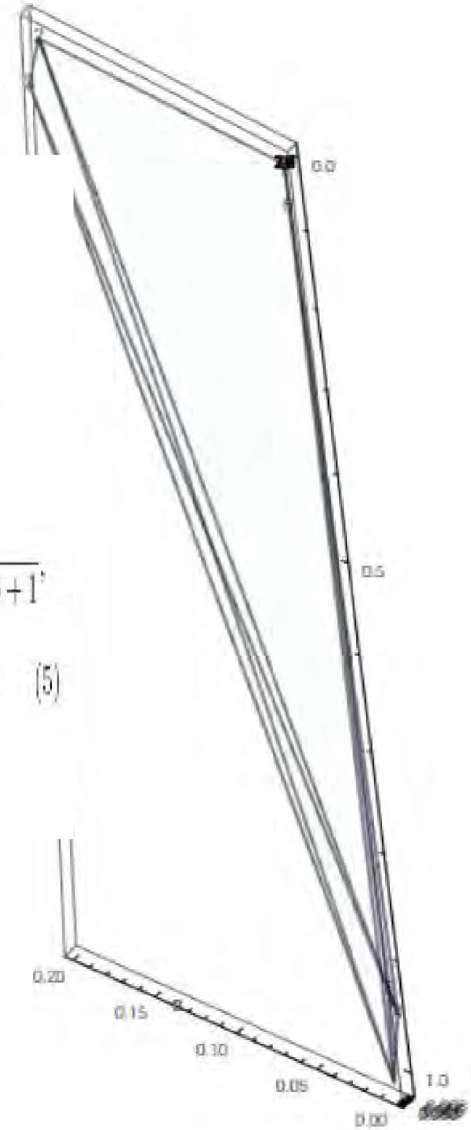
$$\vec{V_\ell} = \left(\frac{\langle V_\ell, 4, 3\rangle}{\langle V_\ell, 4, 3\rangle + \langle V_\ell, \infty, 4\rangle + 1}, \frac{\langle V_\ell, \infty, 4\rangle}{\langle V_\ell, 4, 3\rangle + \langle V_\ell, \infty, 4\rangle + 1}\right)$$

- $(z^6, z^4t^2, z^2t^4, t^6)$ The critical spin is $s_c = 4$, spin-2 is inside the hull. The boundaries are:

$$\langle 4, X, i, i+1\rangle > 0 \text{ for}, \quad i \geq 9, \langle X, 3, 4, 5\rangle > 0, \quad \langle X, 3, 7, 6\rangle > 0, \quad \langle X, 5, 8, 7\rangle > 0, \quad \langle X, 5, 9, 8\rangle > 0, \tag{4}$$

Using the vectors

$$\begin{aligned}\vec{V}_\ell \;&=\; \left(\frac{\langle V_\ell, 3, 4, 5\rangle}{\langle V_\ell, 3, 4, 5\rangle + \langle 3, V_\ell, 6, 7\rangle + \langle 3, V_\ell, \infty_1, \infty_2\rangle + 1}, \frac{\langle 3, V_\ell, 6, 7\rangle}{\langle V_\ell, 3, 4, 5\rangle + \langle 3, V_\ell, 6, 7\rangle + \langle 3, V_\ell, \infty_1, \infty_2\rangle + 1},\right.\\
&\left.\frac{\langle 3, V_\ell, \infty_1, \infty_2\rangle}{\langle V_\ell, 3, 4, 5\rangle + \langle 3, V_\ell, 6, 7\rangle + \langle 3, V_\ell, \infty_1, \infty_2\rangle + 1}\right)\end{aligned} \tag{5}$$



2 Pure Graviton

Consider the configuration (-2, +2, +2, -2) where we have

$$\langle 14\rangle^4[23]^4\left(\sum_{i,j} g_{i,j}z^it^j\right) \tag{8}$$

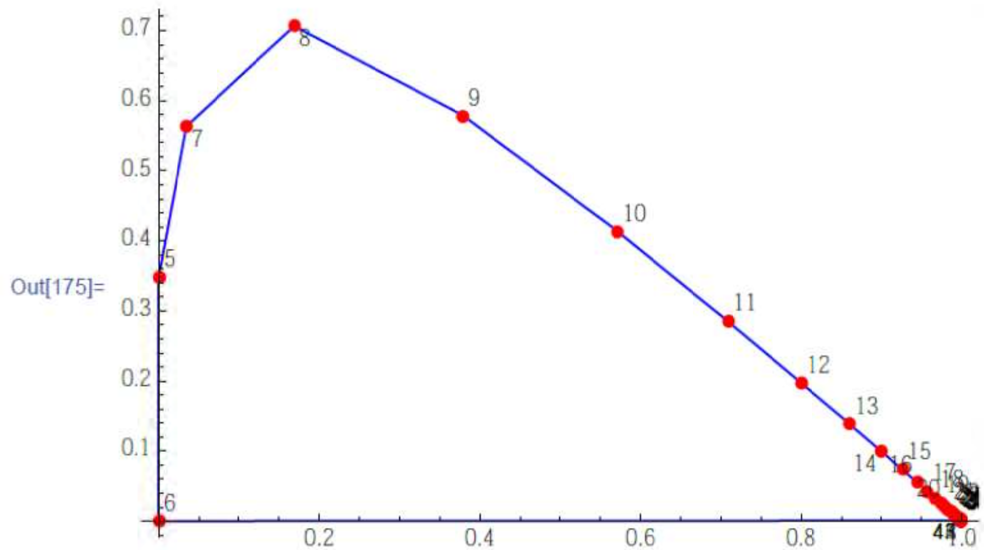
The exchanged spin begins with spin-4

- (z^2,t^2): The space is one-dimensional, and the bound is simply

$$-\frac{11}{36}<\frac{g_{2,0}}{g_{0,2}}$$

- (z^4,z^2t^2,t^4): The critical spin is $s_c=6$, spin-4 is inside the hull, i.e. not a vertex. The boundaries are:

$$\langle X,i,i+1\rangle>0 \text{ for},\quad i\geq 7, \langle X,6,5\rangle>0,\quad \langle X,5,7\rangle>0 \tag{9}$$



(10)

This plot was made by plotting the vectors

$$\vec{V}_\ell=\left(\frac{\langle V_\ell,6,5\rangle}{\langle V_\ell,6,5\rangle+\langle V_\ell,\infty,6\rangle+1},\frac{\langle V_\ell,\infty,6\rangle}{\langle V_\ell,6,5\rangle+\langle V_\ell,\infty,6\rangle+1}\right)$$

The World is Not a Crappy Metal!

Vastly greater constraints on EFT than naively expected. Can we extend to full non-linear constraints of UV Unitarity? Can we prove $\delta\left(M/Q\right)\big|_{extr} < 0$ for weak-gravity?

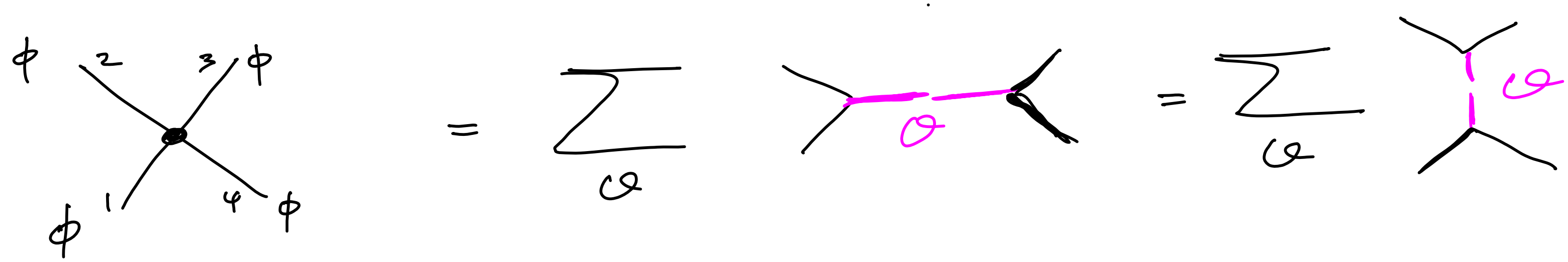
The CFT-hedron

w/ Huang + Shao

* Geometry of Conformal Bootstrap

* Exact constraints on CFT data
  data from positive geometry + combinatorics

Conformal Bootstrap



OPE + Unitarity: $F(z,\bar{z}) = \sum_{\Delta,s} p_{\Delta,s}\, C_{\Delta,s}(z,\bar{z})$, $\quad p_{\Delta,s} \geq 0$

Conformal Blocks

Crossing: $(z\bar{z})^{-\Delta_\phi} F(z,\bar{z})$ is symmetric under $z \to 1-z$

Polyakov Dream: Like Classifying Lie Algebras.

Big Q: What does allowed space of $\{\Delta, s, p_{\Delta,s}\}$ look like?

For simplicity: consider only $SL(2,\mathbb{R})$ blocks + work on $z = \bar{z}$. Then

$$F(z) = \sum_{\Delta} p_{\Delta} \, C_{\Delta}(z) \qquad = z^{\Delta} {}_2F_1(\Delta, \Delta, 2\Delta, z)$$

$$p_{\Delta} \geq 0$$

$$\begin{pmatrix} F(z_1) \\ \vdots \\ F(z_N) \end{pmatrix} = \sum_{\Delta} p_{\Delta} \begin{pmatrix} C_{\Delta}(z_1) \\ \vdots \\ C_{\Delta}(z_N) \end{pmatrix}$$

$$\begin{pmatrix} F^{(0)}(z_*) \\ F^{(1)}(z_*) \\ F^{(2)}(z_*) \\ \vdots \\ F^{(N)}(z_*) \end{pmatrix} = \sum p_{\Delta} \begin{pmatrix} C_{\Delta}^{(0)}(z_*) \\ \vdots \\ C_{\Delta}^{(N)}(z_*) \end{pmatrix}$$

So, obvious Q: what is the shape of Block Polytopes?

Positivity of Conformal Blocks

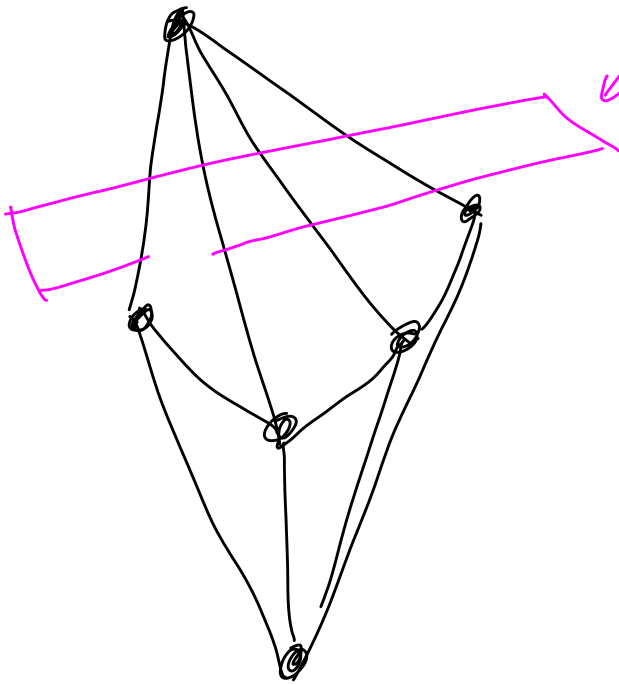
$$\begin{bmatrix} C_{\Delta_1}(z_1) & \cdots & C_{\Delta_N}(z_1) \\ \vdots & & \vdots \\ C_{\Delta_1}(z_N) & & C_{\Delta_N}(z_N) \end{bmatrix}$$

$\Delta_1 < \cdots < \Delta_N$
$z_1 < \cdots < z_N$
is totally positive!

Unitarity : $\vec{F} \in$ Convex Polytope of Blocks

Crossing : $\vec{F} \in$ the "Crossing Plane" X

$$F_0 + F_1 y + \cdots + F_{2N+1} y^{2N+1} = (1+y)^{2\Delta_\phi}\left[f_0 + f_2 y^2 + \cdots + f_{2N} y^{2N}\right], \quad z = \tfrac{1}{2}(1+y)$$



Crossing Plane $X[\Delta_\phi]$

N-dimensional

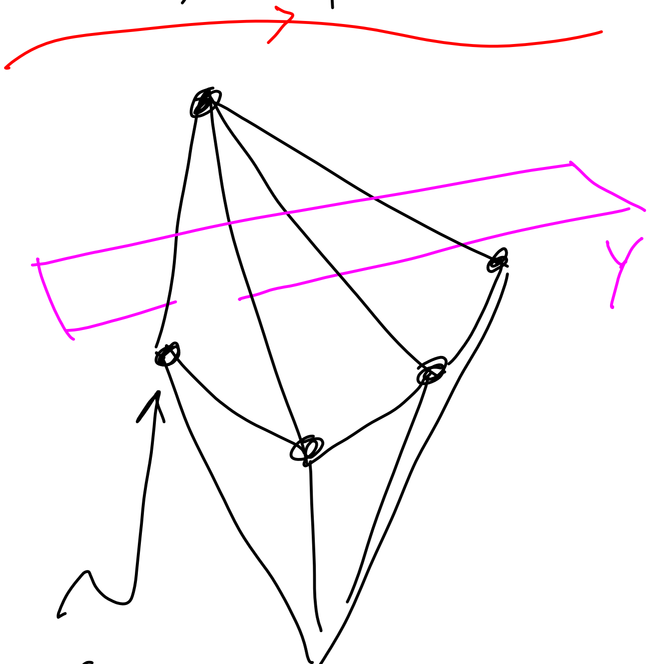
Block Polytope $[\{\Delta_i\}]$

P : 2N+1 dimensional

Q1: What constraints do the Δ's have to satisfy for X to intersect P? [Exact constraints on spectrum]

Q2: What does the intersecting polytope on X look like [Exact constraints on 4-pt function]

(Tree) Amplituhedron

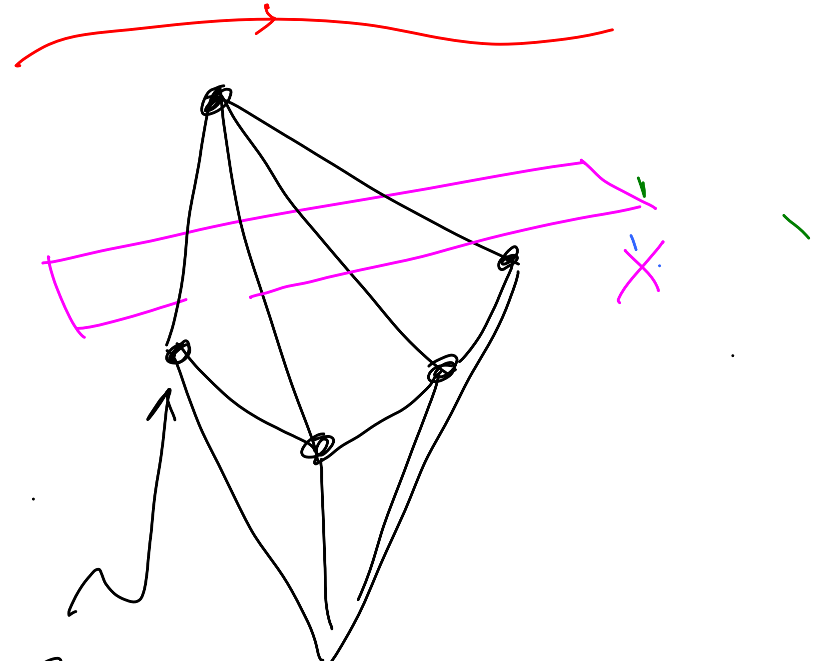


Z's,

positive data; fixed

Y intersects w/ correct winding / "binary code"

CFThedron



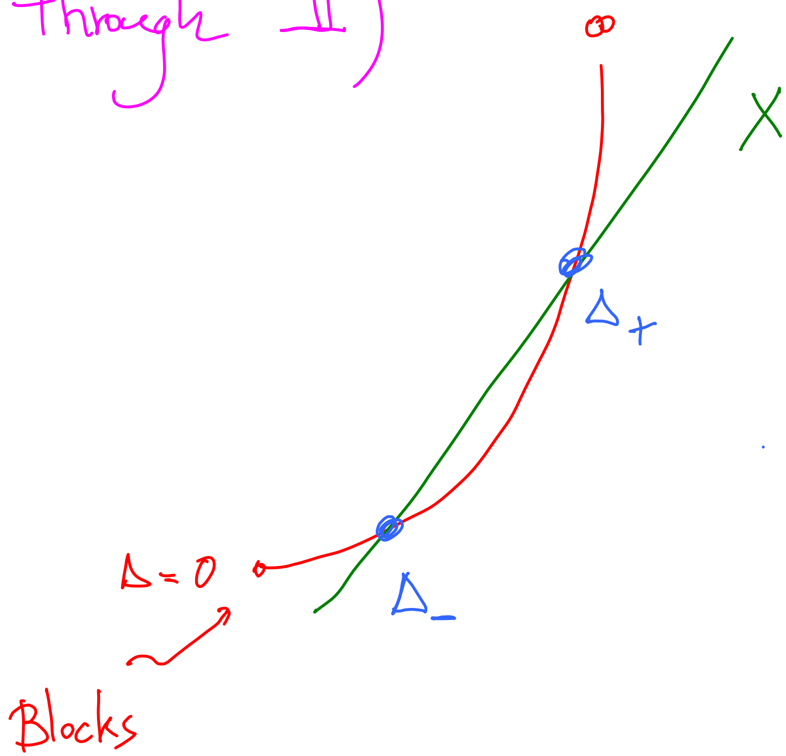
C_Δ's,

positive; change across CFT's

X fixed; need merely intersect

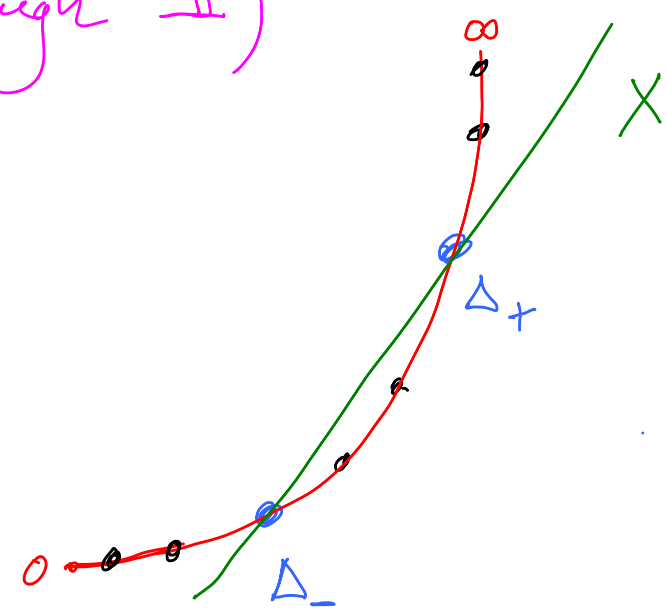
1-d Geometry

(Project through $\mathbb{1}$)



1-d Geometry

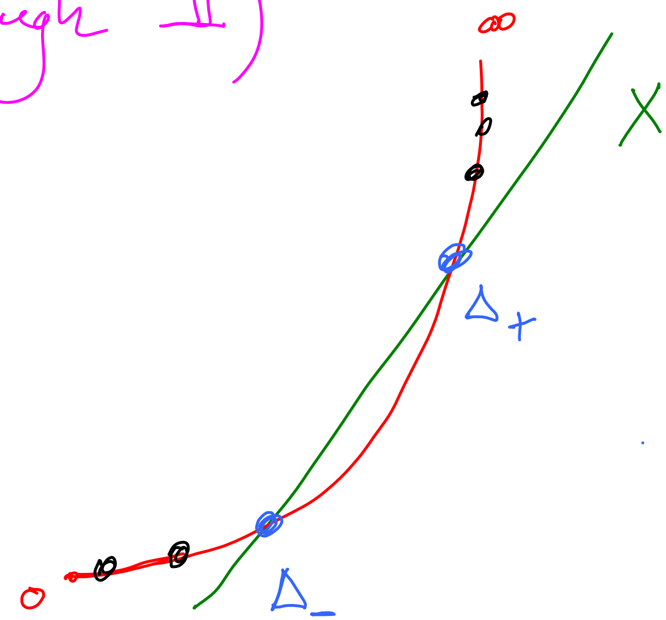
(Project through $\mathbb{1}$)



GOOD

1-d Geometry

(Project through 1)

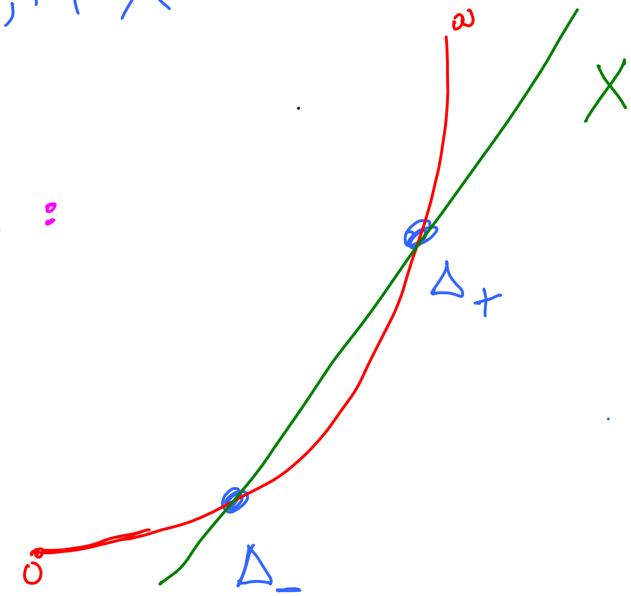


NO GOOD

1-d Geometry

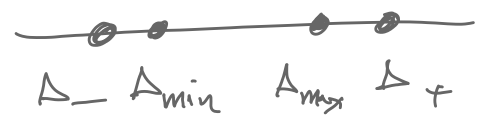
Walls: $(1\ i\ i{+}1)$, int. X in points

Project through 1 :

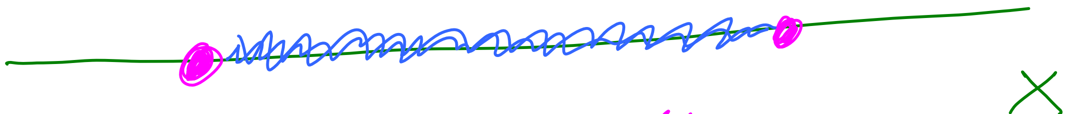


There must exist

$\Delta \in (\Delta_-, \Delta_+)$



Walls of Cyclic Polytope

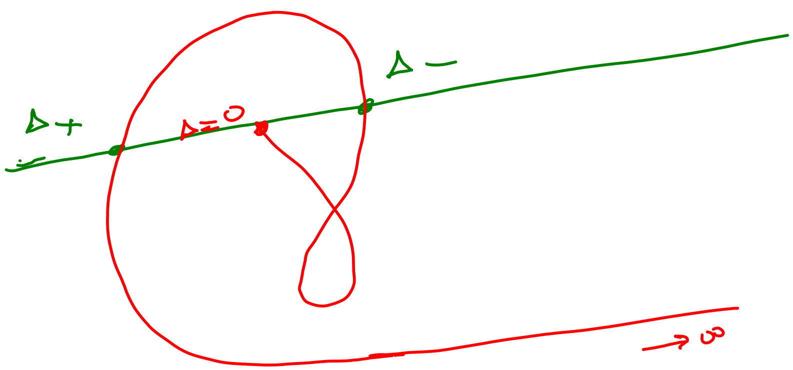


$(1\ \Delta_{min-1}\ \Delta_{min})$ $(1\ \Delta_{max}\ \Delta_{max+1})$

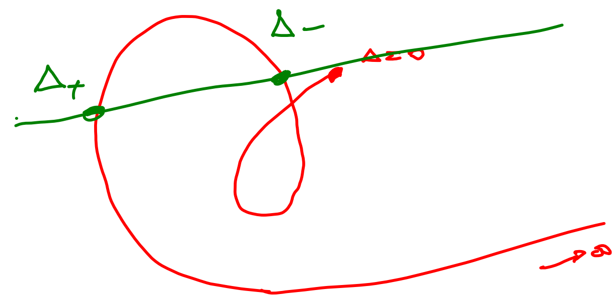
Exact Constraints on 4-pt function

2D - Geometry

Walls $C^{!}_{\infty}$ $ii+1$ $jj+1$) ; project through $\mathbb{1} + X$:

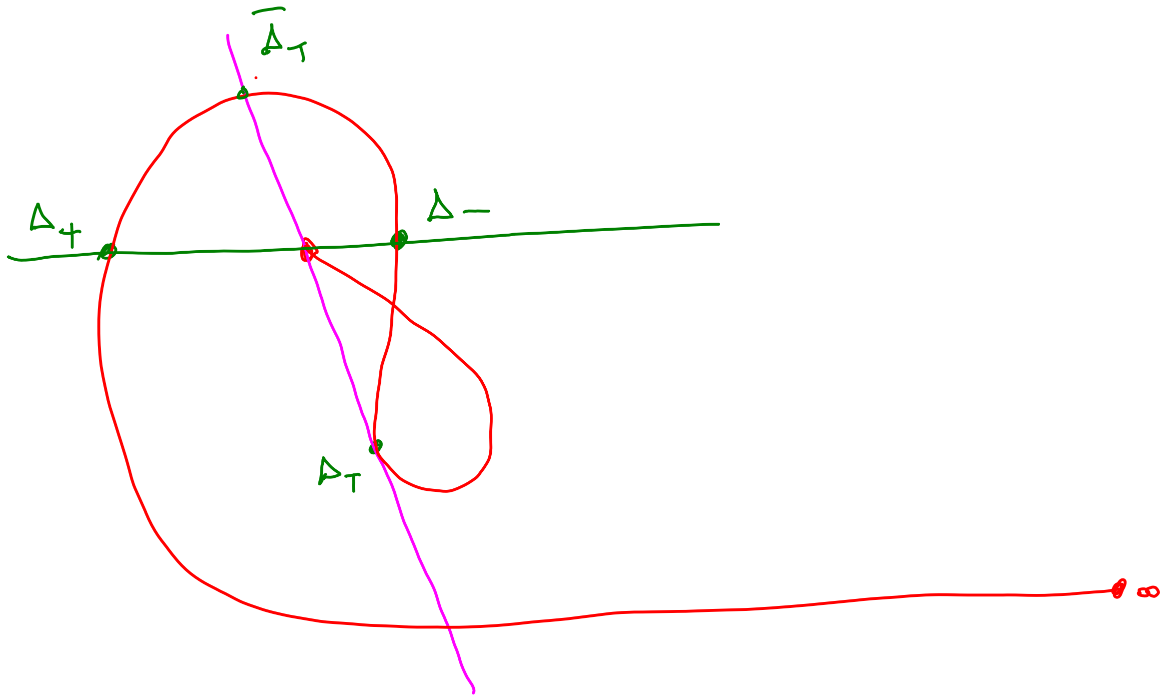


$\Delta_\phi < \Delta_{\phi_*} \sim 1.5$

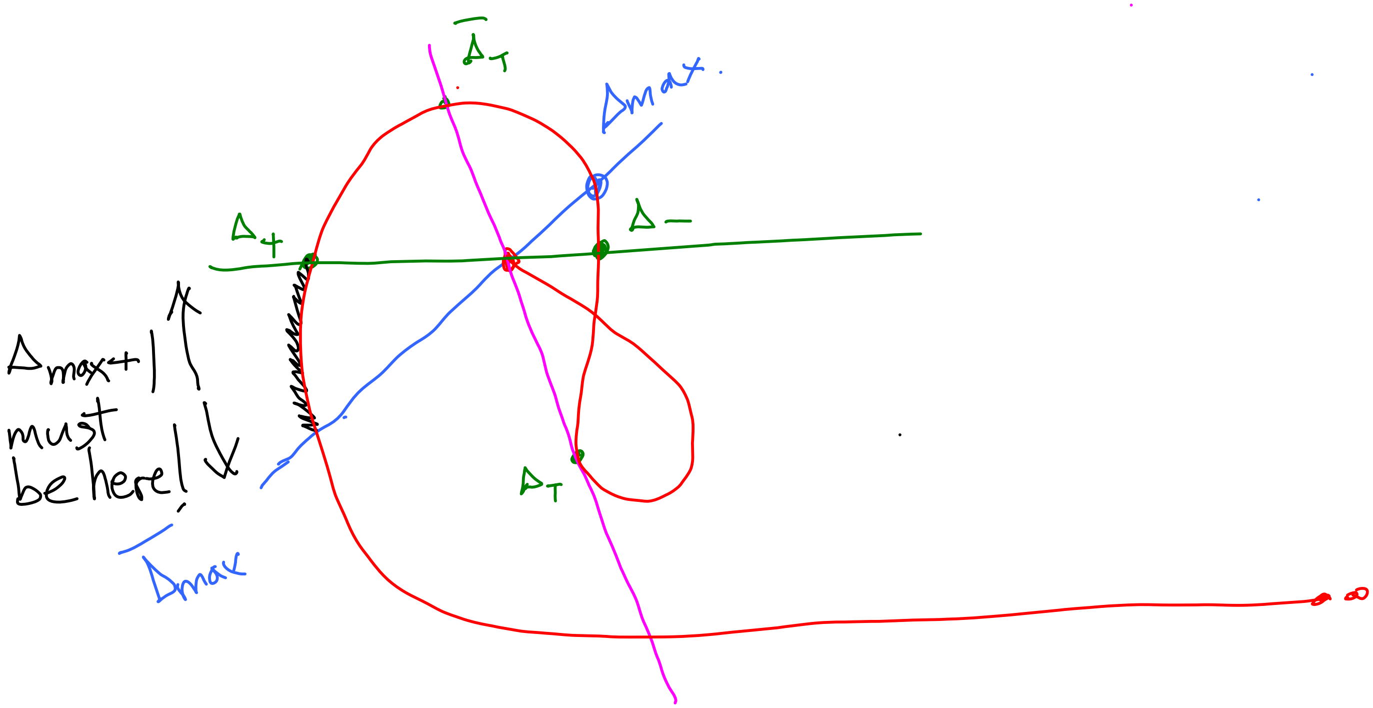


$\Delta_\phi > \Delta_{\phi_*}$

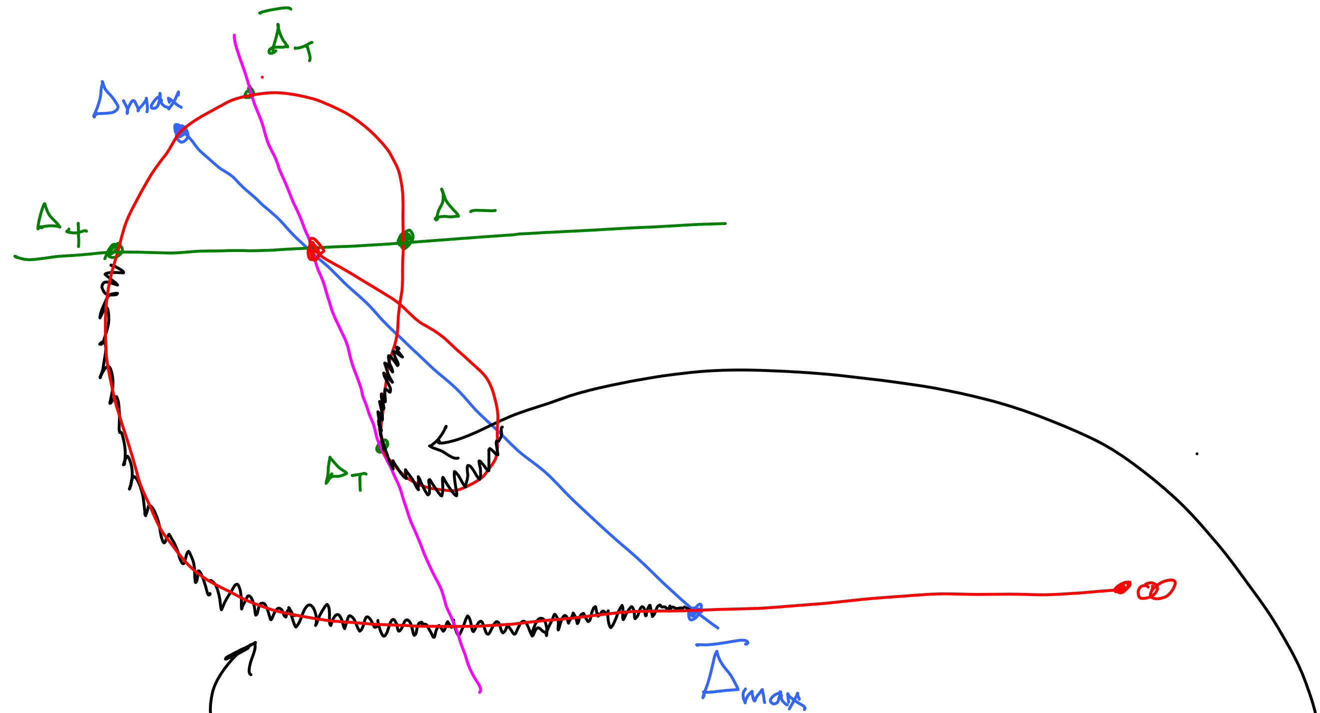
X intersects $\longleftrightarrow$ the spectrum (putting points on the curve) admits a triangle containing the origin



$\bar{\Delta}_T$
Δ_+
Δ_-
Δ_T
∞



$\bar{\Delta}_T$
Δ_{max}
Δ_+
Δ_-
Δ_{max+1} must be here!
Δ_T
Δ_{max}
∞

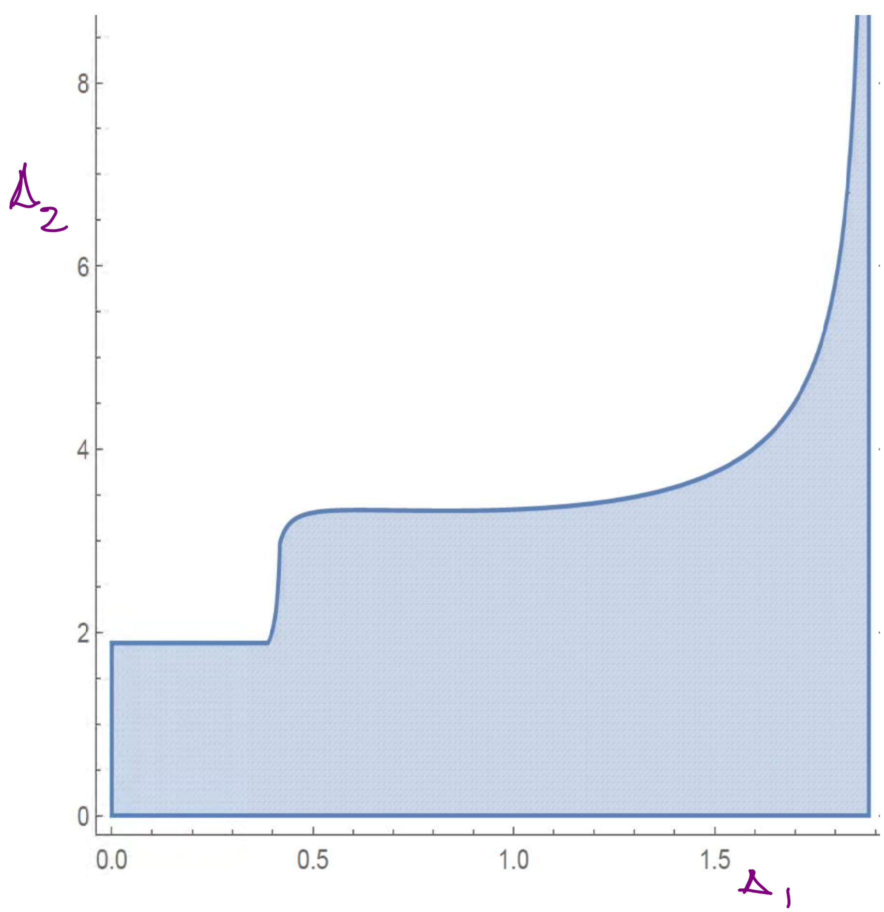


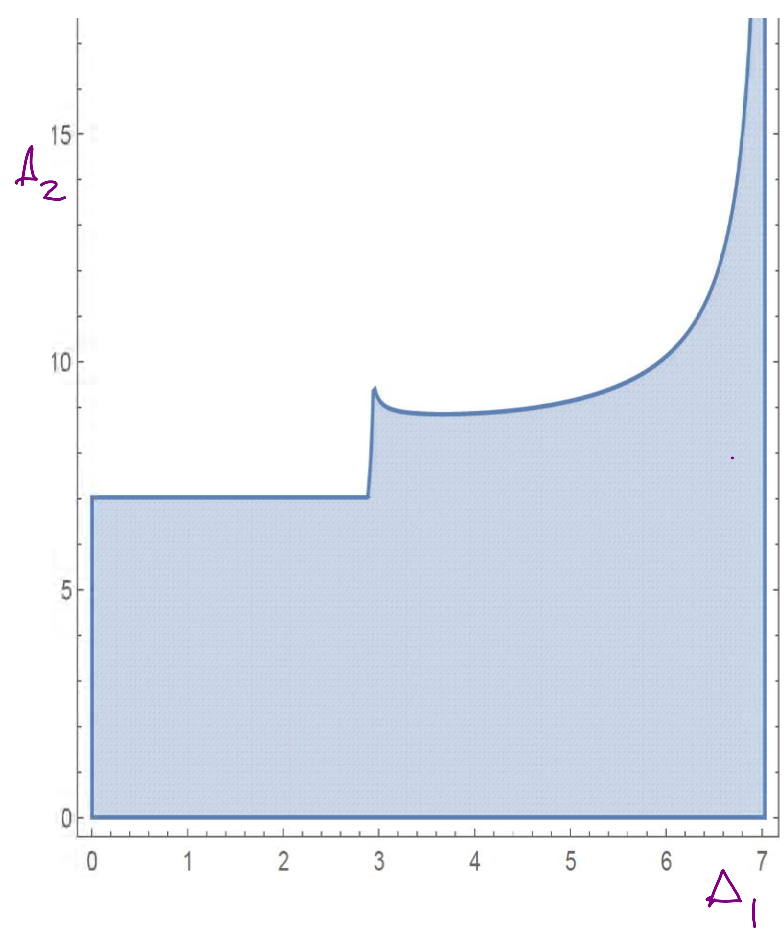
$\Rightarrow$ Either Δ_{max+1}, or if not, there must exist lighter operators

Interesting "Global" constraints on Spectrum

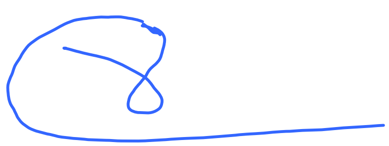
If satisfied, X intersects!

Application: Bounding Lightest 2 Ops

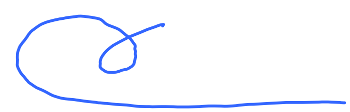




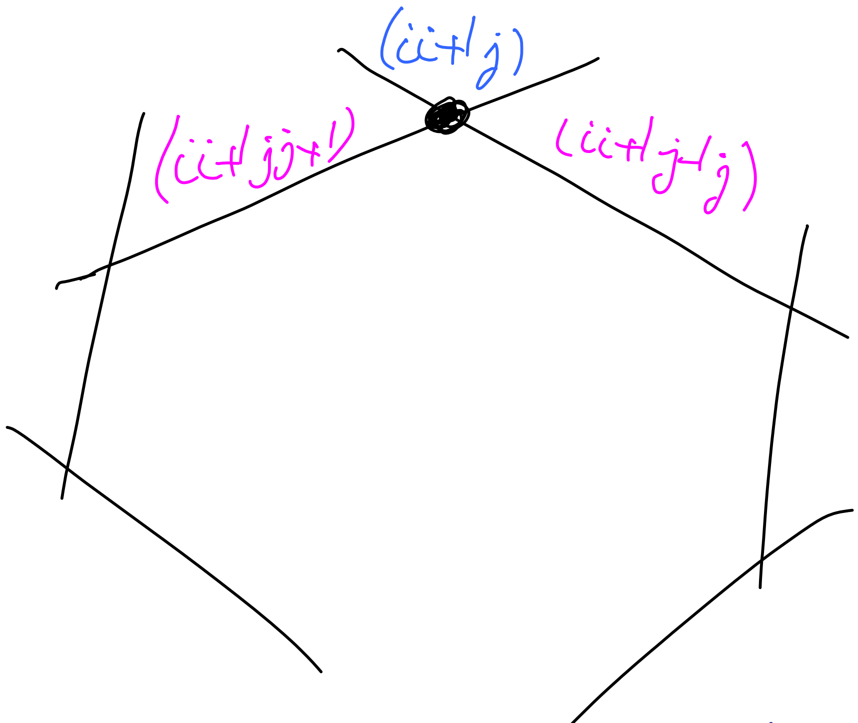
$\Delta_\phi = \frac{1}{8}$



$\Delta_\phi = 3$



Positive (Polygon) Geometry on X

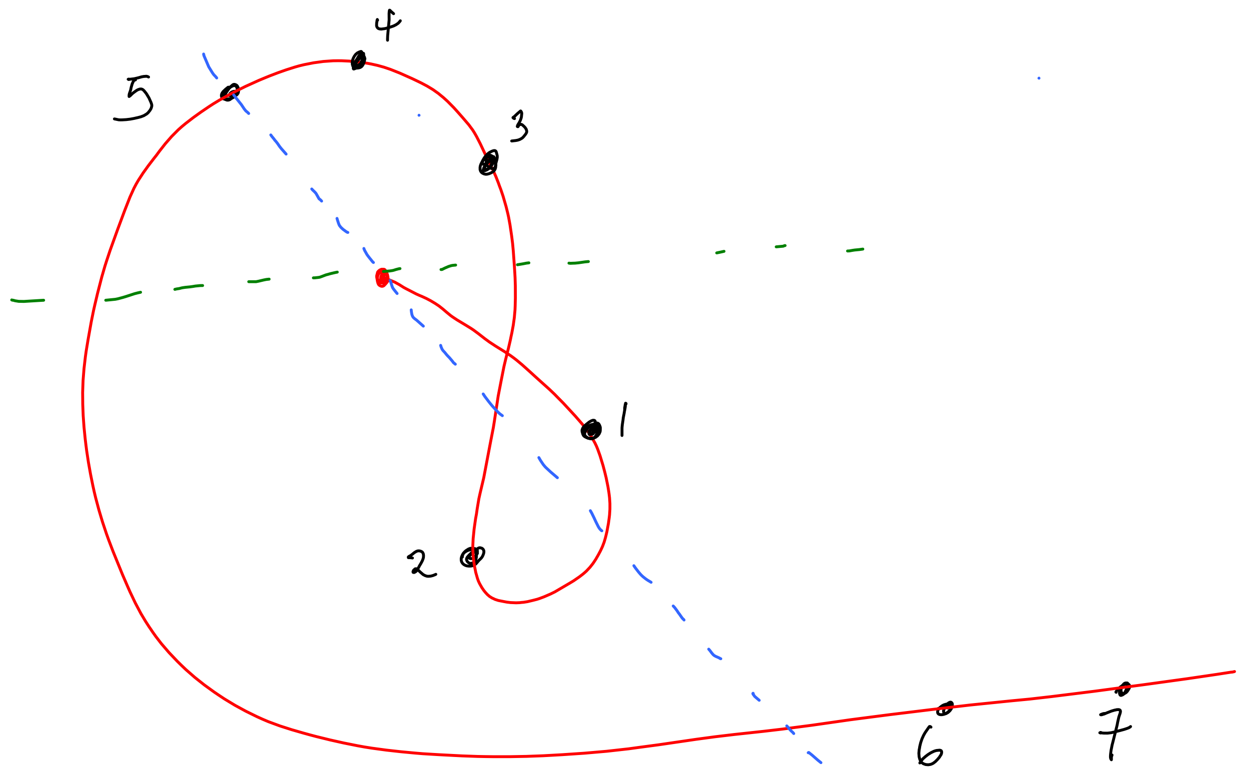


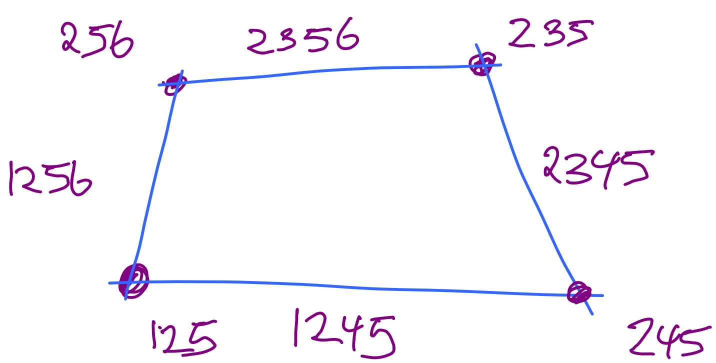
Combinatorial Constraint:

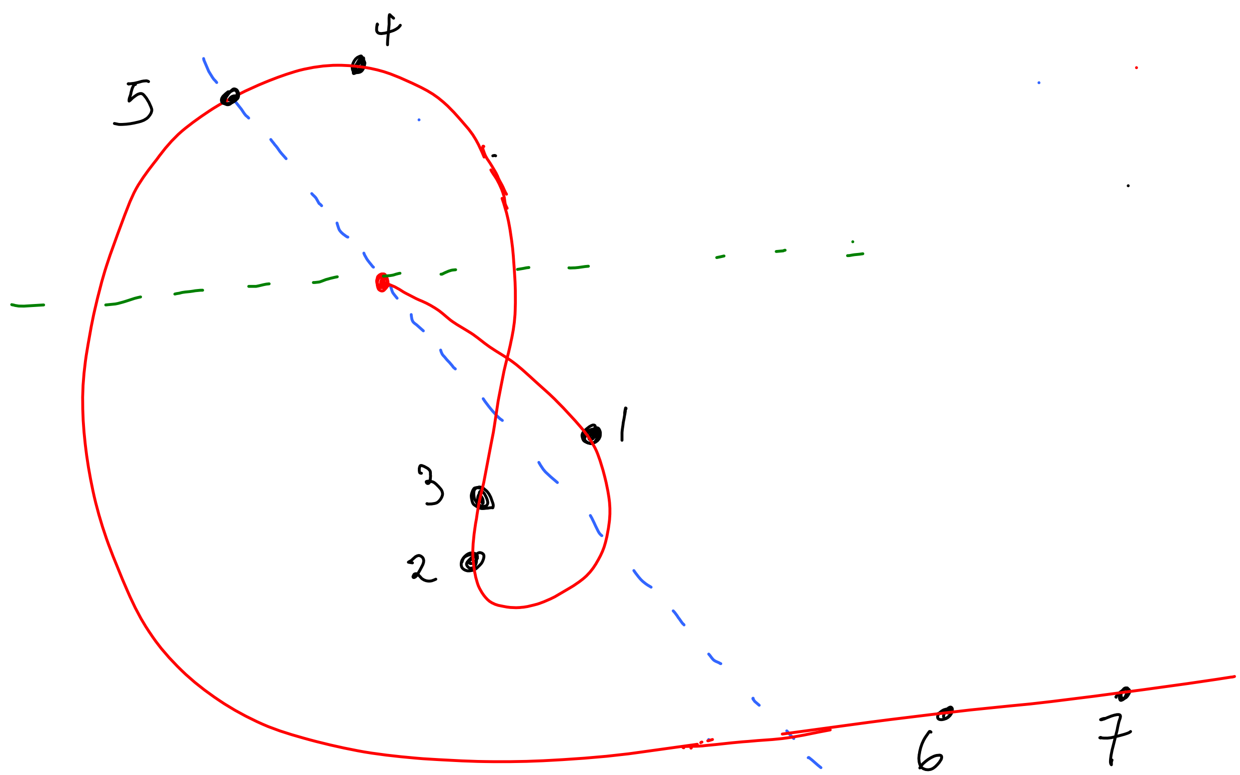
Vertices: $[i\,i{+}1\,j]$'s

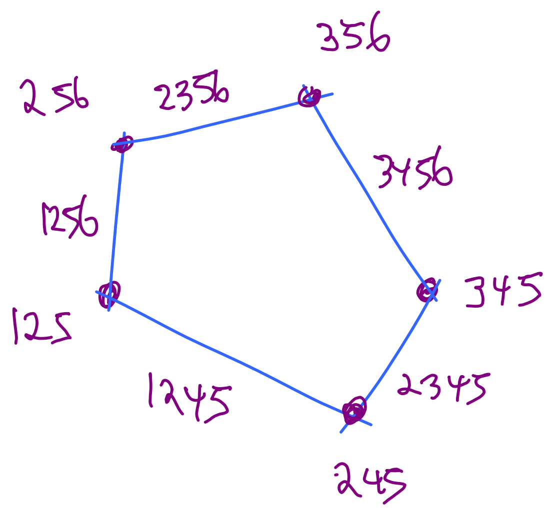
Edges: $[i\,i{+}1\,j\,j{+}1]$'s

Shape of Polygon [= Constraint on 4pt function] depends on the spectrum $\{\Delta_i\}$


4
5
3
1
2
6
7


256
2356
235
1256
2345
125
1245
245


4
5
1
3
2
6
7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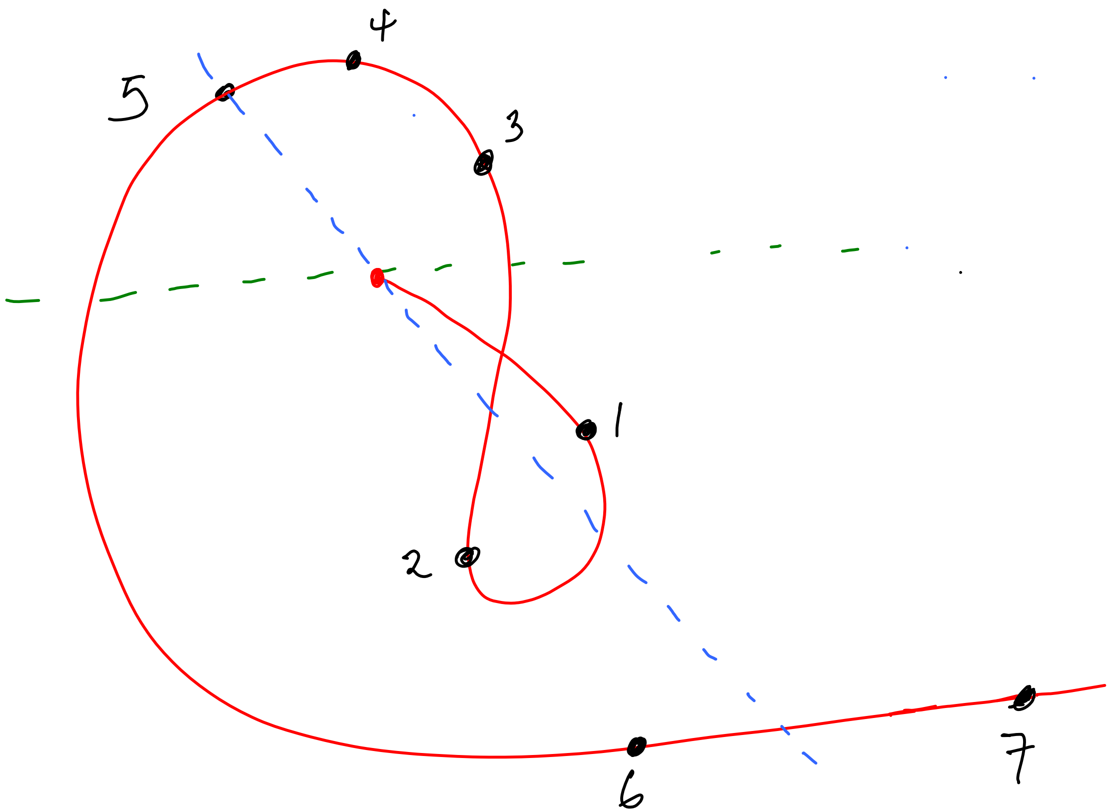
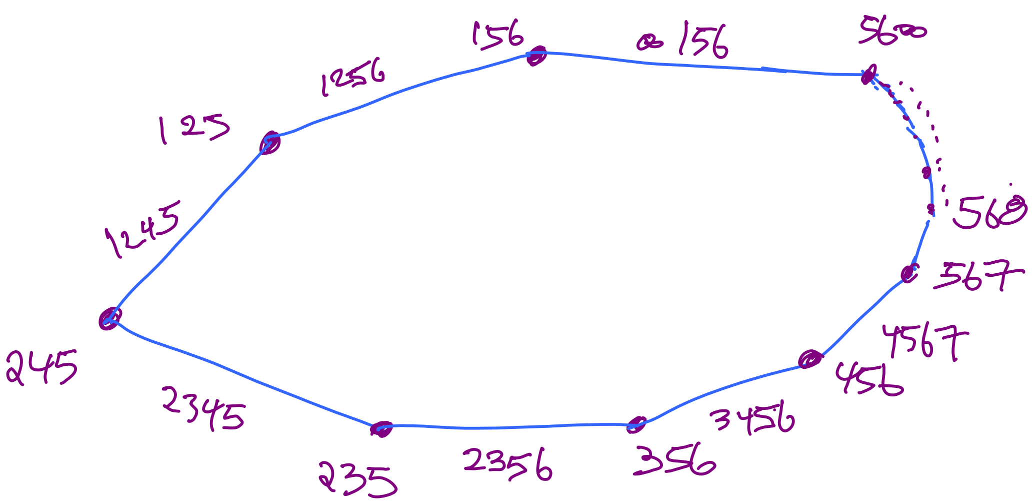
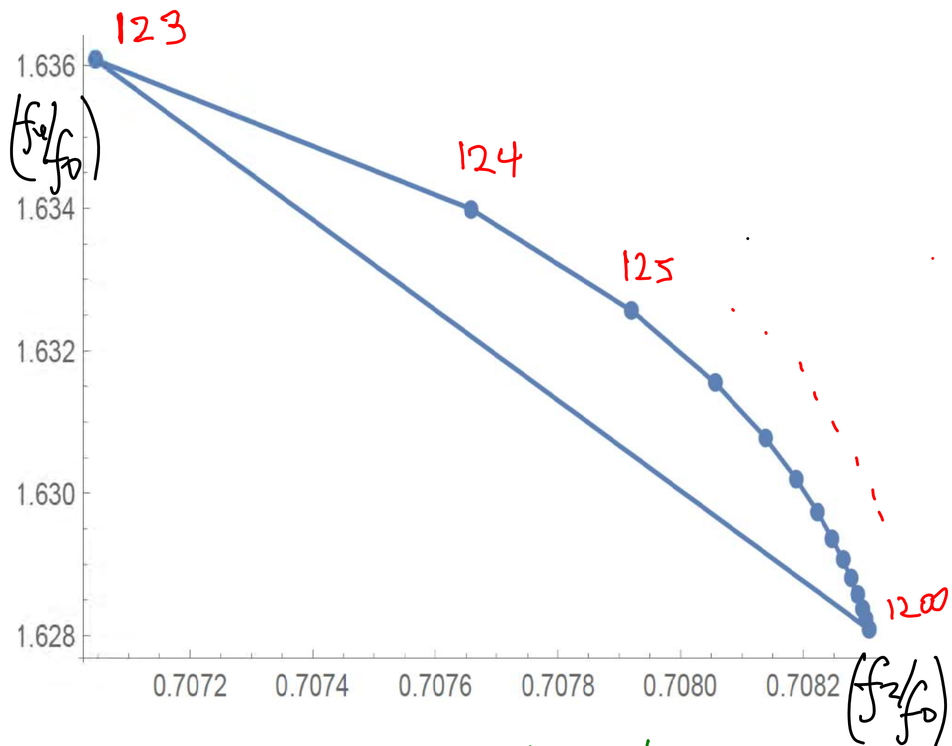
256
2356
3456
1256
345
125
2345
1245
245

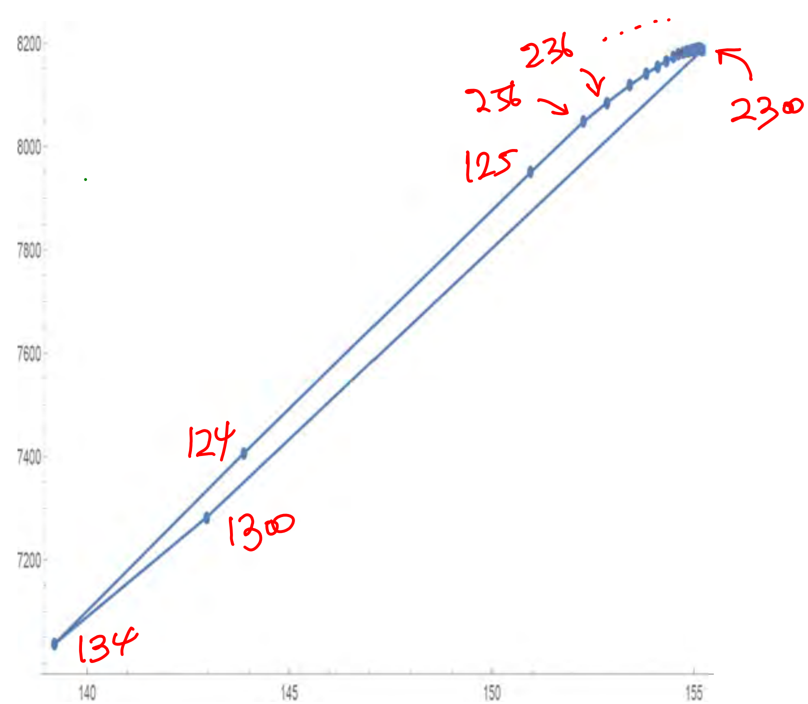


4
5
3
1
2
6
7

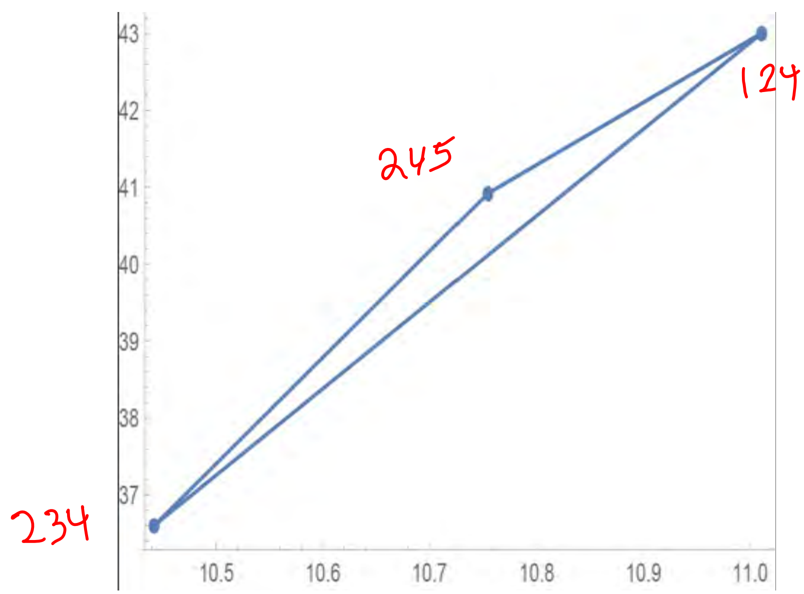


156
156
560
1256
125
568
1245
567
4567
245
456
2345
3456
235
2356
356





Free Fermion, $\Delta_\phi = \frac{1}{8}$ $\longrightarrow$ $\Delta_\phi = 3$



$\Delta_\phi = \frac{6}{5}$

$\Delta = \{ \frac{1}{2}, 1.3, 2.7, 3, 10, 11, 12, \ldots \}$

There is clearly a remarkably rich geometric + combinatorial structure associated with $\{\Delta_i\}$ space, together with the associated polytopes bounding 4pt-function.

We can probably explicitly "solve" the geometries @ low dimensions — hope is that there is some (recursive?) way of systematically going to arbitrarily high dimension.