

# Modular solutions to minimal 6d SCFTs, and how to blow up them all

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ABSTRACT: Minimal 6d SCFTs are constructed by compactification of F-theory on elliptic Calabi-Yau threefolds with noncompact bases which contain a single  $\mathbb{P}^1$  of negative self-intersection number  $-n$ . They include E-string for  $n = 1$ , M-string for  $n = 2$ , and they come with non-Higgsable gauge groups if  $n \geq 3$ . The partition function of a 6d SCFT is a generating function of elliptic genera of non-critical strings arising from D3 branes wrapping curves in the base. We argue on the grounds of modularity and symmetries of SCFTs that the basic building blocks for elliptic genera are weak Jacobi forms, and automorphism invariant Jacobi forms which are multivariable Jacobi forms respecting both Weyl symmetry and automorphism symmetry of the affine Dynkin diagram associated to the Lie algebra of gauge group. We then completely fix elliptic genera for small numbers of strings for minimal 6d SCFTs with  $n = 1, 2, 3, 4$  through top genus formulae of BPS numbers. Finally, we generalize the Gottsche-Nakajima-Yoshioka blowup equations for 4d, 5d supersymmetric gauge theories for the first time to 6d theories, and test the generalized blowup equations against the E-string theory.

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