String-Math 2018

Arxiv 1712.02337, 1802.04802

Mordell-Weil Torsion, Anomalies, Phase Transitions

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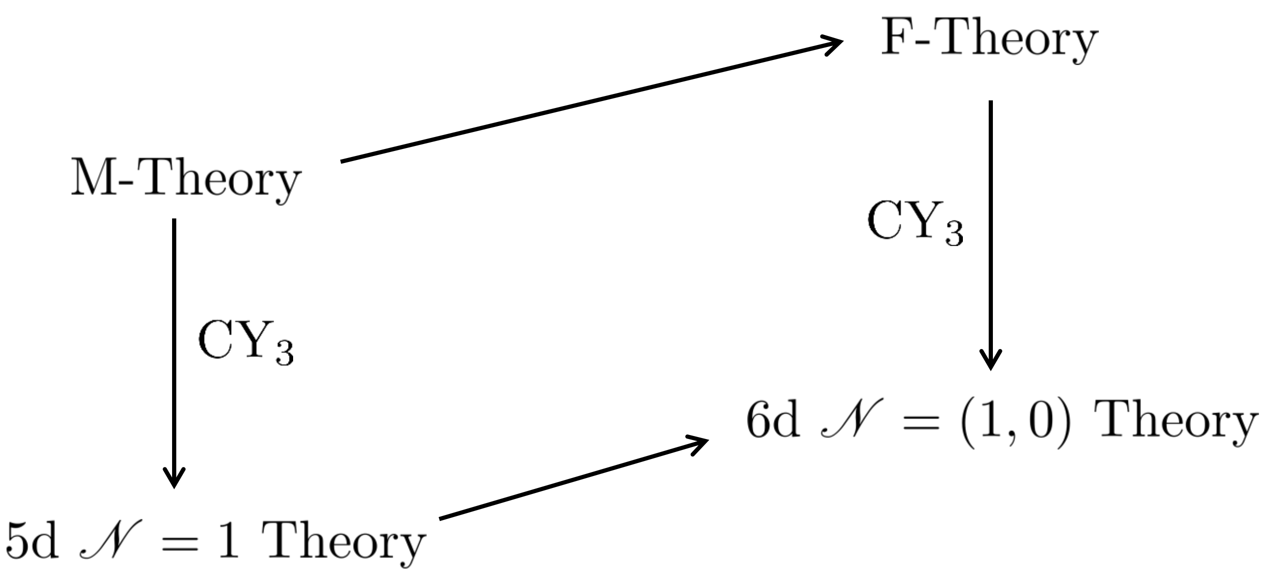
Motivation



- Elliptic fibrations
  - Geometrically engineered gauge theories
  - Captures global aspect of the gauge theory
- Base-free set-up

Elliptic Fibrations and Gauge Theories

- (Semi-simple) Lie group G, Lie algebra $\mathfrak{g}$, Representation $\mathbf{R}$
- Dictionary between the elliptic fibration and the gauge theory

Elliptic Fibration	Gauge Theory
Codimension 1 singularity	Gauge algebra ($\mathfrak{g}$)
Codimension 2 singularity	Representation ($\mathbf{R}$)
Crepant resolution	Coulomb phase
Flop	Phase transition
Triple intersection polynomial	5d prepotential
Mordell-Weil group	The fundamental group of the gauge group ($\pi_1(G)$)

Main Questions

- How the Mordell-Weil group of the elliptic fibration affect these supergravity theories?
  - What is the effect on the Coulomb branch of a 5d gauge theory when a semi-simple group is quotiented by a subgroup of its center?
  - What happens to the extended Mori cone of an elliptically-fibered Calabi-Yau threefold when the Mordell-Weil group is purely torsion?
  - Moreover, what are the 6d uplift of such theories (if any)?

Algorithm to get geometric data

Step 1. Determine a singular Weierstrass model with Kodaira fibers associated to the desired Lie group G.

Step 2. Determine a crepant resolution of the singular Weierstrass model.

Step 3. Compute the pushforward formulas to push the total Chern class of the resolved elliptic fibration to its base.

- The generating function of Euler characteristics is computed.
- For a d-dimensional base, the Euler characteristic is given by the coefficient of t^d in a power series expansion.
- Compute the Euler characteristics for Calabi-Yau threefolds.

Step 4. Compute the Hodge numbers using the fact that the base is a rational surface and Shioda-Tate-Wazir theorem.

Step 5. Determine the fiber structure of the resolved Weierstrass Model.

Step 6. Compute the geometric weights of the irreducible components of the singular fibers over codimension-two points.

Step 7. Compute the triple intersection polynomial.

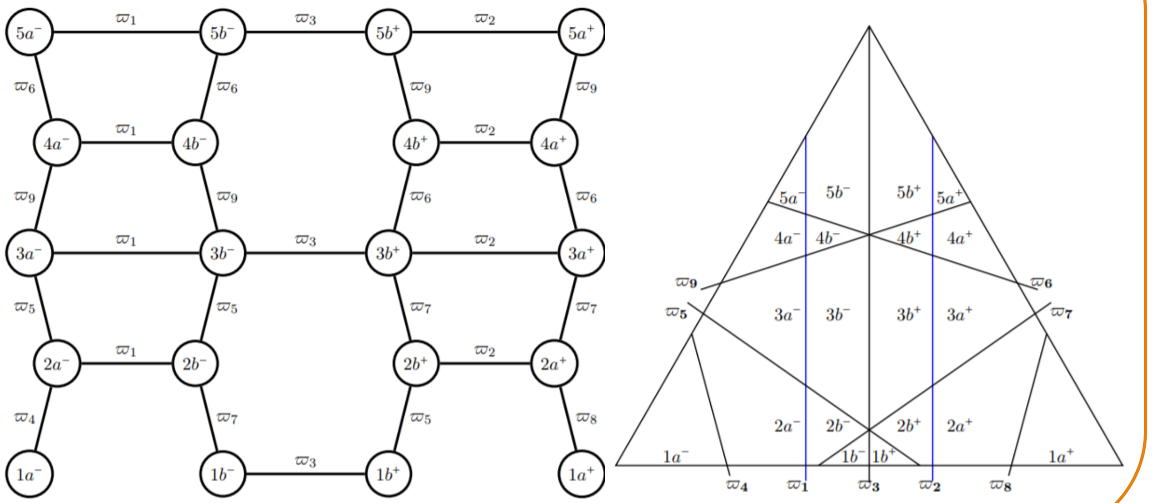
Semi-simple Group with MW Torsion

- Consider two non-trivial models of semi-simple Lie algebra with MW group $\mathbb{Z}_2$ that corresponds to the collision of the Kodaira fibers of type $I_2^{ns}+I_4$.

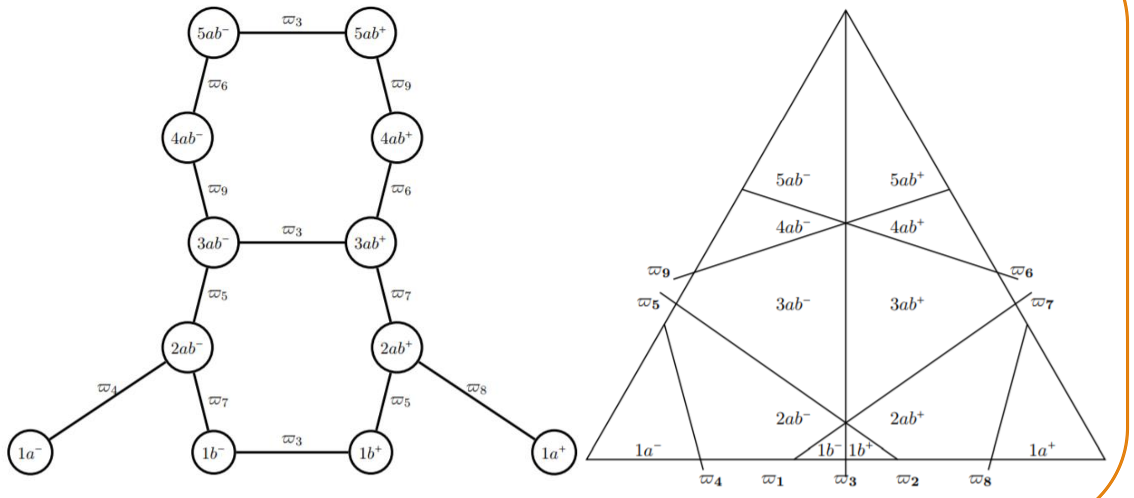
$\mathfrak{g} = A_1 \oplus A_3$	$\mathfrak{g} = A_1 \oplus C_2$
$G = \mathrm{SU}(2)\times\mathrm{SU}(4)$ $\pi_1(G) = \mathbb{Z}_2 \times \mathbb{Z}_4$	$G = \mathrm{SU}(2)\times\mathrm{Sp}(4)$ $\pi_1(G) = \mathbb{Z}_2 \times \mathbb{Z}_2$
Three possibilities for embedding $\mathbb{Z}_2$:	$(\mathbb{Z}_2, 1)$, $(1, \mathbb{Z}_2)$ diagonal $\mathbb{Z}_2$
Possible quotient groups: $\mathrm{SO}(3)\times\mathrm{SU}(4)$, $\mathrm{SU}(2)\times\mathrm{SO}(5)$, $(\mathrm{SU}(2)\times\mathrm{SU}(4))/\mathbb{Z}_2$ Their centers: $\mathbb{Z}_4$, $\mathbb{Z}_2\times\mathbb{Z}_2$, $\mathbb{Z}_2$ Bi-fundamental representation is only compatible with: $(\mathrm{SU}(2)\times\mathrm{SU}(4))/\mathbb{Z}_2$	Possible quotient groups: $\mathrm{SO}(3)\times\mathrm{Sp}(4)$, $\mathrm{SU}(2)\times\mathrm{SO}(6)$, $(\mathrm{SU}(2)\times\mathrm{Sp}(4))/\mathbb{Z}_2$ Their centers: $\mathbb{Z}_2$ Bi-fundamental representation is only compatible with: $(\mathrm{SU}(2)\times\mathrm{Sp}(4))/\mathbb{Z}_2$

Models	Algebraic data	# Flops
$\mathrm{I}_2^{\mathrm{ns}}+\mathrm{I}_4^{\mathrm{ns}}$ MW = $\mathbb{Z}_2$	$F = y^2z - (x^3 + a_2x^2z + st^2xz^2)$ $\Delta = s^2t^4(a_2^2 - 4st^2)$ $G = (\mathrm{SU}(2)\times\mathrm{Sp}(4))/\mathbb{Z}_2$ $\mathbf{R} = (\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})\oplus(\mathbf{2},\mathbf{4})\oplus(\mathbf{1},\mathbf{5})$ $\chi = -4(9K^2 + 8K\cdot T + 3T^2)$	3
$\mathrm{I}_2^{\mathrm{ns}}+\mathrm{I}_4^{\mathrm{ns}}$ MW = $\{1\}$	$F = y^2z - (x^3 + a_2x^2z + \tilde{a}_4st^2xz^2 + \tilde{a}_6s^2t^4z^3)$ $\Delta = s^2t^4(4a_2^3\tilde{a}_6 - a_2^2\tilde{a}_4^2 - 18a_2\tilde{a}_4\tilde{a}_6st^2 + 4a_4^3st^2 + 27\tilde{a}_6^2s^2t^4)$ $G = \mathrm{SU}(2)\times\mathrm{Sp}(4)$ $\mathbf{R} = (\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})\oplus(\mathbf{2},\mathbf{4})\oplus(\mathbf{1},\mathbf{5})\oplus(\mathbf{2},\mathbf{1})\oplus(\mathbf{1},\mathbf{4})$ $\chi = -2(30K^2 + 15K\cdot S + 30K\cdot T + 3S^2 + 8S\cdot T + 10T^2)$	3
$\mathrm{I}_2^{\mathrm{ns}}+\mathrm{I}_4^{\mathrm{s}}$ MW = $\mathbb{Z}_2$	$F = y^2z + a_1xyz - (x^3 + \tilde{a}_2tx^2z + st^2xz^2)$ $\Delta = s^2t^4\left(a_1^4 + 8a_1^2\tilde{a}_2t + 16\tilde{a}_2^2t^2 - 64st^2\right)$ $G = (\mathrm{SU}(2)\times\mathrm{SU}(4))/\mathbb{Z}_2$ $\mathbf{R} = (\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{15})\oplus(\mathbf{2},\mathbf{4})\oplus(\mathbf{2},\bar{\mathbf{4}})\oplus(\mathbf{1},\mathbf{6})$ $\chi = -12\left(3K^2 + 3K\cdot T + T^2\right)$	12
$\mathrm{I}_2^{\mathrm{ns}}+\mathrm{I}_4^{\mathrm{s}}$ MW = $\{1\}$	$F = y^2z + a_1xyz - (x^3 + \tilde{a}_2tx^2z + \tilde{a}_4st^2xz^2 + \tilde{a}_6s^2t^4z^3)$ $\Delta = s^2t^4\left(a_1^4 + 8a_1^2\tilde{a}_2t + 16\tilde{a}_2^2t^2 - 64st^2\right)$ $G = \mathrm{SU}(2)\times\mathrm{SU}(4)$ $\mathbf{R} = (\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{15})\oplus(\mathbf{2},\mathbf{4})\oplus(\mathbf{2},\bar{\mathbf{4}})\oplus(\mathbf{1},\mathbf{6})\oplus(\mathbf{2},\mathbf{1})\oplus(\mathbf{1},\mathbf{4})\oplus(\mathbf{1},\bar{\mathbf{4}})$ $\chi = -2\left(30K^2 + 15K\cdot S + 32K\cdot T + 3S^2 + 8S\cdot T + 10T^2\right)$	20

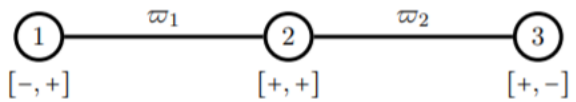
$G = \mathrm{SU}(2)\times\mathrm{SU}(4)$



$G = (\mathrm{SU}(2)\times\mathrm{SU}(4))/\ \mathbb{Z}_2$



$G = \mathrm{SU}(2)\times\mathrm{Sp}(4)$ $\quad G = (\mathrm{SU}(2)\times\mathrm{Sp}(4))/\ \mathbb{Z}_2$



Matter content for each model

F-theory on Y $\downarrow$ 6d $\mathcal{N}=(1,0)$ sugra	M-theory on Y $\downarrow$ 5d $\mathcal{N}=1$ sugra	F-theory on $Y \times S^1$ $\downarrow$ 5d $\mathcal{N}=1$ sugra
$n_V^{(6)} = h^{1,1}(Y) - h^{1,1}(B) - 1$ $n_H^0 = h^{2,1}(Y) + 1$ $n_T = h^{1,1}(B) - 1$	$n_V^{(5)} = n_V^{(6)} + n_T + 1 = h^{1,1}(Y) - 1$ $n_H^0 = h^{2,1}(Y) + 1$	

G	Adjoint	Bifundamental	(Traceless) Antisymmetric, Fundamental
$(SU(2) \times Sp(4))/\mathbb{Z}_2$	$n_{\mathbf{3},\mathbf{1}} = g_S \quad n_{\mathbf{1},\mathbf{10}} = g_T$	$n_{\mathbf{2},\mathbf{4}} = S \cdot T$	$n_{\mathbf{1},\mathbf{5}} = g_T - 1 + \frac{1}{2} T \cdot V(a_2)$
$SU(2) \times Sp(4)$	$n_{\mathbf{3},\mathbf{1}} = g_S \quad n_{\mathbf{1},\mathbf{10}} = g_T$	$n_{\mathbf{2},\mathbf{4}} = S \cdot T$	$n_{\mathbf{1},\mathbf{5}} = g_T - 1 + \frac{1}{2} T \cdot V(a_2)$ $n_{\mathbf{2},\mathbf{1}} = S \cdot V(\tilde{b}_8),\ n_{\mathbf{1},\mathbf{4}} = T \cdot V(\tilde{b}_8)$
$(SU(2) \times SU(4))/\mathbb{Z}_2$	$n_{\mathbf{3},\mathbf{1}} = g_S \quad n_{\mathbf{1},\mathbf{15}} = g_T$	$n_{\mathbf{2},\mathbf{4}} + n_{\mathbf{2},\bar{\mathbf{4}}} = S \cdot T$	$n_{\mathbf{1},\mathbf{6}} = T \cdot V(a_1)$
$SU(2) \times SU(4)$	$n_{\mathbf{3},\mathbf{1}} = g_S \quad n_{\mathbf{1},\mathbf{15}} = g_T$	$n_{\mathbf{2},\mathbf{4}} + n_{\mathbf{2},\bar{\mathbf{4}}} = S \cdot T$	$n_{\mathbf{1},\mathbf{6}} = T \cdot V(a_1)$ $n_{\mathbf{2},\mathbf{1}} = S \cdot V(\tilde{b}_8),\ n_{\mathbf{1},\mathbf{4}} + n_{\mathbf{1},\bar{\mathbf{4}}} = T \cdot V(\tilde{b}_8)$

Charged hypers from triple intersection polynomials

$G = (\mathrm{SU}(2)\times\mathrm{SU}(4))/\mathbb{Z}_2$

$$n_{\mathbf{3},\mathbf{1}} = (2K+T)(3K+2T)+1, \quad n_{\mathbf{2},\mathbf{4}} = n_{\mathbf{2},\bar{\mathbf{4}}} = -T(2K+T),$$

$$n_{\mathbf{1},\mathbf{6}} = -KT, \quad n_{\mathbf{1},\mathbf{15}} = \frac{1}{2}(KT+T^2+2).$$

$G = (\mathrm{SU}(2)\times\mathrm{Sp}(4))/\mathbb{Z}_2$

$$n_{\mathbf{3},\mathbf{1}} = 6L^2-7LT+2T^2+1 = g_S, \quad n_{\mathbf{2},\mathbf{4}} = -2T(T-2L) = 2(-4g_T+T^2+4),$$

$$n_{\mathbf{1},\mathbf{5}} = \frac{1}{2}\left(LT+T^2\right) = -g_T+T^2+1, \quad n_{\mathbf{1},\mathbf{10}} = \frac{1}{2}\left(-LT+T^2+2\right) = g_T.$$

Comparing the triple intersection polynomial with the 5d prepotential completely fixed the number of hypers charged in each irreducible representations when there is a Mordell-Weil group $\mathbb{Z}_2$.

$G = \mathrm{SU}(2)\times\mathrm{SU}(4)$

$$n_{\mathbf{1},\mathbf{4}} + n_{\mathbf{1},\bar{\mathbf{4}}} = -2T(4K+S+2T), \; n_{\mathbf{2},\mathbf{4}} + n_{\mathbf{2},\bar{\mathbf{4}}} = ST,$$

$$n_{\mathbf{2},\mathbf{1}} + 8n_{\mathbf{3},\mathbf{1}} = -2S(2K-S+2T).$$

$G = \mathrm{SU}(2)\times\mathrm{Sp}(4)$

$$n_{\mathbf{1},\mathbf{4}} + n_{\mathbf{1},\mathbf{10}} = -2(2KT+ST-4), \; n_{\mathbf{1},\mathbf{5}} + n_{\mathbf{1},\mathbf{10}} = T^2+1,$$

$$n_{\mathbf{2},\mathbf{1}} + 8n_{\mathbf{3},\mathbf{1}} = -2S(2K-S+2T)+8.$$

While for the cases with a trivial Mordell-Weil group, we are left with some linear relations.

Anomaly Cancellation

- Number of multiplets are given by: $n_V^{(6)} = \dim G, \quad n_T = h^{1,1}(B) - 1 = 9 - K^2,$

$$n_H = n_H^0 + n_H^{ch} = h^{2,1}(Y) + 1 + \sum_i n_{\mathbf{R_i}} \left(\dim \mathbf{R_i} - \dim \mathbf{R_i^{(0)}}\right)$$

- Gravitational Anomalies are canceled when $n_H - n_V^{(6)} + 29 n_T - 273 = 0.$

- For a semi-simple group with two simple components, $G = G_1 + G_2$, the remainder of the anomaly polynomial is given by

$$I_8 = \frac{K^2}{8}(\mathrm{tr}\, R^2)^2 + \frac{1}{6}(X_1^{(2)} + X_2^{(2)})\,\mathrm{tr}\, R^2 - \frac{2}{3}(X_1^{(4)} + X_2^{(4)}) + 4Y_{12}$$

where

$$\left\{ \begin{aligned} X_a^{(2)} &= \left(A_{a,\mathbf{adj}} - \sum_i n_{\mathbf{R_{i,a}}} A_{\mathbf{R_{i,a}}}\right) \mathrm{tr}_{\mathbf{F_a}} F_a^2, \\ X_a^{(4)} &= \left(B_{a,\mathbf{adj}} - \sum_i n_{\mathbf{R_{i,a}}} B_{\mathbf{R_{i,a}}}\right) \mathrm{tr}_{\mathbf{F_a}} F_a^4 + \left(C_{a,\mathbf{adj}} - \sum_i n_{\mathbf{R_{i,a}}} C_{\mathbf{R_{i,a}}}\right) (\mathrm{tr}_{\mathbf{F_a}} F_a^2)^2, \\ Y_{ab} &= \sum n_{\mathbf{R_{i,a}},\mathbf{R_{j,b}}} A_{R_{i,a}} A_{\mathbf{R_{j,b}}}\, \mathrm{tr}_{\mathbf{F_a}} F_a^2\, \mathrm{tr}_{\mathbf{F_b}} F_b^2. \end{aligned} \right\}$$

- If the I_8 factors, then the anomalies are all canceled by Green-Schwartz mechanism.

- We check that all the anomalies are canceled once all the number of hypers in each representation are identified.

Thank you for listening! ☺