A modified Penrose limit and generalized pp-wave solutions

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Introduction

The generalized supergravity [Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

Classical consistency Kappa symmetry of GS action

[Tseytlin-Wulf, 1605.04884]

Quantum consistency Weyl anomaly cancellation

[Sakamoto-Sakatani-Yoshida, 1703.09213]

[Jose J. Fernandez-Melgarejo-Sakamoto-Sakatani-Yoshida, 1811.10600]

⇩

The gSUGRA may give a new consistent b.g. of the superstring

Introduction

A main interest

How to analyze the superstrings on such deformed backgrounds

It is difficult to analyze the superstring on a curved background.

⇩

We assume one can obtain a non-trivial, but solvable background by the Penrose limit

cf. Penrose limit of $AdS_5 \times S^5$ $\longrightarrow$ maximally supersymmetric pp-wave

Motivation

TsT transformed background

EOM w/ periodic b.c.

⟺ [Frolov, 0503201]

Undeformed AdS5 × S5

EOM w/ twisted periodic b.c.

Assumption : A similar relation holds for YB deformed $AdS_5 \times S^5$.

⇓ **Penrose limit**

Such a relation still holds for deformed pp-wave b.g.

We may analyze the superstring on deformed b.g.

Plan of this talk

0. Introduction

1. Review of the standard Penrose limit (Poincare AdS)

2. A (modified) Penrose limit of deformed b.g.

    - A gravity dual of a NCYM
    - A non-unimodular e.g.

3. Summary and discussion

Review of the standard Penrose limit

AdS^5 ✕ S$_5$ metric

$$ds^2_{S^5} = d\psi^2 + \sin^2\psi \, ds^2_{S^4}$$

$$ds^2 = \frac{-dt^2 + dz^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2}{z^2} + ds^2_{S^5}$$

Step1 Expanding the background around a null geodesics on $(t\,,z\,,\psi)$ plane

$$\left(\begin{aligned} r &= \frac{1}{\mu}\sin\mu u \\ t &= -\mu R^2 \cot\mu u - \mu R y^4 + v \\ \psi &= \mu u - \frac{1}{R} y^4 \end{aligned}\right)$$

(And rescale other coordinates like $x^i = R y^i$)

Step2 $R \to \infty$ limit

Review of a standard Penrose limit

By taking $R \to \infty$ limit,

$$ds^2 = -2\mathrm{d}u\mathrm{d}v + \frac{\sin^2(\mu u)}{\mu^2}\sum_{I=1}^{8}(\mathrm{d}y^I)^2$$

$$\Downarrow \quad \left(u = x^+ \quad y^I = \frac{\mu}{\sin(\mu x^+)}x^I \quad v = x^- - \frac{\mu}{2}\cot(\mu x^+)\sum_{I=1}^{8}(x^I)^2 \right)$$

The maximally supersymmetric pp-wave sol.

$$ds^2 = -2\mathrm{d}x^+\mathrm{d}x^- - \mu^2\sum_{I=1}^{8}(x^I)^2(\mathrm{d}x^+)^2 + \sum_{I=1}^{8}(\mathrm{d}x^I)^2$$

$$F_5 = 4\mu\,\mathrm{d}x^+ \wedge \left(\mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 + \mathrm{d}x^5 \wedge \mathrm{d}x^6 \wedge \mathrm{d}x^7 \wedge \mathrm{d}x^8\right)$$

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An example of the Penrose limit of the deformed b.g.

Ex. A gravity dual of a non-commutative gauge theory

$$r = \frac{1}{2} P_1 \wedge P_2$$

[Matsumoto, Yoshida, 1404.3657]

Abelian r-matrix ⟹ A solution to the standard type IIB SUGRA

$$\mathrm{d}s^2 = R^2 \left[\frac{-\mathrm{d}t^2 + (\mathrm{d}x^3)^2 + \mathrm{d}z^2}{z^2} + \frac{z^2[(\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2]}{z^4 + \eta^2} + \mathrm{d}s^2_{S^5} \right]$$

$$B_2 = \frac{\eta R^2}{z^4 + \eta^2} \mathrm{d}x^1 \wedge \mathrm{d}x^2$$

& dilaton F_3, F_5 flux

[Hashimoto, Itzhaki / Maldacena, Russo , 1999]

A modified Penrose limit

■ The standard Penrose limit ⇨ The standard pp-wave sol.

[Fixing the deformation parameter η

AdS radius $R \to \infty$]

No deformation effect

■ The modified Penrose limit ⇨ The generalized pp-wave

(next slide)

Rescaling the deformation parameter η by the radius

$$\eta \to \eta R^n$$

An example of the modified Penrose limit

Results of the modified Penrose limit with $\eta \to \eta R^4$

$$ds^2 = -2dx^+dx^- - \mu^2 W \cdot (dx^+)^2 + \frac{\mu^4\left[(dx^1)^2 + (dx^2)^2\right]}{\mu^4 + \eta^2 \sin^4(\mu x^+)}$$

$$+\frac{2\mu\eta^2 \sin^4(\mu x^+)\cot(\mu x^+)dx^+(x^1dx^1 + x^2dx^2)}{\mu^4 + \eta^2\sin^4(\mu x^+)} + \sum_{i=3}^{8}(dx^i)^2$$

$$\left(\text{here} \quad W = \sum_{i=3}^{8}(x^i)^2 + \left(\frac{1}{\sin^2(\mu x^+)} - \frac{\mu^4\cot^2\mu x^+}{\mu^4 + \eta^2\sin^4(\mu x^+)}\right)\left[(x^1)^2 + (x^2)^2\right]\right)$$

$$B_2 = \frac{\eta\,\mu^2\sin^2(\mu x^+)}{\mu^4 + \eta^2\sin^4(\mu x^+)}\left[dx^1 \wedge dx^2 + \mu\cot\mu x^+\,dx^+ \wedge (x^2dx^1 - x^1dx^2)\right]$$

& dilaton F_3, F_5 flux

The dual metric and the beta field

Let me introduce G_{mn} and β^{mn} such that $G^{mn} - \beta^{mn} = \left(\frac{1}{g+B_2}\right)^{mn}$

$$d\tilde{s}^2 = -2dx^+dx^- - \mu^2 \sum_{i=1}^{8} (x^i)^2 (dx^+)^2 + \sum_{i=1}^{8} (dx^i)^2$$

$$\beta = \frac{\eta}{\mu^2} \left(-\sin \mu x^+ \partial_1 + \mu\, x^1 \cos \mu x^+ \partial_-\right) \wedge \left(-\sin \mu x^+ \partial_2 + \mu\, x^2 \cos \mu x^+ \partial_-\right)$$

$$= \eta\, \hat{M}_{-1} \wedge \hat{M}_{-2}$$

Killing vectors commuting each other

▷ **Beta trans. of the pp-wave**

Comments

- One can also check it including the dilaton and RR-flux.
- The deformed b.g. can be understood as TsT trans. pp-wave.

$$\left(\hat{M}_{-1} = \frac{\partial}{\partial y^1} \qquad \hat{M}_{-2} = \frac{\partial}{\partial y^2} \right)$$

A more complicated example（1/2）

Ex . $r = \frac{1}{2}\left[P_0 \wedge D + P^i \wedge (M_{0i} + M_{1i})\right]$ [Fernandez-Melgarejo, Sakamoto, Sakatani, Yoshida, 1710.06849]

Non-unimodular r-matrix ⟹ A solution to the generalized supergravity

$$\mathrm{d}s^2 = R^2\left[\frac{-\mathrm{d}t^2 + \mathrm{d}z^2}{z^2 - \eta^2} + \frac{z^2\left[(\mathrm{d}x^1)^2 + \mathrm{d}\rho^2\right]}{z^4 + \eta^2\,\rho^2} + \frac{\rho^2\,\mathrm{d}\theta^2}{z^2} + \mathrm{d}s^2_{\mathrm{S}^5}\right]$$

$$B_2 = -\eta R^2\left[\frac{\mathrm{d}t \wedge \mathrm{d}z}{z\,(z^2 - \eta^2)} + \frac{\rho\,\mathrm{d}x^1 \wedge \mathrm{d}\rho}{z^4 + \eta^2\,\rho^2}\right]$$

$$I = -\frac{\eta}{R^2}\,(4\,\partial_t + 2\,\partial_1)$$

& dilaton, F_1, F_3, F_5 flux

Rescale the deformation parameter as $\eta \to \eta R^2$ and take $R \to \infty$ limit.

A more complicated example（2/2）

Results of the modified Penrose limit [S. O. , J. Sakamoto, K. Yoshida, 19xx.xxxxx]

$$ds^2 = -2dx^+dx^- - \left[m_1^2\left((x^1)^2+(x^2)^2+(x^3)^2\right) + m_2^2(x^4)^2 + \mu^2\sum_{i=4}^{8}(x^i)^2\right](dx^+)^2 + \sum_{i=1}^{8}(dx^i)^2$$

$$m_1^2(x^+) = \frac{\mu^2}{\tilde{\mu}^2}\mathrm{sn}^2 \qquad m_2^2(x^+) = \mu^2\left(1-\left(\frac{\mu\,\mathrm{sn}}{\tilde{\mu}\,\mathrm{dn}}\right)^2\right)$$

$$B_2 = -\frac{\eta\,\mathrm{sn}\,\mathrm{cn}}{\tilde{\mu}\,\mathrm{dn}}\,dx^+\wedge dx^-$$

$$I = -4\eta\,\partial_-$$

& dilaton, F_5 flux

here $\mathrm{sn} \equiv \mathrm{sn}\left(\tilde{\mu}x^+, \frac{\eta}{\tilde{\mu}}\right)$, $\tilde{\mu} = \sqrt{\mu^2+\eta^2}$

Comments

- The generalized pp-wave with I^k $\Rightarrow$ A solution to GSE
- cannot be understood as the beta trans. pp-wave

Summary

Concerning YB deformed backgrounds,

- The standard Penrose limit ⇨ The standard pp-wave sol.
- The modified Penrose limit ⇨ The generalized pp-wave

Rescaling the deformation parameter η by the radius : $\eta \to \eta R^n$

- A gravity dual of a NCYM $\Rightarrow$ TsT trans. of pp-wave
- A non-unimodular e.g. $\Rightarrow$ The generalized pp-wave with I^k

Discussion -Future direction-

TsT transformed background | **Undeformed pp-wave**

EOM w/ periodic b.c. ⟺ EOM w/ twisted periodic b.c.

- Light-cone quantization of superstrings on the generalized pp-wave

  HIMR case in [S. O. , J. Sakamoto, K. Yoshida, 19xx.xxxxx]

  light-cone hamiltonian, vertex operator, 2-pt correlation fnc …

- Is it possible to analyze more general cases?

  (including the case of the generalized SUGRA background)

Back up

Introduction

AdS/CFT

Type II superstring on $AdS_5 \times S^5$ ⟷ 4d N=4 SU(N) SYM (Large N)

The integrability has played an important role to analyze this conjecture.

A next issue : Integrable deformation of $AdS_5 \times S^5$ superstring

Yang-Baxter deformations ⇒ Deformed $AdS_5 \times S^5$ background

(as a 2d non-linear sigma model)

[Sakamoto-san's talk]