Poisson-Lie Symmetry and Double Field Theory

Part II

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based on

1810.11446,
1707.08624, 1611.07978,
1502.02428, 1410.6374

and work in progress

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Universidad de Oviedo
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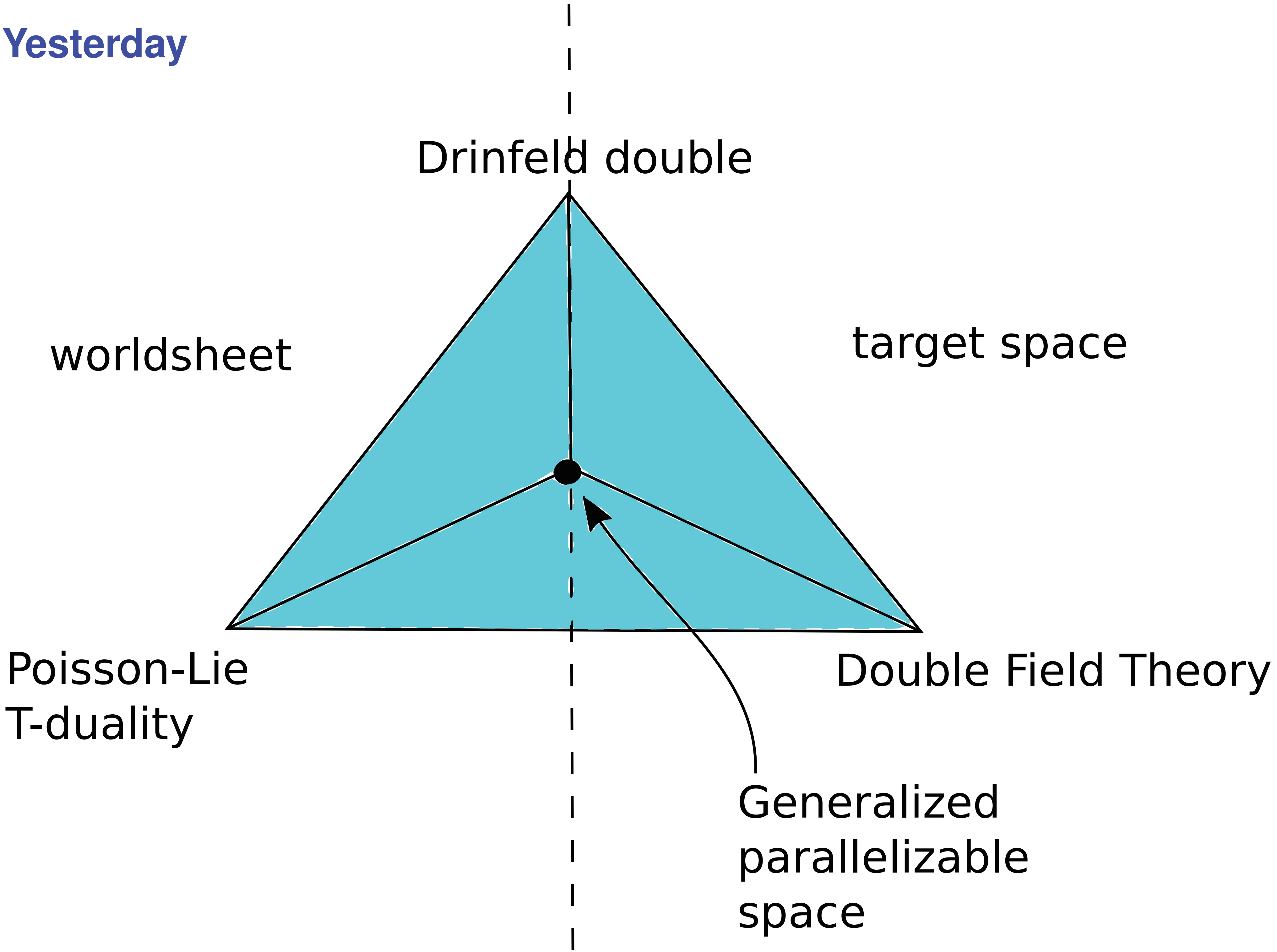
Yesterday



Drinfeld double
worldsheet
target space
Poisson-Lie
T-duality
Double Field Theory
Generalized
parallelizable
space

Ingredients for NS/NS sector of DFT on group manifolds

- Drinfeld double $\mathcal{D}$ with $\boxed{\eta_{AB}\,,\ F_{ABC}\,,\ \mathcal{H}_{AB} \text{ and } d}$
- symmetries of the theory
  1. generalized diffeomorphisms
  2. $2D$ **diffeomorphisms**
  3. global O(D,D) transformations

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- max.isotropic subgroup H of $\mathcal{D}$ $\rightarrow$ SC solution

target space

$$d(X^I) = g(x^i)\tilde{g}(x^{\tilde{i}}) \qquad t_A = \begin{pmatrix} t^a & t_a \end{pmatrix} \qquad \mathcal{D}/\tilde{G}$$

2D-diffeom. — global O(D,D) — PL-TD

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- generalized frame field makes contact with SUGRA fields

Outline

1. Quick reminder
2. Dilaton transformation
3. R/R sector of Double Field Theory on $\mathcal{D}$
4. Application to integrable deformations
5. Outlook

Restrictions on $\mathcal{H}_{AB}$ and d to admit Poisson-Lie T-duality

- in general $\mathcal{H}_{AB}(x^i) \xrightarrow{\text{Poisson-Lie T-duality (}2D\text{-diff.)}} \mathcal{H}_{AB}(x'^i, x'^{\tilde{i}})$
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A doubled space $(\mathcal{D}, \mathcal{H}_{AB}, d)$ admits Poisson-Lie T-dual supergravity descriptions iff

1. $L_\xi \mathcal{H}_{AB} = 0 \quad \forall\, \xi \quad \to \quad D_A \mathcal{H}_{BC} = 0$
2. $L_\xi d = 0 \quad \forall\, \xi \quad \to \quad (D_A - F_A) e^{-2d} = 0$

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Remarks:

- $F_A = D_A \log \left| \det(E^B{}_I) \right|$
- biggest possible isometry group $\mathcal{D}_\mathrm{L} \times \mathcal{D}_\mathrm{R}$
- for Poisson-Lie T-duality just $\mathcal{D}_\mathrm{L}$ required
- if additionally $\mathcal{F} \subset \mathcal{D}_\mathrm{R}$ gauge it $\to$ dressing coset

Dilaton transformation

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- $g = v^T e^T e v \quad \text{with} \quad \begin{cases} (\tilde{B}_0 + \tilde{g}_0)^{ab} = E^{0\,ab} \\ \Pi^{ab} = M^{ac} M^b{}_c \\ e^{-1}e^{-T} = \tilde{g}_0 - (\tilde{B}_0 + \Pi)\tilde{g}_0^{-1}(\tilde{B}_0 + \Pi) \\ \tilde{e}_0^T \tilde{e}_0 = \tilde{g}_0 \\ e^{-T} = \tilde{e}_0 + \tilde{e}_0^{-T}(\tilde{B}_0 + \Pi) \end{cases}$

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- $\phi = \phi_0 + \frac{1}{2}\log|\det e| = \phi_0 - \frac{1}{2}\log|\det \tilde{e}_0| - \frac{1}{2}\log\left|\det\left(1 + \tilde{g}_0^{-1}(\tilde{B}_0 + \Pi)\right)\right|$
- reproduces [Jurco and Vysoky, 2018]

O(D,D) Majorana-Weyl spinor on $\mathcal{D}$

- Γ-matrices: $\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$
- chirality Γ_{2D+1} with $\{\Gamma_{2D+1}, \Gamma_A\} = 0$
- charge conjugation C with $C\Gamma_A C^{-1} = (\Gamma_A)^\dagger$

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- O(D,D) transformation in spinor representation

  $$\mathcal{S}_\mathcal{O}\Gamma_A\mathcal{S}_\mathcal{O}^{-1} = \Gamma_B\mathcal{O}^B{}_A \qquad \mathcal{O}^T\eta\mathcal{O} = \eta$$

R/R sector of DFT on group manifolds

- action $S_{\mathrm{RR}} = \frac{1}{4}\int d^{2d}X \, (\nabla\!\!\!/\,\chi)^\dagger \, S_{\mathcal{H}} \, \nabla\!\!\!/\,\chi$
- covariant derivative $\nabla\!\!\!/\,\chi = \left(\Gamma^A D_A - \frac{1}{12}\Gamma^{ABC} F_{ABC} - \frac{1}{2}\Gamma^A F_A\right)\chi$

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- $\not{\nabla}^2 = 0$ under SC
- χ is chiral (IIB) or anti-chiral (IIA)
- satisfies self duality condition

  $G = -\mathcal{K}G \quad \text{with} \quad G = \not{\nabla}\chi \quad \text{and} \quad \mathcal{K} = C^{-1}S_{\mathcal{H}}$

Symmetries of the action

- $S_{\mathrm{R/R}}$ invariant for $X^I \rightarrow X^I + \xi^A E_A{}^I$ and
  1. $\chi \rightarrow \chi + \mathcal{L}_\xi \chi$ and $\mathcal{H}^{AB} \rightarrow \mathcal{H}^{AB} + \mathcal{L}_\xi \mathcal{H}^{AB}$
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$$\mathcal{L}_\xi \chi = \xi^A \nabla_A \chi + \frac{1}{2} \nabla_A \xi_B \Gamma^{AB} \chi + \frac{1}{2} \nabla_A \xi^A \chi$$

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2. $2D$-diffeomorphisms

$$L_\xi \chi = \xi^A D_A \chi - \tfrac{1}{2} (\xi^A F_A - D_A \xi^A) \chi \quad \text{and} \quad L_\xi \mathcal{H}^{AB} = \xi^C D_C \mathcal{H}^{AB}$$

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3. global O(D,D) transformations ($\mathcal{O}^A{}_C \mathcal{O}^B{}_D \eta^{CD} = \eta^{AB}$)

$$\chi \rightarrow S_{\mathcal{O}} \chi \quad \text{and} \quad \mathcal{H}^{AB} \rightarrow \mathcal{O}^A{}_C \mathcal{H}^{CD} \mathcal{O}^B{}_D$$

- section condition (SC) for f_1, f_2 with weights w_1, w_2

$$(D_A f_1 - w_1 F_A f_1)(D^A f_2 - w_2 F^A f_2) = 0$$

Equivalence to (m)SUGRA: 1. R/R field strengths

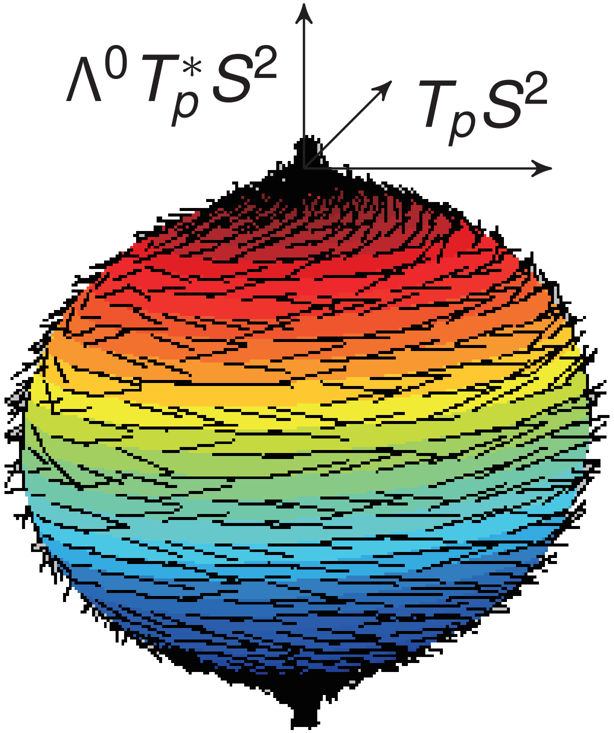
- transport χ to the generalized tangent space:

  $$\widehat{\chi} = |\det \tilde{e}_{ai}|^{-1/2} S_{\widehat{E}} \chi \qquad (t^a \tilde{e}_{ai} = \tilde{g}^{-1} d\tilde{g})$$

- remember generalized metric from yesterday:

  $$\widehat{\mathcal{H}}^{\hat{I}\hat{J}} = \widehat{E}_A{}^{\hat{I}} \mathcal{H}^{AB} \widehat{E}_B{}^{\hat{J}}$$

Remember S^2 is not parallelizable, but generalized parallelizable



Def.: M is parallelizable if $\exists$ d=dim M smooth vector fields providing a basis e_a for T_pM at every point p on M.

- examples: S^3, S^7, Lie groups
- Scherk-Schwarz compactifications on M do not break any SUSY
- counterexample S^2 (hairy ball)


use generalized tangent space instead of TM

- all spheres are generalized parallelizable on $TM \oplus \Lambda^{d-2} T^* M$
- generalized frame field $\hat{E}_A$ fulfilling $\widehat{\mathcal{L}}_{\hat{E}_A} \hat{E}_B = F_{AB}{}^C \hat{E}_C$
- consistent ansätze from compactification with max. SUSY

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- same for covariant derivative

  $$|\det \tilde{e}_{ai}|^{-1/2} S_{\widehat{E}} \nabla\!\!\!/ \chi = \left(\partial\!\!\!/ - \mathbf{X}_{\hat{I}} \widehat{\Gamma}^{\hat{I}} \right) \widehat{\chi} \quad \text{with} \quad \mathbf{X}_{\hat{I}} = \begin{pmatrix} I^i \\ -V_i \end{pmatrix}$$

  $$S_{\widehat{E}} \Gamma^A S_{\widehat{E}}^{-1} \widehat{E}_A{}^{\hat{I}} = \widehat{\Gamma}^{\hat{I}} \quad \text{and} \quad \partial\!\!\!/ = \widehat{\Gamma}^i \partial_i$$

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- $\mathbf{X}_{\hat{I}}$ vanishes if $\tilde{\mathfrak{g}}$ is unimodular
- introduce field strength $\widehat{F} = e^{\phi} S_B \left(\partial\!\!\!/ - \mathbf{X}_{\hat{I}} \widehat{\Gamma}^{\hat{I}} \right) \widehat{\chi}$
- and derivative $\mathbf{d} = e^{\phi} S_B \left(\partial\!\!\!/ - \mathbf{X}_{\hat{I}} \widehat{\Gamma}^{\hat{I}} \right) S_B^{-1} e^{-\phi}$

Equivalence to (m)SUGRA: 2. field equations & Bianchi identity

- DFT R/R field equations: $\not\nabla(\mathcal{K}G) = 0$ remember $G = \not\nabla\chi$
- rewrite them as:

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- action on polyforms

  $$\mathbf{d} \quad \leftrightarrow \quad d + H \wedge - Z \wedge - \iota_I \quad \text{with} \quad Z = d\phi + \iota_I B - V$$

  $$\star \quad \leftrightarrow \quad \star$$

- matches the R/R sector of (m)SUGRA
- some holds for the NS/NS sector

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- $\not\nabla \chi = 0$ for Poisson-Lie symmetric χ is algebraic

  $\not\nabla \chi = \frac{1}{12} F_{ABC} \Gamma^{ABC} \chi$
- finding R/R solutions reduces to linear algebra
- similar for NS/NS sector

  (here field equations are in general quadratic)

Application to integrable deformations

- one parameter deformation of the PCM

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- starting point is solution to (m)CYBE

  $$[\mathcal{R}x, \mathcal{R}y] - \mathcal{R}\left([\mathcal{R}x, y] + [x, \mathcal{R}y]\right) = -c^2[x, y]$$

  1. $c^2 = -1$ Yang-Baxter σ-model or η-deformation
  2. $c^2 = \quad 1$ λ-deformation

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2. $c^2 = 1$ λ-deformation

- generalized metric after global O(D,D) very simple

$$\mathcal{H}^{AB} = \begin{pmatrix} k_{ab} & 0 \\ 0 & k^{ab} \end{pmatrix}$$

- structure coefficients have non-trivial components

$$F_{abc} = 0\,, \quad F_{ab}{}^{c} = \kappa^{-1/2} f_{ab}{}^{c}\,,$$
$$F^{ab}{}_{c} = 0\,, \quad F^{abc} = \kappa^{3/2} c^2 k^{ad} k^{be} f_{de}{}^{c}$$

Application to integrable deformations

- one parameter deformation of the PCM
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$$[\mathcal{R}x,\mathcal{R}y]-\mathcal{R}\left([\mathcal{R}x,y]+[x,\mathcal{R}y]\right)=-c^2[x,y]$$

1. $c^2=-1$ Yang-Baxter σ-model or η-deformation
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- field equations for NS/NS + R/R sector become linear

Field equations: 1. Variation of the NS/NS action

- two contributions

  1. $\delta S_{\text{NS}} = -2 \int d^{2D}X e^{-2d} \mathcal{R} \delta d$
  2. $\delta S_{\text{NS}} = \int d^{2D}X e^{-2d} \mathcal{K}_{AB} \delta \mathcal{H}^{AB}$

$$\begin{aligned}\mathcal{R} = {} & 4\mathcal{H}^{AB}\nabla_A\nabla_B d - \nabla_A\nabla_B\mathcal{H}^{AB} - 4\mathcal{H}^{AB}\nabla_A d\,\nabla_B d + 4\nabla_A d\,\nabla_B\mathcal{H}^{AB} \\ & + \frac{1}{8}\mathcal{H}^{CD}\nabla_C\mathcal{H}_{AB}\nabla_D\mathcal{H}^{AB} - \frac{1}{2}\mathcal{H}^{AB}\nabla_B\mathcal{H}^{CD}\nabla_D\mathcal{H}_{AC} + \frac{1}{6}F_{ACD}F_B{}^{CD}\mathcal{H}^{AB}\end{aligned}$$

$$\begin{aligned}\mathcal{K}_{AB} = {} & \frac{1}{8}\nabla_A\mathcal{H}_{CD}\nabla_B\mathcal{H}^{CD} - \frac{1}{4}\left[\nabla_C - 2(\nabla_C d)\right]\mathcal{H}^{CD}\nabla_D\mathcal{H}_{AB} + 2\nabla_{(A}\nabla_{B)}d \\ & - \nabla_{(A}\mathcal{H}^{CD}\nabla_D\mathcal{H}_{B)C} + \left[\nabla_D - 2(\nabla_D d)\right]\left[\mathcal{H}^{CD}\nabla_{(A}\mathcal{H}_{B)C} + \mathcal{H}^C{}_{(A}\nabla_C\mathcal{H}^D{}_{B)}\right] \\ & + \frac{1}{6}F_{ACD}F_B{}^{CD}\end{aligned}$$

Field equations: 1. Variation of the NS/NS action

- two contributions

  1. $\delta S_{\mathrm{NS}} = -2\int d^{2D}X e^{-2d}\mathcal{R}\delta d$
  2. $\delta S_{\mathrm{NS}} = \int d^{2D}X e^{-2d}\mathcal{K}_{AB}\delta\mathcal{H}^{AB}$

$$\mathcal{R} = 4\mathcal{H}^{AB}\nabla_A\nabla_B d - \nabla_A\nabla_B\mathcal{H}^{AB} - 4\mathcal{H}^{AB}\nabla_A d\,\nabla_B d + 4\nabla_A d\,\nabla_B\mathcal{H}^{AB} + \frac{1}{8}\mathcal{H}^{CD}\nabla_C\mathcal{H}_{AB}\nabla_D\mathcal{H}^{AB} - \frac{1}{2}\mathcal{H}^{AB}\nabla_B\mathcal{H}^{CD}\nabla_D\mathcal{H}_{AC} + \frac{1}{6}F_{ACD}F_B{}^{CD}\mathcal{H}^{AB}$$

$$\mathcal{K}_{AB} = \frac{1}{8}\nabla_A\mathcal{H}_{CD}\nabla_B\mathcal{H}^{CD} - \frac{1}{4}\left[\nabla_C - 2(\nabla_C d)\right]\mathcal{H}^{CD}\nabla_D\mathcal{H}_{AB} + 2\nabla_{(A}\nabla_{B)}d - \nabla_{(A}\mathcal{H}^{CD}\nabla_D\mathcal{H}_{B)C} + \left[\nabla_D - 2(\nabla_D d)\right]\left[\mathcal{H}^{CD}\nabla_{(A}\mathcal{H}_{B)C} + \mathcal{H}^C{}_{(A}\nabla_C\mathcal{H}^D{}_{B)}\right] + \frac{1}{6}F_{ACD}F_B{}^{CD}$$

- $\mathcal{H}_{AB}$ not just symmetric but restricted to $\mathrm{O}(D,D)$ $\rightarrow$ project $\mathcal{K}_{AB}$

Field equations: 2. Poisson-Lie symmetry

- generalized Ricci curvature

$$\mathcal{R}_{AB} = 2P_{(A}{}^{C}\mathcal{K}_{CD}\overline{P}_{B)}{}^{D}$$

$$P_{AB} = \tfrac{1}{2}(\eta_{AB} + \mathcal{H}_{AB}) \quad \text{and} \quad \overline{P}_{AB} = \tfrac{1}{2}(\eta_{AB} - \mathcal{H}_{AB})$$

- finally the field equations are:

$$\mathcal{R} = 0$$

$$\mathcal{H}_A{}^C\mathcal{R}_{CB} = \underbrace{-\frac{1}{8}G^T C\Gamma_{AB}G}_{\text{R/R sector}}$$

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- Poisson-Lie symmetry simplifies $\mathcal{R}$ and $\mathcal{R}_{AB}$

$$\mathcal{R} = \frac{1}{12}F_{ACE}F_{BDF}\left(3\mathcal{H}^{AB}\eta^{CD}\eta^{EF} - \mathcal{H}^{AB}\mathcal{H}^{CD}\mathcal{H}^{EF}\right)$$

$$\mathcal{R}_{AB} = \frac{1}{8}(\mathcal{H}_{AC}\mathcal{H}_{BF} - \eta_{AC}\eta_{BF})(\mathcal{H}^{KD}\mathcal{H}^{HE} - \eta^{KD}\eta^{HE})F_{KH}{}^{C}F_{DE}{}^{F}$$

Generalized frame field and target space fields

- generalized frame field: $\widehat{E}_A{}^{\hat{I}} = \begin{pmatrix} \kappa^{1/2} e^a{}_i & \kappa^{-1/2}(\Pi^{ab} + R^{ab}) e_b{}^i \\ 0 & \kappa^{-1/2} e_a{}^i \end{pmatrix}$

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- metric G and B-field from generalized metric $\widehat{H}^{\hat{I}\hat{J}}$

  $g + B = e^T \left((\kappa k)^{-1} + R + \Pi \right) e \qquad t_a e^a{}_i dx^i = g^{-1} \mathrm{d} g$

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- R/R fields:

$$\widehat{G}^{(1)} = -\frac{1+\kappa^2}{\sqrt{2}} (\Pi + R)^{ab} f_{abc} e^c$$

$$\widehat{G}^{(3)} = \frac{1+\kappa^2}{3\sqrt{2}} f_{abc} e^a \wedge e^b \wedge e^c$$

There are many interesting questions

- translation of all the intriguing results in Poisson-Lie T-duality e.g.
  - implement dressing cosets
  - study global properties
    (non-abelian momentum and winding exchange)
  - D-branes
- better understand supersymmetry
- apply to background with just partial PL-symmetry
- quantization of $\mathcal{E}$-model $\leftrightarrow$ α' corrections
- EFT has similar structure as DFT.
  Can we formulate “Poisson-Lie” U-duality?


PL-TD & DFT