Fibonacci Bloch Functions

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It has been known for a long time that, in general, the Bloch theorem does not apply to quasicrystals [1, 2, and references therein]. Accordingly, single-electron eigenfunctions on typical models of quasicrystals exhibit algebraic decay, as opposed to the extended Bloch eigenfunctions that are observed in similar models of periodic crystals. Nevertheless, it is interesting to explore whether extended quasiperiodic Bloch functions may somehow emerge in realistic physical situations—either as superpositions of eigenfunctions, or even as true eigenfunctions of slightly modified quasiperiodic structures. If so, what is the nature of these quasiperiodic Bloch functions? How are they related to the structure of the underlying quasicrystalline potential? Are they characterized by the same kind of quantum numbers in reciprocal space as their periodic analogs? If so, is there an energy-momentum dispersion curve, or band structure, that is associated with these Bloch functions?

We study these questions using the familiar tight-binding model on the 1-dimensional Fibonacci quasicrystal. Indeed, we find that superpositions of relatively small numbers of nearly degenerate eigenfunctions give rise to extended quasiperiodic Bloch functions. These functions possess the structure of earlier ancestors of the underlying Fibonacci potential, and it is often possible to obtain different ancestors as different superpositions at the same energy. The quantum number that characterizes all of these ancestors is uniquely determined by the average energy of the superimposed eigenfunctions, giving rise to a very clear dispersion curve. We also find that Fibonacci-like Bloch functions emerge as eigenfunctions when a bit of static disorder is introduced into the otherwise perfect Fibonacci potential. These theoretical results may explain certain experimental observations [3,4].

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