Noncommutative Instantons in Operator Formalism

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In this poster, we discuss $U(N)$ instantons in noncommutative (NC) spaces in operator formalism. Noncommutative space is a space on which coordinate ring is noncommutative. Let x^μ be the spacial coordinates. The noncommutativity is represented by the following commutation relation: $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. where $\theta^{\mu\nu}$ is a real antisymmetric tensor and called the noncommutative parameters. When $\theta^{\mu\nu}$ vanishes identically, the coordinate ring is commutative and the underlying space reduces to a commutative one. The commutation relation, like the canonical commutation relation in quantum mechanics, leads to "space-space uncertainty relation." Singularities in commutative space could resolve in noncommutative space thereby. This is one of the prominent features of field theories on noncommutative space and yields various new physical objects such as $U(1)$ instantons. There are two formalism to describe noncommutative gauge theories: the star-product formalism and the operator formalism.

Anti-self-dual (ASD) Yang-Mills equation and the solutions have been studied from the several viewpoints of mathematical physics, particularly, integrable systems, geometry and field theories. Instantons are finite-action solutions of the ASD Yang-Mills equation and become exact solutions of classical Yang-Mills theories. They can reveal non-perturbative aspects of the quantum theories. Actually, the path-integrations, formulating the quantum theories, could reduce to finitedimensional integrations over the instanton moduli spaces. The Atiyah-DrinfeldHitchin-Manin (ADHM) construction is a powerful method to obtain the instantons. Furthermore, via the construction, the instanton moduli space is mapped to the set of quadruple matrices which are solutions of the ADHM equation and called the ADHM data. The aforementioned integration, being thereby an integration over the matrices, becomes tractable. To evaluate the integration, the use of noncommutative instantons is relevant so that a localization formula can be applied to the integration. In the procedures, various formulas and relations of the ADHM construction are required. Hence it is worthwhile to elucidate the one-to one correspondence (reciprocity) between moduli spaces of the noncommutative instantons and the ADHM data and to present all the ingredients in the construction explicitly.

In this poster, we discuss the reciprocity in the operator formalism. We reconsider origin of the instanton number by applying a geometrical formula to the noncommutative situation even for the $U(1)$ case. We give a systematic method to construct exact noncommutative instantons as well.