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Blowup Equations for Refined Topological Strings

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Abstract: Göttsche-Nakajima-Yoshioka K-theoretic blowup equations characterize the Nekrasov partition function of five dimensional $\mathcal{N} = 1$ supersymmetric gauge theories compactified on a circle, which via geometric engineering correspond to the refined topological string theory on $SU(N)$ geometries. We study the K-theoretic blowup equations for general local Calabi-Yau threefolds. We find that both vanishing and unity blowup equations exist for the partition function of refined topological string. These blowup equations are in fact the functional equations for the partition function and each of them results in infinite identities among the refined free energies. Evidences show that they can be used to determine the full refined BPS invariants of local Calabi-Yau threefolds. This serves an independent and sometimes more powerful way to compute the partition function other than the refined topological vertex in the A-model and the refined holomorphic anomaly equations in the B-model. The vanishing blowup equations also result in the compatibility formulae between the exact Nekrasov-Shatashivili quantization conditions and the Grassi-Hatsuda-Mariño conjecture for quantum curves.

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