Aperiodic Crystals: How is that even possible?

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For over a century it was understood that crystals were a form of matter in which the atomic constituents were ordered periodically. Already over 50 years ago it started to become evident that long-range order survives, even when the periodicity is removed via incommensurate modulations, or the intergrowth of two or more crystals with incommensurate periodicities. Yet, it was only about 35 years ago, with the announcement of the discovery of quasicrystals, that the periodicity paradigm had to be fully abandoned and the notion of crystallinity had to be redefined. In this tutorial I shall explain some of the basic physical notions of aperiodic crystals.

Before the age of quasicrystals, it was believed that crystals break the continuous translation and rotation symmetries of the liquid-phase into a discrete lattice of translations, and a finite group of rotations. Quasicrystals, on the other hand, possess no such symmetries— there are no translations, nor, in general, are there any rotations, leaving them invariant. Does this imply that no symmetry is left, or that the meaning of symmetry should be revised? Are quasicrystals even truly ordered, or is their degree of order somewhere in between periodic crystals and amorphous solids? Can we still talk about point groups and space groups? Do we observe systematic extinctions in their diffraction diagrams? Do we still have phonon-like Goldstone modes that may interact with electrons? What is their nature? Assuming that quasicrystals have defects, can we characterize their topological nature?

As time permits, I shall address these and other questions related to the liquid-to-crystal symmetry-breaking transition in the modern context of aperiodic long-range order, using the fundamental notion of indistinguishability [4,5].

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