Higher-dimensional generalizations of the Berry curvature

Lev Spodyneiko*

California Institute of Technology, Pasadena, CA 91125, United States

Abstract

While the definitions of the Berry connection and the Berry curvature do not make an explicit reference to the dimension of space, they are not directly applicable to models of quantum statistical mechanics in spatial dimension $d \geq 1$, or to the models of Quantum Field Theory. We define and study higher-dimensional generalizations of the Berry curvature for gapped lattice systems on $\mathbb{R}^d$. For any family of such systems we define a closed $(d+2)$-form F_d on the parameter space M. The cohomology class of F_d is independent of any choices and is a topological invariant of the family. We show that when all systems in the family admit gapped boundaries locally on M, the integral of F_d over any $(d+2)$-dimensional sphere is an integer. If the gapped boundary condition can be chosen so that it varies continuously with parameters and is defined everywhere on M, the form F_d is exact. Thus the cohomology class of F_d is an obstruction for the existence of such a boundary condition. This is analogous to the fact that the cohomology class of the ordinary Berry curvature is an obstruction for the existence of a normalized ground-state which varies continuously with parameters.

*Electronic address: lionspo@caltech.edu