S-Duality, Quadratic Reciprocity, and Janus Configurations

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Abstract

Quadratic reciprocity in number theory maps the question "does the equation $x^2 \equiv q \pmod{p}$, for given odd primes p and q, have an integer solution x?" to a mirror question with p and q interchanged, and numbers of solutions are encoded in the *quadratic residue*. We first show that this reciprocity is a direct consequence of T-duality, by recasting the quadratic Gauss sum as the partition function of $N = 4$ $U(1)$ SYM on a mapping-torus $\mathcal{M}_3$ with coupling constant g^2 and θ-angle varying along the base S^1. With an R-symmetry twist, $\mathcal{M}_3$ preserves SUSY, and an Olive-Montonen $SL(2, Z)$ twist on S^1 connects g^2 and θ smoothly.

In our setting, we compactify Gaiotto-Witten Janus of $N = 4$ SYM on T^2 to obtain $2d$ SUSY σ-models with target space T^2 with complex structure varying along one worldsheet direction and Kähler modulus varying along another. For these double-Janus configurations, topological partition functions can be written as quadratic Gauss sums. The limit where the Janus circle is much larger than the base of $\mathcal{M}_3$ and the opposite limit provide two ways of calculating the same partition function. For gauge group $G = U(1)$, the equivalence of the two methods leads to the L-S relation, from which quadratic reciprocity follows; for more complicated $SL(2, Z)$ twists, we obtain 2- and 3-variable generalizations of L-S relation, whose analytical proof will be sketched. More abielian number-theoretic identities can be systematically generated. Since quadratic reciprocity is generalized by Artin reciprocity, which is the starting point of Langlands' conjectures, we may probe some features in arithmetic Langlands program.

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