A local-to-global principle for stability conditions on Fukaya categories of surfaces

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Abstract

It is known from the work of Haiden, Katzarkov and Kontsevich that there is a close relation between Bridgeland stability conditions on Fukaya categories of surfaces and the geometry of measured foliations; in particular their work constructs stability conditions starting from a meromorphic quadratic differential. However it is not clear that this captures all the possible stability conditions, mostly due to the possible complications introduced by the global topology of the surface. In this paper I present a local-to-global principle which relates global stability conditions to purely local data. Using these tools we can show that such complications do not arise for the case of fully stopped Fukthe measured foliations do in fact recover all the desired stability conditions. I conjecture that some version of these tools should exist for more complicated classes of examples, e.g. Fukaya categories of higher-dimensional spaces.