Special geometry on the 101 dimensional moduli space of the quintic threefold.

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Special geometry

Special Kähler geometry is a mathematical structure describing 4d supergravity theories, e.g. target spaces of scalars in vector multiplets in type II. When supergravity is a low-energy limit of the superstring theory it is described in terms of geometry of the underlying Calabi-Yau manifold.

Explicit knowledge of the Kähler metric (Weil-Petersson metric) of the target space allows to compute vevs after moduli stabilization, enters Holomorphic Anomaly Equations to compute higher genus B-model partition function, allow to compute distances in the moduli space which is important for phenomenology (e.g. Refined Swampland Distance Conjecture).

What was done

Until now it was computed only in a few cases with small dimension of moduli spaces. We propose a new method which allows to write explicitly the moduli space metric at least for hypersurfaces in weighted projective spaces.

Our method works for arbitrary number of moduli, in particular, we wrote the metric on the 101-dimensional moduli space of the Quintic threefolds as a series around the orbifold point (mirror to FJRW theory).

Our approach is based on a version of CY/LG correspondence. Namely, we show that Weil-Petersson metric is equal to certain restriction of the tt^* metric of the corresponding LG model. The latter one can be easily computed.

As a bonus we get expression for all the (or almost all) periods of the holomorphic form in integral symplectic bases which gives D-branes central charges.

Some formulae

LG/CY type correspondence:

$(X_\phi, W\mathbb{P}^4)$	$(W(x,\phi), \mathbb{C}^5)$
$H^3(X_\phi) = F^3H^3 \supset \cdots \supset F^0H^3 = \langle\Omega\rangle$	R : invariant Milnor ring $R = R^{\leq 3d} \supset \cdots \supset R^{\leq 0} = \langle 1\rangle$
Complex conjugation on $H^3(X_\phi)$: $\overline{\chi_\mu} \to M_\mu^\nu \chi_\nu$	Landau-Ginzburg real structure on R
Period integrals: $\int_\gamma \Omega$	Complex oscillating integrals: $\int_{\Gamma^+} e^{-W(x,\phi)}\,\mathrm{d}^5x$
Weil-Petersson metric potential: $e^{-K} = \int_{X_\phi} \Omega \wedge \overline{\Omega}$	tt^* metric Kähler potential: $\sum_a \int_{\Gamma_+^a} e^{-W(x,\phi)}\,\mathrm{d}^5x\, \overline{\int_{\Gamma_a^-} e^{W(x,\phi)}\,\mathrm{d}^5x}$

Formula for the Quintic:

$$e^{-K} = \sum_{(k_1,\ldots,k_5),\ k_i<4, \sum k_i = 0,5,10,15} (-1)^{\sum k_i/5} \prod_{i=1}^{5} \frac{\Gamma\left((k_i+1)/5\right)}{\Gamma\left((4-k_i)/5\right)} \ |\sigma_{\bar{k}}(\phi)|^2,$$

where $\sigma_{\bar{k}}(\phi) = O(\phi^{\sum k_i/5})$ are explicit series in ϕ with rational coefficients.

THANK YOU FOR YOUR ATTENTION!