Abstract: Semi-infinite Hodge structure originates from K. Saito's study of primitive forms in singularity theory and has been greatly extended to Calabi-Yau geometries by Barannikov-Kontsevich. In the first part of the talk, I will explain how semi-infinite Hodge theory extends Kodaira-Spencer gravity (Bershadsky-Cecotti-Ooguri-Vafa theory of B-twisted closed topological string field theory) into a full solution of Batalin-Vilkovisky master equation. This allows us to formulate quantum B-model via a rigorous BV quantization method and construct integrable hierarchies arising naturally from the background symmetry of BCOV theory. In the second part of the talk, I will explain the recent discovery of the connection between K.Saito's primitive form and 4d N=2 Seiberg-Witten geometry arising from singularity theory.