Introduction to higher-dimensional description of quasicrystal structures

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Quasicrystals (QCs) are quasiperiodic crystals that show a quasiperiodic long-range order. Of these, icosahedral QCs display icosahedral symmetry in their diffraction patterns and have no periodicity in three-dimensional (3D) space. In this tutorial lecture, staring from the 1D analogue of QC, namely Fibonacci sequence, how icosahedral QCs can be described as a periodic crystal in 6D space will be explained. In 6D space, the structure of an icosahedral QC is described by an ordered arrangement of occupation domains (or atomic surfaces), which are extended along the 3D internal (or complementary, perpendicular) space that is perpendicular to the 3D external (or physical, parallel) space, in a unit cell. An occupation domain is a 3D object whose shape is compatible with its site symmetry. The most successful atomic structure description of icosahedral QCs to date is given by a model based on atomic clusters with icosahedral symmetry placed on a subset of the twelve-fold vertices of Ammann-Kramer-Neri tiling with appropriate edge length [1]. In this model, the structure of an icosahedral QC is described by means of an atomic decoration of the three building blocks, namely, rhombic triacontahedron (RTH), acute rhombohedron (AR) and obtuse rhombohedron (OR) [2]. The RTH describes an atomic cluster, and the multiple combinations of AR and/or OR fill gaps existing in between clusters. The occupation domains of the corresponding 6D crystal are derived by the atomic decoration of the building blocks.

[1] P. Kramer & R. Neri, *Acta Cryst.*, A **40** (1984) 580.
[2] H. Takakura, *Phil. Mag.*, **88** (2008) 1905.