Testing inflation with small-scale anisotropies

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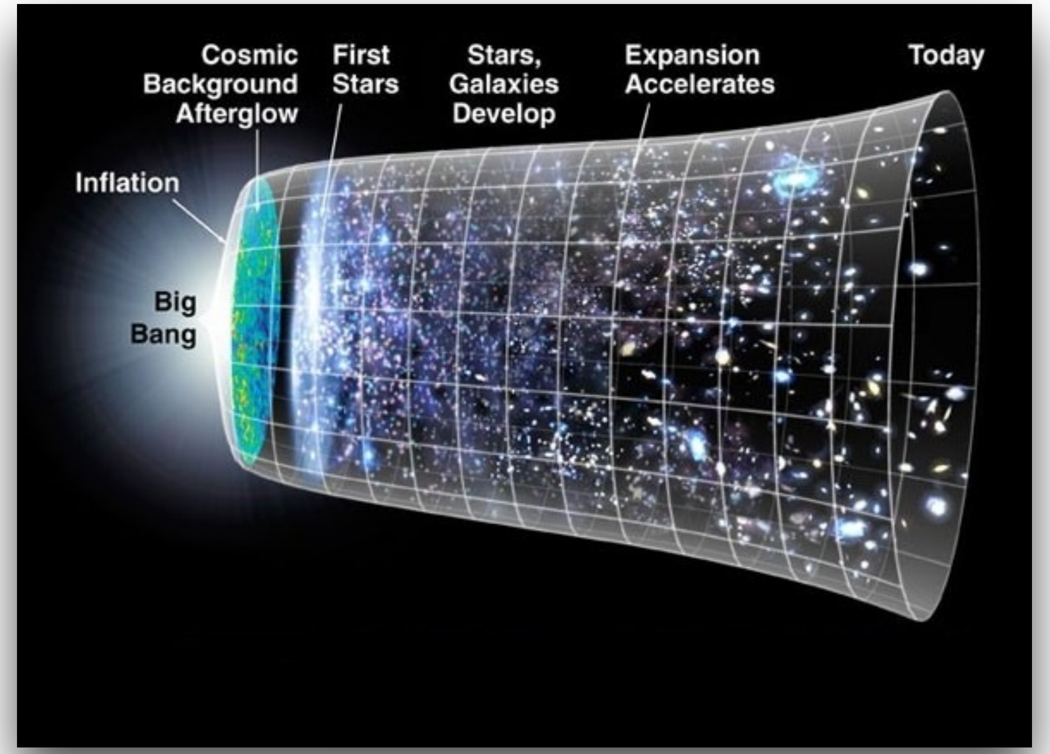
Dawn of Gravitational-wave Cosmology and Theory of Gravity

Tohoku Forum for Creativity, March 2nd 2022



Inflation predicts a stochastic gravitational wave background

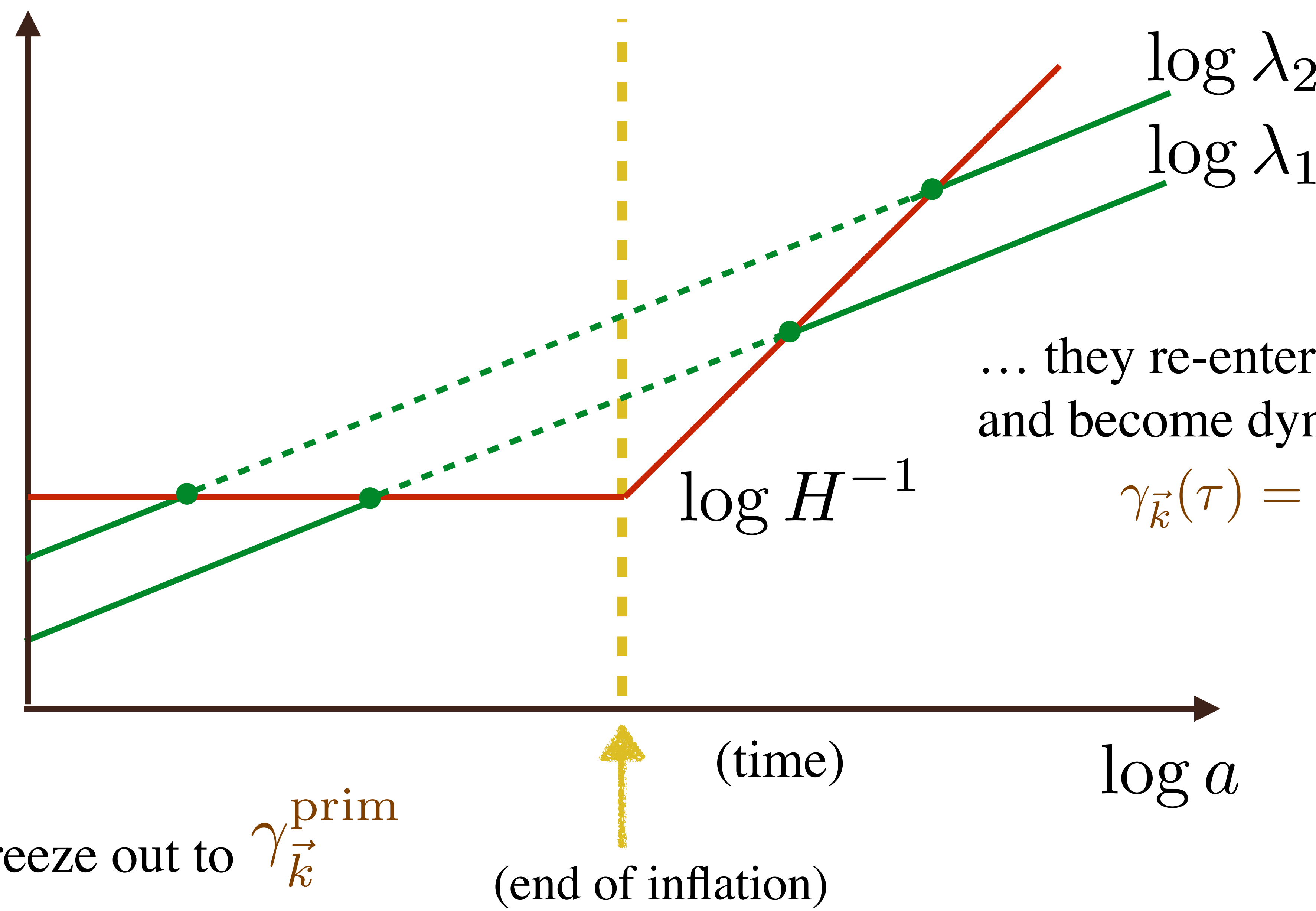
- How does it look like?
- What info does it provide on inflation?
- How do we characterise it?

SGWB anisotropies from primordial non-Gaussianity

Scales

wavenumber → e-folding → time of re-entry

$k \sim f$ $\quad$ N_k

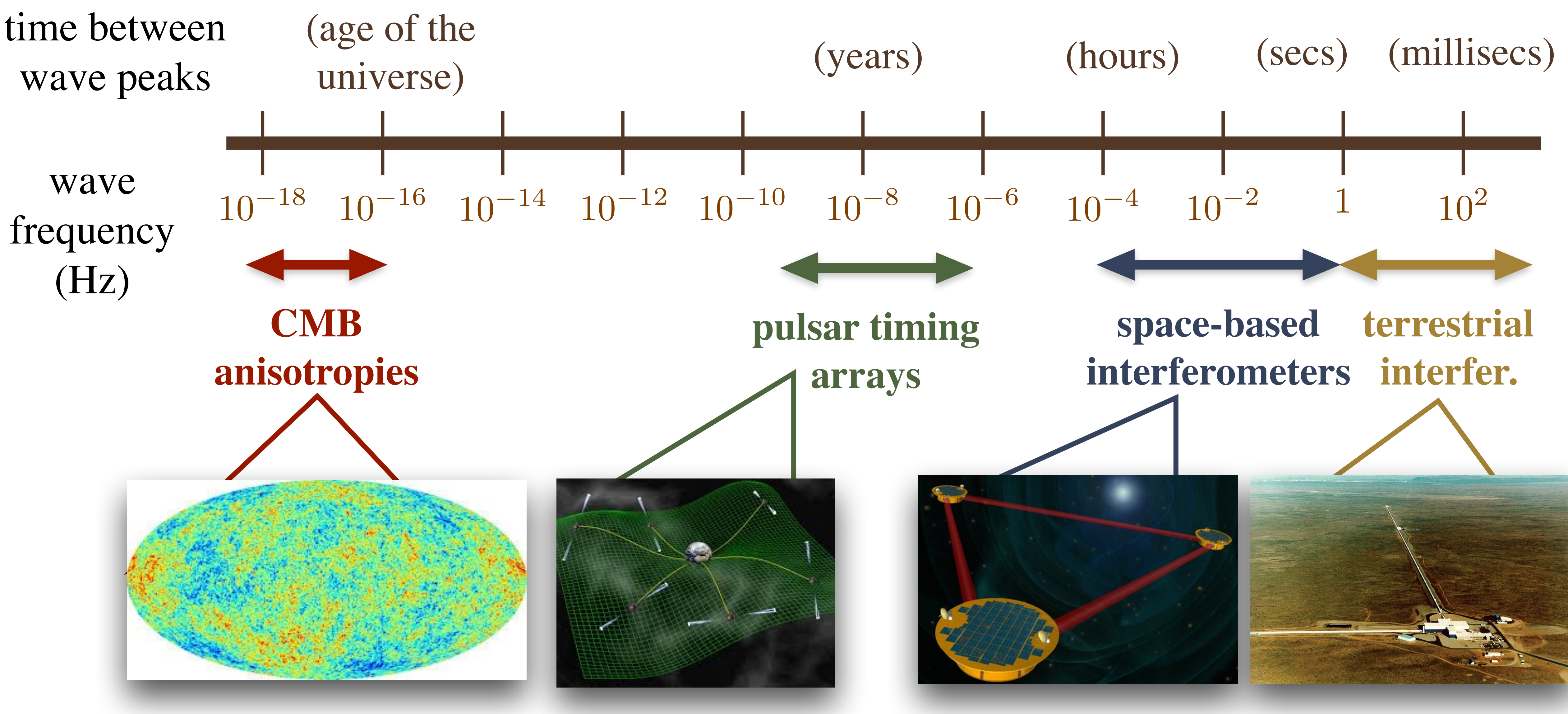


Perturbation modes leave horizon during inflation and freeze out to $\gamma_{\vec{k}}^{\mathrm{prim}}$

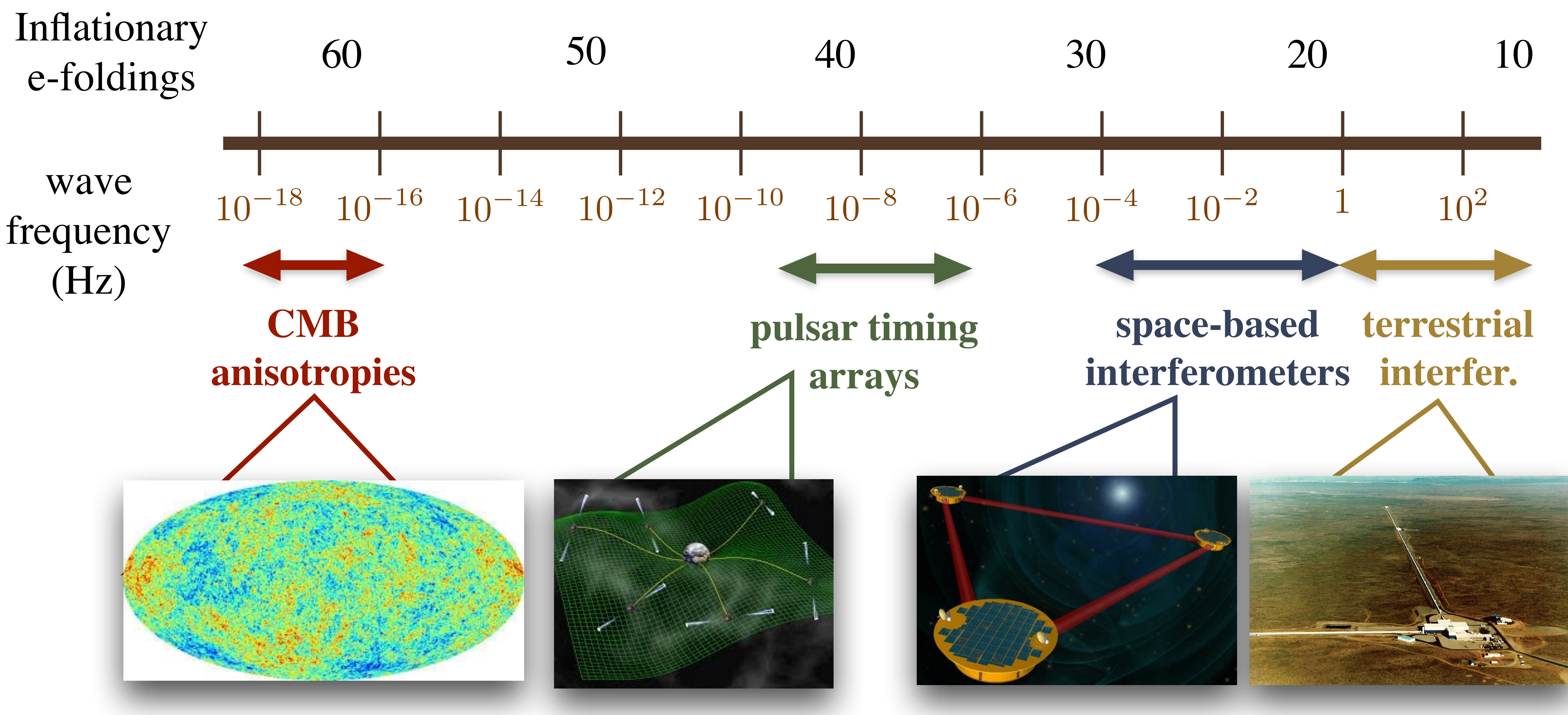
… they re-enter after inflation and become dynamical again

$\gamma_{\vec{k}}(\tau) = T(\tau, k)\gamma_{\vec{k}}^{\mathrm{prim}}$

Scales — Experiments



Scales — Experiments



GW can tell us a whole lot about inflation:

- GW from the amplification of vacuum fluctuations

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 0$$

Production of gravitons out of the vacuum
in an expanding universe!

GW can tell us a whole lot about inflation:

- GW from the amplification of vacuum fluctuations
- Generation of GW from additional fields during inflation

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

anisotropic stress-energy tensor

- Axion-gauge field models $\frac{\lambda\chi}{4f}F\tilde{F}$
  [Anber - Sorbo 2009, Cook - Sorbo 2011, Barnaby - Peloso 2011, Adshead - Wyman 2011, Maleknejad - Sheikh-Jabbari, 2011, ED - Fasiello - Tolley 2012, ED - Peloso 2012, Namba - ED - Peloso 2013, Adshead - Martinec -Wyman 2013, ED - Fasiello - Fujita 2016 Agrawal - Fujita - Komatsu 2017, Caldwell - Devulder 2017, Domcke et al. 2018, …]

- GW from extra non-minimally coupled spin-2 field (EFT formulation)
  [Bordin et al, 2018; …]

- Spectator fields with small sound speed

$$\ddot{\gamma}_{ij} + 3H\dot{\gamma}_{ij} + k^2\gamma_{ij} = 16\pi G\,\Pi_{ij}^{TT} \propto \partial_i\sigma\partial_j\sigma$$

  [Biagetti, Fasiello, Riotto 2012, Biagetti, ED, Fasiello, Peloso 2014, …]

- . . .

GW can tell us a whole lot about inflation:

- GW from the amplification of vacuum fluctuations
- Generation of GW from additional fields during inflation
- GW production in models with alternative spacetime symmetry breaking patterns
  - Solid inflation (broken space-diffeomorphisms)

    [Endlich et al, 2014; …]
  - Supersolid inflation (broken time + space diffs)

    [Bartolo et al, 2016; Celoria et al, 2017; Tasinato et al, 2017; …]
  - . . .

GW can tell us a whole lot about inflation:

- GW from the amplification of vacuum fluctuations
- Generation of GW from additional fields during inflation
- GW production in models with alternative spacetime symmetry breaking patterns
- Second order GW from peaks in the scalar power spectrum
  - Potentials with inflection points
    [Garcia-Bellido, Morales, 2017; …]
  - Turning trajectories in the inflationary landscape
    [Fumagalli et al, 2020; …]
  - Inflation with axion and gauge fields
    [Garcia-Bellido, Peloso, Unal , 2017; …]
  - . . .

Inflationary GW from vacuum fluctuations (SFSR)

- **Energy scale** of inflation: $V_{\rm inf}^{1/4} \simeq 10^{16}\mathrm{GeV}(r/0.01)^{1/4}$

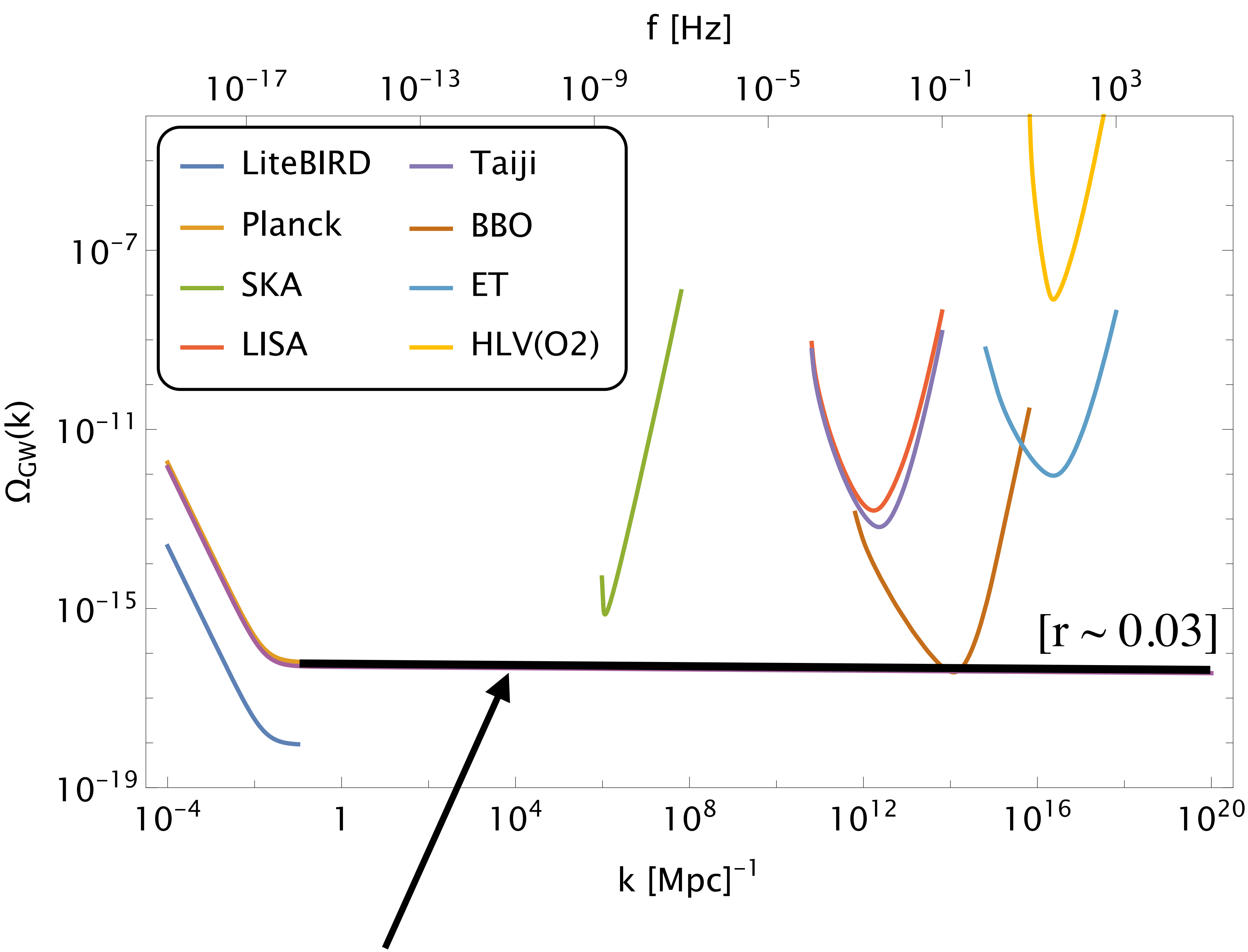
  $H \simeq 2 \times 10^{13}\mathrm{GeV}(r/0.01)^{1/2}$

- Red **tilt**: $n_T \simeq -2\epsilon = -r/8$

- Non-**chiral**: $P_L = P_R$

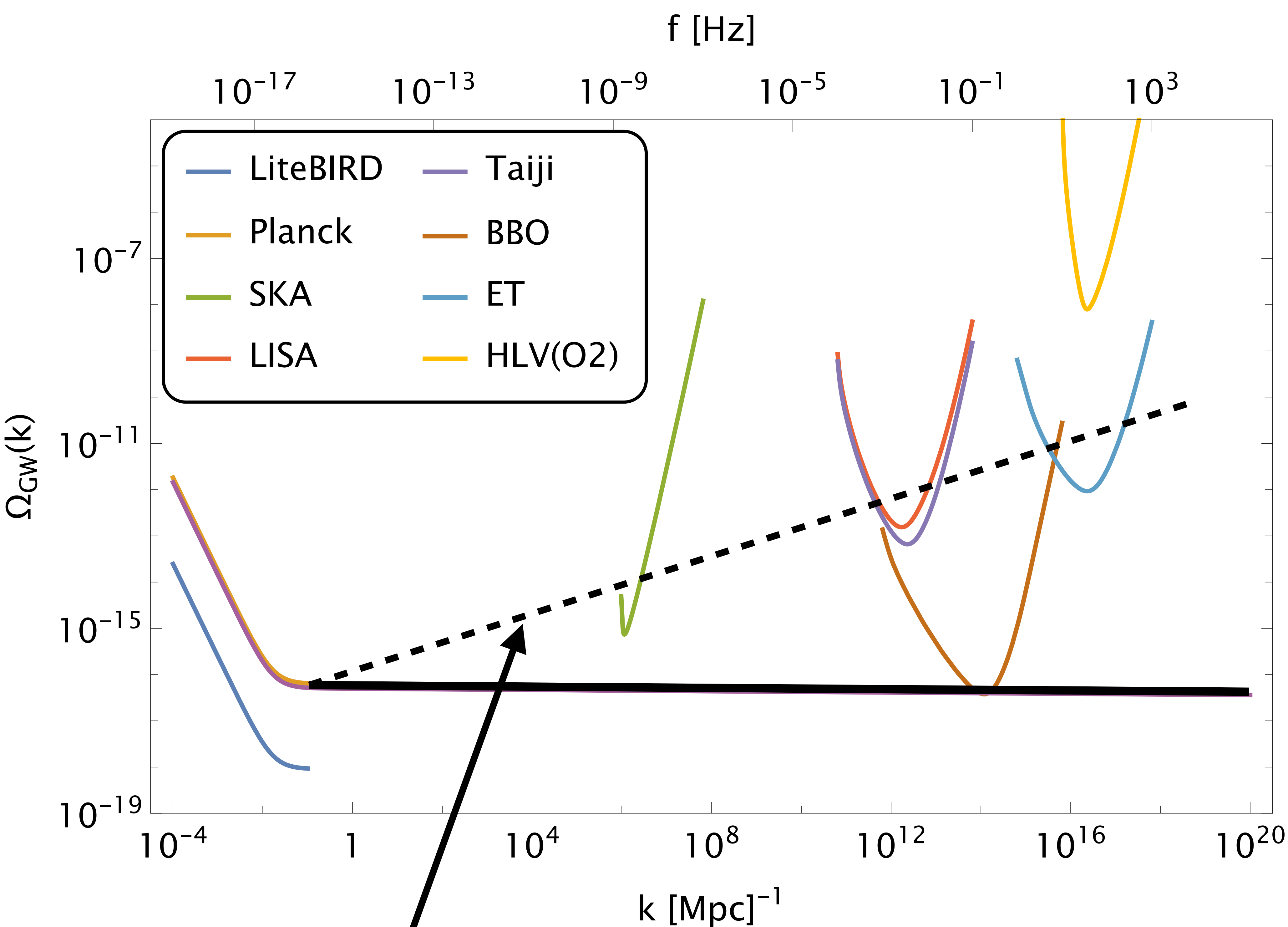
- Nearly **Gaussian**: $f_{\rm NL} \ll 1$

Prediction and sensitivity limits



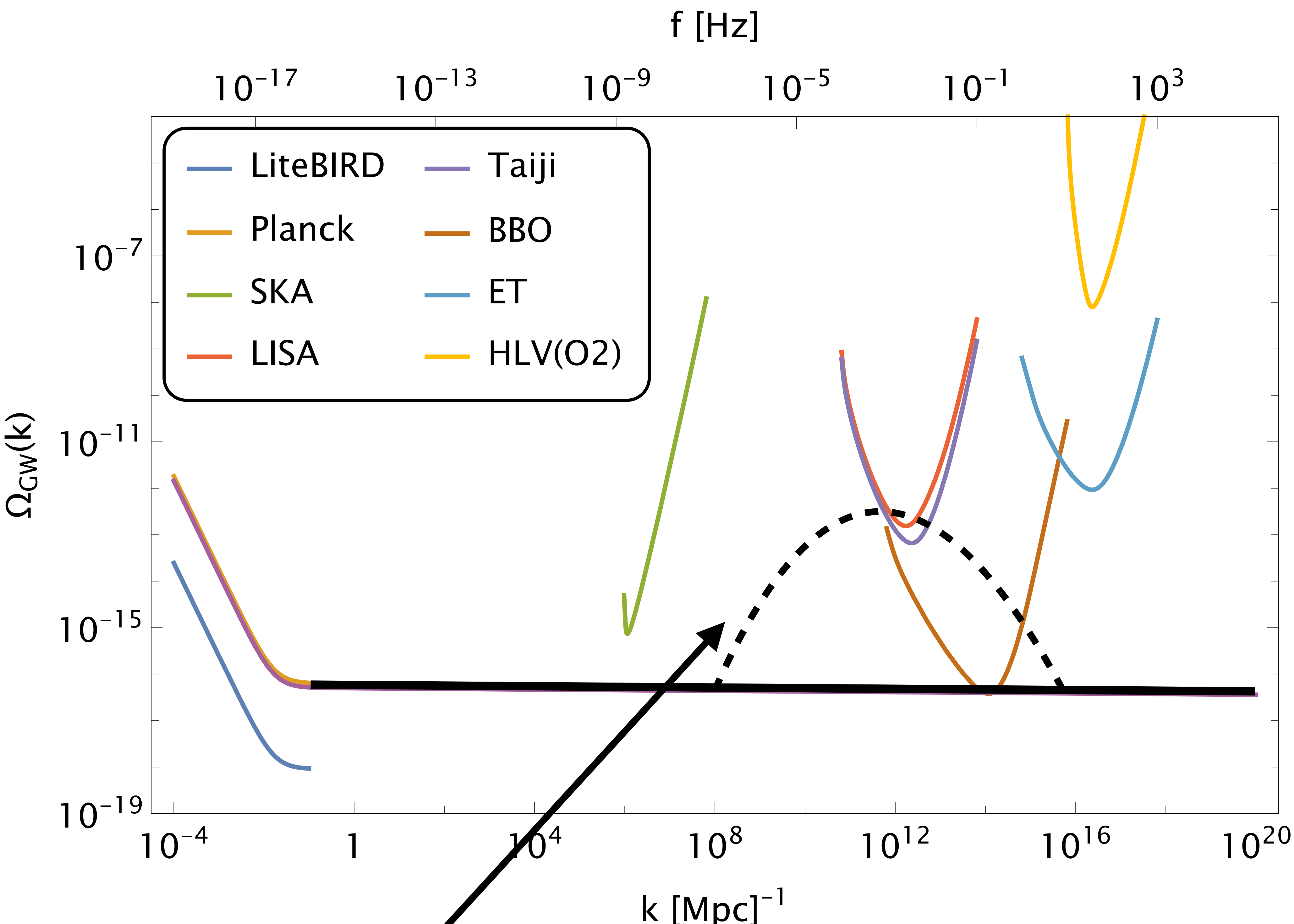
Standard SFSR would go undetected at small scales (**red tilt**)

Prediction and sensitivity limits



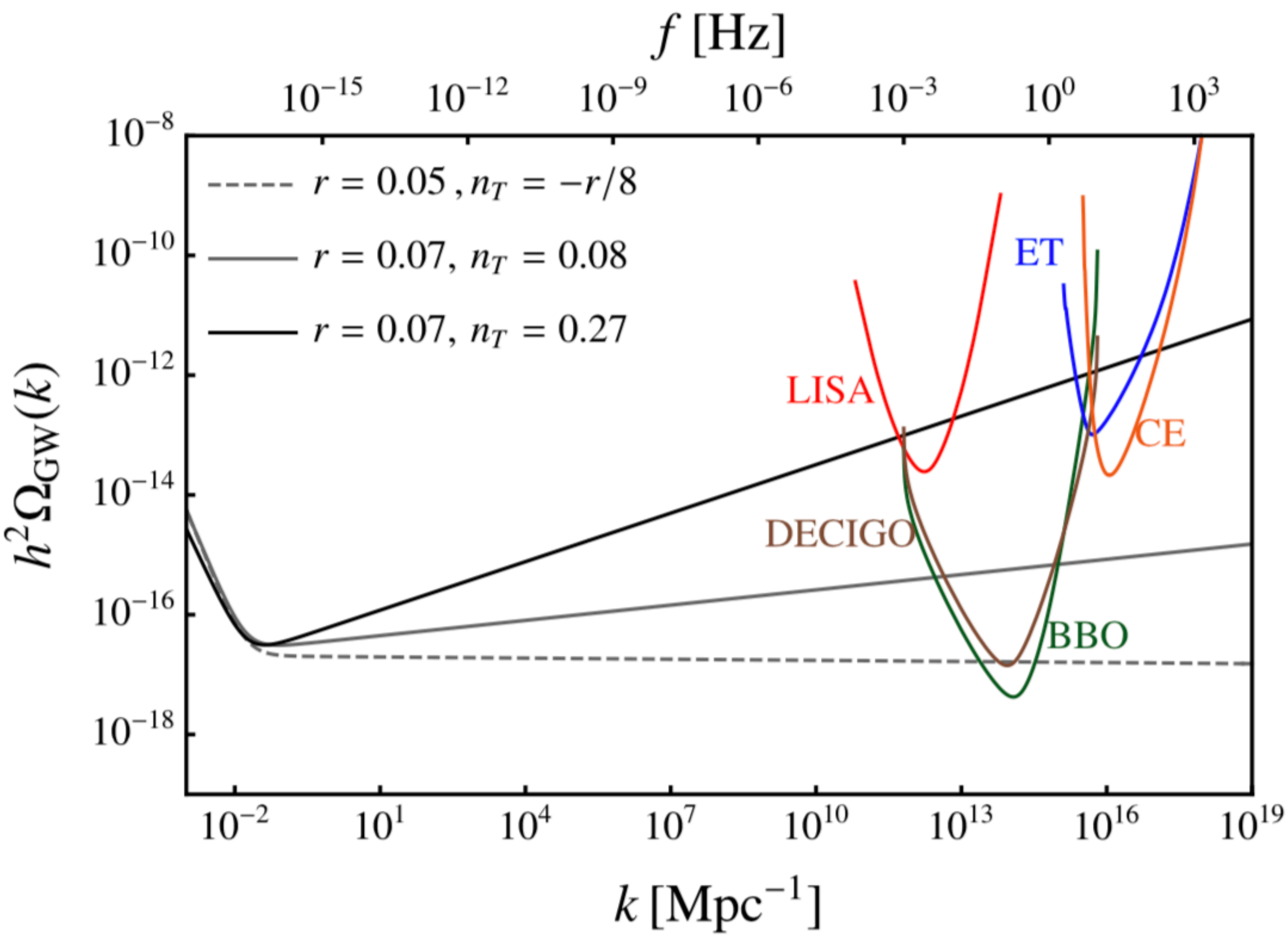
Power spectrum larger at small scales: e.g. **blue tilt**

Prediction and sensitivity limits



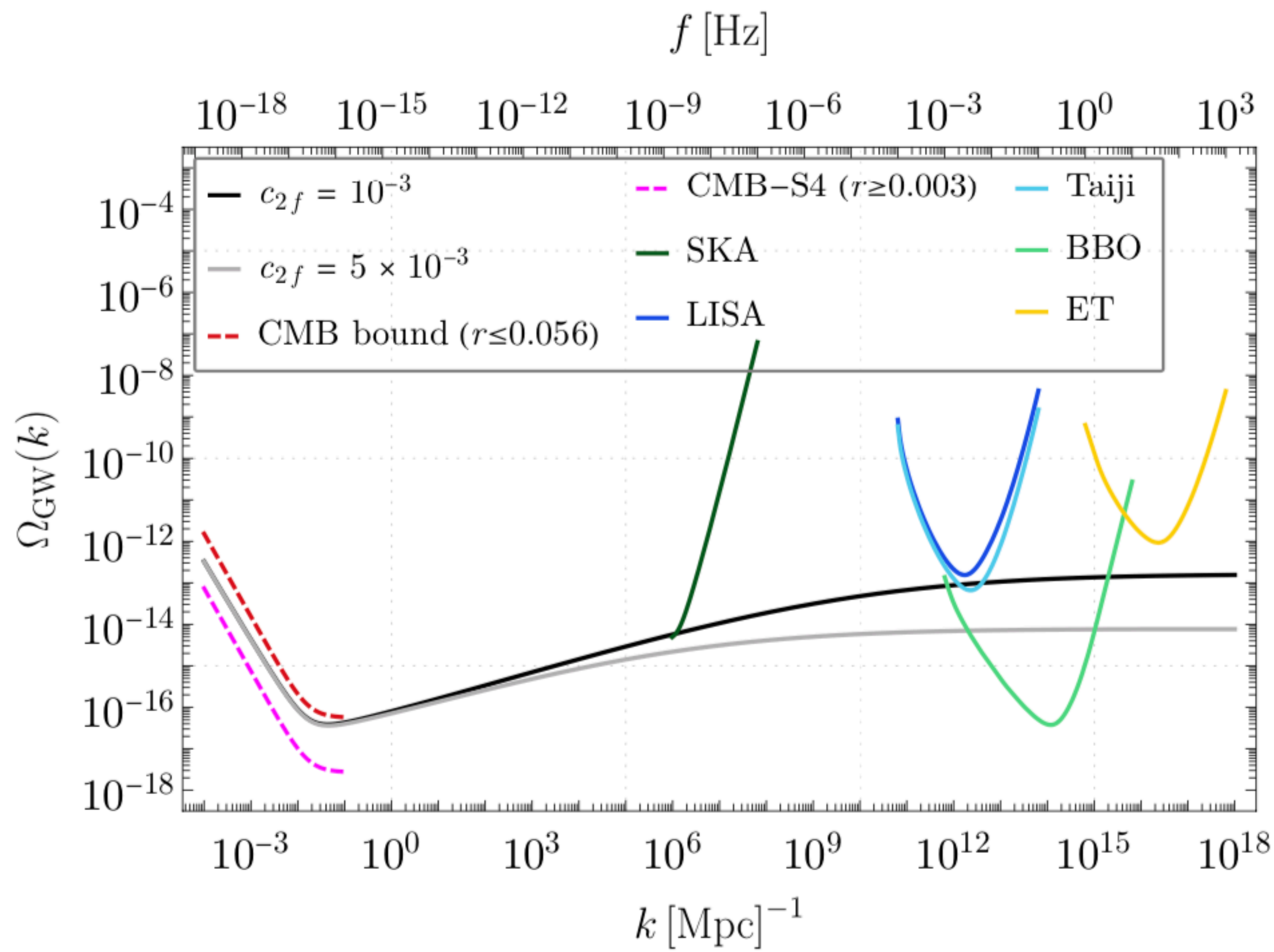
Power spectrum larger at small scales: e.g. **bump**

- Example: solid inflation vs SFSR



[Figure from Malhotra, ED, Fasiello, Shiraishi 2020]

- Example: EFT with non-minimally coupled spin-2 field



[Figure from ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Inflationary GW from vacuum fluctuations (SFSR)

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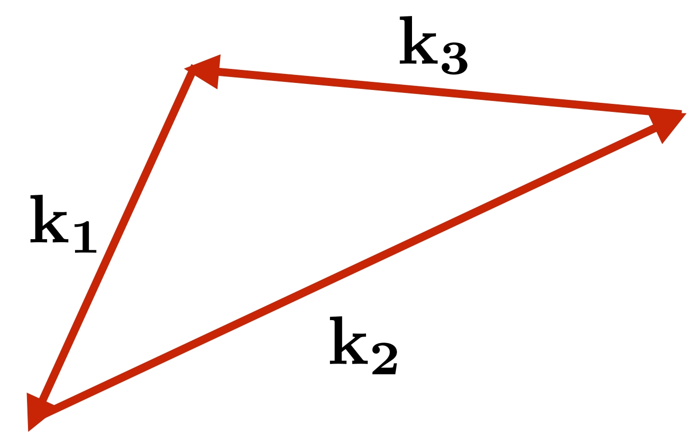
SGWB: inflation or what else?

Other possible cosmological sources:

* Reheating
* First order phase transitions
* Cosmic strings
* Alternatives to inflation
* ???

Astrophysical sources:
SGWB expected due to the superposition of signals from a large number of astrophysical sources (e.g. mergers of black holes, neutron stars,…)

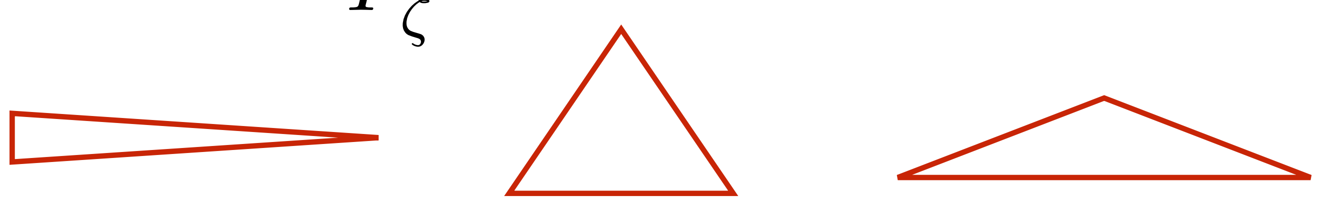
Non-Gaussianity



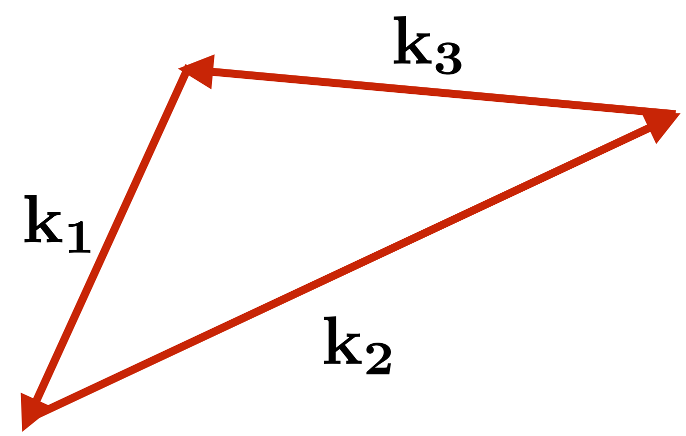
$$\langle \gamma^{\lambda_1}_{\mathbf{k_1}} \gamma^{\lambda_2}_{\mathbf{k_2}} \gamma^{\lambda_3}_{\mathbf{k_3}} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) B^{\lambda_1 \lambda_2 \lambda_3}_{\gamma}(k_1, k_2, k_3)$$

tensor bispectrum

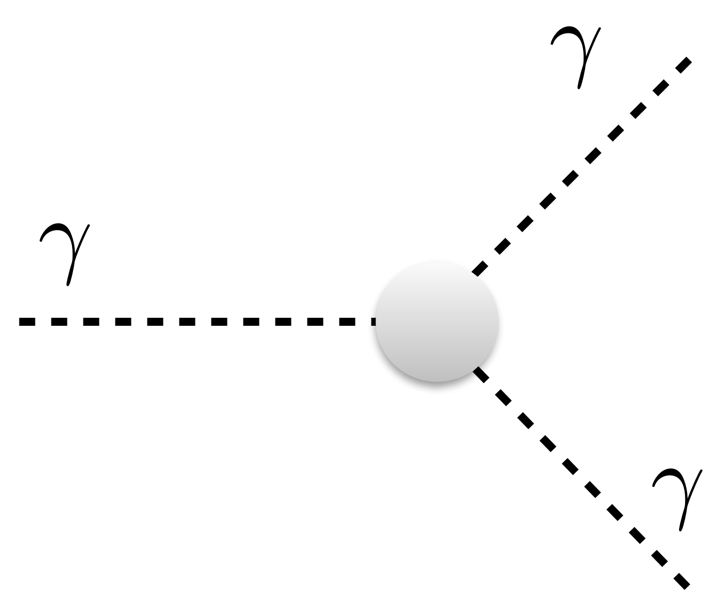
amplitude: $f_{NL} = \dfrac{B}{P_\zeta^2}$

shape: 

Tensor non-Gaussianity



from interactions of the tensors with other fields or from self-interactions



key probe of the fields and interactions during inflation,
as well as of the theory of gravity

Non-Gaussianity (tensor / mixed): CMB constraints

- We do have constraints from CMB anisotropies and future B mode observations are expected to bring important improvements

Example: LiteBIRD-like experiment could detect an O(1) signal for

$$f_{\rm NL}^{tss,{\rm sq}} \quad f_{\rm NL}^{ttt,{\rm sq}} \quad f_{\rm NL}^{ttt,{\rm eq}}$$

[Shiraishi, 2019]

- The formalism for constraining non-Gaussianity with CMB anisotropies is by now well developed

Can we constrain tensor (and mixed) non-Gaussianity with interferometers?

Non-Gaussianity at interferometers

Shapiro time delay:

$$\gamma'' + 2\mathcal{H}\gamma' - [1 + (12/5)\zeta]\,\gamma_{,kk} = 0$$

GW propagating in FRW background
+ long-wavelength perturbations

$$\gamma_{ij} = A_{ij}\, e^{ik\tau + ik\cdot 2\int^{\tau} d\tau'\, \zeta[\tau', (\tau' - \tau_0)\hat{k}]}$$

GW from different directions
undergo different phase shift
due to intervening structure

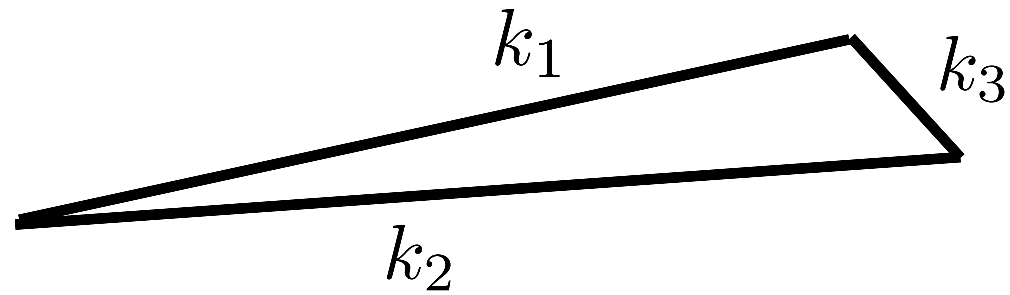
$\longrightarrow$ decorrelation $\longrightarrow$ cannot measure bispectrum directly with interferometers

[Bartolo, De Luca, Franciolini, Lewis, Peloso, Riotto 2018]

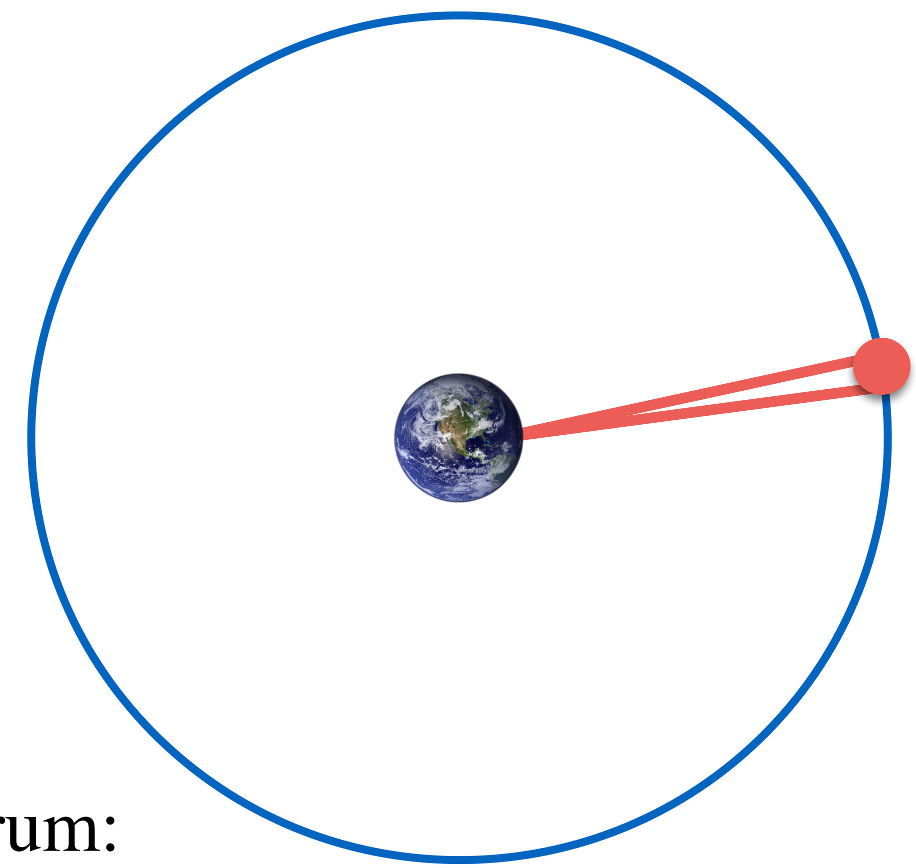
Note: signal measured by an interferometer arises from the superposition of signals from a large number of Hubble patches (CLT)

[Adshead, Lim 2009 — Caprini, Figueroa 2018 — Bartolo, De Luca, Franciolini, Lewis, Peloso, Riotto 2018]

Ultra squeezed non-Gaussianity



Correlation among two short-wavelength modes (e.g. interferometer scale) and 1 very long-wavelength mode: the latter has not undergone propagation!



Signals originate from the same patch!

How do we constrain this ultra-squeezed bispectrum:

Look for anisotropies in the SGWB!

$$\Omega_{\rm GW}(k) = \bar{\Omega}_{\rm GW}(k) \left[1 + \frac{1}{4\pi} \int d^2\hat{n}\, \delta_{\rm GW}(k, \hat{n}) \right]$$

[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

SGWB anisotropies from primordial non-Gaussianity

$$\Omega_{\rm GW}(k) = \bar{\Omega}_{\rm GW}(k)\left[1 + \frac{1}{4\pi}\int d^2\hat{n}\,\delta_{\rm GW}(k,\hat{n})\right]$$

isotropic component

anisotropic component

$$\Omega_{\rm GW}(k) \equiv \frac{1}{\rho_{\rm cr}}\frac{d\rho_{\rm GW}}{d\ln k}$$

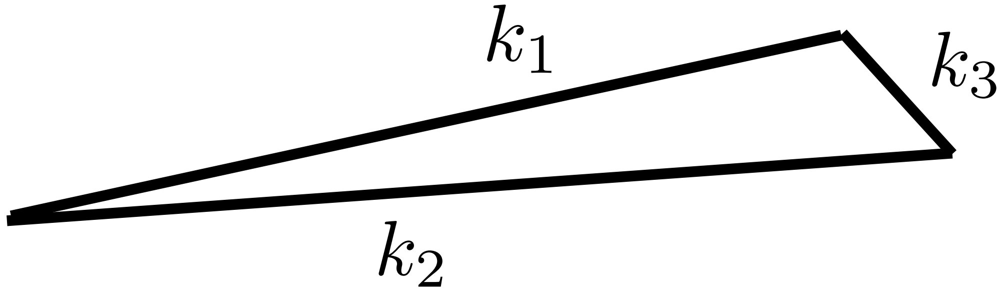
energy density spectrum
for the stochastic GW background

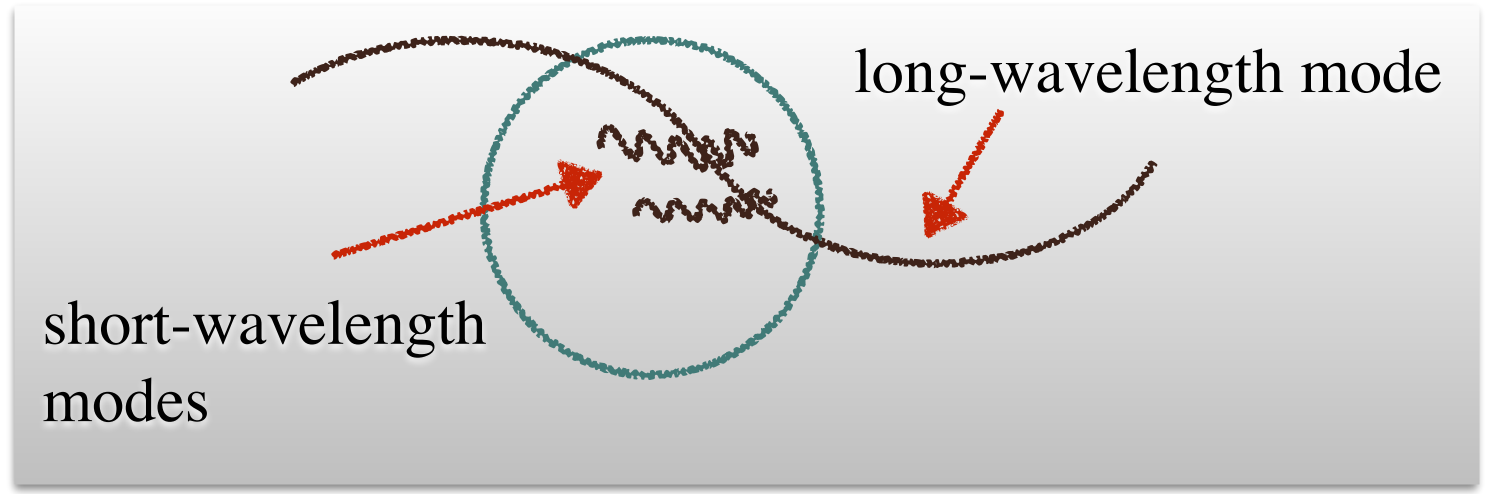
k= comoving wavenumber (proportional to the observed frequency)

$\hat{n}$= direction of incoming graviton

How do SGWB anisotropies relate to non-Gaussianity?

$k_1 \simeq k_2 \gg k_3$





long wavelength modes introduces a modulation
in the primordial power spectrum of the short wavelength modes

$$B^{F\gamma\gamma} \equiv \langle F_L \gamma_S \gamma_S \rangle' \sim F_L \cdot \langle \gamma_S \gamma_S \rangle'_{F_L}$$

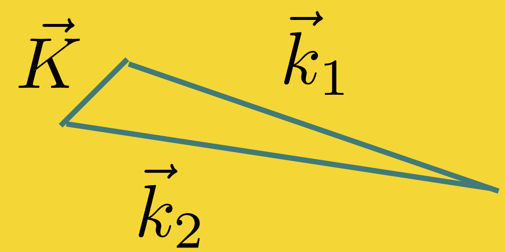
$f_{\rm NL}^{F\gamma\gamma}$

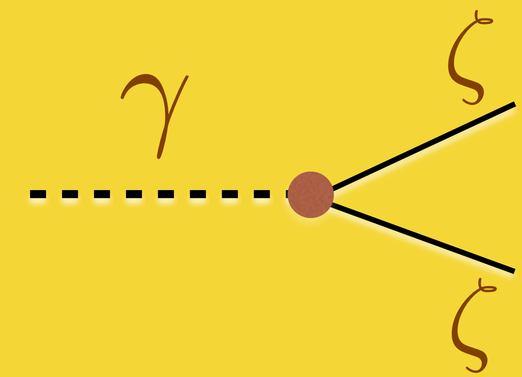
$$\delta\langle \gamma_S \gamma_S \rangle \equiv \langle \gamma_S \gamma_S \rangle_{F_L} \sim \frac{B^{F\gamma\gamma}}{P_F(k_3)} \cdot F_L^* = P_\gamma(k_1) \cdot \frac{B^{F\gamma\gamma}}{P_F(k_3) P_\gamma(k_1)} \cdot F_L^*$$

$$\boxed{\langle \gamma_S \gamma_S \rangle'_{\rm total} = P_\gamma(k_1) \left(1 + f_{\rm NL}^{F\gamma\gamma} \cdot F_L^*\right)}$$

[ED, Fasiello, Jeong, Kamionkowski - 2014, ED, Fasiello, Kamionkowski - 2015, …]

Soft limits and fossils

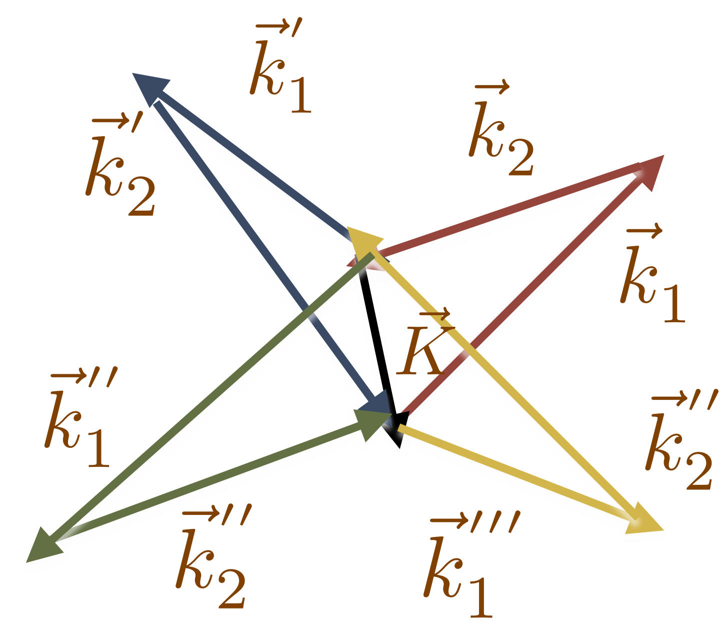
Given 

from 

<u>super-Hubble K</u>:
constrain tensor modes amplitude/interactions
with induced quadrupole anisotropy

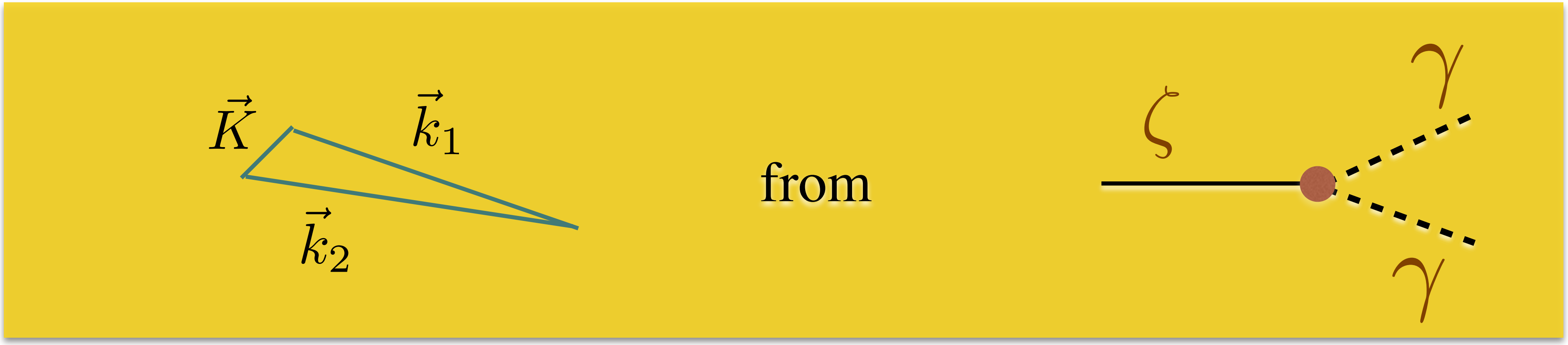
$$P_\zeta(\mathbf{k}, \mathbf{x}_c)|_{\gamma_L} = P_\zeta(k)\Big(1 + \mathcal{Q}_{\ell m}(\mathbf{x}_c, \mathbf{k})\hat{k}_\ell \hat{k}_m\Big)$$

<u>sub-Hubble K</u>:
estimate tensor modes amplitude
from off-diagonal correlations



[ED, Fasiello, Jeong, Kamionkowski - 2014, ED, Fasiello, Kamionkowski - 2015]

Soft limits and fossils



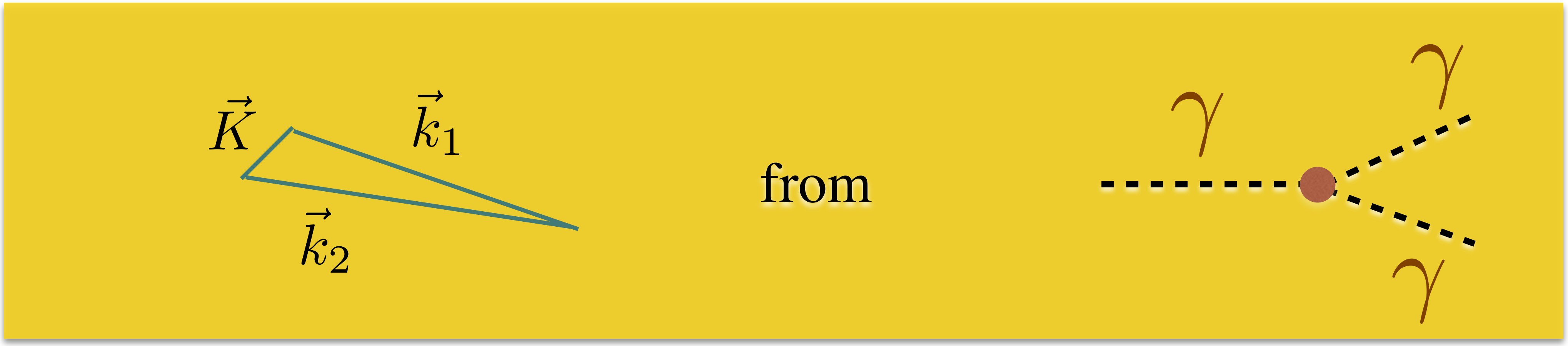
$$\delta_{\mathrm{GW}}(k,\hat{n}) = \int \frac{d^3q}{(2\pi)^3} e^{-id\hat{n}\cdot\mathbf{q}} \zeta(\mathbf{q}) F_{\mathrm{NL}}^{\mathrm{stt}}(\mathbf{k},\mathbf{q})$$

$$\mathbf{d} = -(\eta_0 - \eta_{\mathrm{in}})\hat{n}$$

$$\Omega_{\mathrm{GW}}(k) = \bar{\Omega}_{\mathrm{GW}}(k)\left[1 + \frac{1}{4\pi}\int d^2\hat{n}\,\delta_{\mathrm{GW}}(k,\hat{n})\right]$$

[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

Soft limits and fossils



$$P_{\gamma}^{\rm mod}(\mathbf{k},\mathbf{x}) = P_{\gamma}(k)\left[1 + \mathcal{Q}_{\ell m}(\mathbf{k},\mathbf{x})\,\hat{n}_{\ell}\hat{n}_{m}\right]$$

$$\int \frac{d^3 q}{(2\pi)^3}\, e^{i\mathbf{q}\cdot\mathbf{x}} \sum_{\lambda_3} h_{\ell m}^{\lambda_3}(\mathbf{q})\, F_{\rm NL}^{\rm ttt}(\mathbf{k},\mathbf{q})$$

$$\boxed{\delta_{\rm GW}(k,\hat{n}) = \mathcal{Q}_{\ell m}(\mathbf{k},\mathbf{d})\,\hat{n}_{\ell}\hat{n}_{m}}$$

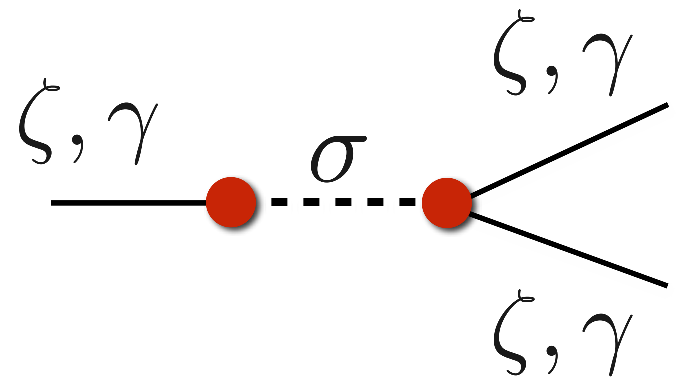
$$\mathbf{d} = -(\eta_0 - \eta_{\rm in})\hat{n}$$

$$\Omega_{\rm GW}(k) = \bar{\Omega}_{\rm GW}(k)\left[1 + \frac{1}{4\pi}\int d^2\hat{n}\,\delta_{\rm GW}(k,\hat{n})\right]$$

[ED, Fasiello, Tasinato, PRL 124(2020)6 061302]

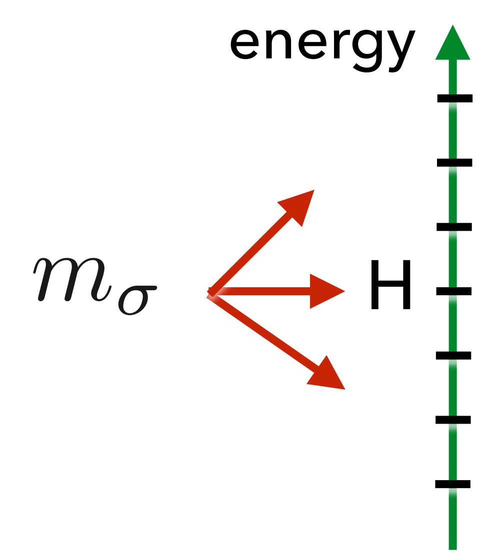
Soft limits in inflation

- ***Extra*** fields



Soft limits reveal (extra) fields mediating inflaton or graviton interactions

squeezed bispectrum delivers info on mass spectrum!!!



Soft limits in inflation

- ***Extra*** *fields* / superhorizon ***evolution***

  [Chen - Wang 2009, Baumann - Green 2011, Chen et al 2013,
  ED - Fasiello - Kamionkowski 2015, …]

- ***Non-Bunch Davies*** initial states

  [Holman - Tolley 2007, Ganc - Komatsu 2012, Brahma - Nelson - Shandera 2013, …]

- ***Broken space diffs***
  (e.g. space-dependent background)

  [Endlich et al. 2013, ED - Fasiello - Jeong - Kamionkowski 2014, …]

Ideal probe for (extra) fields, pre-inflationary dynamics, (non-standard) symmetry patterns

SGWB anisotropies from primordial non-Gaussianity

- Typical amplitude of these anisotropies:

$$\delta_{\rm GW}^{\rm tss} \sim F_{\rm NL}^{\rm tss} \sqrt{A_{\rm S}}$$

$$\delta_{\rm GW}^{\rm ttt} \sim F_{\rm NL}^{\rm ttt} \sqrt{r\, A_{\rm S}}$$

amplitude of the scalar power spectrum at CMB scales

tensor-to-scalar ratio

Ameek Malhotra
(UNSW Sydney)

[Malhotra, ED, Fasiello, Shiraishi 2020 -
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

SGWB anisotropies from primordial non-Gaussianity

- Typical amplitude of these anisotropies:

$$\delta_{\rm GW}^{\rm tss} \sim F_{\rm NL}^{\rm tss}\sqrt{A_{\rm S}}$$

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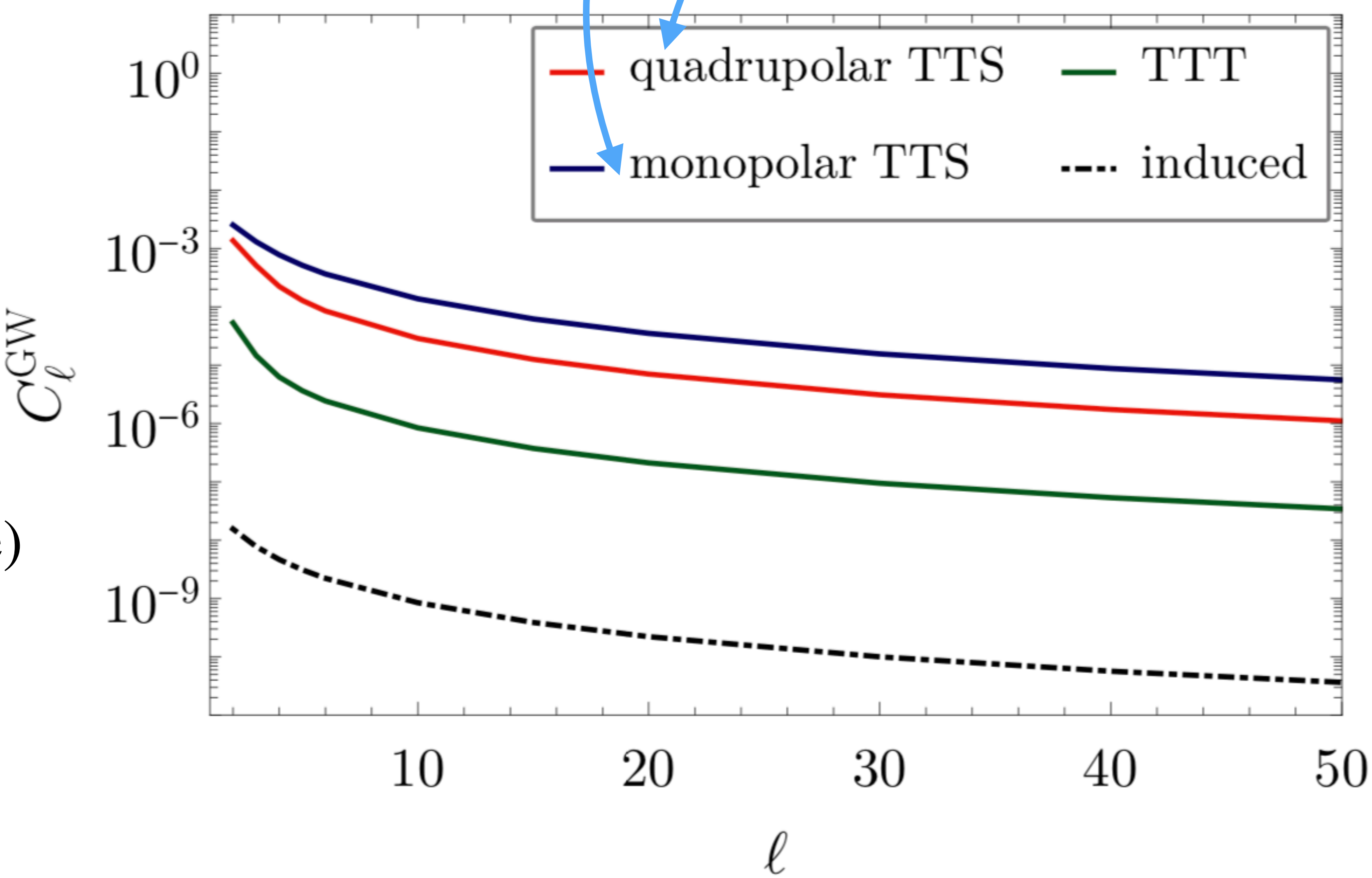
Referring to the angular dependence of $F_{\rm NL}(\mathbf{k},\mathbf{q})$

(scale-invariant case)



$$\langle \delta_{{\rm GW},\ell_1 m_1}\delta_{{\rm GW},\ell_2 m_2}\rangle = \delta_{\ell_1\ell_2}\delta_{m_1 m_2}\mathcal{C}_{\ell_1}^{\rm GW}$$

[Malhotra, ED, Fasiello, Shiraishi 2020 -
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Cross-correlations of GW and CMB anisotropies

$$\boxed{\delta_{\rm GW}^{\rm stt} \sim F_{\rm NL}^{\rm stt} \cdot \zeta_L}$$

$$\left.\begin{array}{l} \delta_{\rm GW}^{\rm stt} \sim F_{\rm NL}^{\rm stt} \cdot \zeta_L \\ \dfrac{\Delta T}{T} \sim \zeta_L \end{array}\right\} C_\ell^{\text{GW-T}} \sim F_{\rm NL}^{\rm stt} \cdot C_\ell^{TT}$$

[Adshead, Afshordi, ED, Fasiello, Lim, Tasinato 2020]

Cross-correlations of GW and CMB anisotropies

- SGWB anisotropy from ttt bispectrum:

$$\delta^{\rm GW}_{\ell m} = (2\pi)(-i)^\ell \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \sum_{s=\pm 2} \int \frac{d^3q}{(2\pi)^3} F^{\rm ttt}_{\rm NL}(k,q)\gamma^s_{\mathbf{q}} \frac{j_\ell(qd)}{(qd)^2} {}_{-s}Y^*_{\ell m}(\hat{q})$$

- CMB temperature anisotropy from tensor modes:

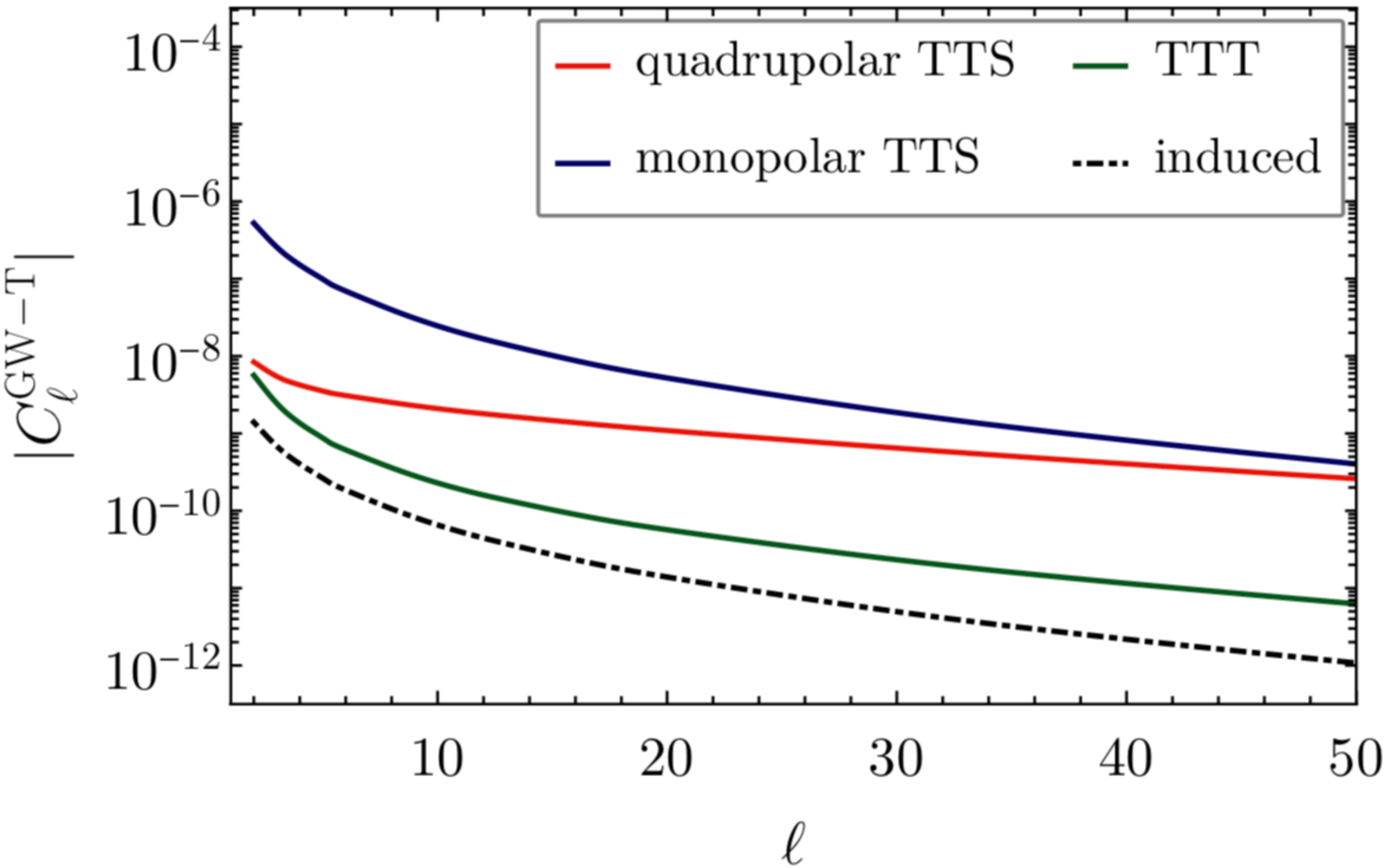
$$\delta^{\rm T}_{\ell m} = \pi\,(-i)^\ell \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \sum_{s=\pm 2} \int d\eta\, \frac{d^3q}{(2\pi)^3} \frac{\partial\gamma^s_{\mathbf{q}}}{\partial\eta} \frac{j_\ell(q\chi(\eta))}{(q\chi(\eta))^2} {}_{-s}Y^*_{LM}(\hat{q})\,. \qquad \chi(\eta) = \eta_0 - \eta$$

- Cross-correlation of the two:

$$C_\ell^{\rm GW-T,ttt} = \frac{(\ell-1)\ell(\ell+1)(\ell+2)}{4\pi} \sum_{s=\pm 2} \int_{q\ll k} q^2 dq\, F^{ttt}_{\rm NL}(\mathbf{k},\mathbf{q}) P^s_\gamma(q) \frac{j_\ell(qd)}{(qd)^2}$$
$$\times \int d\eta\, \frac{\partial\mathcal{T}(k,\eta)}{\partial\eta} \frac{j_\ell(q\chi(\eta))}{(q\chi(\eta))^2}\,.$$

[Malhotra, ED, Fasiello, Shiraishi 2020 -
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Cross-correlations of GW and CMB anisotropies



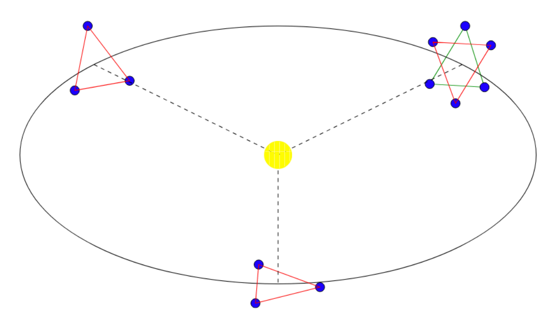
$F_{\rm NL}(\mathbf{k}, \mathbf{q})$ angular dependence: amplitude of the cross-correlation smaller for the quadrupole compared to monopole: the GW anisotropy is locally a quadrupole, whereas the source term for the CMB is a monopole

[Malhotra, ED, Fasiello, Shiraishi 2020 -
ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Projected constraints on $F_{\rm NL}^{\rm tss}$

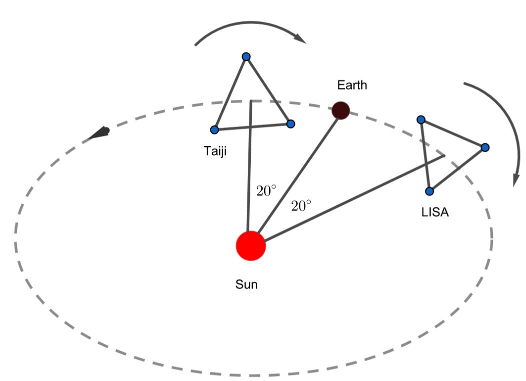
$$F_{ij} = \sum_{XY} \sum_{\ell=\ell_{\rm min}}^{\ell_{\rm max}} \frac{\partial C_\ell^X}{\partial \theta_i} \left(\mathcal{C}_\ell^{XY}\right)^{-1} \frac{\partial C_\ell^Y}{\partial \theta_j} \qquad X,Y = \{\mathrm{TT,GW,GW\text{-}T}\}$$

$$\mathscr{C}_\ell = \frac{2}{2\ell+1} \begin{bmatrix} (C_\ell^{\rm TT})^2 & (C_\ell^{\rm GW-T})^2 & C_\ell^{\rm TT} C_\ell^{\rm GW-T} \\ (C_\ell^{\rm GW-T})^2 & (C_\ell^{\rm GW})^2 & C_\ell^{\rm GW} C_\ell^{\rm GW-T} \\ C_\ell^{\rm TT} C_\ell^{\rm GW-T} & C_\ell^{\rm GW} C_\ell^{\rm GW-T} & \frac{1}{2}(C_\ell^{\rm GW-T})^2 + \frac{1}{2} C_\ell^{\rm TT} C_\ell^{\rm GW} \end{bmatrix}$$



[Crowder - Cornish, 2005]

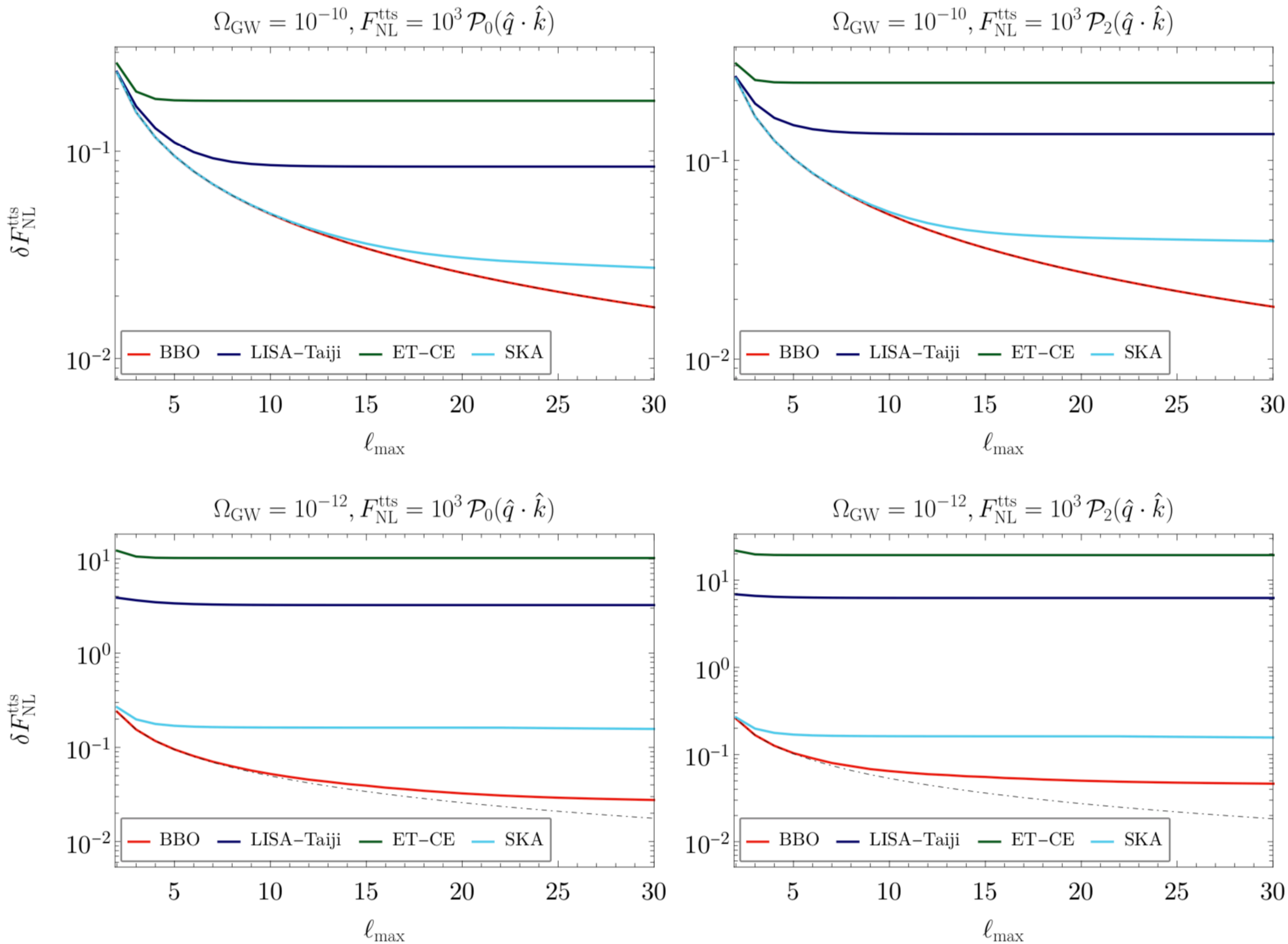
- BBO: 4 LISA-like constellations
- LISA+Taiji



[Ruan et al, 2020]

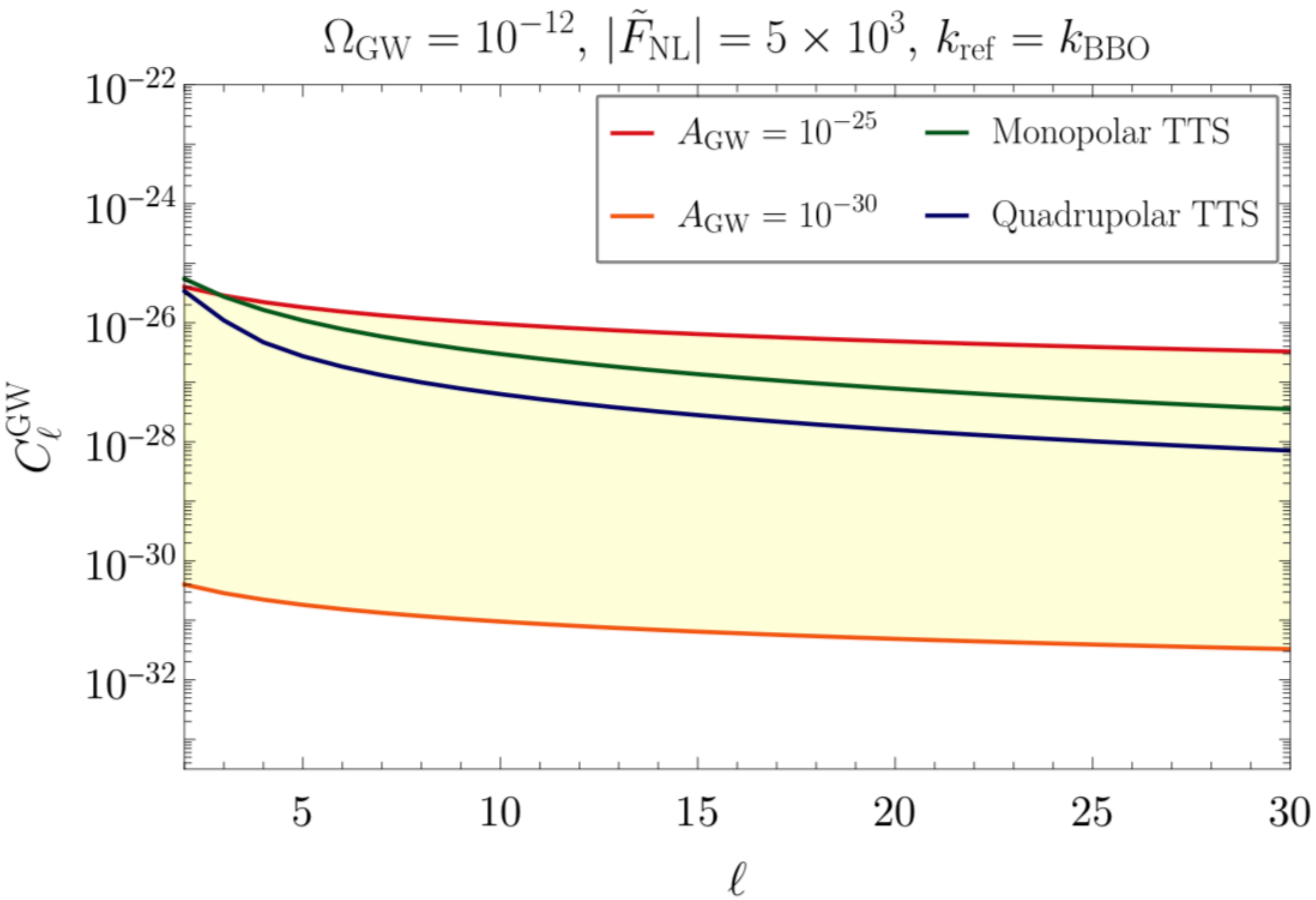
- ET + CE
- SKA (assumed 50 identical pulsars)

Projected constraints on $F_{\rm NL}^{\rm tss}$



[ED, Fasiello, Malhotra, Meerburg, Orlando 2021]

Astrophysical foregrounds

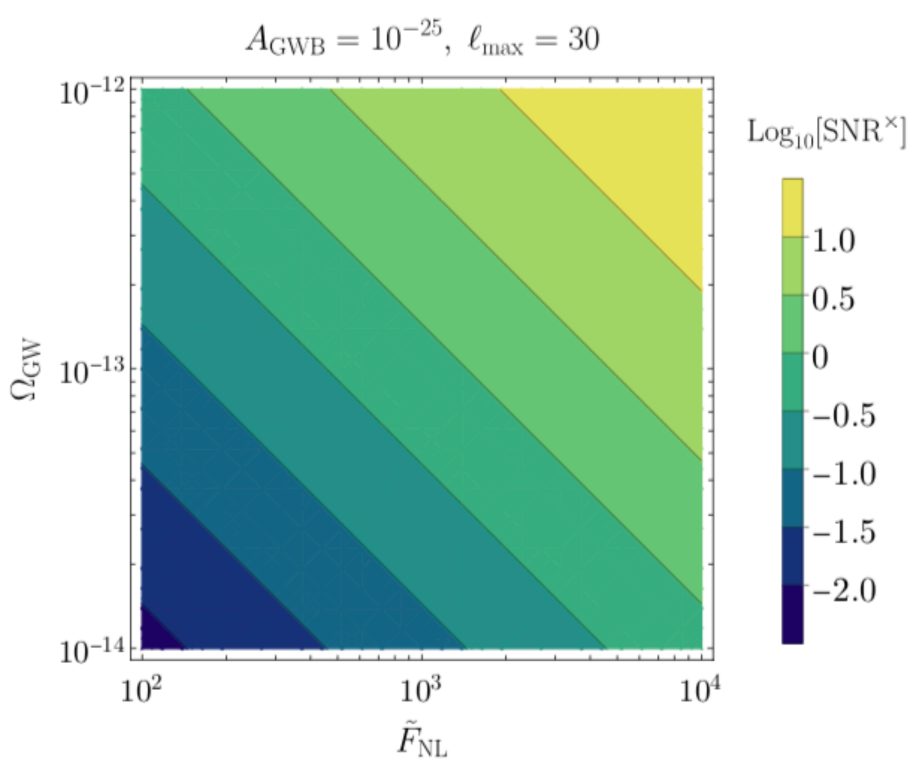


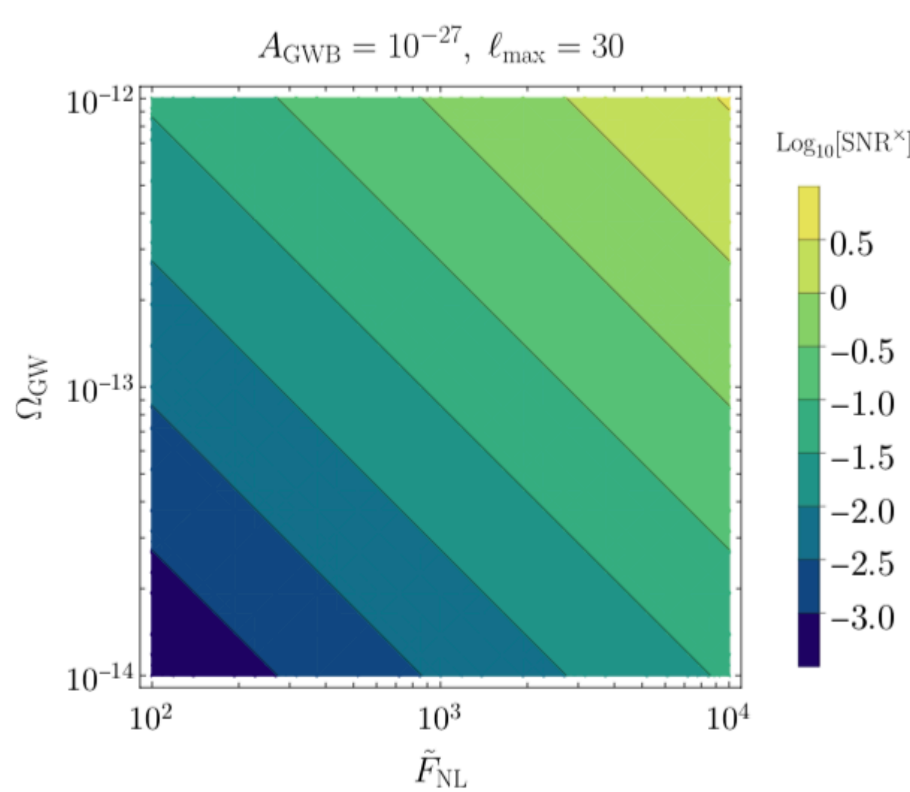
SNR for GW-CMB cross-correlations:

$$\mathrm{SNR}^\times = \left[\sum_{\ell_{\rm min}}^{\ell_{\rm max}} (2\ell+1)\frac{\left(C_\ell^{\rm GW-T,signal}\right)^2}{\left(C_\ell^{\rm GW-T,total}\right)^2 + C_\ell^{\rm GW,total} C_\ell^{\rm TT}}\right]^{1/2}$$

$$C_\ell^{\rm GW-T,signal} = C_\ell^{\rm GW-T,tts}$$
$$C_\ell^{\rm GW-T,total} = C_\ell^{\rm GW-T,signal} + C_\ell^{\rm GW-T,induced},$$
$$C_\ell^{\rm GW,total} = C_\ell^{\rm GW,tts} + C_\ell^{\rm GW,induced} + C_\ell^{\rm GW,astro} + N_\ell^{\rm GW}$$

Monopolar stt bispectrum





Quadrupolar stt bispectrum

Modeling of astrophysical background using results in [Cusin, Dvorkin, Pitrou, Uzan 2018-2019]

SGWB anisotropies: astrophysical sources

- SGWB from superposition of signals from black holes, neutron star binaries
- ASGWB also expected to be anisotropic due to the distribution of sources
- Anisotropies in the ASGWB can inform us about many things (e.g. start formation model, mass distribution, etc) **[see e.g. Cusin et al, 2018-19-20]**
- Anisotropies in the ASGWB do not correlate strongly with CMB, much more strongly with LSS observables **[Ricciardone et al, 2021]**

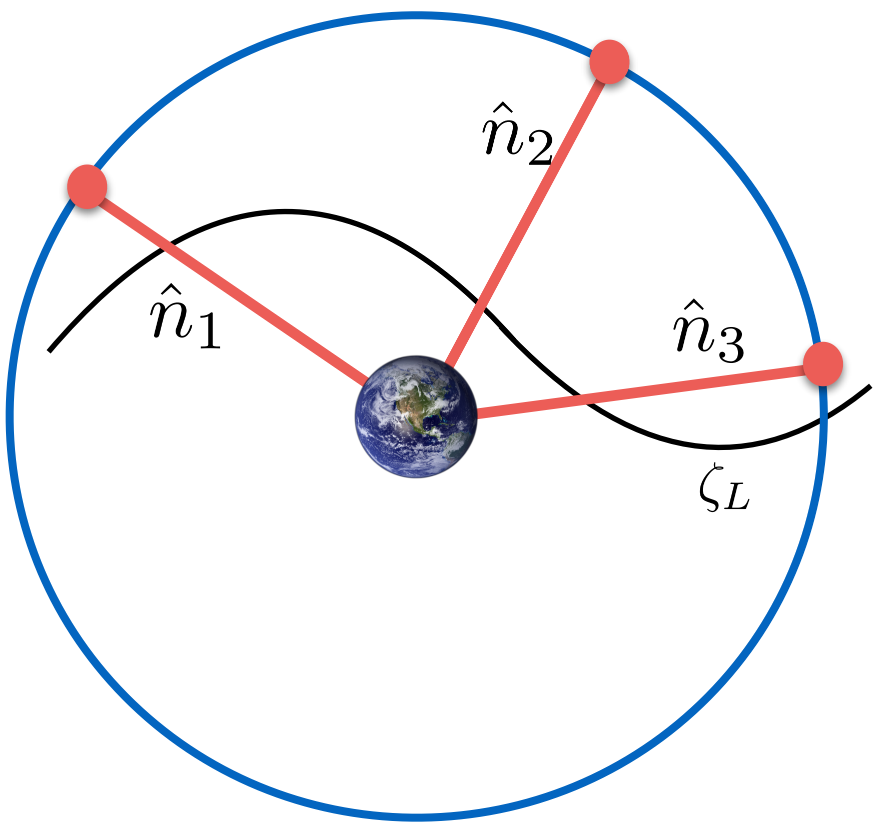
$\downarrow$

GW-CMB correlation excellent probe of cosmological SGWB!

Any other kinds of anisotropies expected in the SGWB?

GW propagate through the perturbed universe, so they are subject to Sachs-Wolfe / integrated Sachs-Wolfe, … , just like CMB photons

Simplified treatment in **[Alba, Maldacena 2015]**:
large-scale gravitational potential $\longrightarrow$ SW dominates



$$\boxed{\frac{\delta f(\hat{n})}{f} = \frac{1}{5}\left[\zeta_{L(\text{today})} - \zeta_L(\hat{n}\cdot\eta_0)\right]}$$

Gravitational redshift/blueshift of gravitons

$$\zeta_L(\hat{n}_1\cdot\eta_0) \neq \zeta_L(\hat{n}_2\cdot\eta_0)$$

$\downarrow$

Direction-dependent frequency shift

$\downarrow$

Anisotropy in the GW energy density $\delta_{\rm GW}(f,\,\hat{n}) \sim \frac{\alpha}{5}\cdot\zeta_{\rm L}(\hat{n}\cdot\eta_0)$

$\bar{\Omega}_{\rm GW}(f) \sim \left(\frac{f}{f_0}\right)^{\alpha}$

[See: Contaldi, 2017 — Bartolo, Bertacca, Matarrese, Peloso, Ricciardone, Riotto, Tasinato, 2019 — for full Boltzmann treatment of GW anisotropies]

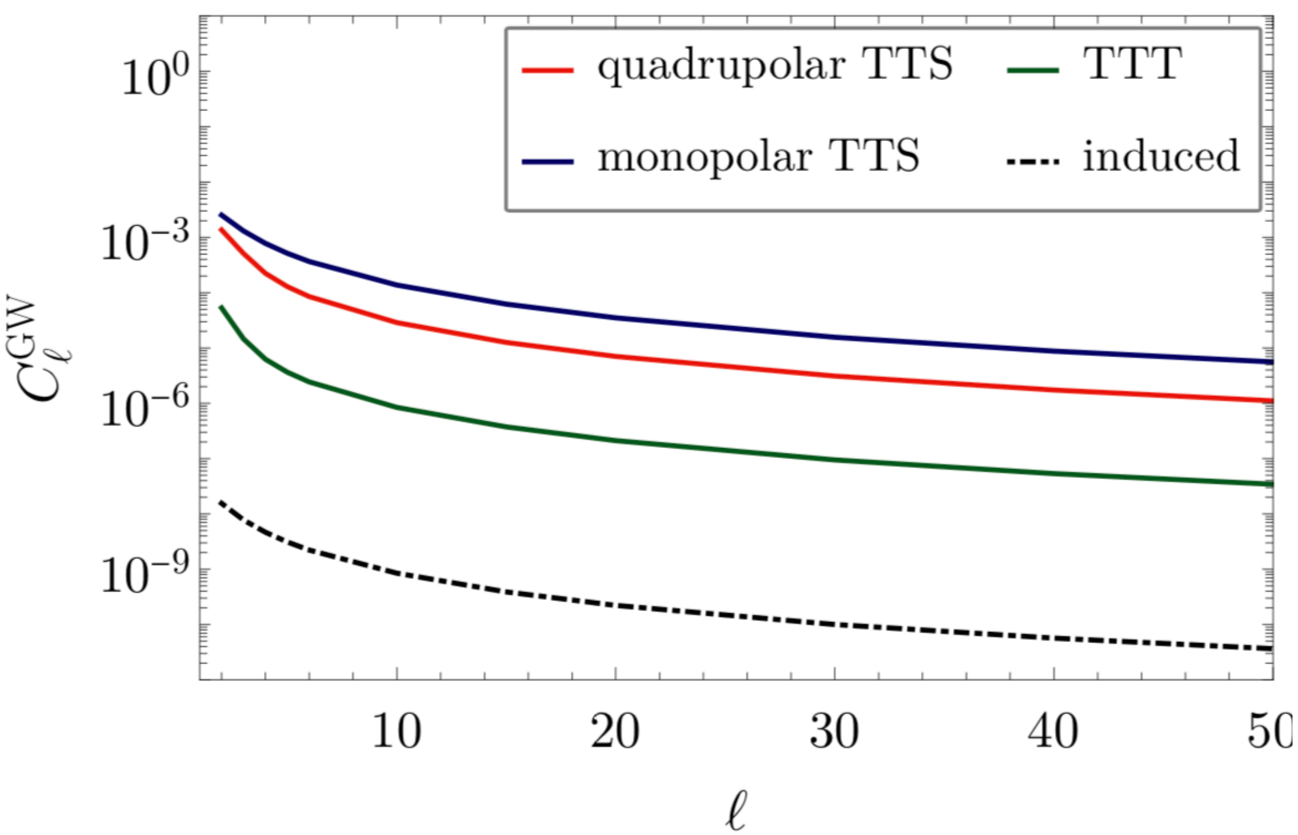
SGWB anisotropies from propagation

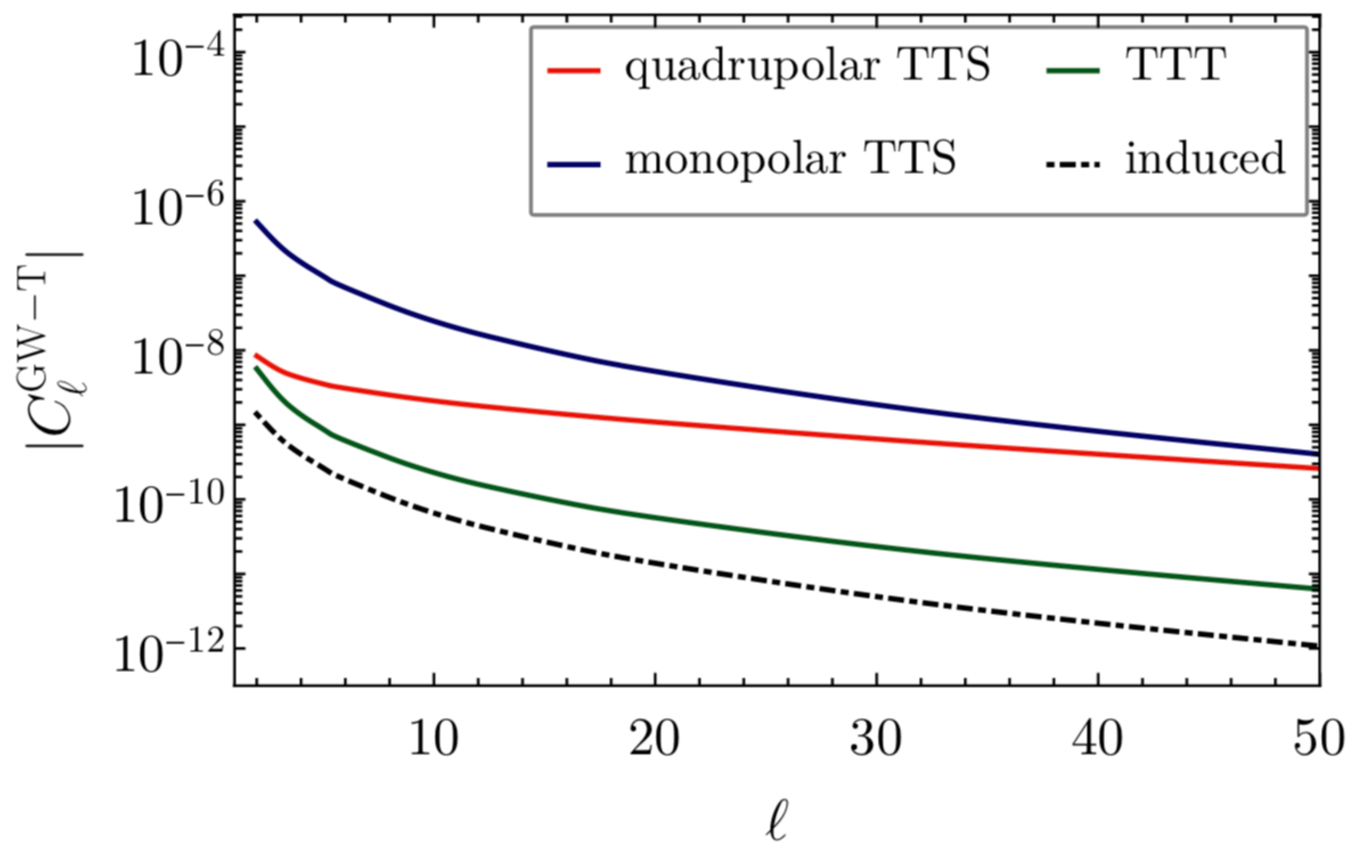
large-scale gravitational potential $\longrightarrow$ SW dominates

$$\delta_{\rm GW}^{\rm propagation} \sim \zeta_L$$

$$\delta_{\rm GW}^{\rm stt} \sim F_{\rm NL}^{\rm stt} \cdot \zeta_L$$

$$\frac{\delta f}{f} \sim -\frac{\zeta_L}{5} \longrightarrow \boxed{\delta_{\rm GW} \sim \zeta_L \sim 10^{-5}}$$





Anisotropies from propagation: affect all types of cosmological backgrounds!

Gravitational waves

- Extremely useful in testing inflation, also at interferometer scales

- A variety of classes of models (beyond the vanilla scenario) generate interesting gravitational waves signatures

- SGWB anisotropies powerful for disentangling inflationary SGWB from the one due to other cosmological sources and from the astrophysical background