Geometry of covariant double field theory from supergeometry

Noriaki Ikeda

Ritsumeikan University, Kyoto, Japan

DFT in supermanifold formulation and group manifold as background geometry,
U. Carow-Watamura, NI, T. Kaneko and S. Watamura,
arXiv:1812.03464. etc.

§1. Introduction

To do

- understand T-duality geometry

- geometry of double field theory
section conditions, closure conditions, generalized Bianchi identities, etc.

$\rightarrow$ We use super symplectic geometry (BRST-BV formalism of TFT)

- coordinate independent and $GL(2D)$ covariant formalism of DFT

- simplify DFT computation methods and formulas

Plan of Talk

2. $O(D,D)$ covariant double field theory (notation)

3. Supergeometry (graded manifold and pre-QP structure)

4. Section conditions, generalized fluxes and generalized Bianchi identity via supergeometry

5. $GL(2D)$ covariant DFT

§2. Double field theory

Siegel '93, Hull-Zwiebach '09

M: an original D-dimensional spacetime
$\widetilde{M}$: a D-dimensional T-dual spacetime

2D-dimensional doubled space $\widehat{M} = \widetilde{M} \times M$,

$X^{\hat{M}} = (\tilde{X}_M, X^M)$: coordinates of the doubled space

hat index: 2D dimensional indices,

unhat index: D dimensional indices

$M, N, \cdots$: spacetime indices,

$A, B, \cdots$: tangent flat space indices

We assume $O(D,D)$ an invariant metric $\eta_{\hat{M}\hat{N}}$.

Generalized Lie derivative and section condition (closure condition)

The generalized Lie derivative for $V^{\hat{M}}$ with a gauge parameter $\Lambda^{\hat{M}}$:

$$\mathcal{L}_\Lambda V^{\hat{M}} = \Lambda^{\hat{N}}\partial_{\hat{N}}V^{\hat{M}} + (\eta^{\hat{M}\hat{P}}\eta_{\hat{N}\hat{Q}}\partial_{\hat{P}}\Lambda^{\hat{Q}} - \partial_{\hat{N}}\Lambda^{\hat{M}})V^{\hat{N}},$$

$\mathcal{L}_\Lambda$ does not satisfy the Leibniz rule,

$$\Delta^M(\Lambda_1,\Lambda_2,V) = \mathcal{L}_{\mathcal{L}_{\Lambda_1}\Lambda_2}V^M - [\mathcal{L}_{\Lambda_1},\mathcal{L}_{\Lambda_2}]V^M \neq 0.$$

The condition of $\Delta^M(\Lambda_1, \Lambda_2, V) = 0$ is called the **closure condition** (**section condition**). The closure condition is

$$\eta^{\hat{M}\hat{N}}(\partial_{\hat{M}}\Phi)(\partial_{\hat{N}}\Psi) = 0.$$

We choose a D dimensional physical spacetime in the $2D$ dimensional doubled spacetime, on which $\Delta^M(\Lambda_1, \Lambda_2, V) = 0$ is satisfied.

Generalized metric and generalized vielbein

$\mathcal{H}_{\hat{M}\hat{N}}$: a generalized metric,

$$\mathcal{H}_{\hat{M}\hat{N}} = \begin{pmatrix} g^{MN} & -g^{MP}b_{PN} \\ b_{MP}g^{PN} & g_{MN} - b_{MP}g^{PQ}b_{QN} \end{pmatrix}.$$

$E_{\hat{\hat{A}}}{}^{\hat{M}}$: we use the generalized vielbein,

$$E_{\hat{\hat{A}}}{}^{\hat{M}} = \begin{pmatrix} E_A{}^M & E_{BM} \\ E^{AN} & E^B{}_N \end{pmatrix} = \begin{pmatrix} e_A{}^M & e_B{}^L B_{LM} \\ e^A{}_L \beta^{LN} & e^B{}_N + e^B{}_L B_{NK} \beta^{KL} \end{pmatrix}.$$

$\eta^{\hat{\hat{A}}\hat{\hat{B}}}$: $O(D,D)$ invariant metric.

The generalized Lie derivative is

$$\mathcal{L}_\Lambda E^{\hat{M}}_{\bar{A}} = \Lambda^{\hat{N}}\partial_{\hat{N}}E^{\hat{M}}_{\bar{A}} + (\eta^{\hat{M}\hat{P}}\eta_{\hat{N}\hat{Q}}\partial_{\hat{P}}\Lambda^{\hat{Q}} - \partial_{\hat{N}}\Lambda^{\hat{M}})E^{\hat{N}}_{\bar{A}}.$$

Lorentz frame

$S_{\hat{\hat{A}}\hat{\hat{B}}}$: an $O(1, D-1)\times O(1, D-1)$ invariant double Lorentz metric.
The generalized metric $\mathcal{H}_{\hat{M}\hat{N}}$ are written as

$$\mathcal{H}_{\hat{M}\hat{N}} = E^{\hat{\hat{A}}}{}_{\hat{M}} S_{\hat{\hat{A}}\hat{\hat{B}}} E^{\hat{\hat{B}}}{}_{\hat{N}}\ .$$

§3. Supergeometry of double field theory

Deser, Stasheff, '14, Deser, Saemann '16, Heller, NI, Watamura, '16

Graded manifold

A **graded manifold** is a 'manifold' with Z–graded coordinates.

Grading is called **degree**.

Note: (doubled) spacetime is a degree 0 part of graded manifold.

pre-QP-manifold

A graded manifold is called a pre-QP-manifold of degree n if it has the following structure.

1, (nondegenerate) graded Poisson bracket $\{-,-\}$ of degree $-n$.
2, Q: a graded vector field of degree $+1$

Note: We take a generator 'Hamiltonian' function $\Theta \in C^\infty(\mathcal{M})$ of degree $n+1$ such that $Q(-) = \{\Theta, -\}$.

If $Q^2 = 0$, a pre-QP-manifold is called a **QP-manifold**. $Q^2 = 0$ is equivalent to $\{\Theta, \Theta\} = 0$, the classical master equation.

Note: Θ corresponds to a BRST charge.

Example: Derived bracket construction of Courant algebroid

Roytenberg '99

Local coordinate of a graded manifold $T^*[2]T[1]M$ is (x^i, ξ_i, q^i, p_i): canonical conjugate pair coordinates of degree $(0, 2, 1, 1)$.

$$\{x^i, \xi_j\} = -\{\xi_j, x^i\} = \delta^i_j, \quad \{q^i, p_j\} = \{p_j, q^i\} = \delta^i_j.$$

We consider a Hamiltonian function Θ of degree 3, which gives an odd vector field $Q(-) = \{\Theta_0, -\}$,

$$\Theta_0 = \xi_i q^i.$$

A degree 1 function, $X^i(x)p_i + \alpha_i(x)q^i$, is identified to $X + \alpha = X^i(x)\partial_i + \alpha_i(x)dx^i \in \Gamma(TM \oplus T^*M)$.

The Dorfman bracket for $X + \alpha$ and $Y + \beta$, is

$$[X + \alpha, Y + \beta]_D = -\{\{X + \alpha, \Theta_0\}, Y + \beta\}$$
$$= [X, Y] + \mathcal{L}_X\beta - \iota_Y\alpha,$$

The anchor map is $\rho(X + \alpha)f = -\{\{X + \alpha, \Theta_0\}, f\} = Xf$.

All the identities of a Courant algebroid are given from $\{\Theta_0, \Theta_0\} = 0$.

Derived bracket construction of generalized Lie derivative

Take 2D dimensional doubled spacetime $\widehat{M}$.

We take a pre-QP-manifold $(\mathcal{M} = T^*[2]T[1]\widehat{M}, \{-,-\}, Q = \{\Theta, -\})$. **Note:** We do not require $Q^2 = 0$.

A generalized Lie derivative is defined by a **derived bracket**,

$$\mathcal{L}_V V' = [V, V']_D = [V, V'] \equiv -\{\{V, \Theta\}, V'\},$$

for generalized vector fields V, V', which are functions of degree 1.

Note: A graded Poisson bracket has automatically $O(D, D)$ str.

Closure condition

In general, $\{\Theta, \Theta\} \neq 0$ on a pre-QP-manifold.

We obtain the following identity of the derived bracket for any $f, g, h \in C^\infty(\mathcal{M})$,

$$\begin{aligned}[f,[g,h]] =& \{\{f,\Theta\},\{\{g,\Theta\},h\}\} \\ =& [[f,g],h] + (-1)^{(|f|+n+1)(|g|+n+1)}[g,[f,h]] \\ &+ (-1)^{|g|+n}\frac{1}{2}\{\{\{\{\Theta,\Theta\},f\},g\},h\}.\end{aligned}$$

Case 1, If $\{\Theta, \Theta\} = 0$, the derived bracket $[\cdot, \cdot]$ satisfies the following Leibniz identity of degree $-n+1$,

$$[f, [g, h]] = [[f, g], h] + (-1)^{(|f|-n+1)(|g|-n+1)}[g, [f, h]].$$

$[-, -] = \{\{-, \Theta\}, -\}$: the Dorfman bracket of a Courant algebroid.

Case 2, We can relax the condition to

$$\{\{\{\{\Theta, \Theta\}, f\}, g\}, h\} = 0,$$

which is sufficient for closure of the derived bracket. We call the condition the weak master equation. It is the section condition in DFT! Deser-Saemann '16, Bruce-Grabowski '16

Local coordinates

$X^{\hat{M}} = (\tilde{X}_M, X^M)$ is a general coordinate on doubled spacetime $\widehat{M} = \widetilde{M} \times M$.

$(X^{\hat{M}}, \Xi_{\hat{M}}, Q^{\hat{M}}, P_{\hat{M}})$: local coordinates of degree $(0, 2, 1, 1)$ on a graded manifold.

The Poisson brackets are

$$\{X^{\hat{M}}, \Xi_{\hat{N}}\} = -\{\Xi_{\hat{N}}, X^{\hat{M}}\} = \delta^{\hat{M}}_{\hat{N}},$$
$$\{Q^{\hat{M}}, P_{\hat{N}}\} = \{P_{\hat{N}}, Q^{\hat{M}}\} = \delta^{\hat{M}}_{\hat{N}}.$$

DFT basis (Clifford basis) and $O(D, D)$ metric

We can take basis $(Q'^{\hat{M}}, P'_{\hat{M}})$ of degree 1 coordinates such that

$$\{Q'^{\hat{M}}, Q'^{\hat{N}}\} = \eta^{\hat{M}\hat{N}}, \quad \{P'_{\hat{M}}, P'_{\hat{N}}\} = \eta_{\hat{M}\hat{N}}, \quad \{Q'^{\hat{M}}, P'_{\hat{N}}\} = 0,$$

which defines a Clifford algebra. We have

$$Q'^{\hat{M}} := \frac{1}{\sqrt{2}}(Q^{\hat{M}} - \eta^{\hat{M}\hat{N}} P_{\hat{N}}) \quad , \quad P'_{\hat{M}} := \frac{1}{\sqrt{2}}(P_{\hat{M}} + \eta_{\hat{M}\hat{N}} Q^{\hat{N}}),$$

A generalized vector field is identified to

$$V^{\hat{M}} \partial_{\hat{M}} \sim V^{\hat{M}} P'_{\hat{M}}.$$

Hamiltonian and generalized Lie derivative

We require $O(D,D)$ invariance. The simplest choice is

$$\Theta_0 = \eta^{\hat{M}\hat{N}} \Xi_{\hat{M}} P'_{\hat{N}}.$$

A derived bracket using this Θ_0 gives the correct generalized Lie derivative

$$\begin{aligned}\mathcal{L}_\Lambda V = [\Lambda, V]_D = & -\{\{\Lambda, \Theta_0\}, V\} \\ = & \Lambda^{\hat{N}} \partial_{\hat{N}} V^{\hat{M}} + (\eta^{\hat{M}\hat{P}} \eta_{\hat{N}\hat{Q}} \partial_{\hat{P}} \Lambda^{\hat{Q}} - \partial_{\hat{N}} \Lambda^{\hat{M}}) V^{\hat{N}}.\end{aligned}$$

Closure condition

$\{\Theta_0, \Theta_0\} = 0$ is not satisfied,

$$\{\Theta_0, \Theta_0\} = \eta^{\hat{M}\hat{N}} \Xi_{\hat{M}} \Xi_{\hat{N}} \neq 0.$$

The closure condition is $\{\{\{\{\Theta_0, \Theta_0\}, V_1\}, V_2\}, V_3\} = 0$, which is

$$2(\partial^{\hat{M}} V_1^{\hat{N}} V_{2\hat{N}} \partial_{\hat{M}} V_3^{\hat{Q}} - 2\partial^{\hat{M}} V_1^{[\hat{P}} \partial_{\hat{M}} V_2^{\hat{Q}]} V_{3\hat{P}}) P'_{\hat{Q}} = 0.$$

This condition holds if V satisfies the section condition,

$$\partial^{\hat{M}} V_i^{\hat{P}} \partial_{\hat{M}} V_j^{\hat{Q}} = 0.$$

§4. Twist and generalized fluxes

We introduce **fluxes** by a canonical transformation called **twist**.

Twist

$$e^{\delta_\alpha} f = f + \{f, \alpha\} + \frac{1}{2}\{\{f, \alpha\}, \alpha\} + \cdots ,$$

for $f \in C^\infty(\mathcal{M})$. Here α is a local function of **degree** 2, corresponding to a gerbe connection (a stack of groupoids). It is degree-preserving and obeys

$$\{e^{\delta_\alpha} f, e^{\delta_\alpha} g\} = e^{\delta_\alpha}\{f, g\},$$

for all functions f, g on graded manifold.

Closure condition

If a Hamiltonian function Θ is twisted by α, $\Theta \to \Theta' = e^{\delta_\alpha}\Theta$, the closure condition is changed to

$$\{\{\{\{\Theta', \Theta'\}, f\}, g\}, h\} = 0,$$

i.e.

$$e^{\delta_\alpha}\{\{\{\{\Theta, \Theta\}, e^{-\delta_\alpha} f\}, e^{-\delta_\alpha} g\}, e^{-\delta_\alpha} h\} = 0.$$

• A twist does not change a D-dimensional physical spacetime $M \subset \widetilde{M}$.

• A twist introduces flux terms to the section condition for a generalized vector field.

Local Lorentz frame

$\bar{Q}^{\hat{\bar{A}}}, \bar{P}_{\hat{\bar{A}}}$: flat tangent and cotangent coordinates of degree 1 corresponding to the local Lorentz frame. The DFT basis is

$$\bar{Q}'^{\hat{\bar{A}}} := \frac{1}{\sqrt{2}}(\bar{Q}^{\hat{\bar{A}}} - \eta^{\hat{\bar{A}}\hat{\bar{B}}}\bar{P}_{\hat{\bar{B}}}) \quad , \quad \bar{P}'_{\hat{\bar{A}}} := \frac{1}{\sqrt{2}}(\bar{P}_{\hat{\bar{A}}} + \eta_{\hat{\bar{A}}\hat{\bar{B}}}\bar{Q}'^{\hat{\bar{B}}})$$

DFT has the following three twists,

$$\begin{aligned} E &:= E_{\hat{\bar{A}}}{}^{\hat{M}}(X)\eta^{\hat{\bar{A}}\hat{\bar{B}}}P'_{\hat{M}}\bar{P}'_{\hat{\bar{B}}}, \\ u &:= u_{\hat{P}}{}^{\hat{M}}(X)\eta^{\hat{N}\hat{P}}P'_{\hat{M}}P'_{\hat{N}}, \quad \bar{u} := \bar{u}_{\hat{\bar{A}}}{}^{\hat{\bar{B}}}(X)\eta^{\hat{\bar{C}}\hat{\bar{A}}}\bar{P}'_{\hat{\bar{B}}}\bar{P}'_{\hat{\bar{C}}}. \end{aligned}$$

Twist in DFT

Then, the Hamiltonian function twisted by E becomes

$$\begin{aligned}\Theta_F =& e^{\frac{\pi}{2}\delta_E}\Theta_0 \\ =& E_{\hat{\hat{A}}}{}^{\hat{M}}\Xi_{\hat{M}}\bar{P}'^{\hat{\hat{A}}} + \frac{1}{3!}\mathcal{F}_{\hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}}\bar{P}'^{\hat{\hat{A}}}\bar{P}'^{\hat{\hat{B}}}\bar{P}'^{\hat{\hat{C}}} + \frac{1}{2}\Phi_{\hat{\hat{C}}\hat{M}\hat{N}}P'^{\hat{M}}P'^{\hat{N}}\bar{P}'^{\hat{\hat{C}}}\end{aligned}$$

where

$$\mathcal{F}_{\hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}} = 3\Omega_{[\hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}]}, \qquad \Phi_{\hat{\hat{C}}\hat{M}\hat{N}} = -\Omega_{\hat{\hat{C}}\hat{\hat{A}}\hat{\hat{B}}}E^{\hat{\hat{A}}}{}_{\hat{M}}E^{\hat{\hat{B}}}{}_{\hat{N}}.$$

Here $\Omega_{\hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}} := E_{\hat{\hat{A}}}{}^{\hat{M}}\partial_{\hat{M}}E_{\hat{\hat{B}}}{}^{\hat{N}}E_{\hat{\hat{C}}\hat{N}}$ is a generalized Weitzenböck connection and $\Omega_{\hat{M}\hat{N}\hat{P}} = E^{\hat{\hat{A}}}{}_{\hat{M}} E^{\hat{\hat{B}}}{}_{\hat{N}} E^{\hat{\hat{C}}}{}_{\hat{P}} \Omega_{\hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}}$. Aldazabal, Baron, Marques, Nunez, '11

We obtain the correct forms of a generalized flux and a generalized Weitzenböck connection.

General form

The most general degree 3 Hamiltonian which consist of $(X^{\hat{M}}, \Xi_{\hat{M}}, P'^{\hat{M}}, \bar{P}'^{\hat{\bar{C}}})$.

$$\begin{aligned}\Theta_F =& \bar{\rho}_{\hat{\bar{A}}}{}^{\hat{M}}(X)\Xi_{\hat{M}}\bar{P}'^{\hat{\bar{A}}} + \rho_{\hat{N}}{}^{\hat{M}}(X)\Xi_{\hat{M}}P'^{\hat{N}} + \frac{1}{3!}\mathcal{F}_{\hat{\bar{A}}\hat{\bar{B}}\hat{\bar{C}}}(X)\bar{P}'^{\hat{\bar{A}}}\bar{P}'^{\hat{\bar{B}}}\bar{P}'^{\hat{\bar{C}}} \\ &+ \frac{1}{2}\Phi_{\hat{\bar{C}}\hat{M}\hat{N}}(X)P'^{\hat{M}}P'^{\hat{N}}\bar{P}'^{\hat{\bar{C}}} \\ &+ \frac{1}{2}\Delta_{\hat{\bar{A}}\hat{\bar{B}}\hat{M}}(X)P'^{\hat{M}}\bar{P}'^{\hat{\bar{A}}}\bar{P}'^{\hat{\bar{B}}} + \frac{1}{3!}\Psi_{\hat{M}\hat{N}\hat{P}}(X)P'^{\hat{M}}P'^{\hat{N}}P'^{\hat{P}},\end{aligned}$$

We obtain more general fluxes.

§5. Generalized Bianchi identity via pre-QP-manifold

Bianchi identity of fluxes in supergravity

In SUGRA, Bianchi identities of fluxes are equivalent to the classical master equation $\{\Theta, \Theta\} = 0$, where Θ is a Hamiltonian twisted by fluxes. Heller, NI, Watamura '16

$$\Theta = \rho^M{}_N(x)\xi_M q^N + \pi^{MN}(x)\xi_M p_N + \frac{1}{3!}H_{MNP}(x)q^L q^M q^N$$
$$+ \frac{1}{2}F^N_{LM}(x)q^L q^M p_N + \frac{1}{2}Q^{MN}_L(x)q^L p_M p_N + \frac{1}{3!}R^{LMN}(x)p_L p_M p_N.$$

Examples

1. Original Neveu-Schwarz H-flux

$H = dB$, $F = 0$, $Q = 0$, $R = 0$.

$$\Theta_1 = e^{\delta_B}\Theta_0 = \xi_M q^M + \frac{1}{3!}H_{LMN}(x)q^L q^M q^N,$$

where $B = \frac{1}{2}B_{MN}(x)q^M q^N$.

$\{\Theta_1, \Theta_1\} = 0$ is equivalent to $dH = 0$.

2. Fluxes with metric

Blumenhagen-Deser-Plauschinn-Rennecke '12

$$
\begin{aligned}
H &= \nabla B \\
F &= T + \beta^{\sharp} H \\
Q &= \nabla \beta + \wedge^2 \beta^{\sharp} H, \\
R &= [\beta, \beta]_S^{\nabla} + \wedge^3 \beta^{\sharp} H,
\end{aligned}
$$

where ∇ is a covariant derivative with respect to the Riemannian connection and T is a torsion tensor.

Let

$$B = \frac{1}{2} B_{MN}(x) q^M q^N, \quad \beta = \frac{1}{2} \beta^{MN}(x) p_M p_N,$$
$$e = e_A{}^M(x) q^A p_M, \quad e^{-1} = e^A{}_M(x) q^M p_A.$$

and consider twist $\Theta_2 = e^{-\delta_e} e^{\delta_{e^{-1}}} e^{-\delta_e} e^{-\delta_\beta} \Theta_1$.

From Θ_2, we obtain forms H, F, Q, R in the previous page, and

$$\{\Theta_2, \Theta_2\} = 0,$$

gives the correct Bianchi identity of H, F, Q, R.

3. Flux on Poisson manifold background

Asakawa-Muraki-Sasa-Watamura '15, Bessho-Heller-NI-Watamura '15

Let M be a Poisson manifold with a nondegenerate Poisson bivector $\pi \in \Gamma(\wedge^2 TM)$.

$$H = F = Q = 0, \qquad R = \frac{1}{2}[\pi, \beta]_S.$$

where β is a β-field.

We take $\pi = \frac{1}{2}\pi^{ij}(x) p_i p_j$, $\pi^{-1} = \frac{1}{2}\pi^{-1}_{ij}(x) q^i q^j$, and $\{\Theta_4, \Theta_4\} = 0$ under the twisting,

$$\Theta_4 = e^{\delta_\beta} e^{\delta_\pi} e^{-\delta_{\pi^{-1}}} e^{\delta_\pi} e^{-\delta_{-B}} \Theta_1.$$

DFT: pre-Bianchi identity

Carow-Watamura, NI, Kaneko and Watamura, '18

In a pre-QP-manifold, $\{\Theta_F, \Theta_F\} \neq 0$.

We propose a weak version of the Bianchi identity equation,

$$\mathcal{B}(\Theta_F, \Theta_0, \alpha) = \{\Theta_F, \Theta_F\} - e^{\delta_\alpha}\{\Theta_0, \Theta_0\} = 0.$$

where α is a twist and Θ_0 is a Hamiltonian function without fluxes. Here the Hamiltonian function with generalized fluxes is

$$\Theta_F = E_{\hat{\hat{A}}}{}^{\hat{M}} \Xi_{\hat{M}} \bar{P}'^{\hat{\hat{A}}} + \frac{1}{3!} \mathcal{F}_{\hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}} \bar{P}'^{\hat{\hat{A}}} \bar{P}'^{\hat{\hat{B}}} \bar{P}'^{\hat{\hat{C}}} + \frac{1}{2} \Phi_{\hat{\hat{C}}\hat{M}\hat{N}} P'^{\hat{M}} P'^{\hat{N}} \bar{P}'^{\hat{\hat{C}}}.$$

For example, a twist by $\alpha = E = E_{\hat{\bar{A}}}{}^{\hat{M}} \eta^{\hat{\bar{A}}\hat{\bar{B}}} P'_{\hat{M}} \bar{P}'_{\hat{\bar{B}}}$ gives

$$
\begin{aligned}
&\mathcal{B}(\Theta_F, \Theta_0, E) \\
=&\big(2\partial_{\hat{N}} E_{\hat{\bar{C}}}{}^{\hat{M}} E_{\hat{\bar{D}}}{}^{\hat{N}} + \eta^{\hat{\bar{A}}\hat{\bar{B}}} E_{\hat{\bar{A}}}{}^{\hat{M}} \mathcal{F}_{\hat{\bar{B}}\hat{\bar{C}}\hat{\bar{D}}} - \eta^{\hat{M}\hat{N}} \Omega_{\hat{N}\hat{Q}\hat{U}} E_{\hat{\bar{C}}}{}^{\hat{Q}} E_{\hat{\bar{D}}}{}^{\hat{U}}\big) \Xi_{\hat{M}} \bar{P}'^{\hat{\bar{C}}} \bar{P}'^{\hat{\bar{D}}} \\
&+ \big(\eta^{\hat{\bar{A}}\hat{\bar{B}}} E_{\hat{\bar{A}}}{}^{\hat{M}} \Phi_{\hat{\bar{B}}\hat{N}\hat{P}} + \eta^{\hat{M}\hat{Q}} \Omega_{\hat{Q}\hat{N}\hat{P}}\big) \Xi_{\hat{M}} P'^{\hat{N}} P'^{\hat{P}} \\
&+ \Big(- \frac{2}{3!} E_{\hat{\bar{A}}}{}^{\hat{M}} \partial_{\hat{M}} \mathcal{F}_{\hat{\bar{B}}\hat{\bar{C}}\hat{\bar{D}}} + \frac{3}{4} \eta^{\hat{\bar{E}}\hat{\bar{F}}} \mathcal{F}_{\hat{\bar{E}}\hat{\bar{A}}\hat{\bar{B}}} \mathcal{F}_{\hat{\bar{F}}\hat{\bar{C}}\hat{\bar{D}}} - \frac{1}{4} \eta^{\hat{\bar{E}}\hat{\bar{F}}} \Omega_{\hat{\bar{E}}\hat{\bar{A}}\hat{\bar{B}}} \Omega_{\hat{\bar{F}}\hat{\bar{C}}\hat{\bar{D}}} \Big) \bar{P}'^{\hat{\bar{A}}} \bar{P}'^{\hat{\bar{B}}} \bar{P}'^{\hat{\bar{C}}} \bar{P}'^{\hat{\bar{D}}} \\
&+ \Big(- E_{\hat{\bar{A}}}{}^{\hat{P}} \partial_{\hat{P}} \Phi_{\hat{\bar{B}}\hat{M}\hat{N}} + \frac{1}{2} \eta^{\hat{\bar{C}}\hat{\bar{D}}} \mathcal{F}_{\hat{\bar{A}}\hat{\bar{B}}\hat{\bar{C}}} \Phi_{\hat{\bar{D}}\hat{M}\hat{N}} \\
&- \eta^{\hat{Q}\hat{R}} \Phi_{\hat{\bar{A}}\hat{Q}\hat{M}} \Phi_{\hat{\bar{B}}\hat{R}\hat{N}} + \frac{1}{2} \eta^{\hat{P}\hat{R}} \Omega_{\hat{P}\hat{M}\hat{N}} \Omega_{\hat{R}\hat{Q}\hat{U}} E_{\hat{\bar{A}}}{}^{\hat{Q}} E_{\hat{\bar{B}}}{}^{\hat{U}} \Big) P'^{\hat{M}} P'^{\hat{N}} \bar{P}'^{\hat{\bar{A}}} \bar{P}'^{\hat{\bar{B}}} \\
&+ \frac{1}{4} \big(\eta^{\hat{R}\hat{S}} \Phi_{\hat{R}\hat{M}\hat{N}} \Phi_{\hat{S}\hat{P}\hat{Q}} - \eta^{\hat{R}\hat{S}} \Omega_{\hat{R}\hat{M}\hat{N}} \Omega_{\hat{S}\hat{P}\hat{Q}}\big) P'^{\hat{M}} P'^{\hat{N}} P'^{\hat{P}} P'^{\hat{Q}}.
\end{aligned}
$$

The pre-Bianchi identity gives

$$2\partial_{\hat{N}} E_{[\hat{\hat{C}}}{}^{\hat{M}} E_{\hat{\hat{D}}]}{}^{\hat{N}} + E_{\hat{\hat{A}}}^{\hat{\hat{A}}\hat{M}} \mathcal{F}_{\hat{\hat{B}}\hat{\hat{C}}\hat{\hat{D}}} - \Omega^{\hat{M}}{}_{\hat{Q}\hat{U}} E_{\hat{\hat{C}}}{}^{\hat{Q}} E_{\hat{\hat{D}}}{}^{\hat{U}} = 0,$$

$$E^{\hat{\hat{A}}\hat{M}} \Phi_{\hat{\hat{A}}\hat{N}\hat{P}} + \Omega^{\hat{M}}{}_{\hat{N}\hat{P}} = 0,$$

$$-\frac{2}{3!} E_{[\hat{\hat{A}}}{}^{\hat{M}} \partial_{\hat{M}} \mathcal{F}_{\hat{\hat{B}}\hat{\hat{C}}\hat{\hat{D}}]} + \frac{3}{4} \mathcal{F}_{\hat{\hat{E}}[\hat{\hat{A}}\hat{\hat{B}}} \mathcal{F}^{\hat{\hat{F}}}{}_{\hat{\hat{C}}\hat{\hat{D}}]} - \frac{1}{4} \Omega_{\hat{\hat{E}}[\hat{\hat{A}}\hat{\hat{B}}} \Omega^{\hat{\hat{E}}}{}_{\hat{\hat{C}}\hat{\hat{D}}]} = 0,$$

$$- E_{[\hat{\hat{A}}}{}^{\hat{P}} \partial_{\hat{P}} \Phi_{\hat{\hat{B}}]\hat{M}\hat{N}} + \frac{1}{2} \mathcal{F}^{\hat{\hat{C}}}{}_{\hat{\hat{A}}\hat{\hat{B}}} \Phi_{\hat{\hat{C}}\hat{M}\hat{N}} - \Phi_{[\hat{\hat{A}}[\hat{M}}{}^{\hat{Q}} \Phi_{\hat{\hat{B}}]\hat{N}]\hat{Q}}$$

$$+ \frac{1}{2} \Omega^{\hat{P}}{}_{\hat{M}\hat{N}} \Omega_{\hat{P}\hat{Q}\hat{U}} E_{\hat{\hat{A}}}{}^{\hat{Q}} E_{\hat{\hat{B}}}{}^{\hat{U}} = 0,$$

$$\Phi_{\hat{R}[\hat{M}\hat{N}} \Phi^{\hat{R}}{}_{\hat{P}\hat{Q}]} - \Omega_{\hat{R}[\hat{M}\hat{N}} \Omega^{\hat{R}}{}_{\hat{P}\hat{Q}]} = 0.$$

1st and 2nd: local expressions of $\mathcal{F}_{\hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}}$ and $\Phi_{\hat{\hat{A}}\hat{N}\hat{P}}$.

3rd: the **generalized Bianchi identity** in DFT in

Aldazabal, Marques, Nunez, '13, Geissbühler, Marques, Nunez, Penas, '13.

4th: New generalized Bianchi identity for $\Phi_{\hat{\hat{A}}\hat{M}\hat{N}}$!

5th: trivially satisfied.

General form

The most general degree 3 Hamiltonian which consist of $(X^{\hat{M}}, \Xi_{\hat{M}}, P'^{\hat{M}}, \bar{P}'^{\hat{\bar{C}}})$.

$$
\begin{aligned}
\Theta_F =& \bar{\rho}_{\hat{\bar{A}}}{}^{\hat{M}}(X)\Xi_{\hat{M}}\bar{P}'^{\hat{\bar{A}}} + \rho_{\hat{N}}{}^{\hat{M}}(X)\Xi_{\hat{M}}P'^{\hat{N}} + \frac{1}{3!}\mathcal{F}_{\hat{\bar{A}}\hat{\bar{B}}\hat{\bar{C}}}(X)\bar{P}'^{\hat{\bar{A}}}\bar{P}'^{\hat{\bar{B}}}\bar{P}'^{\hat{\bar{C}}} \\
&+ \frac{1}{2}\Phi_{\hat{\bar{C}}\hat{M}\hat{N}}(X)P'^{\hat{M}}P'^{\hat{N}}\bar{P}'^{\hat{\bar{C}}} \\
&+ \frac{1}{2}\Delta_{\hat{\bar{A}}\hat{\bar{B}}\hat{M}}(X)P'^{\hat{M}}\bar{P}'^{\hat{\bar{A}}}\bar{P}'^{\hat{\bar{B}}} + \frac{1}{3!}\Psi_{\hat{M}\hat{N}\hat{P}}(X)P'^{\hat{M}}P'^{\hat{N}}P'^{\hat{P}}, \\
\Theta_0 =& \eta^{\hat{M}\hat{N}}\Xi_{\hat{M}}P'_{\hat{N}}.
\end{aligned}
$$

We obtain more general generalized Bianchi identity.

§6. $GL(2D)$ Covariant DFT from pre-QP-manifold

We generalize the formalism to a covariant pre-QP formulation.

Differential geometry of DFT   Jeon, Lee, Park, '11, Hohm, Zwiebach '13

Let $X^{\hat{M}} = (\tilde{X}_M, X^M)$ be local coordinates of a $2D$-dimensional curved doubled spacetime with $GL(2D)$ indices, $\hat{M}, \hat{N}, \cdots$.

$\hat{I}, \hat{J}, \cdots$ for the indices of an $O(D,D)$ frame with an $O(D,D)$ metric $\eta^{\hat{I}\hat{J}}$.

$\hat{\hat{A}}, \hat{\hat{B}}, \cdots$ for the indices of local double Lorentz frame with an $O(D-1,1) \times O(1,D-1)$ metric $\eta^{\hat{\hat{A}}\hat{\hat{B}}}$.

An ansatz of the total vielbein is

$$\mathcal{E}_{\hat{\hat{A}}}{}^{\hat{M}} = \widehat{E}_{\hat{\hat{A}}}{}^{\hat{I}} E_{\hat{I}}{}^{\hat{M}} \ ,$$

where $E_{\hat{I}}{}^{\hat{M}}$ is the background vielbein and $\widehat{E}_{\hat{\hat{A}}}{}^{\hat{I}}$ is the fluctuation vielbein.

Covariantization

We define a basis $\Xi^{\nabla}_{\hat{M}}$ of degree 2, corresponding to the covariant derivative $\nabla_{\hat{M}}$, with affine connection Γ and spin connection W,

$$\Xi^{\nabla}_{\hat{M}} := \Xi_{\hat{M}} + \Gamma_{\hat{M}\hat{N}}{}^{\hat{P}}(X) Q^{\hat{N}} P_{\hat{P}} + W_{\hat{M}\hat{I}}{}^{\hat{J}}(X) \widehat{Q}^{\hat{I}} \widehat{P}_{\hat{J}}.$$

The Poisson bracket $\{-, \Xi^{\nabla}_{\hat{M}}\}$ with the vector fields $V^{\hat{M}} P_{\hat{M}}, \widehat{V}^{\hat{I}} \widehat{P}_{\hat{I}}$ give their covariant derivative:

$$\{V^{\hat{M}}(X) P_{\hat{M}}, \Xi^{\nabla}_{\hat{N}}\} = \nabla_{\hat{N}} V^{\hat{M}}(X) P_{\hat{M}},$$
$$\{\widehat{V}^{\hat{I}}(X) \widehat{P}_{\hat{I}}, \Xi^{\nabla}_{\hat{N}}\} = \nabla_{\hat{N}} \widehat{V}^{\hat{I}}(X) \widehat{P}_{\hat{I}}.$$

Vielbein and metric condition

Require the condition (the vielbein postulate),

$$\{E_{\hat{I}}{}^{\hat{N}} P_{\hat{N}} \widehat{Q}^{\hat{I}}, \Xi^{\nabla}_{\hat{M}}\} = 0,$$

then, $\nabla_{\hat{M}} E_{\hat{I}}{}^{\hat{N}} = 0$, $\nabla_{\hat{M}} \eta_{\hat{I}\hat{J}} = 0$, and $\nabla_{\hat{M}} \eta_{\hat{N}\hat{P}} = 0$. and we obtain conditions of generalized connections,

$$W_{\hat{M}\hat{I}}{}^{\hat{J}} E^{\hat{I}}{}_{\hat{N}} E_{\hat{J}}{}^{\hat{P}} - \Omega_{\hat{M}\hat{N}}{}^{\hat{P}} - \Gamma_{\hat{M}\hat{N}}{}^{\hat{P}} = 0,$$

$$W_{\hat{M}\hat{J}\hat{K}} + W_{\hat{M}\hat{K}\hat{J}} = 0.$$

Covariant Hamiltonian function and generalized Lie derivative

A Hamiltonian function is covariantized as

$$\Theta_0^{\nabla} = \eta^{\hat{M}\hat{N}} \Xi^{\nabla}_{\hat{M}} P'_{\hat{N}}.$$

The generalized Lie derivative is defined by the derived bracket,

$$\mathcal{L}^{\nabla}_{\Lambda} V = -\{\{\Lambda, \Theta_0^{\nabla}\}, V\}.$$

The covariant generalized Lie derivative is given by replacing the derivative in the generalized Lie derivative by $\nabla_{\hat{M}}$:

$$\mathcal{L}^{\nabla}_{\Lambda} V^{\hat{M}} = \Lambda^{\hat{N}} \nabla_{\hat{N}} V^{\hat{M}} + (\nabla^{\hat{M}} \Lambda_{\hat{N}} - \nabla_{\hat{N}} \Lambda^{\hat{M}}) V^{\hat{N}}.$$

Closure condition

The covariantized closure condition is

$$\{\{\{\{\widehat{\Theta}^{\nabla}_0, \widehat{\Theta}^{\nabla}_0\}, \widehat{V}_1\}, \widehat{V}_2\}, \widehat{V}_3\} = 0.$$

This condition leads to the following conditions for the spin connection $W_{\hat{M}\hat{I}}{}^{\hat{J}}$ and arbitrary generalized vectors $\widehat{V}_1, \widehat{V}_2$ and $\widehat{V}_3$, The following conditions are sufficient for the closure condition,

$$\partial^{\hat{M}}\widehat{V}_1^{\hat{J}}\widehat{V}_{2\hat{J}}\partial_{\hat{M}}\widehat{V}_3^{\hat{I}} - 2\partial^{\hat{M}}\widehat{V}_1^{[\hat{J}}\partial_{\hat{M}}\widehat{V}_2^{\hat{I}]}\widehat{V}_{3\hat{J}} = 0,$$

$$(-2\Omega_{[\hat{I}\hat{J}]\hat{K}} + 3W_{[\hat{I}\hat{J}\hat{K}]})E^{\hat{K}\hat{M}}\partial_{\hat{M}}\widehat{V}_1^{\hat{L}} = 0,$$

$$2R_{[\hat{I}\hat{J}\hat{K}\hat{L}]} - W_{\hat{H}[\hat{I}\hat{J}}W^{\hat{H}}{}_{\hat{K}\hat{L}]} - 2(2W_{[\hat{I}\hat{J}}{}^{\hat{H}} - 2\Omega_{[\hat{I}\hat{J}}{}^{\hat{H}})W_{\hat{H}\hat{K}\hat{L}]} = 0.$$

Solutions: two examples

1) **DFT on group manifold** The second condition is satisfied by taking,

$$3W_{[\hat{I}\hat{J}\hat{K}]} = 2\Omega_{[\hat{I}\hat{J}]\hat{K}}.$$

Then, the third condition becomes

$$R_{[\hat{I}\hat{J}\hat{K}\hat{L}]} + W^{\hat{H}}{}_{[\hat{I}\hat{J}}W_{|\hat{H}|\hat{K}\hat{L}]} = 0.$$

1) corresponds to DFT on group manifold, where W is a structure constant, $3W_{[\hat{I}\hat{J}\hat{K}]} = F_{\hat{I}\hat{J}\hat{K}}$. Blumenhagen, Hassler, Luest, '14

2) **generalized Scherk-Schwarz compactification** The conditions can be satisfied by

$$W_{\hat{I}[\hat{J}\hat{K}]} = \Omega_{\hat{I}[\hat{J}\hat{K}]},$$
$$\Omega_{\hat{K}\hat{I}\hat{J}} E^{\hat{K}\hat{M}} \partial_{\hat{M}} \widehat{V}_1^{\hat{N}} = E_{\hat{J}\hat{Q}} \eta^{\hat{P}\hat{M}} \partial_{\hat{P}} E_{\hat{I}}{}^{\hat{Q}} \partial_{\hat{M}} \widehat{V}_1^{\hat{N}} = 0.$$

With this choice, $R_{\hat{I}\hat{J}\hat{K}\hat{L}} = 0$, so the third condition becomes

$$\eta^{\hat{M}\hat{N}} \partial_{\hat{M}} E_{[\hat{I}}{}^{\hat{P}} \partial_{\hat{N}} E_{\hat{J}}{}^{\hat{Q}} E_{\hat{K}\hat{P}} E_{\hat{L}]\hat{Q}} = 0.$$

2) corresponds to the generalized Scherk-Schwarz compactification.

Aldazabal, Baron, Marques, Nunez '11, Grana, Marques, '12, Berman, Lee '13

Twist

Possible twist functions made from $P', \widehat{P}'$ and $\bar{P}'$ are

$$A := A^{\hat{I}\hat{M}} P'_{\hat{M}} \widehat{P}'_{\hat{I}}, \quad \widehat{A} := \widehat{A}^{\hat{\bar{A}}\hat{J}} \widehat{P}'_{\hat{J}} \bar{P}'_{\hat{\bar{A}}}, \quad \mathcal{A} := \mathcal{A}^{\hat{\bar{A}}\hat{M}} P'_{\hat{M}} \bar{P}'_{\hat{\bar{A}}},$$

$$u := u_{\hat{P}}{}^{\hat{M}} \eta^{\hat{N}\hat{P}} P'_{\hat{M}} P'_{\hat{N}}, \quad \widehat{u} := \widehat{u}_{\hat{I}}{}^{\hat{J}} \eta^{\hat{K}\hat{I}} \widehat{P}'_{\hat{J}} \widehat{P}'_{\hat{K}}, \quad \bar{u} := \bar{u}_{\hat{\bar{A}}}{}^{\hat{\bar{B}}} \eta^{\hat{\bar{C}}\hat{\bar{A}}} \bar{P}'_{\hat{\bar{B}}} \bar{P}'_{\hat{\bar{C}}}.$$

Here $A^{\hat{I}\hat{M}}$, $\widehat{A}^{\hat{\bar{A}}\hat{J}}$ and $\mathcal{A}^{\hat{\bar{A}}\hat{M}}$ are $GL(2D)$ matrices and we can take them as vielbein $E_{\hat{I}}{}^{\hat{M}}, \widehat{E}_{\hat{\bar{A}}}{}^{\hat{I}}$ and $\mathcal{E}_{\hat{\bar{A}}}{}^{\hat{M}}$.

We obtain twist of the Hamiltonian function,

$$\begin{aligned}\Theta_F =& e^{\frac{\pi}{2}\delta_{\widehat{E}}}\widehat{\Theta}_0^{\nabla} \\ =& \mathcal{E}_{\hat{\bar{A}}}{}^{\hat{M}}\Xi^{\nabla}_{\hat{M}}\bar{P}'^{\hat{\bar{A}}} + \frac{1}{3!}\mathscr{F}_{\hat{\bar{A}}\hat{\bar{B}}\hat{\bar{C}}}\bar{P}'^{\hat{\bar{A}}}\bar{P}'^{\hat{\bar{B}}}\bar{P}'^{\hat{\bar{C}}} + \frac{1}{2}\mathscr{G}_{\hat{\bar{A}}\hat{I}\hat{J}}\bar{P}'^{\hat{\bar{A}}}\widehat{P}'^{\hat{I}}\widehat{P}'^{\hat{J}}.\end{aligned}$$

Pre-Bianchi identities

We consider the pre-Bianchi identity for DFT on covariantized pre-QP-manifold.

$$\mathcal{B}(\Theta_F, \Theta_0, \alpha) := \{\Theta_F, \Theta_F\} - e^{\delta_\alpha}\{\Theta_0, \Theta_0\} = 0.$$

gives the generalized Bianchi identities.

$$2\partial_{\hat{N}}\mathcal{E}_{\hat{\hat{A}}}{}^{\hat{M}}\mathcal{E}_{\hat{\hat{B}}}{}^{\hat{N}} + \mathcal{E}^{\hat{\hat{C}}\hat{M}}\mathscr{F}_{\hat{\hat{C}}\hat{\hat{A}}\hat{\hat{B}}} + 2\Gamma_{\hat{I}\hat{K}\hat{J}}E^{\hat{K}\hat{M}}\widehat{E}_{\hat{\hat{A}}}{}^{\hat{I}}\widehat{E}_{\hat{\hat{B}}}{}^{\hat{J}} - \widehat{\Omega}^{\nabla\hat{M}}{}_{\hat{\hat{A}}\hat{\hat{B}}} = 0,$$

$$\mathcal{E}^{\hat{\hat{C}}\hat{M}}\mathscr{G}_{\hat{\hat{C}}\hat{I}\hat{J}} + \widehat{\Omega}^{\nabla\hat{M}}{}_{\hat{\hat{D}}\hat{\hat{C}}}\widehat{E}^{\hat{\hat{D}}}{}_{\hat{I}}\widehat{E}^{\hat{\hat{C}}}{}_{\hat{J}},$$

$$\frac{1}{3}\mathcal{E}_{\hat{\hat{A}}}{}^{\hat{M}}\partial_{\hat{M}}\mathscr{F}_{\hat{\hat{B}}\hat{\hat{C}}\hat{\hat{D}}} - \frac{1}{4}\mathscr{F}^{\hat{\hat{E}}}{}_{\hat{\hat{A}}\hat{\hat{B}}}\mathscr{F}_{\hat{\hat{E}}\hat{\hat{C}}\hat{\hat{D}}} - \Gamma_{\hat{J}\hat{I}\hat{K}}\widehat{\Omega}^{\nabla}_{\hat{N}\hat{\hat{A}}\hat{\hat{B}}}E^{\hat{I}\hat{N}}\widehat{E}^{\hat{J}}{}_{\hat{\hat{C}}}\widehat{E}^{\hat{K}}{}_{\hat{\hat{D}}} + \frac{1}{4}\widehat{\Omega}^{\nabla}_{\hat{M}\hat{\hat{A}}\hat{\hat{B}}}\widehat{\Omega}^{\nabla\hat{M}}{}_{\hat{\hat{C}}\hat{\hat{D}}} = 0,$$

$$\mathcal{E}_{\hat{\hat{A}}}{}^{\hat{M}}\nabla_{\hat{M}}\mathscr{G}_{\hat{\hat{B}}\hat{I}\hat{J}} + \frac{1}{2}\mathscr{F}^{\hat{\hat{E}}}{}_{\hat{\hat{A}}\hat{\hat{B}}}\mathscr{G}_{\hat{\hat{E}}\hat{I}\hat{J}} - \mathscr{G}_{\hat{\hat{A}}\hat{I}}{}^{\hat{K}}\mathscr{G}_{\hat{\hat{B}}\hat{J}\hat{K}},$$

$$-\Gamma_{\hat{\hat{A}}\hat{\hat{C}}\hat{\hat{B}}}\mathcal{E}^{\hat{\hat{C}}\hat{N}}\widehat{\Omega}^{\nabla}_{\hat{N}\hat{\hat{D}}\hat{\hat{E}}}\widehat{E}^{\hat{\hat{D}}}{}_{\hat{I}}\widehat{E}^{\hat{\hat{E}}}{}_{\hat{J}} + \frac{1}{2}\widehat{\Omega}^{\nabla}_{\hat{M}\hat{\hat{A}}\hat{\hat{B}}}\widehat{\Omega}^{\nabla\hat{M}}{}_{\hat{\hat{C}}\hat{\hat{D}}}\widehat{E}^{\hat{\hat{C}}}{}_{\hat{I}}\widehat{E}^{\hat{\hat{D}}}{}_{\hat{J}} = 0,$$

$$\mathscr{G}^{\hat{\hat{A}}}{}_{\hat{I}\hat{J}}\mathscr{G}_{\hat{\hat{A}}\hat{K}\hat{L}} - \tilde{\Omega}^{\nabla}_{\hat{M}\hat{\hat{A}}\hat{\hat{B}}}\tilde{\Omega}^{\nabla\hat{M}}{}_{\hat{\hat{C}}\hat{\hat{D}}}\tilde{E}^{\hat{\hat{A}}}{}_{\hat{I}}\tilde{E}^{\hat{\hat{B}}}{}_{\hat{J}}\tilde{E}^{\hat{\hat{C}}}{}_{\hat{K}}\tilde{E}^{\hat{\hat{D}}}{}_{\hat{L}} = 0.$$

§. Summary and outlook

• We formulated DFT geometry in coordinate independent form using pre-QP-manifold.

A generalized Lie derivative is defined by a derived bracket,

$$\mathcal{L}_V V' = -\{\{V, \Theta\}, V'\}.$$

The closure condition (the weak master equation) is the weak master equation,

$$\{\{\{\{\Theta, \Theta\}, f\}, g\}, h\} = 0.$$

Generalized fluxes are introduced by twist,

$$\Theta_F = e^{\delta_\alpha}\Theta_0,$$

A generalized Bianchi identity is a pre-Bianchi identity,

$$\mathcal{B}(\Theta_F, \Theta_0, \alpha) = \{\Theta_F, \Theta_F\} - e^{\delta_\alpha}\{\Theta_0, \Theta_0\} = 0.$$

We confirmed this formulation gave known results and new completions in the GSS compactification and DFT on group manifold.

Outlook

• Inclusion of a dilaton

• Nonabelian/Poisson-Lie T-duality

• Physics: action, quantization, etc.

• exceptional field theory

• Characteristic classes of T^d bundles and nongeometric fluxes. (Q defines a complex and cohomology.)

Thank you for your attention!