Poisson-Lie Symmetry and Double Field Theory

Part I

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based on

1810.11446,
1707.08624, 1611.07978,
1502.02428, 1410.6374

and work in progress

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Motivation: I) $\mathcal{E}$-Model ...

$$S = S_{\mathrm{WZW}} - \frac{1}{2}\int \langle l^{-1}\partial_\sigma l, \mathcal{E} l^{-1}\partial_\sigma l\rangle$$

$$S_{\mathrm{WZW}} = \frac{1}{2}\int \mathrm{d}\sigma\,\mathrm{d}\tau \langle l^{-1}\partial_\sigma l, l^{-1}\partial_\tau l\rangle + \frac{1}{12}\int \langle [l^{-1}\mathrm{d}l, l^{-1}\mathrm{d}l], l^{-1}\mathrm{d}l\rangle$$

- target space Lie group $\mathcal{D} \ni l$ with maximal isotropic subgroup $\tilde{G}$
- Poisson-Lie symmetry and T-duality are manifest symmetries
- integrate out 1/2 of the degrees of freedom $\to$ σ-model on $\mathcal{D}/\tilde{G}$
- $\mathcal{D}$ captures the phase space of this σ-model

$$J = T_A J^A = l^{-1}\partial_\sigma l$$

$$H = \frac{1}{2}\int \mathrm{d}\sigma \langle J, \mathcal{E} J\rangle$$

$$\{J^A(\sigma), J^B(\sigma')\} = F^{AB}{}_C J^C(\sigma)\delta(\sigma - \sigma') + \eta^{AB}\partial_\sigma\delta(\sigma - \sigma')$$

Motivation: …and integrable deformations of the PCM

- field equations from $\partial_\tau J^A = \{H, J^A\}$
- example principle chiral model (PCM) $J = j_0 + j_1$

$$\begin{aligned} \partial_\tau j_0 - \partial_\sigma j_1 &= 0 \\ \partial_\tau j_1 - \partial_\sigma j_0 - [j_0, j_1] &= 0 \end{aligned}$$

- Zakharov-Mikhailov field equations $\rightarrow$ Lax pair
- Lax pair $\rightarrow$ infinite number of conserved charges $\rightarrow$ integrable
- new integrable model
  1. deform $\{\ ,\ \}$ and keep H
  2. such that field equations do not change

Because both $\{\ ,\ \}$ and H are manifest in the $\mathcal{E}$-model it is perfectly suited to explore these deformations.

¿Low energy effective target space theory?

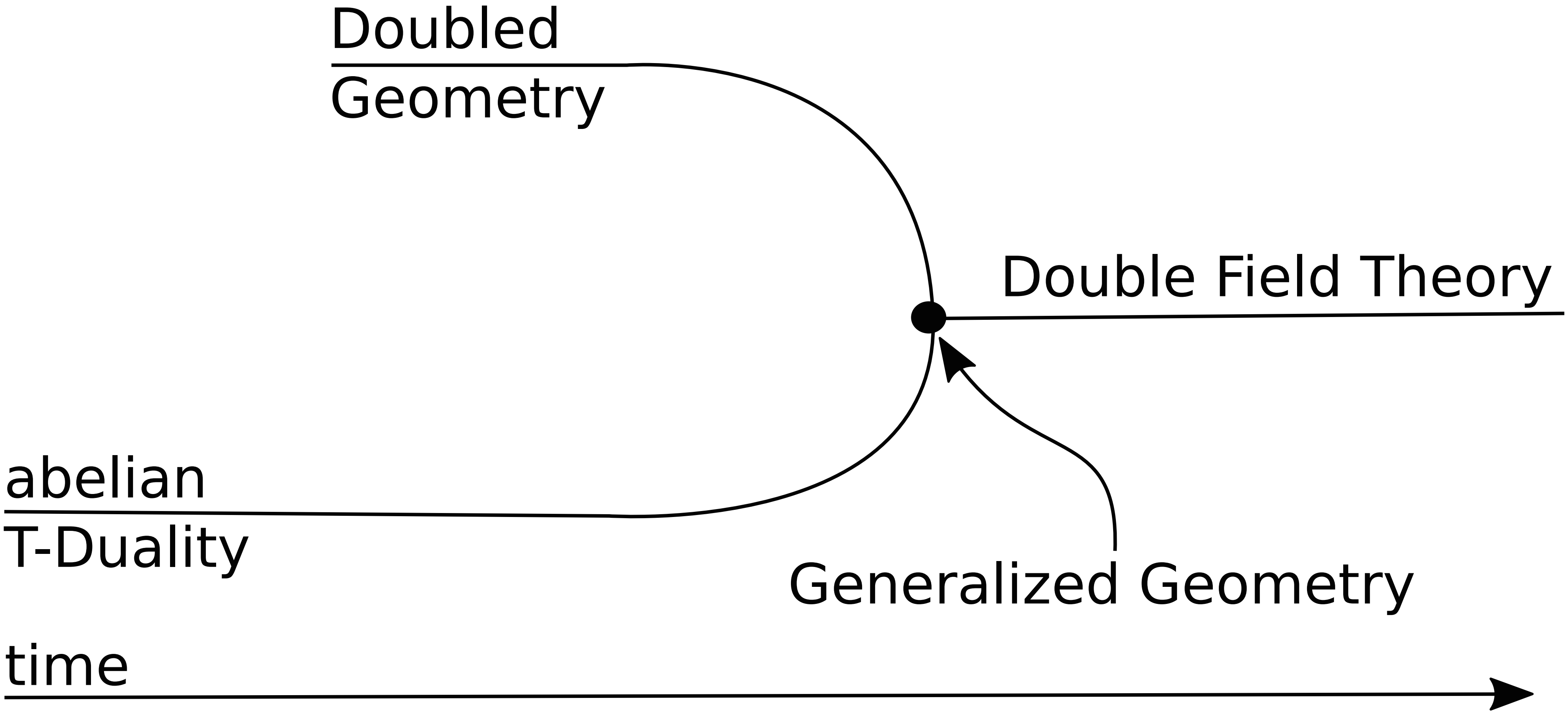
- $\mathcal{E}$-model = σ-model on $\mathcal{D}/\tilde{G} \to$ (g)SUGRA
- but then we loose all the nice structure on $\mathcal{D}$
- $\mathcal{E}$-model = doubled σ-model $\to$ Double Field Theory?

  CHALLENGES
  1. doubled space = winding + normal coordinates $\neq \mathcal{D}$
  2. abelian T-duality is manifest $\subset$ Poisson-Lie T-duality

$\to$ standard DFT does not work

Today, I will show you how to change the standard DFT framework to overcome these challenges. The result is called DFT on group manifolds (abbreviated DFT$_{\text{WZW}}$) and will meet all our expectations.

Motivation: II) Incorporate Poisson-Lie T-duality into DFT



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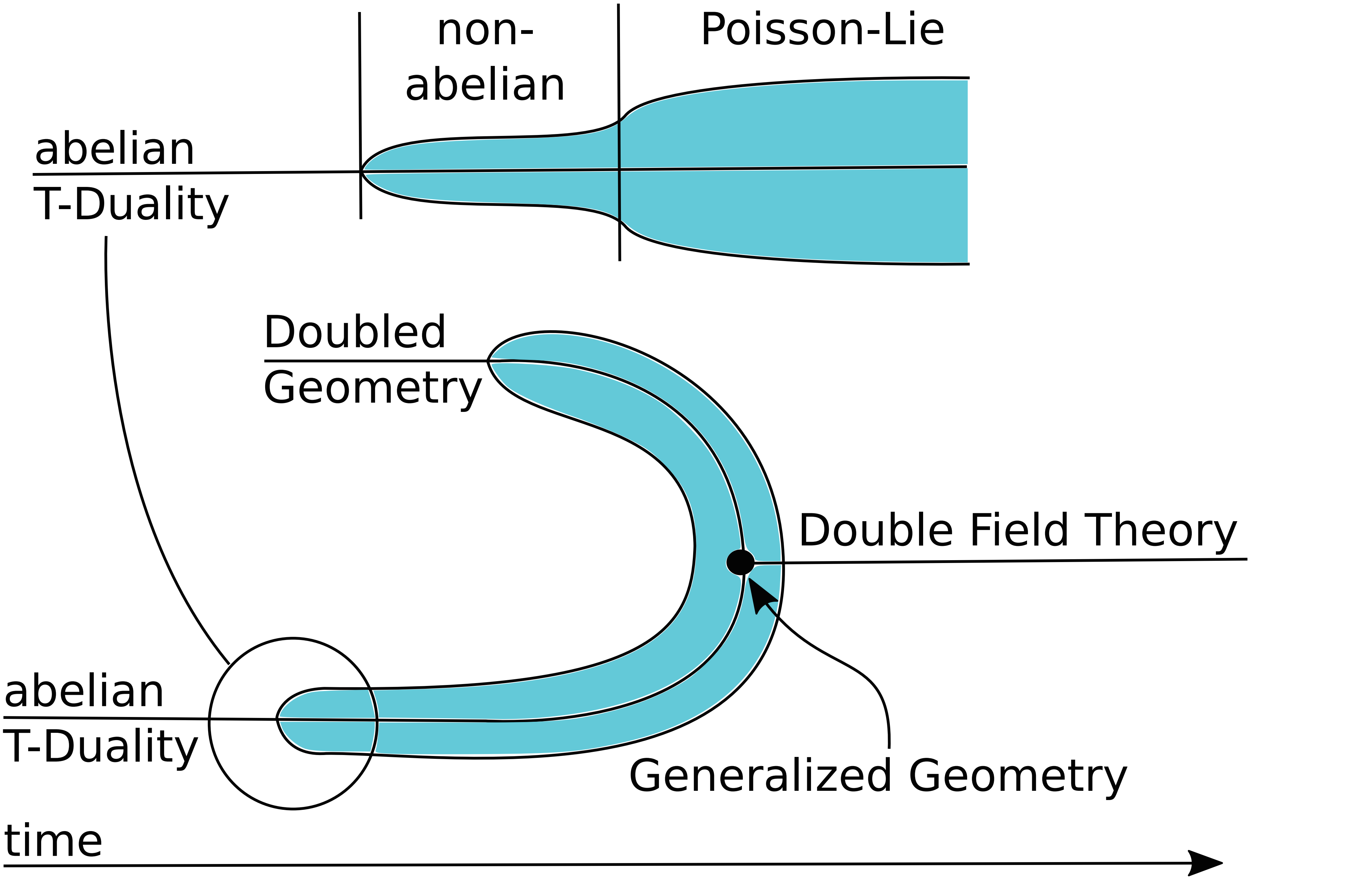

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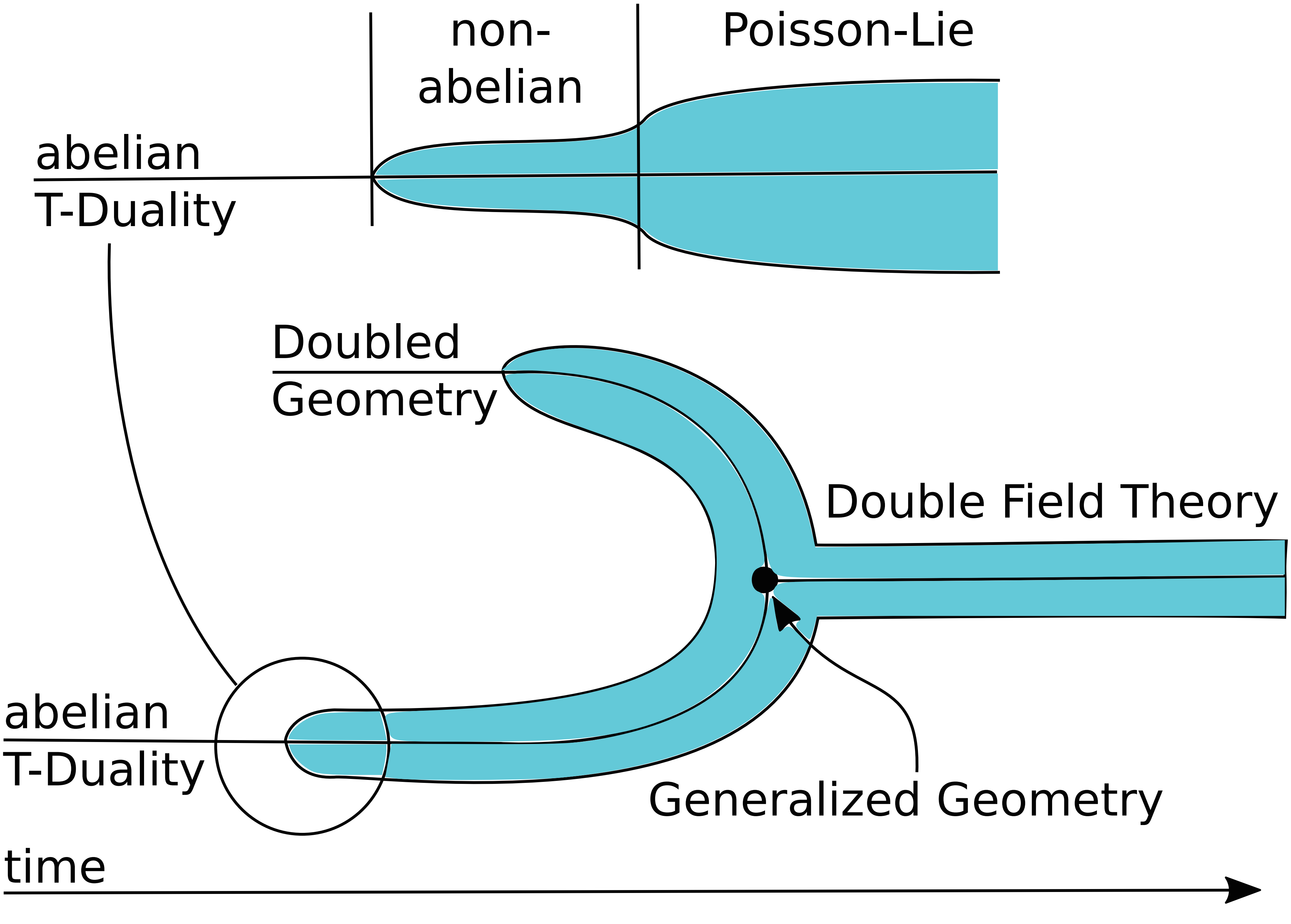

What do we gain?

- a target space description with manifest Poisson-Lie symmetry
- captures the dilaton
- captures the R/R sector
  - first derivation of R/R sector transformation for full Poisson-Lie T-duality
  - before only for abelian and non-abelian T-duality known
- modified SUGRA automatically build in
- simplified handling of integrable deformations
- consistent truncations in SUGRA

Outline

1. Motivation

2. Poisson-Lie T-duality

3. Double Field Theory on group manifolds

4. Summary

Drinfeld double

Definition: A **Drinfeld double** is a $2D$-dimensional Lie group $\mathcal{D}$, whose Lie-algebra $\mathfrak{d}$

1. has an ad-invariant bilinear for $\langle\,\cdot\,,\,\cdot\rangle$ with signature (D, D)
2. admits the decomposition into two maximal isotropic subalgebras $\mathfrak{g}$ and $\tilde{\mathfrak{g}}$

- $\begin{pmatrix} t^a & t_a \end{pmatrix} = T_A \in \mathfrak{d}\,, \quad t_a \in \mathfrak{g}$ and $t^a \in \tilde{\mathfrak{g}}$
- $\langle T_A, T_B\rangle = \eta_{AB} = \begin{pmatrix} 0 & \delta^a_b \\ \delta_a^b & 0 \end{pmatrix}$
- $[T_A, T_B] = F_{AB}{}^C T_C$ with non-vanishing commutators

$$[t_a, t_b] = f_{ab}{}^c t_c \qquad [t_a, t^b] = \tilde{f}^{bc}{}_a t_c - f_{ac}{}^b t^c$$

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$$[t_a, t_b] = f_{ab}{}^c t_c + f'_{abc} t^c \qquad [t_a, t^b] = \tilde{f}^{bc}{}_a t_c - f_{ac}{}^b t^c$$

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Poisson-Lie T-duality: 1. Definition

- 2D σ-model on target space M with action

  $S(E, M) = \int dz d\bar{z}\, E_{ij} \partial x^i \bar{\partial} x^j$
- $E_{ij} = g_{ij} + B_{ij}$ captures metric and two-from field on M
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- adjoint action of $g \in G$ on $t_A \in \mathfrak{d}$: $\mathrm{Ad}_g\, t_A = g t_A g^{-1} = M_A{}^B t_B$
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Definition: $S(E, \mathcal{D}/\tilde{G})$ and $S(\tilde{E}, \mathcal{D}/G)$ are **Poisson-Lie T-dual** if

$$E^{ij} = v_c{}^i M_a{}^c (M^{ae} M^b{}_e + E_0^{ab}) M_b{}^d v_d{}^j$$
$$\tilde{E}^{ij} = \tilde{v}^{ci} \tilde{M}^a{}_c (\tilde{M}_{ae} \tilde{M}_b{}^e + E_{0\,ab}) \tilde{M}^b{}_d \tilde{v}^{dj}$$

holds, where E_0^{ab} is constant and invertible with the inverse $E_{0\,ab}$.

Remark: The $\mathcal{E}$-model looks much nicer

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- $\mathcal{E} : \mathfrak{d} \to \mathfrak{d}$ is captured by the *generalized metric*

$$\mathcal{H}_{AB} = \langle T_A, \mathcal{E} T_B\rangle = \begin{pmatrix} G^{ab} & G^{ac}B_{cb} \\ -B_{ac}G^{cb} & G_{ab} + B_{ac}G^{cd}G_{db} \end{pmatrix}$$

- with $\boxed{G_{ab} + B_{ab} = E_{0\,ab}}$

Poisson-Lie T-duality: 2. Properties

- captures $\begin{cases} \text{abelian T-d.} & G \text{ abelian} & \text{and} & \tilde{G} \text{ abelian} \\ \text{non-abelian T-d.} & G \text{ non-abelian} & \text{and} & \tilde{G} \text{ abelian} \end{cases}$

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- dual σ-models related by canonical transformation

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- preserves conformal invariance at one-loop

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- dilaton transformation

$$\phi = -\tfrac{1}{2}\log\left|\det\left(1+\tilde{g}_0^{-1}(\tilde{B}_0+\Pi)\right)\right|$$
$$\tilde{\phi} = -\tfrac{1}{2}\log\left|\det\left(1+g_0^{-1}(B_0+\tilde{\Pi})\right)\right|$$

details tomorrow

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2D σ-model perspective

(modified) SUGRA perspective

Additional structure on the Drinfeld double

- *right* invariant vector $E_A{}^I$ field on $\mathcal{D}$ is the inverse transposed of *left* invariant Maurer-Cartan form $t_A E^A{}_I dX^I = g^{-1} dg$

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- two η-compatible, covariant derivatives[1]
  1. flat derivative

     $$D_A V^B = E_A{}^I \partial_I V^B$$

  2. convenient derivative

     $$\nabla_A V^B = D_A V^B + \tfrac{1}{3} F_{AC}{}^B V^C - w F_A V^B, \qquad F_A = D_A \log \left| \det(E^B{}_I) \right|$$

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  $$\mathcal{H}_{AB} = \mathcal{H}_{(AB)}, \qquad \mathcal{H}_{AC} \eta^{CD} \mathcal{H}_{DB} = \eta_{AB}$$
- generalized dilaton d with e^{-2d} scalar density of weight $w = 1$

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- generalized dilaton d with e^{-2d} scalar density of weight $w = 1$
- triple $(\mathcal{D}, \mathcal{H}_{AB}, d)$ captures the doubled space of DFT

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Double Field Theory for $(\mathcal{D}, \mathcal{H}_{AB}, d)$

- action ($\nabla_A d = -\frac{1}{2} e^{2d} \nabla_A e^{-2d}$)

$$S_{\mathrm{NS}} = \int_{\mathcal{D}} d^{2D}X e^{-2d} \Big(\frac{1}{8} \mathcal{H}^{CD} \nabla_C \mathcal{H}_{AB} \nabla_D \mathcal{H}^{AB} - \frac{1}{2} \mathcal{H}^{AB} \nabla_B \mathcal{H}^{CD} \nabla_D \mathcal{H}_{AC} - 2 \nabla_A d \nabla_B \mathcal{H}^{AB} + 4 \mathcal{H}^{AB} \nabla_A d \nabla_B d + \frac{1}{6} F_{ACD} F_B{}^{CD} \mathcal{H}^{AB} \Big)$$

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$$L_\xi V^A = \xi^B D_B V^A + w D_B \xi^B V^A$$

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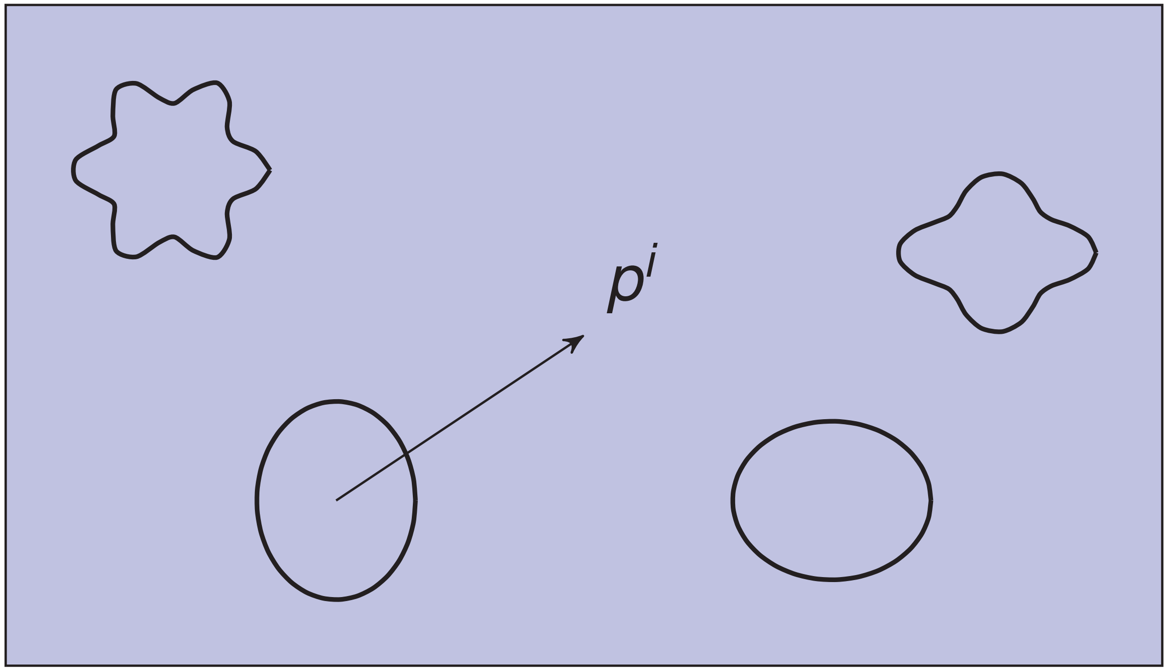
- section condition (SC)

$$\eta^{AB} D_A \cdot D_B \cdot = 0$$

How we got this action?

- closed strings in D-dim. flat space
- truncate all massive excitations
- match scattering amplitudes of strings with EFT

$$S_{\mathrm{NS}} = \int \mathrm{d}^D x \sqrt{g} e^{-2\phi} \left(\mathcal{R} + 4\partial_i \phi \partial^i \phi - \frac{1}{12} H_{ijk} H^{ijk} \right)$$



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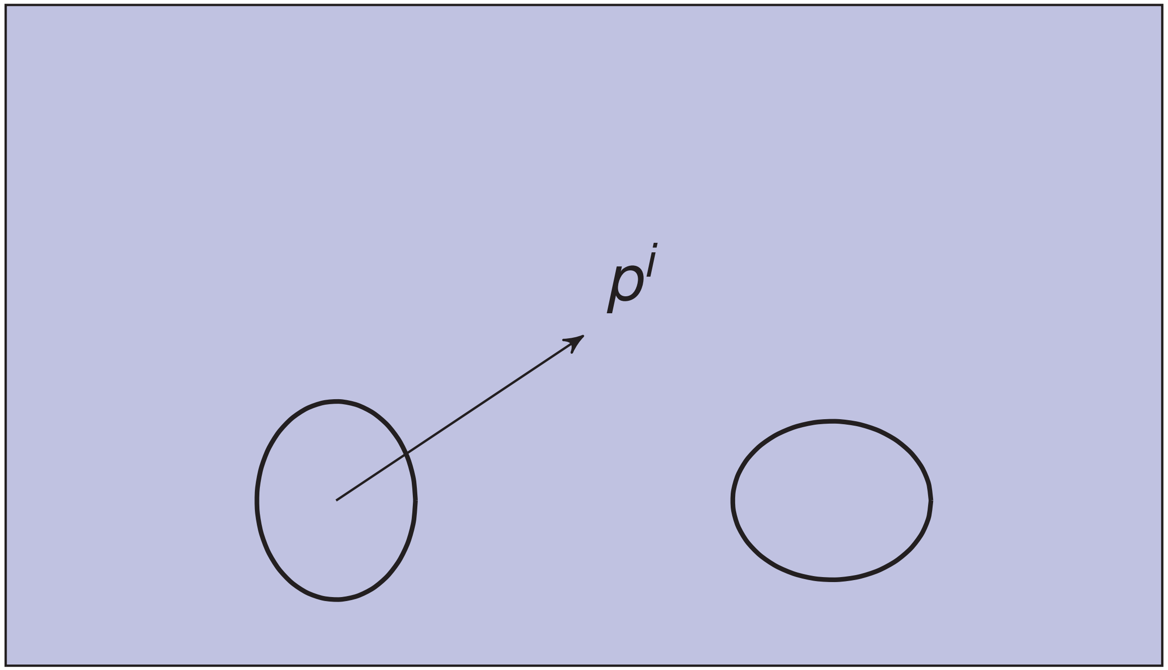

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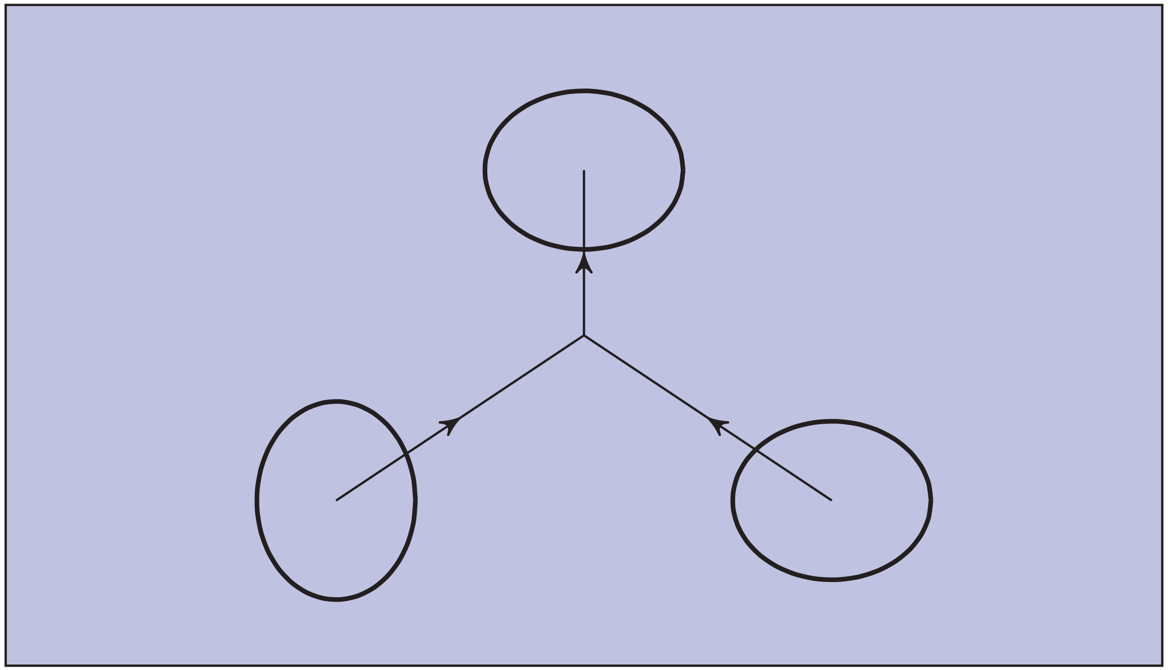
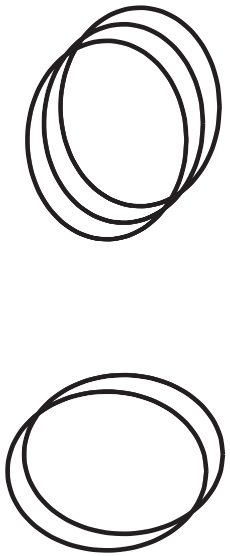




$g_{ij}(p^k)$

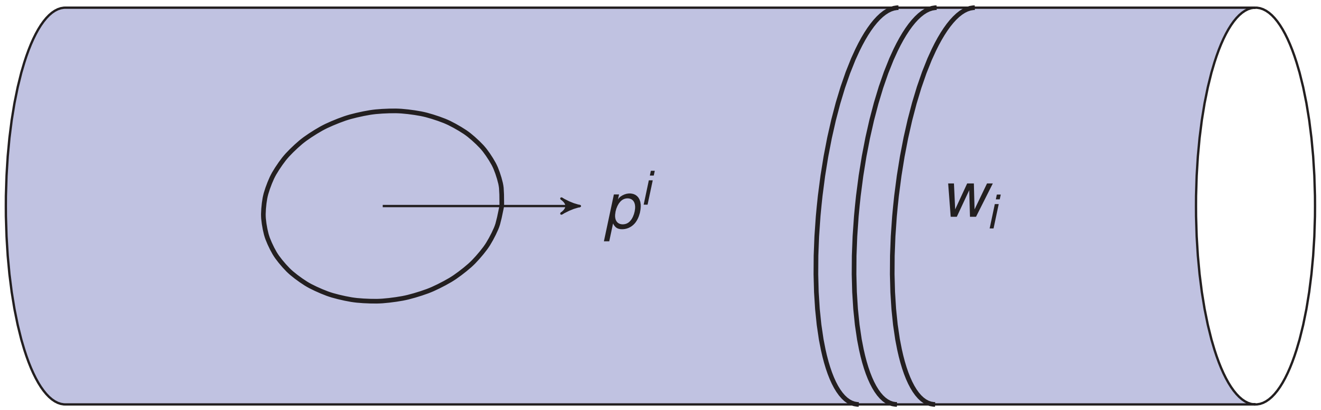
$\phi(p^i)$

Double Field Theory

- closed strings on a flat torus
- combine conjugated variables x_i and $\tilde{x}^i$ into $X^M = (\tilde{x}_i \quad x^i)$
- repeat steps from SUGRA derivation

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D}X\, e^{-2d}\mathcal{R}(\mathcal{H}_{MN}, d)$$

- fields are constrained by strong constraint $\boxed{\partial_M \partial^M \cdot = 0}$



DFT on group manifolds = DFT$_{\text{WZW}}$

Use group manifold (Wess-Zumino-Witten model) instead of a torus to derive DFT!

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TASKS

- Derive cubic action and gauge transformations (CSFT)
- Rewrite in terms of η_{AB}, F_{ABC} and $\mathcal{H}_{AB}$
- Figure out that $\mathcal{D}$ does not have to be $G_{\mathrm{L}} \times G_{\mathrm{R}}$

not trivial :-)

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$$\eta^{AB} D_A \cdot D_B \cdot = 0$$

Symmetries of the action

- S_{NS} invariant for $X^I \to X^I + \xi^A E_A{}^I$ and
  1. $\mathcal{H}^{AB} \to \mathcal{H}^{AB} + \mathcal{L}_\xi \mathcal{H}^{AB}$ and $e^{-2d} \to e^{-2d} + \mathcal{L}_\xi e^{-2d}$
  2. $\mathcal{H}^{AB} \to \mathcal{H}^{AB} + L_\xi \mathcal{H}^{AB}$ and $e^{-2d} \to e^{-2d} + L_\xi e^{-2d}$

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object	gen.-diffeomorphisms	$2D$-diffeomorphisms	global O(D,D)
$\mathcal{H}_{AB}$	tensor	scalar	tensor
$\nabla_A d$	not covariant	scalar	1-form
e^{-2d}	scalar density (w=1)	scalar density (w=1)	invariant
η_{AB}	invariant	invariant	invariant
$F_{AB}{}^C$	invariant	invariant	tensor
$E_A{}^I$	invariant	vector	1-form
S_{NS}	invariant	invariant	invariant
SC	invariant	invariant	invariant
D_A	not covariant	covariant	covariant
∇_A	not covariant	covariant	covariant

manifest

Poisson-Lie T-duality: 1. Solve SC

- fix D physical coordinates x^i from $X^I = \begin{pmatrix} x^i & x^{\tilde{i}} \end{pmatrix}$ on $\mathcal{D}$

  such that $\eta^{IJ} = E_A{}^I \eta^{AB} E_B{}^J = \begin{pmatrix} 0 & \cdots \\ \cdots & \cdots \end{pmatrix} \to$ SC is solved
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2D-diffeom. global O(D,D) PL-TD

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- same structure as in the original paper [Klimčík and Ševera, 1995]
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Poisson-Lie T-duality is a manifest symmetry of DFT

Equivalence to supergravity: 1. Generalized parallelizable spaces

- generalized tangent space element $V^{\widehat{I}} = \begin{pmatrix} V^i & V_i \end{pmatrix}$
- generalized Lie derivative

$$\widehat{\mathcal{L}}_\xi V^{\widehat{I}} = \xi^{\widehat{J}} \partial_{\widehat{J}} V^{\widehat{I}} + \left(\partial^{\widehat{I}} \xi_{\widehat{J}} - \partial_{\widehat{J}} \xi^{\widehat{I}} \right) V^{\widehat{J}} \qquad \text{with} \qquad \partial_{\widehat{I}} = \begin{pmatrix} 0 & \partial_i \end{pmatrix}$$

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Definition: A manifold M which admits a globally defined generalized frame field $\widehat{E}_A{}^{\hat{I}}(x^i)$ satisfying

1. $\widehat{\mathcal{L}}_{\widehat{E}_A} \widehat{E}_B{}^{\hat{I}} = F_{AB}{}^C \widehat{E}_C{}^{\hat{I}}$

   where $F_{AB}{}^C$ are the structure constants of a Lie algebra $\mathfrak{h}$
2. $\widehat{E}_A{}^{\hat{I}} \eta^{AB} \widehat{E}_B{}^{\hat{J}} = \eta^{\hat{I}\hat{J}} = \begin{pmatrix} 0 & \delta_i^j \\ \delta^i_j & 0 \end{pmatrix}$

is a **generalized parallelizable space** $(M, \mathfrak{h}, \widehat{E}_A{}^{\hat{I}})$.

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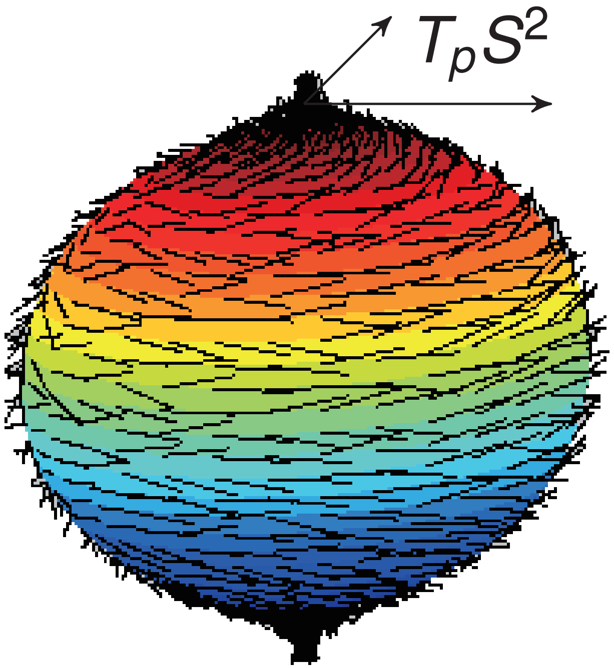
S^2 is not parallelizable



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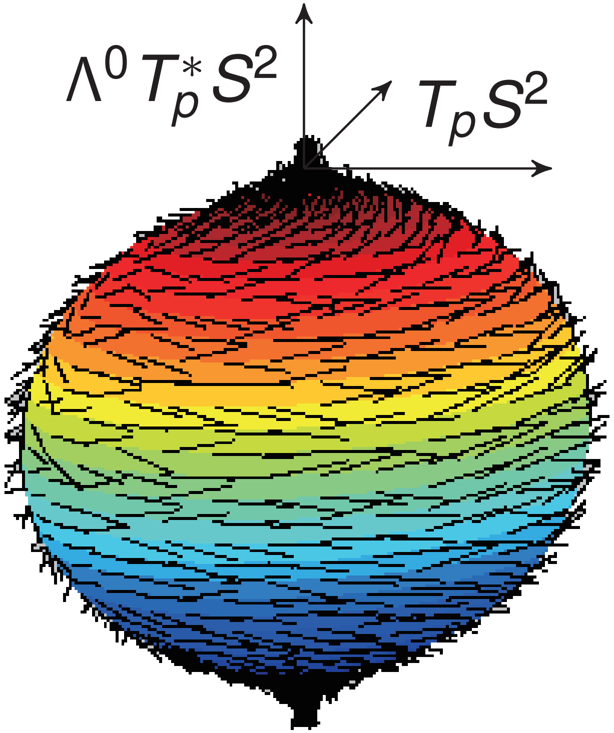

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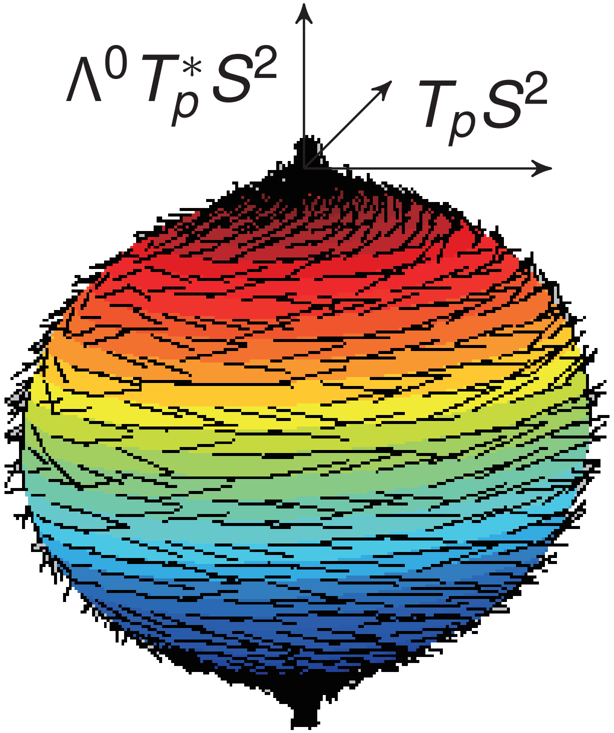

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- all spheres are generalized parallelizable on $TM \oplus \Lambda^{d-2} T^* M$
- generalized frame field $\hat{E}_A$ fulfilling $\widehat{\mathcal{L}}_{\hat{E}_A} \hat{E}_B = F_{AB}{}^C \hat{E}_C$
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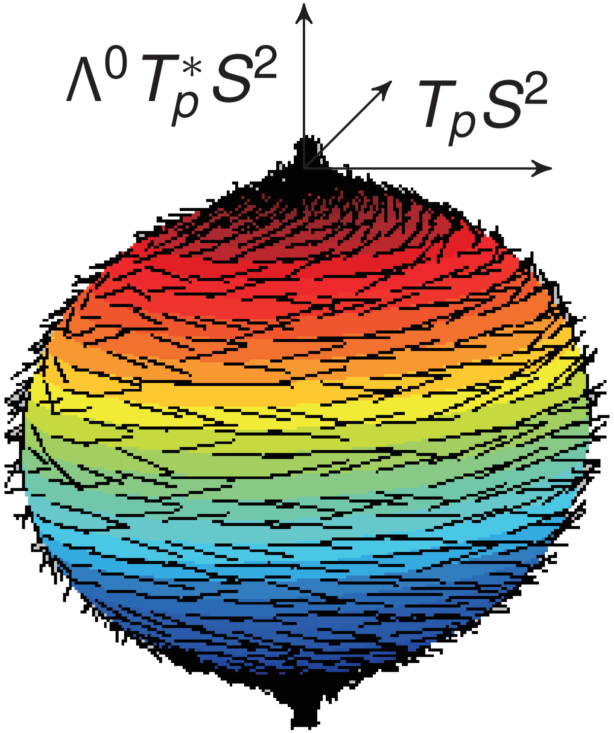

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¿ Is there a systematic way to construct them ?

Equivalence to supergravity: 2. Generalized metric and dilaton

- Drinfeld double $\mathcal{D} \to$ two generalized parallelizable spaces:

$$(D/\tilde{G}, \mathfrak{d}, \widehat{E}_A{}^{\widehat{I}}) \qquad \text{and} \qquad (D/G, \mathfrak{d}, \widetilde{\widehat{E}}_A{}^{\widehat{I}})$$

$$\widehat{E}_A{}^{\widehat{I}} = M_A{}^B \begin{pmatrix} v^b{}_i & 0 \\ 0 & v_b{}^i \end{pmatrix}_B{}^{\widehat{I}} \qquad \widetilde{\widehat{E}}_A{}^{\widehat{I}} = \tilde{M}_{AB} \begin{pmatrix} \tilde{v}_{bi} & 0 \\ 0 & \tilde{v}^{bi} \end{pmatrix}^{B\widehat{I}}$$

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- express $\mathcal{H}^{AB}$ in terms of the generalized $\widehat{\mathcal{H}}^{\hat{I}\hat{J}}$ on $TD/\tilde{G} \oplus T^*D/\tilde{G}$

$$\mathcal{H}^{AB} = \widehat{E}^A{}_{\hat{I}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \widehat{E}^B{}_{\hat{J}} \qquad \text{with} \qquad \widehat{\mathcal{H}}^{\hat{I}\hat{J}} = \begin{pmatrix} g_{ij} - B_{ik} g^{kl} B_{lk} & -B_{ik} g^{kl} \\ g^{ik} B_{kj} & g^{ij} \end{pmatrix}$$

- express d in terms of the standard generalized dilaton $\widehat{d}$

$$d = \widehat{d} - \frac{1}{2} \log |\det \tilde{v}_{ai}|$$

$$\widehat{d} = \phi - 1/4 \log \left|\det g_{ij}\right|$$

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- plug into the DFT action S_{NS}

Equivalence to supergravity: 3. IIA/B bosonic sector action

- if G and $\tilde{G}$ are unimodular

$$S_{\mathrm{NS}} = V_{\tilde{G}} \int d^D x\, e^{-2\widehat{d}} \Big(\frac{1}{8} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{K}} \widehat{\mathcal{H}}_{\hat{I}\hat{J}} \partial_{\hat{L}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - 2 \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} - \frac{1}{2} \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{J}} \widehat{\mathcal{H}}^{\hat{K}\hat{L}} \partial_{\hat{L}} \widehat{\mathcal{H}}_{\hat{I}\hat{K}} + 4 \widehat{\mathcal{H}}^{\hat{I}\hat{J}} \partial_{\hat{I}} \widehat{d} \partial_{\hat{J}} \widehat{d} \Big)$$

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- holds for all $\mathcal{H}_{AB}(x^i)$ / $\widehat{\mathcal{H}}^{\hat{I}\hat{J}}(x^i)$
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- similar story for R/R sector (tomorrow)

Restrictions on $\mathcal{H}_{AB}$ and d to admit Poisson-Lie T-duality

- in general $\mathcal{H}_{AB}(x^i) \xrightarrow{\text{Poisson-Lie T-duality (}2D\text{-diff.)}} \mathcal{H}_{AB}(x'^i, x'^{\tilde{i}})$
- $x'^{\tilde{i}}$ part not compatible with ansatz for SUGRA reduction $\rightarrow$ avoid it

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A doubled space $(\mathcal{D}, \mathcal{H}_{AB}, d)$ admits Poisson-Lie T-dual supergravity descriptions iff

1. $L_\xi \mathcal{H}_{AB} = 0 \quad \forall \xi \quad \to \quad D_A \mathcal{H}_{BC} = 0$
2. $L_\xi d = 0 \quad \forall \xi \quad \to \quad (D_A - F_A) e^{-2d} = 0$ (explore tomorrow)

Summary

- DFT, Poisson-Lie T-duality and Drinfeld doubles fit together naturally
- interpretation of doubled space does not require winding modes anymore (phase space perspective instead)

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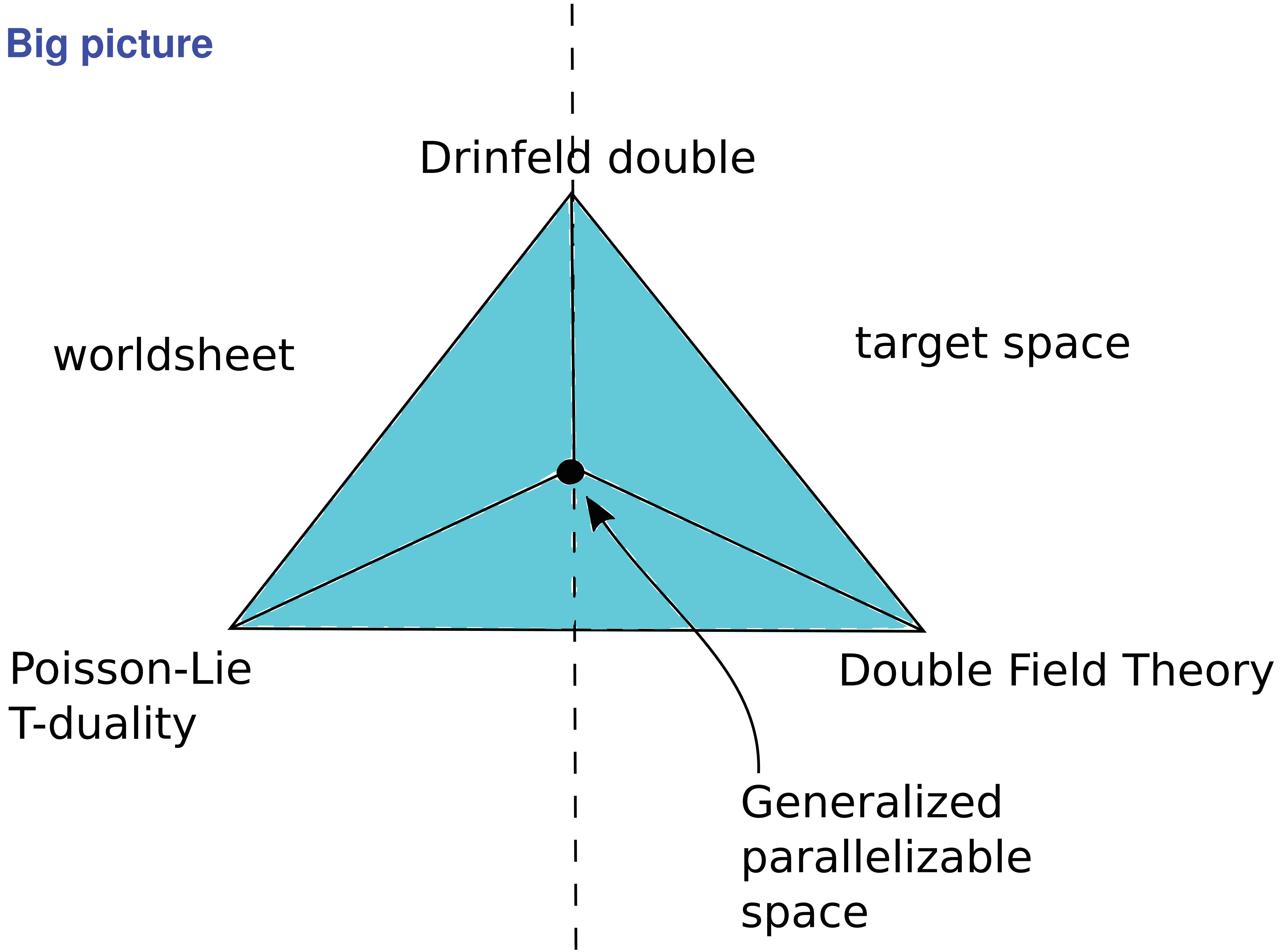
- DFT, Poisson-Lie T-duality and Drinfeld doubles fit together naturally
- interpretation of doubled space does not require winding modes anymore (phase space perspective instead)

- plan for tomorrow
  - dilaton transformation
  - R/R sector transformation
  - modified SUGRA
  - integrable deformations
  - dressing coset construction

Big picture


Drinfeld double
worldsheet
target space
Poisson-Lie
T-duality
Double Field Theory
Generalized
parallelizable
space