Constrained systems and Lie algebroids, their BV and BFV formalisms

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We observe that a system of irreducible, fiber-linear, first class constraints on $T^*M$ is equivalent to the definition of a foliation Lie algebroid over $M$. The BFV formulation of the constrained system is given by the Hamiltonian lift of the Vaintrob description $(E[1], Q)$ of the Lie algebroid to its cotangent bundle $T^*E[1]$. Adding a Hamiltonian to the system corresponds to a metric $g$ on $M$. Evolution invariance of the constraint surface introduces a connection $\nabla$ on $E$ and one obtains the compatibility of $g$ with $(E, \rho, \nabla)$. We discuss a relation of a BFV-Hamiltonian to a Cartan-Lie algebroid. The BV formulation of the system is obtained from BFV by a (time-dependent) AKSZ procedure. This observation can be applied to field theories and string theories with Lie algebroid structures.