T-folds from YB deformations

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arXiv : 1710.06849 JHEP 1712 (2017) 108

arXiv : 1803.05903 JHEP 1806 (2018) 147

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String: T-duality, Integrability and Geometry   2019/03/05@Tohoku Univ.

Introduction

■ YB deformation [Klimcik, 2002,2008]

- A systematic way that describes integrable deformations of 2d non-linear sigma models.
- A deformation is characterized by specifying an r-matrix.

  (a solution of the (m)CYBE)
- Application to the $\mathrm{AdS}_5 \times S^5$ superstring

  [Delduc-Magro-Vicedo, 1309.5850] [Kawaguchi-Matsumoto-Yoshida,1401.4855]

Q. What is the physical meaning of the deformations?

YB deformations based on the CYBE = String duality transf.

[Matsumoto-Yoshida, 1404.1838,1404.3657, 1502.00740] [Orlando-Reffert-J.S.-Yoshida , 1607.00795]
[Hoare-Tseytlin, 1609.02550] [Borsato-Wulff, 1609.09834, 1706.10169]
[J.S.-Sakatani-Yoshida, 1703.09213, 1705.07116] etc.

Introduction

■ Relations to string duality transformations

➢ The first observation [Matsumoto-Yoshida, 1404.1838,1404.3657,1502.00740] [Osten-Tongeren, 1608.08504]

> A class of YB deformations = TsT transformations

e.g. gravity dual of NCSYM, Lunin-Maldacena background
[Hashimoto-Itzhaki, Maldacena-Russo, 1999] [Lunin-Maldacena, 2005]

➢ General case

> YB deformations = (a class of) Non-Abelian T-dualities

Conjecture: [Hoare-Tseytlin, 1609.02550]
Proof: [Borsato-Wulff, 1609.09834, 1706.10169]

Introduction

➢ Another reformulation in terms of DFT

[J.S.-Sakatani-Yoshida, 1703.09213, 1705.07116]
[J.S.-Sakatani, 1803.05903]

YB deformations are equal to (local) β-transformations

The transf. is generated by $e^{\beta} = \begin{pmatrix} \delta^m_n & \beta^{mn} \\ 0 & \delta^n_m \end{pmatrix} \in O(10,10)$

e.g. Constant β : TsT transformations

➢ Some of YB deformed b.g. have non-geometric Q-fluxes.

YB deformed backgrounds may be regarded as T-folds.

[Fernandez-Melgarejo-J.S.-Sakatani-Yoshida, 1710.06849]

Talk Plan

0. Introduction

1. YB deformations of the $\mathrm{AdS}_5 \times S^5$ superstring

2. YB deformations as local β-transformations

3. T-folds from YB deformations

4. Summery and discussion

1. YB deformations of the $AdS_5 \times S^5$ superstring

The $\mathrm{AdS}_5 \times S^5$ superstring

The $\mathrm{AdS}_5 \times S^5$ superstring can be described by using the supercoset

$$\frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

■ GS action of the $\mathrm{AdS}_5 \times S^5$ sueperstring [Metsaev-Tseytlin, 9805028]

$$S = -\frac{T}{4}\int \mathrm{d}^2\sigma(\gamma^{\alpha\beta} - \epsilon^{\alpha\beta})\mathrm{Str}\left[g^{-1}\partial_\alpha g\, d(g^{-1}\partial_\beta g)\right]$$

$$g = g(X,\Theta) \in SU(2,2|4) \qquad d = P_1 + 2P_2 - P_3$$

$P_i\,(i = 0,1,2,3)$: projections to the $\mathbb{Z}_4$-grading components of su(2,2|4)

➡ existence of a Lax pair (classically integrable)

[Bena-Polchinski-Roiban, hep-th/0305116]

YB deformations of the $\mathrm{AdS}_5 \times S^5$ superstring

- The action of the YB sigma model for $\mathrm{AdS}_5 \times S^5$

[Delduc-Magro-Vicedo, 1309.5850] [Kawaguchi-Matsumoto-Yoshida,1401.4855]

$$S_{\mathrm{YB}} = -\frac{T}{4}\int \mathrm{d}^2\sigma\, (\gamma^{\alpha\beta} - \epsilon^{\alpha\beta})\, \mathrm{Str}\left[g^{-1}\partial_\alpha g\, d \circ \frac{1}{1 - \eta\, R_g \circ d} g^{-1}\partial_\beta g\right]$$

- A deformation parameter η
- The skew-symmetric linear operator R

$$R_g(x) = g^{-1}R(gxg^{-1})g \qquad x \in \mathfrak{su}(2,2|4)$$

The action of R is specified by taking an r-matrix

YB deformations of the $\mathrm{AdS}_5 \times S^5$ superstring

- The action of R-operator

r-matrix : $r = \frac{1}{2} r^{ij} T_i \wedge T_j$ ➡ $R(x) = r^{ij} T_i \, \mathrm{Str}(T_j \, x)$

$$r^{ij} = -r^{ji} = \text{const.} \qquad T_i \in \mathfrak{su}(2,2|4)$$

Condition : The classical Yang-Baxter equation (CYBE)

$$[R(x), R(y)] - R([R(x), y] + [x, R(y)]) = c[x, y]$$

(I) $c = \pm 1$ (mCYBE) (II) $c = 0$ (CYBE)

- The existence of a Lax pair ➡ Integrable deformations
- κ -symmetry

A derivation of YB deformed $AdS_5 \times S^5$

Strategy

expand $S_{\rm YB}$ up to second order in Θ_I ,
and compare it with the canonical form of the GS action
[Arutyunov-Borsato-Frolov, 1507.04239] [Kyono-Yoshida,1605.02519]

The canonical form of the GS action

[Cvetic-Lu-Pope-Stelle, 9907202]

$$
\begin{aligned}
S_{\rm YB} = -T\int \mathrm{d}^2\sigma\, \Big[& P_-^{\alpha\beta}(g_{mn}+B_{mn})\partial_\alpha X^m \partial_\beta X^n && \Big\}\ \mathcal{O}(\Theta^0) \\
& + i\,P_+^{\alpha\beta}\bar\Theta_1 e_\alpha{}^a \Gamma_a D_{+\beta}\Theta_1 + i\,P_-^{\alpha\beta}\bar\Theta_2 e_\alpha{}^a \Gamma_a D_{-\beta}\Theta_2 && \\
& - i\frac{1}{8}P_+^{\alpha\beta}\bar\Theta_1 e_\alpha{}^a \Gamma_a\, e^{\Phi}\boldsymbol{F}\, e_\beta{}^b \Gamma_b \Theta_2 \Big] + \mathcal{O}(\Theta^4) && \Big\}\ \mathcal{O}(\Theta^2)
\end{aligned}
$$

➢ $P_\pm^{\alpha\beta} = \dfrac{\gamma^{\alpha\beta}\pm\epsilon^{\alpha\beta}}{2}$, $\boldsymbol{F} \equiv \displaystyle\sum_{n=1,3,5,7,9}\frac{1}{n!}F_{a_1\cdots a_n}\Gamma^{a_1\cdots a_n}$

We can write down the general formulas for deformed b.g.

[J.S.-Sakatani, 1803.05903]

The general formula for YB deformed $\mathrm{AdS}_5 \times S^5$

Assumption: $r = \frac{1}{2} r^{ij} T_i \wedge T_j \quad T_i \in \mathfrak{so}(2,4) \times \mathfrak{so}(6)$

Cf. [Borsato-Wulff,1608.03570] for the most general case.

■ Metric and B-field

$$g'_{mn} + B'_{mn} = \left[(g^{-1} - \beta)\right]^{-1}_{mn} ,$$

- g_{mn} : $\mathrm{AdS}_5 \times S^5$ metric
- β-field : $\beta^{mn}(x) = 2\eta\, r^{ij} \hat{T}_i^m \hat{T}_j^n$ ⬅ Killing vector for T_i

[Araujo-Bakhmatov-O Colgain-J.S.-Sheikh Jabbari-Yoshida, 1702.02861, 1705.02063]
[J.S.-Sakatani-Yoshida, 1703.09213, 1705.07116] [J.S.-Sakatani, 1803.05903]

The general formula for YB deformed $AdS_5 \times S^5$

- Dilaton and R-R fields

[Borsato-Wulff,1608.03570]
[J.S.-Sakatani-Yoshida, 1703.09213, 1705.07116]
[J.S.-Sakatani, 1803.05903]

$$\Phi' = \Phi + \frac{1}{2}\log\left(\sqrt{|g'|}\right) - \frac{1}{2}\log\left(\sqrt{|g|}\right)$$

$$F' = e^{-B'_2 \wedge} e^{-\beta \vee} F_5$$ (in differential form)

➢ $F' = \sum_{p=1,3,5,7,9} F'_p \qquad F_5 = 4(\omega_{\mathrm{AdS}_5} + \omega_{\mathrm{S}^5})$

➢ $\beta \vee F_5 \equiv \frac{1}{2}\beta^{mn}\iota_m\iota_n F_5 \qquad \iota_m$: Interior product along x^m

The unimodularity condition

Q. Is a deformed background always a solution of SUGRA ?

A. NO

■ The unimodularity condition [Borsato-Wulff 1608.03570]

$r^{ij}[T_i, T_j] = 0$ ➡ SUGRA solutions

- Abelian r-matrix : $[T_i, T_j] = 0$ (TsT transformation)

  e.g. gravity dual of NCSYM, Lunin-Maldacena background

  [Matsumoto-Yoshida, 1404.1838] [Matsumoto-Yoshida, 1404.3657]

- Non-Abelian r-matrix : $[T_i, T_j] \neq 0$

  e.g. $r = a_1 \, P_3 \wedge M_{12} + a_2 \, P_1 \wedge P_2$

  [Borsato-Wulff 1608.03570]

Non-unimodular deformations and the GSE

Non-unimodular case

YB deformations give solutions
to the generalized supergravity equations (GSE)

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795] [Tseytlin-Wulff, 1605.04884]

Sakatani-san's and Yoshida-san's talks

NOTE: A non-dynamical Killing vector field I is contained.

For YB deformed b.g. , the Killing vector I is given by

$$\frac{1}{\sqrt{|g|}}\partial_n(\sqrt{|g|}\beta^{nm}) = I^m$$

[Araujo-Bakhmatov-O Colgain-J.S.-Sheikh Jabbari-Yoshida, 1702.02861]

The β-field characterizes all fields on deformed backgrounds.

2. YB deformations as local β-transformations

YB deformations as local β-transformations

YB deformations are a kind of O(d,d) duality transformations

[J.S.-Sakatani, 1803.05903] [J.S.-Sakatani-Yoshida, 1703.09213, 1705.07116]

■ The generalized metric (A bosonic field in DFT)

$$\mathcal{H}_{MN} = \begin{pmatrix} (g - B\, g^{-1}\, B)_{mn} & B_{mk}\, g^{kn} \\ -g^{mk}\, B_{kn} & g^{mn} \end{pmatrix} ,$$

➢ Covariant under the O(10,10) duality transformations

$$\mathcal{H}' = h^{\mathrm{T}} \mathcal{H} h\,, \quad \underline{h \in O(10,10)}$$

➡ B-shift, T-dualities, β-transformations

YB deformations as local β-transformations

- (local) β-transformation

$$\mathcal{H}' = e^{\beta^{\mathrm{T}}} \mathcal{H} e^{\beta} \qquad e^{\beta} = \begin{pmatrix} \delta_n^m & \beta^{mn} \\ 0 & \delta_m^n \end{pmatrix} \in O(10,10)$$

- If we take the generalized metric and the β-field as

$$\mathcal{H}_{MN} = \begin{pmatrix} g_{mn} & 0 \\ 0 & g^{mn} \end{pmatrix} \qquad \beta^{mn}(x) = 2\eta\, r^{ij} \hat{T}_i^m \hat{T}_j^n$$

we can reproduce the previous formula for YB deformed b.g.:

$$g'_{mn} + B'_{mn} = [(g^{-1} - \beta)]^{-1}_{mn} \,,$$

- The transformations of RR fields and dilaton can also be understood as the (local) β-transformation.

Comments on local β-transformations

Comment 1

If β-fields are not composed of r-matrices satisfying the CYBE, the associated β-transf. generally do not give solutions to the SUGRA eq. and the GSE.

Comment 2

We can apply β-transformations to almost all backgrounds.

e.g. Minkowski spacetime, [Matsumoto-Orlando-Reffert-J.S.-Yoshida, 1505.04553] [Fernandez-Melgarejo-J.S.-Sakatani-Yoshida, 1710.06849]

pp-wave, [Okumura-J.S.-Yoshida, 19XX.XXXXX]

$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{T}^4$ with H-flux [J.S.-Sakatani, 1803.05903] [Borsato-Wulff, 1812.07287] [Araujo-Ó Colgáin-Sakatani-Sheikh-Jabbari-Yavartanoo, 1811.03050]

It is not straightforward to define YB sigma models for these b.g.

Example

■ Abelian r-matrix [Matsumoto-Yoshida, 1404.3657]

$$r = \frac{1}{2} P_1 \wedge P_2 \qquad [P_1, P_2] = 0$$

P_μ : Translation generators of so(2,4)

We take a coordinate system of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ background as

$$\mathrm{d}s^2 = \frac{\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}z^2}{z^2} + \mathrm{d}s^2{}_{\mathrm{S}^5} \qquad (\mu\,,\nu = 0,1,2,3)$$

$$F_5 = 4\left(\omega_{\mathrm{AdS}_5} + \omega_{\mathrm{S}^5}\right) \qquad \omega_{\mathrm{AdS}_5} = -\frac{\mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}z}{z^5}\,,$$

The associated β−field is

$$\beta = \eta\,\hat{P}_1 \wedge \hat{P}_2 = \eta\,\partial_1 \wedge \partial_2$$

Example

For the NS sector, we consider the following matrix

$$(g^{-1}-\beta)^{mn} = \begin{pmatrix} -z^2 & 0 & 0 & 0 & 0 \\ 0 & z^2 & -\eta & 0 & 0 \\ 0 & \eta & z^2 & 0 & 0 \\ 0 & 0 & 0 & z^2 & 0 \\ 0 & 0 & 0 & 0 & z^2 \end{pmatrix},$$

$$[\, x^0, x^1, x^2, x^3, z \,]$$

By taking the inverse of the matrix, we obtain

$$ds^2 = \frac{dz^2 - (dx^0)^2 + (dx^3)^2}{z^2} + \frac{z^2[(dx^1)^2 + (dx^2)^2]}{z^4+\eta^2} + ds^2_{S^5},$$

$$B_2 = \frac{\eta}{z^4+\eta^2}\, dx^1 \wedge dx^2, \qquad \Phi = \frac{1}{2}\log\left[\frac{z^4}{z^4+\eta^2}\right].$$

This is the NS sector of a gravity dual of NC SYM with $[\hat{x}^1, \hat{x}^2] = i\,\eta$.

[Hashimoto-Itzhaki, Maldacena-Russo, 1999]

Example

The undeformed RR 5-form

$$F_5 = 4\left(\omega_{\mathrm{AdS}_5} + \omega_{\mathrm{S}^5}\right) \qquad \omega_{\mathrm{AdS}_5} = -\frac{\mathrm{d}x^0\wedge\mathrm{d}x^1\wedge\mathrm{d}x^2\wedge\mathrm{d}x^3\wedge\mathrm{d}z}{z^5}\,,$$

- STEP1

$$\eta\,\iota_{x^1}\iota_{x^2}\,\omega_{\mathrm{AdS}_5}$$

$$e^{-\beta\vee}F_5 = 4\left(\omega_{\mathrm{AdS}_5} + \omega_{\mathrm{S}^5}\right) - 4\,\beta\vee\omega_{\mathrm{AdS}_5}$$

$$= 4\left(\omega_{\mathrm{AdS}_5} + \omega_{\mathrm{S}^5}\right) - 4\eta\,\frac{\mathrm{d}x^0\wedge\mathrm{d}x^3\wedge\mathrm{d}z}{z^5}$$

- STEP2

$$F' = e^{-B_2'\wedge}e^{-\beta\vee}F_5$$

$$= -4\,\eta\,\frac{\mathrm{d}x^0\wedge\mathrm{d}x^3\wedge\mathrm{d}z}{z^5} + 4\left(\frac{z^4}{z^4+\eta^2}\,\omega_{\mathrm{AdS}_5} + \omega_{\mathrm{S}^5}\right) - 4\,B_2'\wedge\omega_{\mathrm{S}^5}$$

3-form 5-form 7-form

3. T-folds from YB deformations

3. T-folds from YB deformations

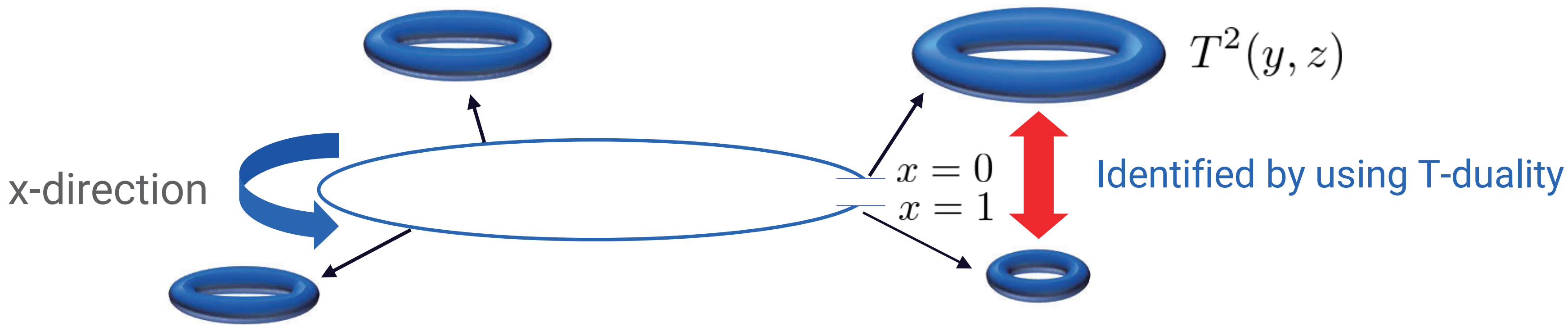
The structure group contains the T-duality group

[Hull, 0406102]

Toy example of T-fold

- A simple example of T-fold [Hull, 0406102]

$$ds^2 = dx^2 + \frac{dy^2 + dz^2}{1 + k^2 x^2}\,, \qquad B_2 = \frac{k\,x}{1 + k^2 x^2} dy \wedge dz\,.$$



Glue using diff. and T-dualities.

O(d,d; Z) T-duality monodromy

The background has the T-duality monodromy

($\because$) The generalized metric on this background is

$$\mathcal{H}_{MN}(x) = \begin{pmatrix} \delta_m^p & 0 \\ -2k\,x\,\delta_y^{[m}\delta_z^{p]} & \delta_p^m \end{pmatrix} \begin{pmatrix} \delta_{pq} & 0 \\ 0 & \delta^{pq} \end{pmatrix} \begin{pmatrix} \delta_n^q & 2k\,x\,\delta_y^{[q}\delta_z^{n]} \\ 0 & \delta_q^n \end{pmatrix} .$$

$\Rightarrow$ $\beta^{yz} = k\,x$

This expression implies

$$\mathcal{H}_{MN}(x+1) = \left[\Omega^{\mathrm{T}}\,\mathcal{H}(x)\Omega\right]_{MN} ,$$

where $\Omega^M{}_N = \begin{pmatrix} \delta_n^m & 2k\,\delta_y^{[m}\delta_z^{n]} \\ 0 & \delta_m^n \end{pmatrix} \in O(3,3;\mathbb{Z})\,.\quad k \in \mathbb{Z}$

O(3,3; Z) T-duality monodromy

Non-geometric Q-flux and O(d, d; Z) monodromy

- The O(3, 3; Z) monodromy matrix = a constant shift in the β field

$$\beta^{yz} = k\,x \qquad \beta^{yz} \to \beta^{yz} + k$$

- Non-geometric Q-flux

$$\boxed{Q_p{}^{mn} = \partial_p \beta^{mn}}$$

Then, the constant shift in the β field is rewritten as

$$\beta^{mn}(x+1) - \beta^{mn}(x) = \int_x^{x+1} \mathrm{d}x'^p\,\partial_p\beta^{mn}(x') = \int_x^{x+1} \mathrm{d}x'^p\,Q_p{}^{mn}(x')\,.$$

The non-geometricity is measured by the Q-flux.

T-folds from YB deformations

We can obtain T-folds from YB deformations

(∵)

YB b.g.: $\mathcal{H}' = e^{\beta^{\mathrm{T}}}\mathcal{H}e^{\beta}$ $\quad e^{\beta} = \begin{pmatrix} \delta^m_n & \beta^{mn} \\ 0 & \delta_m^n \end{pmatrix} \in O(10,10)$

Assumption: $\beta^{mn}(y) = \underline{\beta^{mn}_y}\, y + \underline{\bar{\beta}^{mn}}$ (present such examples later)

Constant bi-vectors

By performing the shift $y \to y+1$, we can obtain

Q-flux: $Q^{mn}_y = \beta^{mn}_y$ (constant)

Monodromy: $\Omega^M{}_N = \begin{pmatrix} \delta^m_n & \beta^{mn}_y \\ 0 & \delta_m^n \end{pmatrix} \in O(10,10;\mathbb{Z})\,.$

Examples (unimodular)

- Unimodular r-matrix [Borsato-Wulff 1608.03570]

$$r = \frac{1}{2\,\eta}\left[\eta_1\,(D + M_{+-}) \wedge P_+ + \eta_2\,M_{+2} \wedge P_3\right], \qquad D : \text{The dilatation op.}$$

➢ β-field : $\beta = \eta_1\left(z\,\partial_z + 2\,x^-\,\partial_- + x^2\,\partial_2 + x^3\,\partial_3\right) \wedge \partial_+ + \eta_2\left(x^2\,\partial_+ + x^-\,\partial_2\right) \wedge \partial_3\,.$

➢ The deformed b.g. is a solution of type IIB SUGRA

$$\begin{aligned}
\mathrm{d}s^2 &= \frac{\mathrm{d}z^2}{z^2} + \frac{z^2\,[(\mathrm{d}x^2)^2 + (\mathrm{d}x^3)^2]}{z^4 + (\eta_2\,x^-)^2} - \frac{2\,z^2\,\mathrm{d}x^+\,\mathrm{d}x^- - 4\,\eta_1^2\,z^{-1}\,x^-\,\mathrm{d}z\,\mathrm{d}x^-}{z^4 - (2\,\eta_1\,x^-)^2} \\
&\quad + \frac{2\,\{[x^2(2\,\eta_1^2 + \eta_2^2) - \eta_1\,\eta_2\,x^3]\,z^2\,x^-\,\mathrm{d}x^2 + \eta_1\,(2\,\eta_1\,x^3 - \eta_2\,x^2)\,\mathrm{d}x^3\}\,\mathrm{d}x^-}{[z^4 - (2\,\eta_1\,x^-)^2]\,[z^4 + (\eta_2\,x^-)^2]} \\
&\quad - \frac{(\eta_1^2 + \eta_2^2)\,(z\,x^2)^2 - 2\,\eta_1\,\eta_2\,z^2\,x^2\,x^3 + \eta_1^2\,[z^4 + (z\,x^3)^2 + (\eta_2\,x^-)^2]}{[z^4 - (2\,\eta_1\,x^-)^2]\,[z^4 + (\eta_2\,x^-)^2]}\,(\mathrm{d}x^-)^2 + \mathrm{d}s^2_{\mathrm{S}^5}\,, \\
B_2 &= -\left[\frac{\eta_1\,\{x^2\,[z^4 + 2\,(\eta_2\,x^-)^2] - 2\,\eta_1\,\eta_2\,(x^-)^2\,x^3\}\,\mathrm{d}x^2 + \{\eta_1\,z^4\,x^3 - \eta_2\,x^2\,[z^4 - 2\,(\eta_1\,x^-)^2]\}\,\mathrm{d}x^3}{[z^4 - (2\,\eta_1\,x^-)^2]\,[z^4 + (\eta_2\,x^-)^2]}\right. \\
&\quad \left. + \frac{\eta_1\,(z\,\mathrm{d}z - 2\,x^-\,\mathrm{d}x^+)}{z^4 - (2\,\eta_1\,x^-)^2}\right] \wedge \mathrm{d}x^- + \frac{\eta_2\,x^-\,\mathrm{d}x^2 \wedge \mathrm{d}x^3}{z^4 + (\eta_2\,x^-)^2}\,, \\
\Phi &= \frac{1}{2}\ln\left[\frac{z^8}{[z^4 - (2\,\eta_1\,x^-)^2]\,[z^4 + (\eta_2\,x^-)^2]}\right],
\end{aligned}$$

(+ RR 1, 3, 5, 7, 9-form field strengths)

Examples (unimodular)

- Q-flux : $Q_z{}^{z+} = \eta_1\,, \quad Q_-{}^{-+} = 2\,\eta_1\,, \quad Q_2{}^{2+} = \eta_1\,, \quad Q_3{}^{3+} = \eta_1\,, \quad Q_2{}^{+3} = \eta_2\,, \quad Q_-{}^{23} = \eta_2\,.$

- The monodromy matrix corresponding to the shift $x^3 \sim x^3 + \eta_1^{-1}$:

$$\mathcal{H}_{MN}(x^3 + \eta_1^{-1}) = \left[\Omega^{\mathrm{T}}\mathcal{H}(x)\,\Omega\right]_{MN}\,, \quad \Omega^M{}_N \equiv \begin{pmatrix} \delta^m_n & 2\,\delta_3^{[m}\,\delta_+^{n]} \\ 0 & \delta^n_m \end{pmatrix} \in \mathrm{O}(10,10;\mathbb{Z})\,.$$

We can regarded the deformed b.g. as a T-fold.

NOTE : Until now, there were very few T-folds that can be used as backgrounds of string theory.
In particular, T-folds with RR fields was obtained for the first time.

The YB deformation is a systematic procedure to obtain solutions with Q-fluxes in DFT !

Examples (non-unimodular)

- Non-unimodular r-matrix

[Orlando-Reffert-J.S.-Yoshida, 1607.00795]

$$r = \frac{1}{2} P_0 \wedge D \qquad [D, P_0] = P_0$$

➡ Non-unimodular

- The β-field :

$$\beta = \eta\, \hat{P}_0 \wedge \hat{D} = \eta\, \partial_0 \wedge (z\partial_z + x^1\partial_1 + x^2\partial_2 + x^3\partial_3)$$

- The divergence of the β-field is non-zero

$$I^0 = \frac{1}{\sqrt{|g|}} \partial_n (\sqrt{|g|}\beta^{0n}) = -\eta$$

Examples (non-unimodular)

➢ The background is a solution of the GSE: [Orlando-Reffert-J.S.-Yoshida, 1607.00795]

$$ds^2 = \frac{z^2\,[\mathrm{d}z^2 - (\mathrm{d}x^0)^2 + \mathrm{d}\rho^2] - \eta^2\,(\mathrm{d}\rho - \rho\, z^{-1}\,\mathrm{d}z)^2}{z^4 - \eta^2\,(z^2+\rho^2)} + \frac{\rho^2\,(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)}{z^2} + \mathrm{d}s^2_{\mathrm{S}^5}\,,$$

$$B_2 = -\eta\,\frac{\mathrm{d}x^0 \wedge (z\,\mathrm{d}z + \rho\,\mathrm{d}\rho)}{z^4 - \eta^2\,(z^2+\rho^2)}\,, \qquad \Phi = \frac{1}{2}\ln\biggl[\frac{z^4}{z^4 - \eta^2\,(z^2+\rho^2)}\biggr]\,, \qquad I = -\eta\,\partial_0\,,$$

(+ RR 3,5,7 fields) $\quad x^1 = \rho\sin\theta\cos\phi\,, \qquad x^2 = \rho\sin\theta\sin\phi\,, \qquad x^3 = \rho\cos\theta\,,$

➢ Q-flux : $\quad Q_z{}^{0z} = Q_1{}^{01} = Q_2{}^{02} = Q_3{}^{03} = \eta\,.$

➢ The monodromy matrix corresponding to the shift $x^1 \sim x^1 + \eta^{-1}$:

$$\mathcal{H}_{MN}(x^1 + \eta^{-1}) = \bigl[\Omega^{\mathrm{T}}\mathcal{H}(x^1)\,\Omega\bigr]_{MN}\,, \quad \Omega^M{}_N \equiv \begin{pmatrix} \delta^m_n & 2\,\delta_0^{[m}\,\delta_1^{n]} \\ 0 & \delta_m^n \end{pmatrix}\,.$$

➡ This background is also regarded as a T-fold.

Examples (non-unimodular)

Q. Is there a difference between T- folds of the GSE and those of the standard SUGRA?

From the divergence formula, the following expression holds:

$$I^0 = \frac{1}{\sqrt{|g|}}\partial_n(\sqrt{|g|}\beta^{0n}) = Q_n{}^{0n} + \Gamma^0_{nq}\beta^{qn} + \Gamma^n_{nq}\beta^{0q}$$

This might imply that the extra vector I is a source of the trace of the Q-flux.

YB deformations with non-unimodular r-matrices lead to non-geometric backgrounds.

Summery

- We mainly considered YB def. of the $\mathrm{AdS}_5 \times S^5$ superstring.
- We directly obtained the general formula of YB deformed backgrounds from the GS action.
- YB deformations can be understood as local β-transformations.
  (= A kind of O(d,d) duality transformations)
- We also discussed the global structures of YB deformed backgrounds based on the CYBE.
- Some YB deformed backgrounds can be regarded as T-folds.
  In the case of the generalized supergravity backgrounds,
  the Killing vector I is related to the trace of the Q-flux.

Discussion

- What is the condition for Poisson bi-vectors (β-fields) to give SUGRA solutions?

  In the case of YB def., the β-field is a Poisson bi-vector:

  $$\beta^{mn}(x) = 2\eta\, r^{ij}\hat{T}_i^m \hat{T}_j^n \quad \beta^{[m|q}\,\partial_q \beta^{|np]} = 0$$ (→ Poisson bi-vec.)

  However, there exists Poisson bi-vectors that are not associated with r-matrices.

- Generalization to the membrane sigma models

- Can YB deformed backgrounds be interpreted as T-folds even after receiving higher order α'-corrections?

Thank you

Appendix

The cases of non-Abelian r-matrices

- Unimodular r-matrix: [Borsato-Wulff 1608.03570]

$$r = a_1 P_3 \wedge L_{12} + a_2 P_1 \wedge P_2$$

- The resulting deformed background is given by

$$\begin{aligned}
ds^2 =& z^{-6}\left(1+\frac{\eta^2(a_1^2\rho^2+a_2^2)}{z^4}\right)^{-1}\left[z^4(d\rho^2+(dx^3)^2+\rho^2 d\theta^2)+\eta^2(a_1\rho d\rho - a_2 dx^3)^2\right] \\
&+\frac{-(dx^0)^2+dz^2}{z^2}+ds^2_{S^5}\,, \\
B =& \frac{\eta(a_1\rho^2 dx^3 + a_2\rho d\rho)\wedge d\theta}{z^4+\eta^2(a_1^2\rho^2+a_2^2)}\,, \qquad\qquad x^1 = \rho\cos\theta\,, x^2 = \rho\sin\theta \\
F_3 =& -\frac{4\eta}{z^5}dx^0\wedge(a_1\rho d\rho - a_2 dx^3)\wedge dz\,, \\
F_5 =& 4(e^{2\Phi}\omega_{\mathrm{AdS}_5}+\omega_{S^5})\,, \qquad \Phi = \frac{1}{2}\log\left[\frac{z^4}{z^4+\eta^2(a_1^2\rho^2+a_2^2)}\right]
\end{aligned}$$

The background is a solution of the usual type IIB SUGRA.

The cases of non-Abelian r-matrices

The above deformed background is reproduced by the following sequences:

1. TsT transformation

$$\text{T-dual:} \quad x^3 \to \tilde{x}^3 \,, \qquad \theta \to \theta + a_1 \eta \tilde{x}^3 \,, \quad \text{T-dual:} \quad \tilde{x}^3 \to x^3$$

2. Coordinate transformation

$$\rho = \sqrt{(x^1)^2 + (x^2)^2} \,, \qquad \theta = \mathrm{Arctan}(x^2/x^1)$$

3. TsT transformation

$$\text{T-dual:} \quad x^1 \to \tilde{x}^1 \,, \quad \tilde{x}^2 \to x^2 + a_2 \eta \tilde{x}^1 \,, \quad \text{T-dual:} \quad \tilde{x}^1 \to x^1$$

The generalized supergravity equations

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795] [Tseytlin-Wulff, 1605.04884]

The generalized supergravity equations (GSE)

$$R_{MN} - \frac{1}{4} H_{MKL} H_N{}^{KL} + D_M X_N + D_N X_M = T_{MN} \,,$$

$$\frac{1}{2} D^K H_{KMN} + \frac{1}{2} \mathcal{F}^K \mathcal{F}_{KMN} + \frac{1}{12} \mathcal{F}_{MNKLP} \mathcal{F}^{KLP} = X^K H_{KMN} + D_M X_N - D_N X_M \,,$$

$$R - \frac{1}{12} H_3^2 + 4 D_M X^M - 4 X_M X^M = 0 \,,$$

$$\mathrm{d} * \mathcal{F}_p - Z \wedge * \mathcal{F}_p + *(I \wedge \mathcal{F}_{p-2}) - H_3 \wedge * \mathcal{F}_{p+2} = 0 \,, \qquad \mathcal{F}_{n_1 n_2 \dots} = \mathrm{e}^{\Phi} F_{n_1 n_2 \dots}$$

Here, we have ignored dilatino, and gravitino.

The modifications are characterized by *I* and *Z*.

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Here, we have ignored dilatino, and gravitino.

The modifications are characterized by *I* and *Z*.

The relation between I and Z

$$D_m I_n + D_n I_m = 0 \qquad \text{(Killing equations)}$$

$$I^p H_{pmn} + 2\,\partial_{[m} Z_{n]} = 0 \qquad Z_m I^m = 0$$

By fixing the gauge of *B* as $\mathcal{L}_I B = 0$,

$$\Rightarrow \quad Z_m = \partial_m \Phi + I^n B_{nm}$$

The GSE can be characterized by the Killing vector *I*