Gravitational waves from primordial fluctuations

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JRP Tohoku "Dawn of Gravitational-wave Cosmology and Theory of Gravity"
March 3rd, 2022

Check my review paper: 2109.01398!

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https://domenechcosmo.netlify.app

FELLINI — MSCA, G.A.: 754496

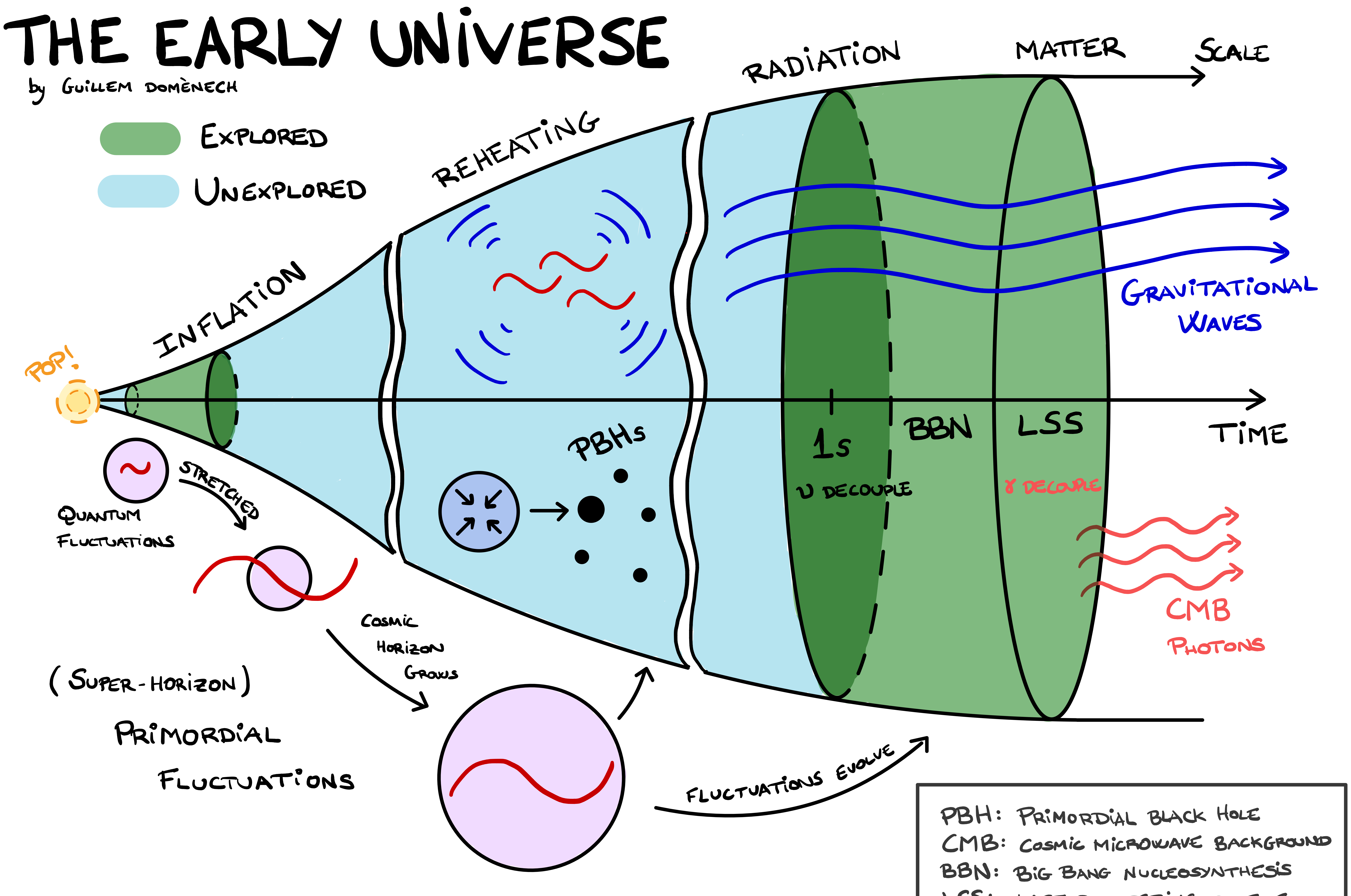


Image from: https://domenechcosmo.netlify.app/post/induced-gws/

Cosmological sources of GWs

For more see e.g.
Guzzetti+1605.01615
Caprini+1801.04268
Kuroyanagi+1807.00786

New physics during inflation:

Quantum fluctuations:

- Primordial tensor fluctuations
- GWs induced by primordial scalar fluctuations ("guaranteed" signal from the CMB)

Additional sources:

- SU(2) gauge fields
- Axion fields

Also… New physics after inflation (but before BBN):

Phase transitions:

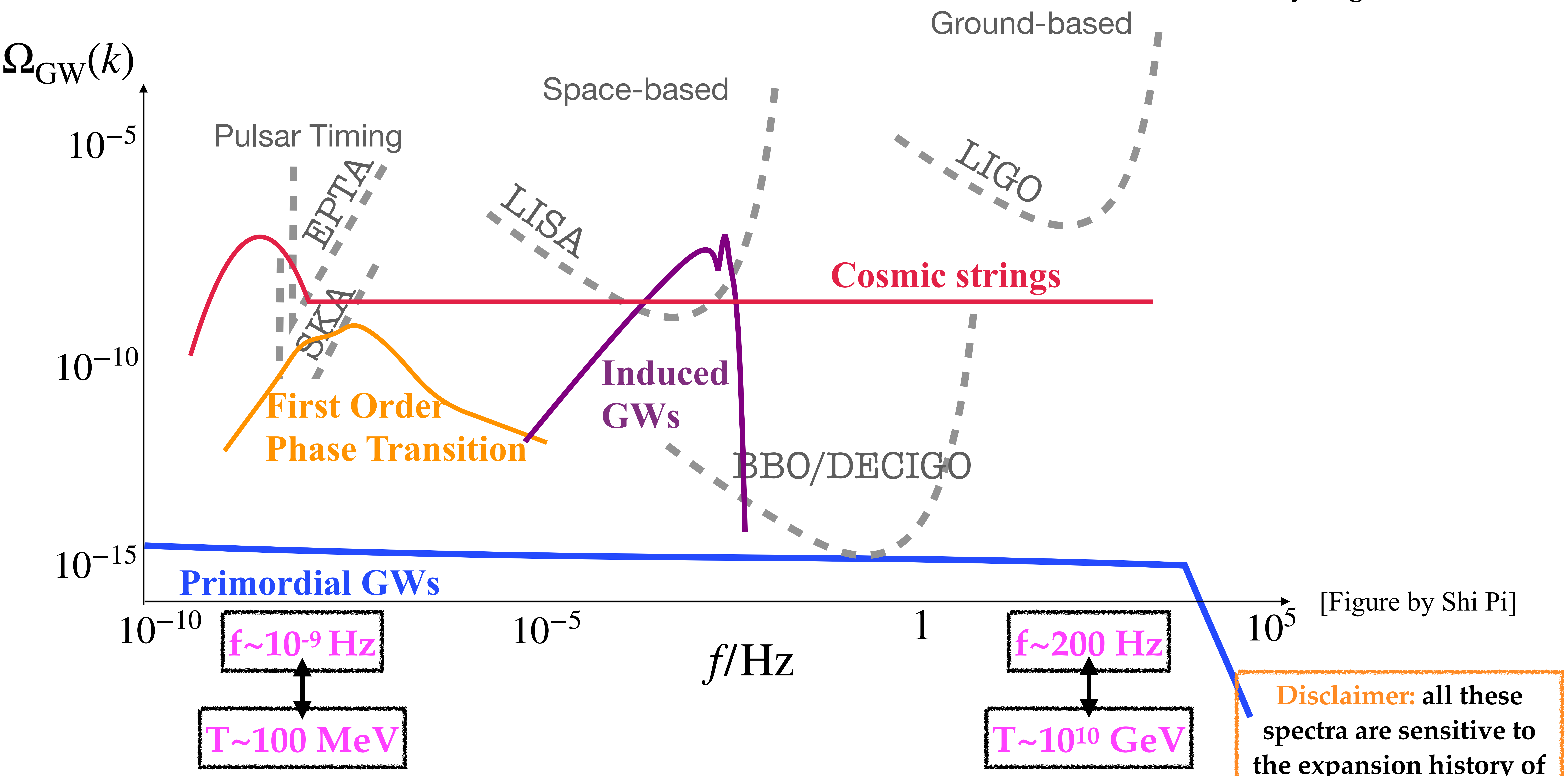
- Strong first order phase transitions
- Topological defects like cosmic strings

Preheating and reheating:

- Parametric resonances

Cosmological Stochastic GW background

For more see e.g.
Guzzetti+1605.01615
Caprini+1801.04268
Kuroyanagi+1807.00786



Disclaimer: all these spectra are sensitive to the expansion history of the early universe

Overview

- Gravitational Waves induced by primordial fluctuations

  (Review of induced GWs in "standard" (radiation dominated) cosmology)

- Induced GWs and early universe's expansion histories

  (Go beyond "standard" cosmology with general expansion histories)

- Induced GWs and early isocurvature fluctuations

  (Go beyond "standard" cosmology with initial conditions)

Talk based on:

[GD, 1912.05583]

[GD, S.Pi, M.Sasaki, 2005.12314]

[GD and S.Pi, 2010.03976]

[GD and C.Lin, M.Sasaki, 2012.08151]

[V.Atal, GD: 2103.01056]

[GD and V.Takhistov, M.Sasaki, 2105.06816]

[GD, 2109.01398]

[L.Witkowski, GD, J.Fumagalli, S.Renaux-Petel, 2110.09480]

[GD, S.Passaglia, S.Renaux-Petel, 2112.10163]

GWs induced by primordial fluctuations

Non-Linear Theory of Gravitational Instability in the Expanding Universe

Kenji TOMITA

Research Institute for Theoretical Physics
Hiroshima University, Takehara, Hiroshima-ken

(Received January 5, 1967)

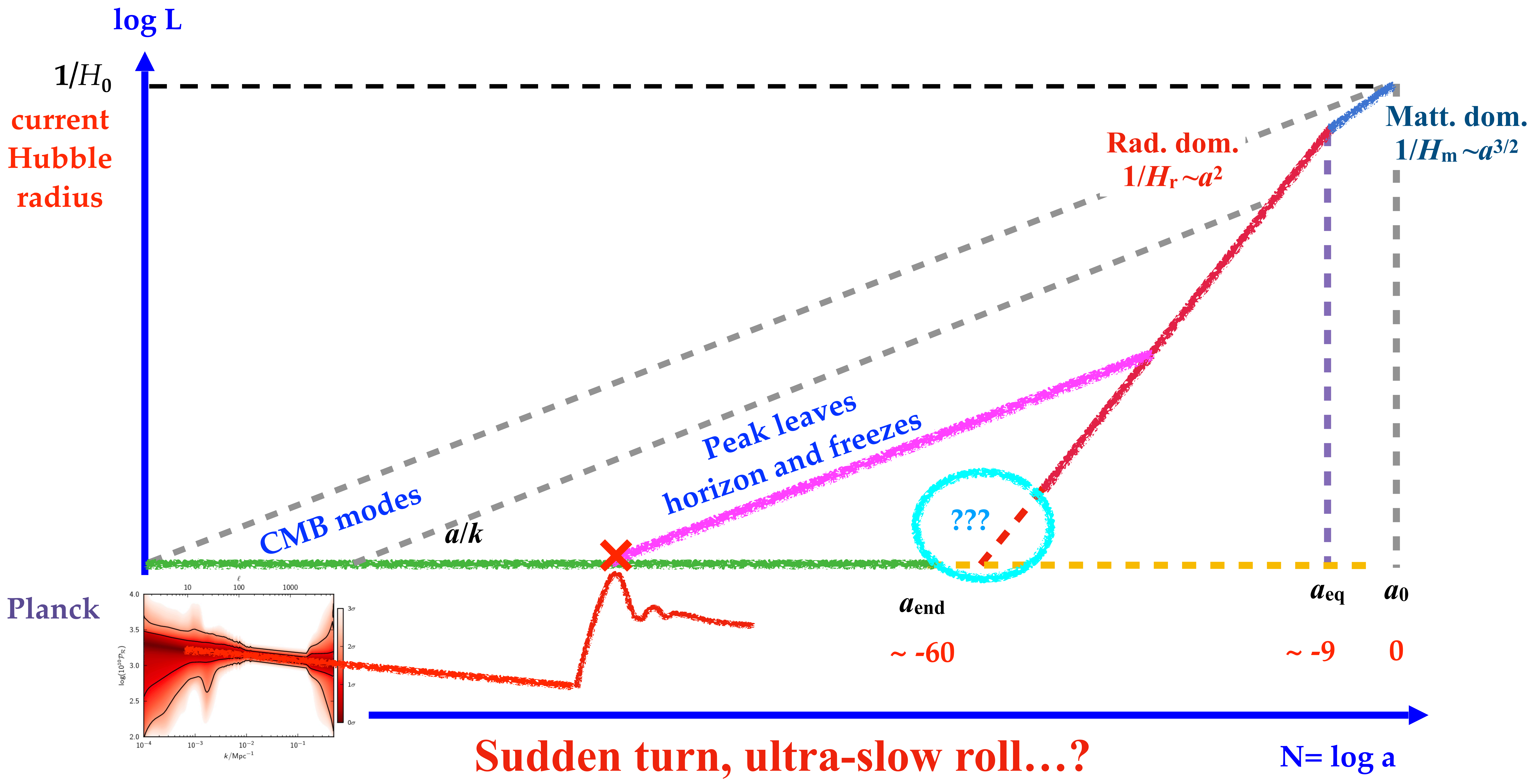
The gravitational instability in the expanding universe is studied in the second-order approximation. This work is an extension of Lifshitz's linearized theory on the basis of general relativity. Basic equations are formulated generally, but their analysis is confined to a special case where pressure effects are negligible and the spatial curvature of the unperturbed model universe is zero. The results show that the second-order density contrast tends to accentuate the increase of the first-order density contrast with time, unless the linear dimension of the perturbation is too great. Moreover it is shown that gravitational wave is induced by deformed density perturbations even if the first-order metric perturbation includes no part of gravitational wave. If time is reversed, our results will be applicable to the problem of the gravitational instability in the contracting universe or in the collapsing star.

Induced GWs history

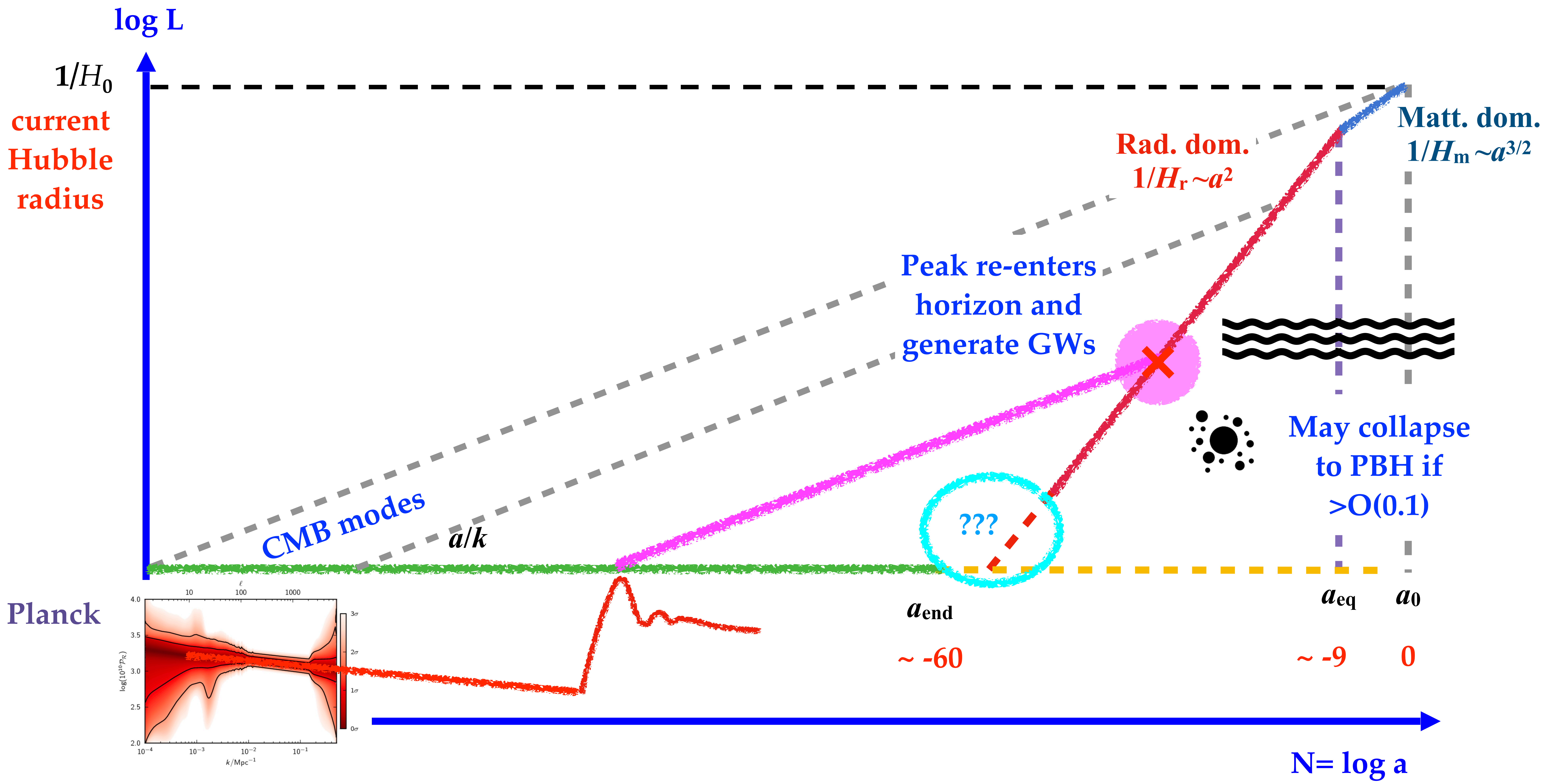
- First pointed out by K. Tomita in 1971 [*Prog. Theor. Phys.* 45, 1747 (1971)]
- Followed by Matarrese, Pantano, Saez in 1993 [*Phys.Rev.Lett.* 72 (1994) 320-323]
- Also Matarrese, Mollerach, Bruni in 1997 [*Phys.Rev.D* 58 (1998) 043504]
- Then Ananda, Clarkson and Wands in 2006 [gr-qc/0612013]
- And Baumann, Ichiki, Steinhardt and Takahashi in 2007 [hep-th/0703290]
- Saito and Yokoyama in 2008: **induced GWs <=> PBHs!** [0812.4339]
- …After the first LIGO detection the publication number keeps growing!

[Sorry for missing all the other works… they don't fit here, not even mines! More in my review paper: 2109.01398]

New physics during inflation

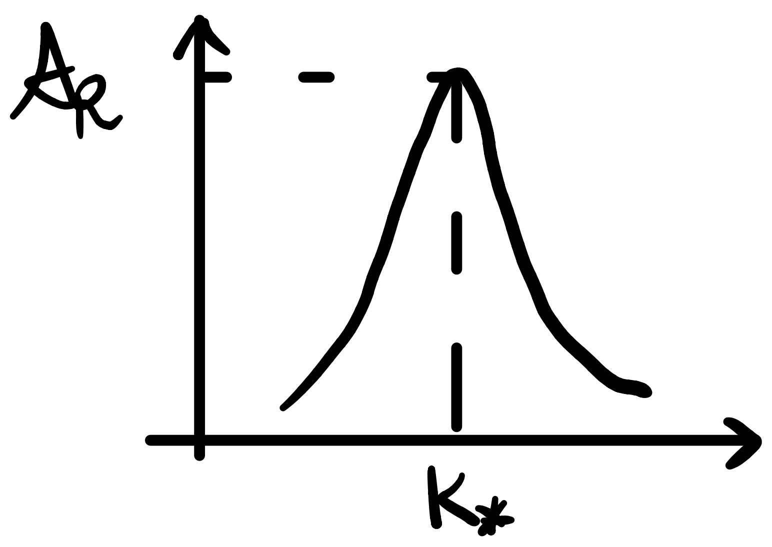


New physics during inflation



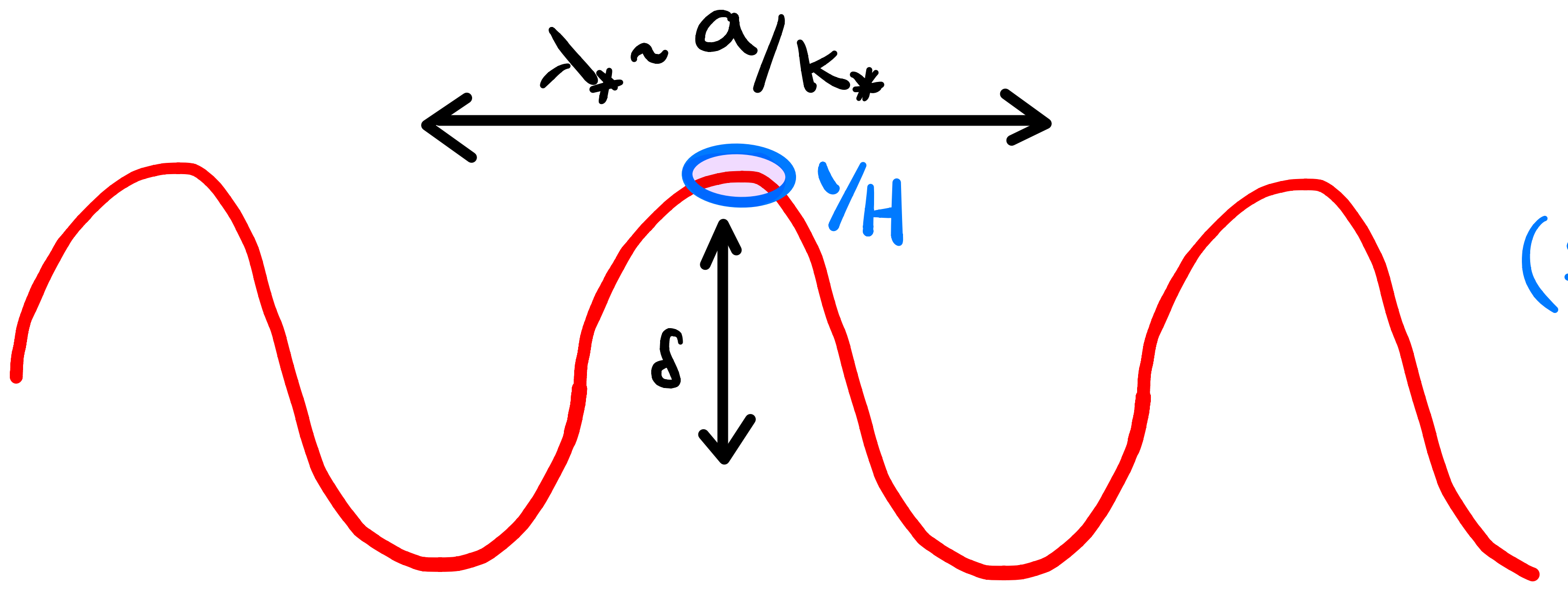
GENERATION OF GWs AND PBHs

(SHARP) PEAK
IN POWER SPECTRUM



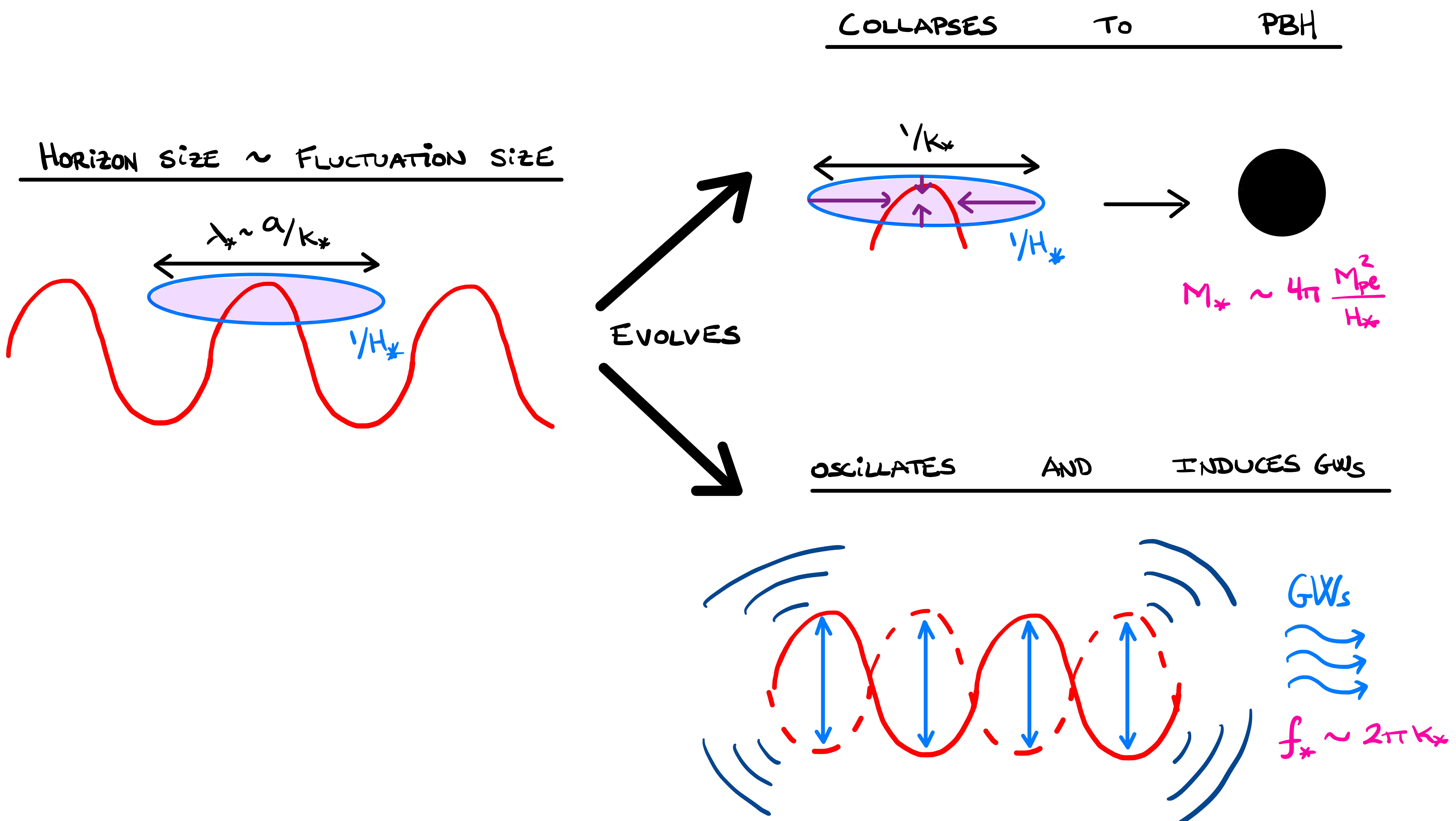
$\approx\!>$

(INITIAL) FLUCTUATIONS ON SCALES λ_*
WITH $P(\delta) \sim e^{-\frac{\delta^2}{2A_R}}$

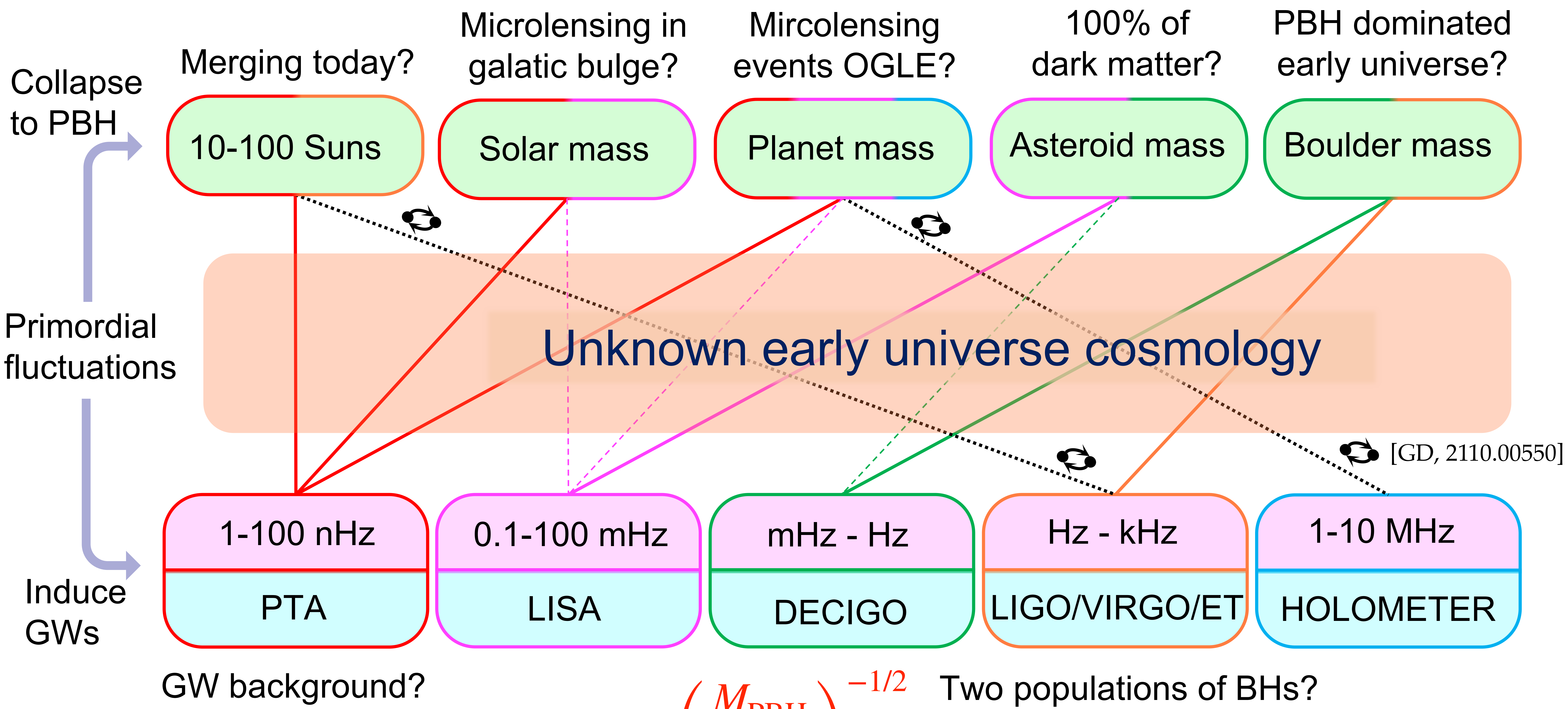


(SUPERHORIZON)

GENERATION OF GWs AND PBHs



The PBH — induced GW connection



[De Luca et al 2102.03809]

Planet mass [GD and S.Pi, 2010.03976]
or dark matter PBH [De Luca+, Sugiyama+]
or solar mass PBH [Vaskonen+, Kohri+, Inomata+]

$$f_{\rm GW} \sim {\rm Hz} \left(\frac{M_{\rm PBH}}{10^{16}{\rm g}} \right)^{-1/2}$$

(in RD)

Induced GWs amplitude

After inflation: **1st order:** **Free wave propagating** $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} = 0$

Curvature perturbation

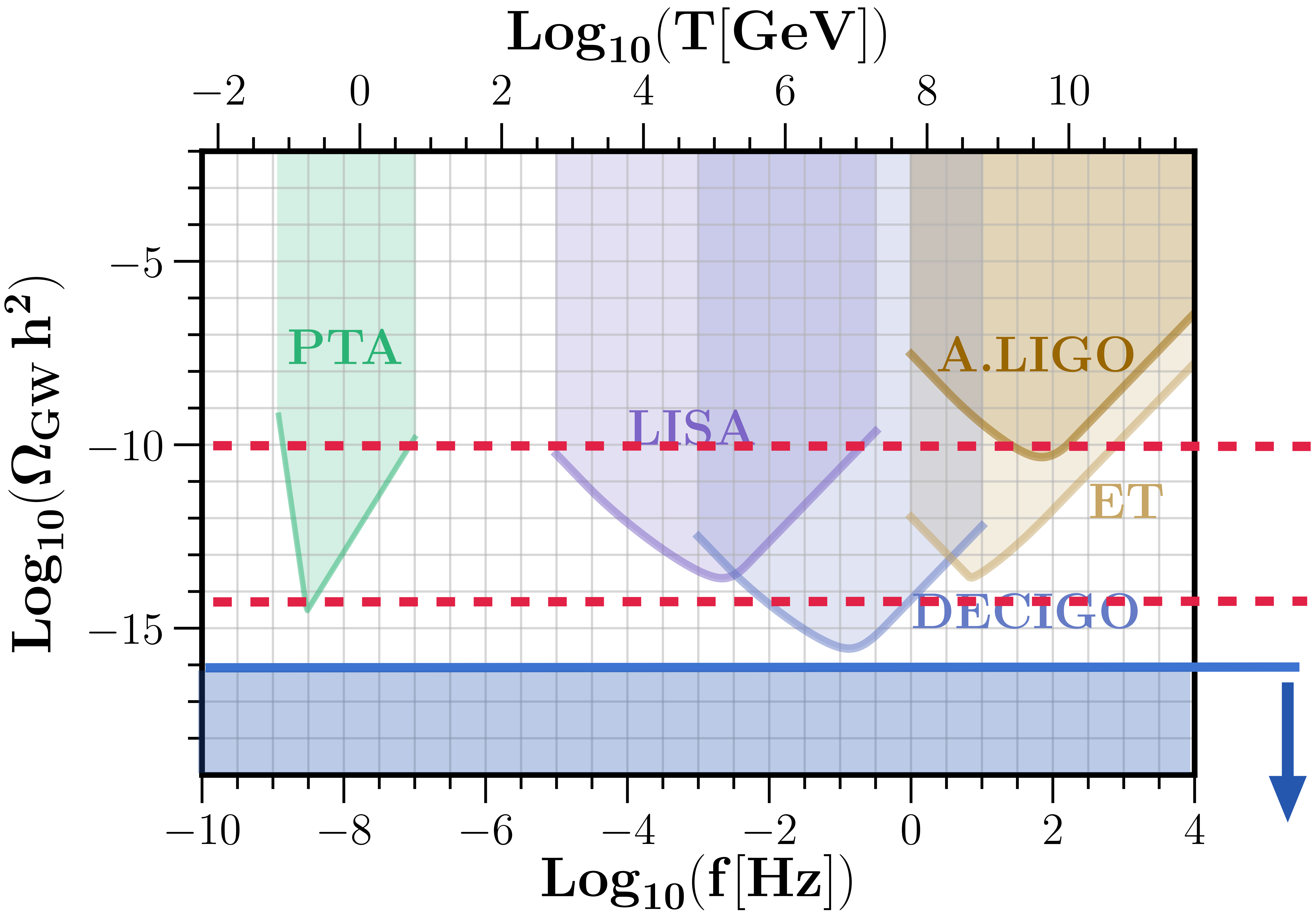
2nd order: **Massless field with source** $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{\;ab}(\partial_a\Phi\partial_b\Phi)$

$$\Omega_{\rm GW}(k) = \frac{d\rho_{GW}}{d\ln k} = \frac{k^2}{12\mathcal{H}^2}\mathcal{P}_h(k,\tau)$$

$$\Omega_{\rm GW}^{\rm induced} \sim \frac{1}{12}\Omega_{r,0}\mathcal{P}_{\mathcal{R}}^2 \sim 10^{-6}\mathcal{P}_{\mathcal{R}}^2(k \gg k_{\rm CMB})$$

Density ratio of radiation today $\Omega_{r,0} \sim 4\times10^{-5}$

Primordial fluctuations — Induced GWs amplitudes



$\Omega_{GW}^{\text{induced}} \sim 10^{-6}\mathcal{P}_{\mathcal{R}}^2(k \gg k_{\text{CMB}})$

$\mathcal{P}_{\mathcal{R}} > 10^{-2}$ **+ Detectable PBH**

$\mathcal{P}_{\mathcal{R}} > 10^{-4}$ **Detectable GWs**

$\Omega_{GW}^{\text{inflation}}$

Undetectable?

Power-law integrated sensitivity curves: Thrane & Romano 1310.5300

Induced GWs and the early universe's expansion histories

Induced GWs spectrum

After inflation: **1st order:** **Free wave propagating** $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} = 0$

2nd order: **Massless field with source** $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{ab}(\partial_a\Phi\partial_b\Phi)$

Effects from "unknown" cosmological phases:

1. Equation of state parameter w: **A. Expansion History (H)**
B. Evolution of Φ

2. Propagation speed of fluctuations c_s: **C. Resonances (f_h~$2f_\Phi$)**

$k \sim 2c_sk_p$

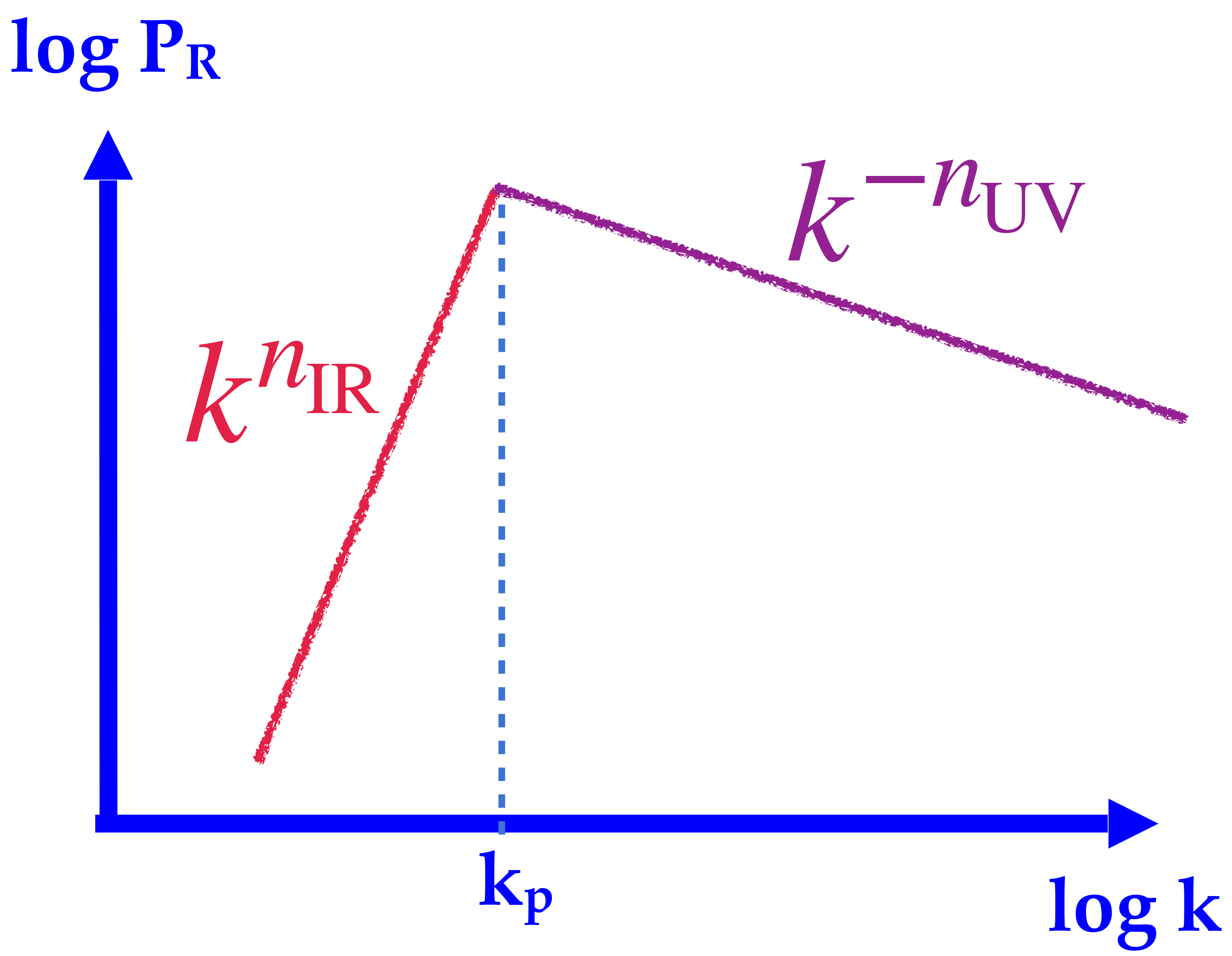
$$(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{ab}(\partial_a\Phi\partial_b\Phi)$$

1st example:

Broken power-law

Induced GWs spectrum

[V.Atal & GD: 2103.01056]



$n_{UV} \propto f_{NL}$ (= local non-Gaussianity, or η parameter)

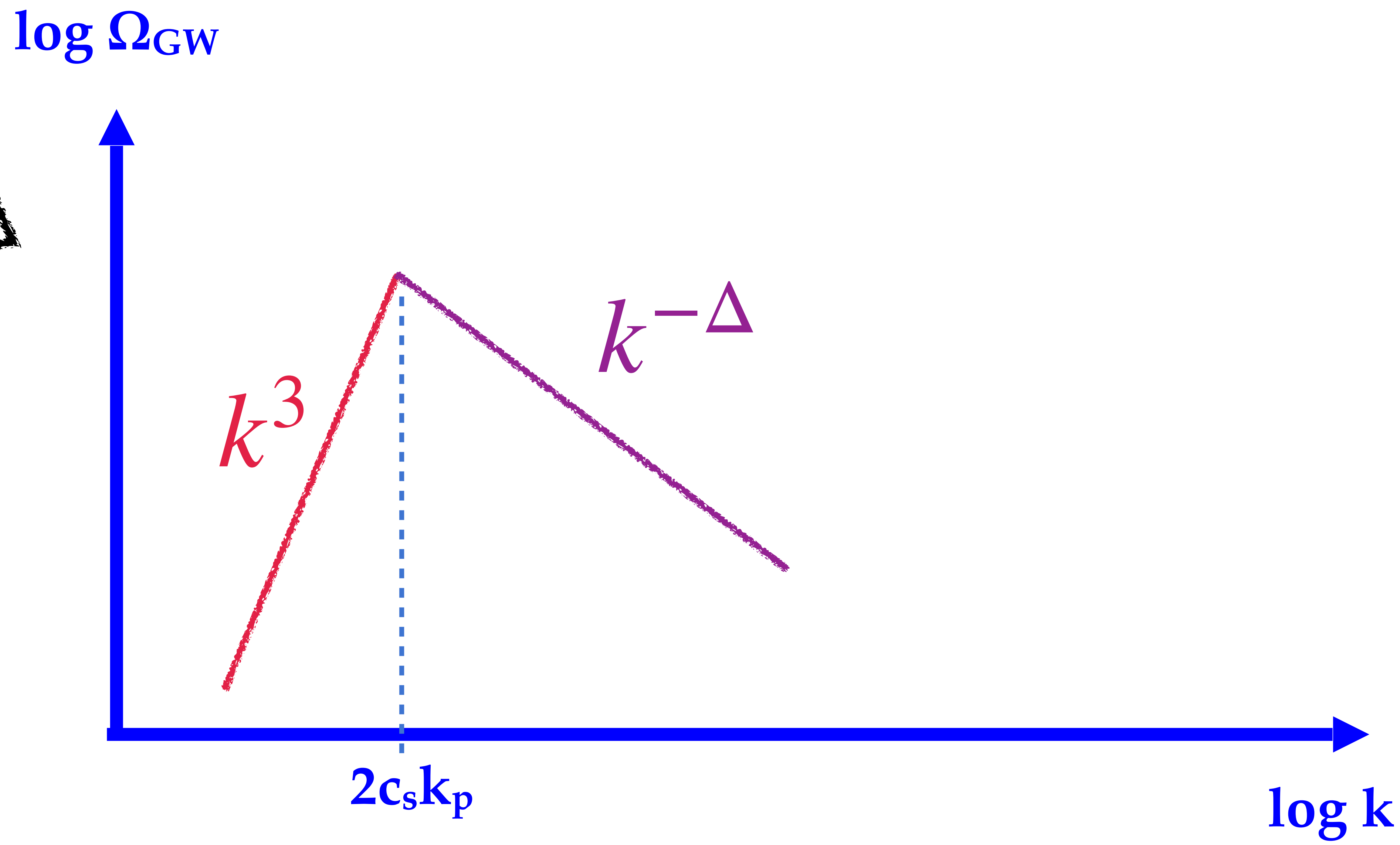
$$\Delta = \begin{cases} 2n_{UV} & n_{UV} < 4 \\ 4 + n_{UV} & n_{UV} > 4 \end{cases}$$

Broken-power law

E.g. single field inflation models

$w = c_s^2 = 1/3$

Radiation



Broken-power law

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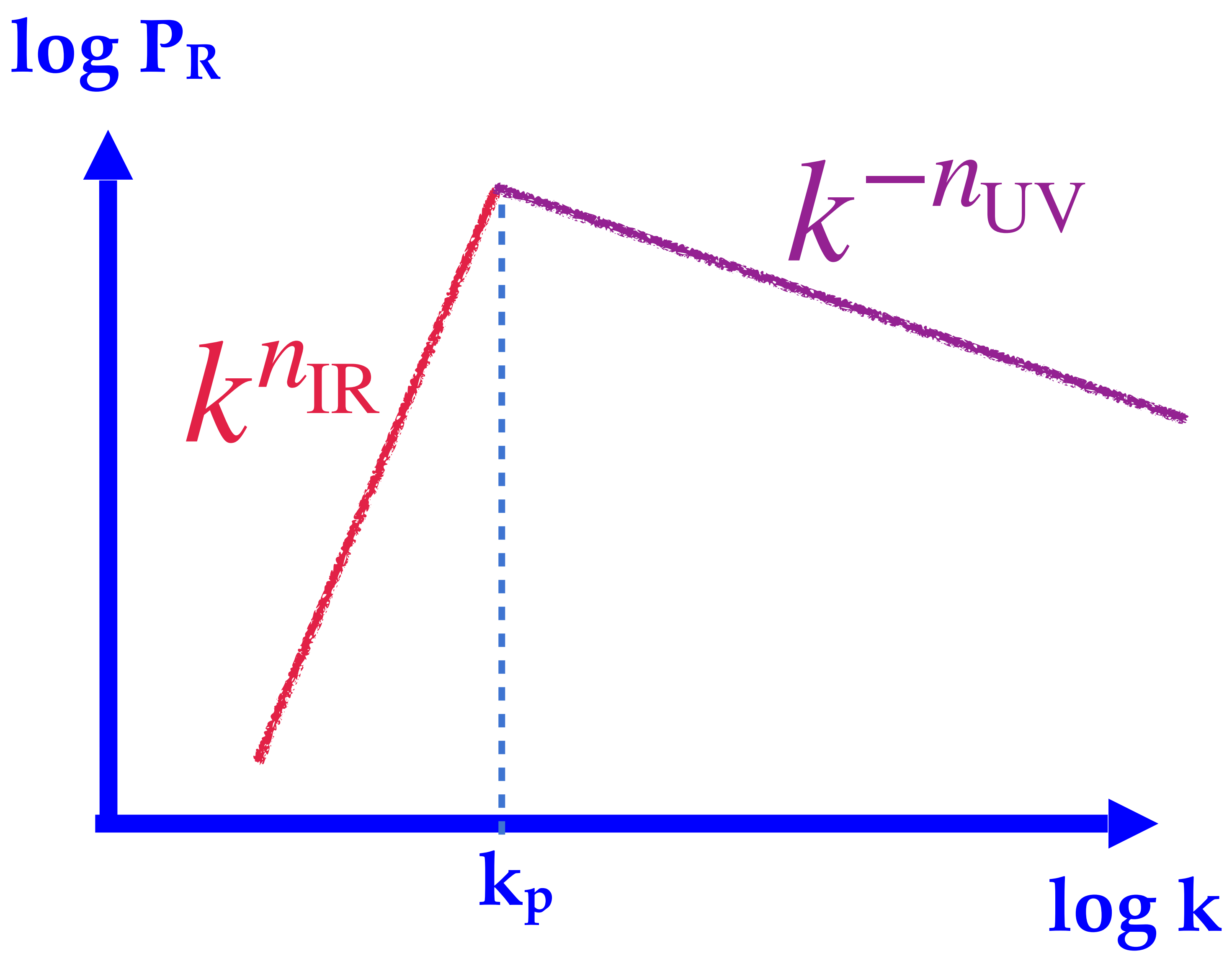
[V.Atal & GD: 2103.01056]



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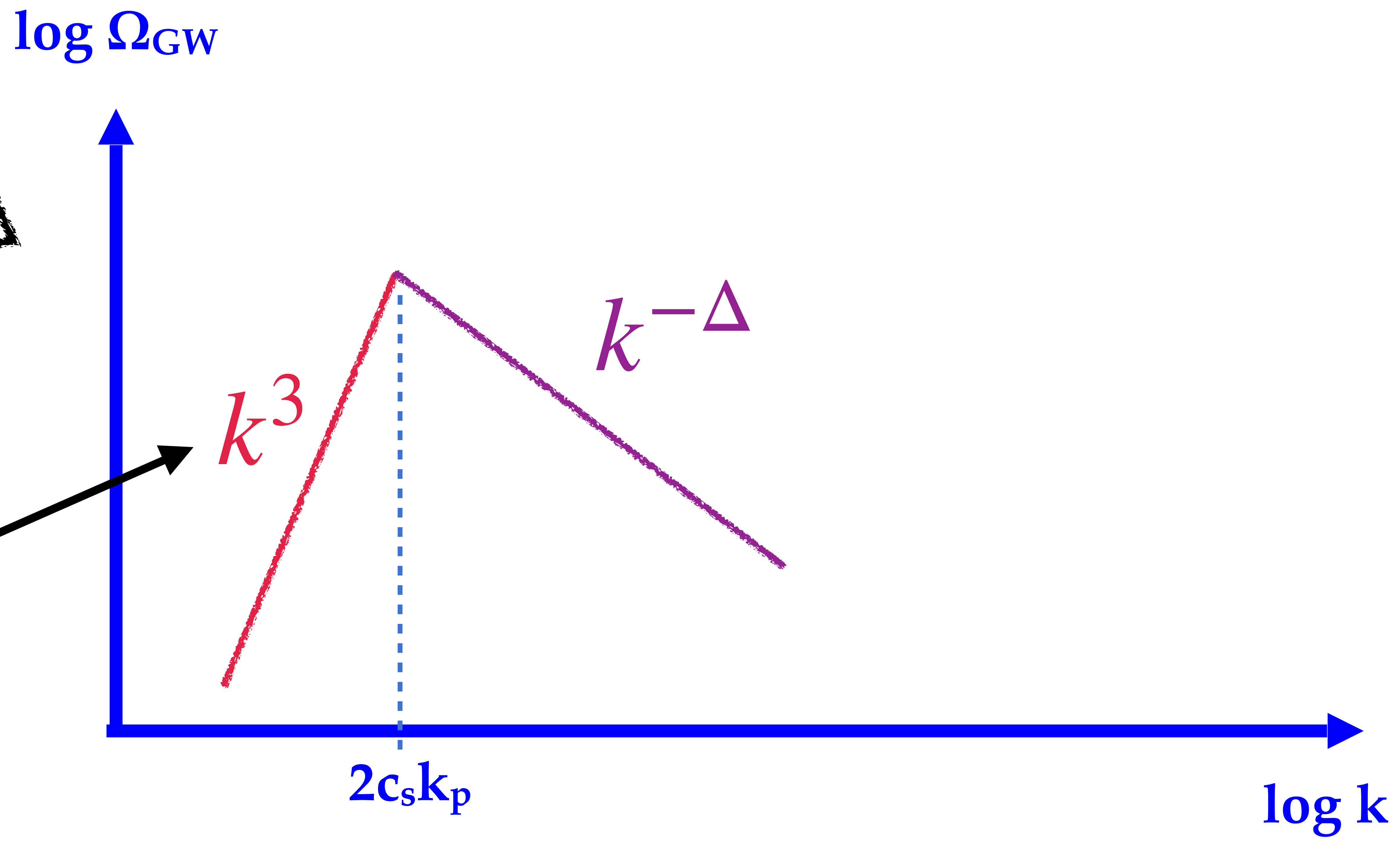
Radiation



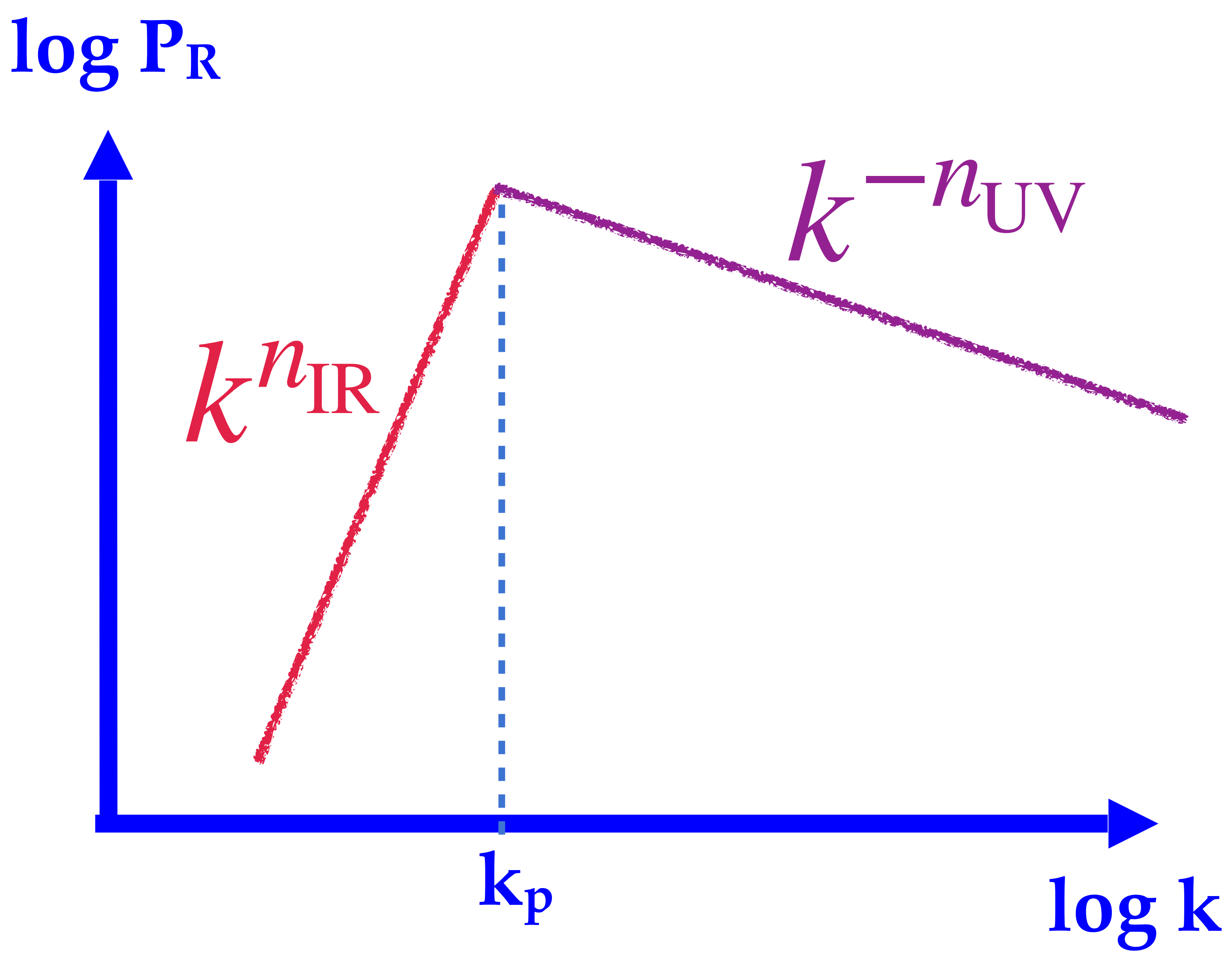
Broken-power law

"Universal" IR slope

[Cai, Pi & Sasaki,1909.13728]

Induced GWs spectrum

[V.Atal & GD: 2103.01056]



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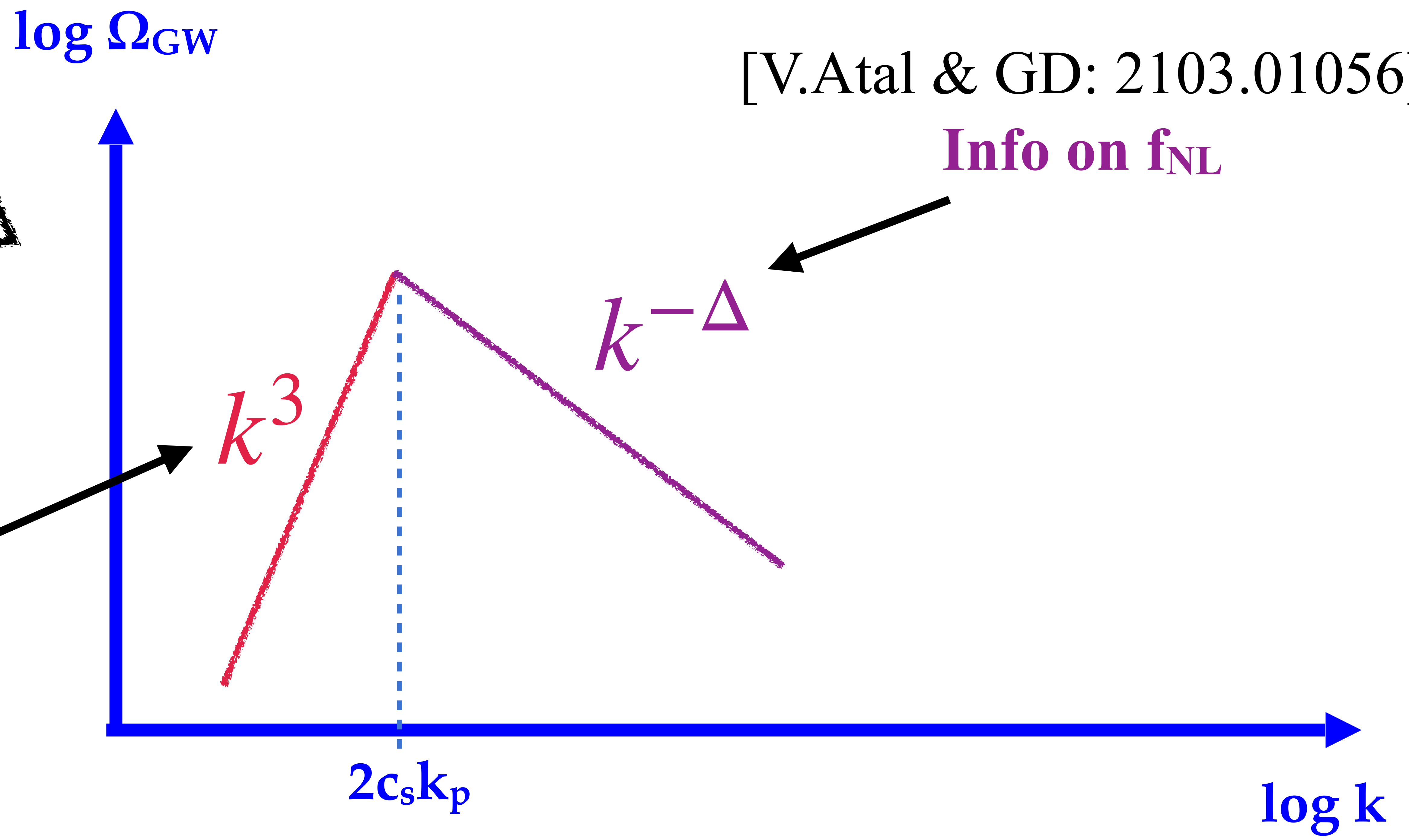
Radiation

[V.Atal & GD: 2103.01056]

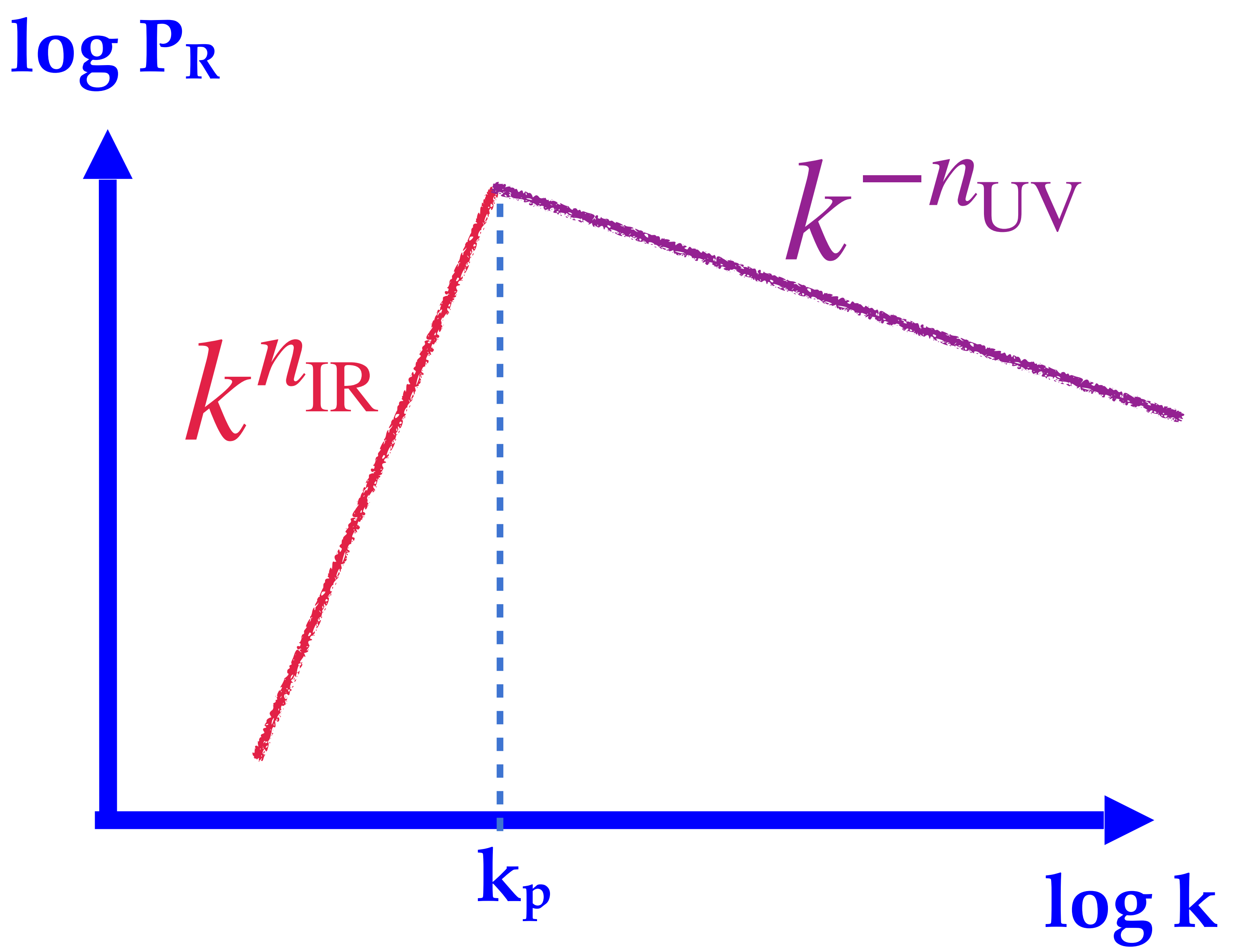


"Universal" IR slope

[Cai, Pi & Sasaki,1909.13728]

Broken-power law

Induced GWs spectrum



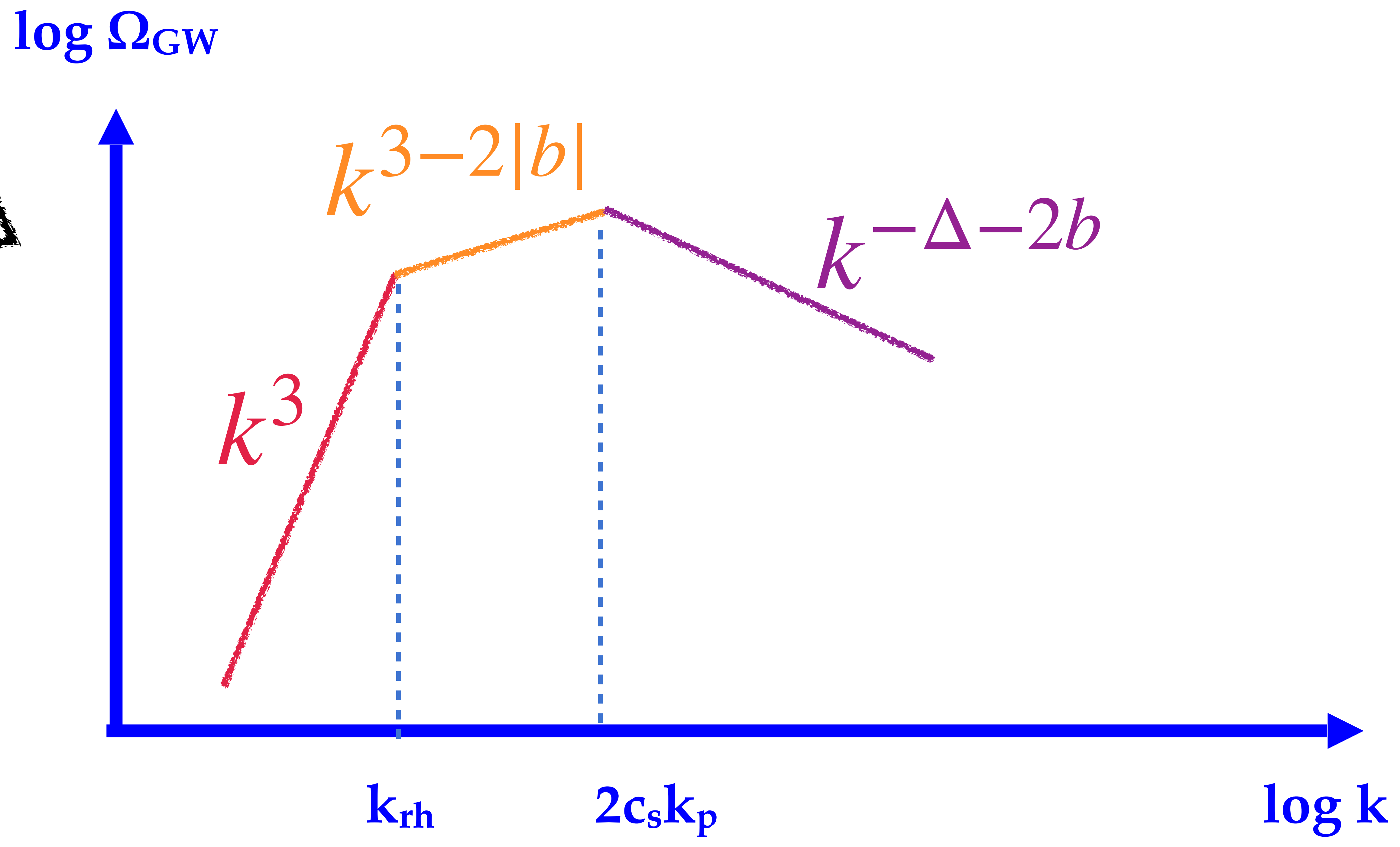
Broken-power law

$n_{UV} \propto f_{NL}$

$w \neq 1/3$

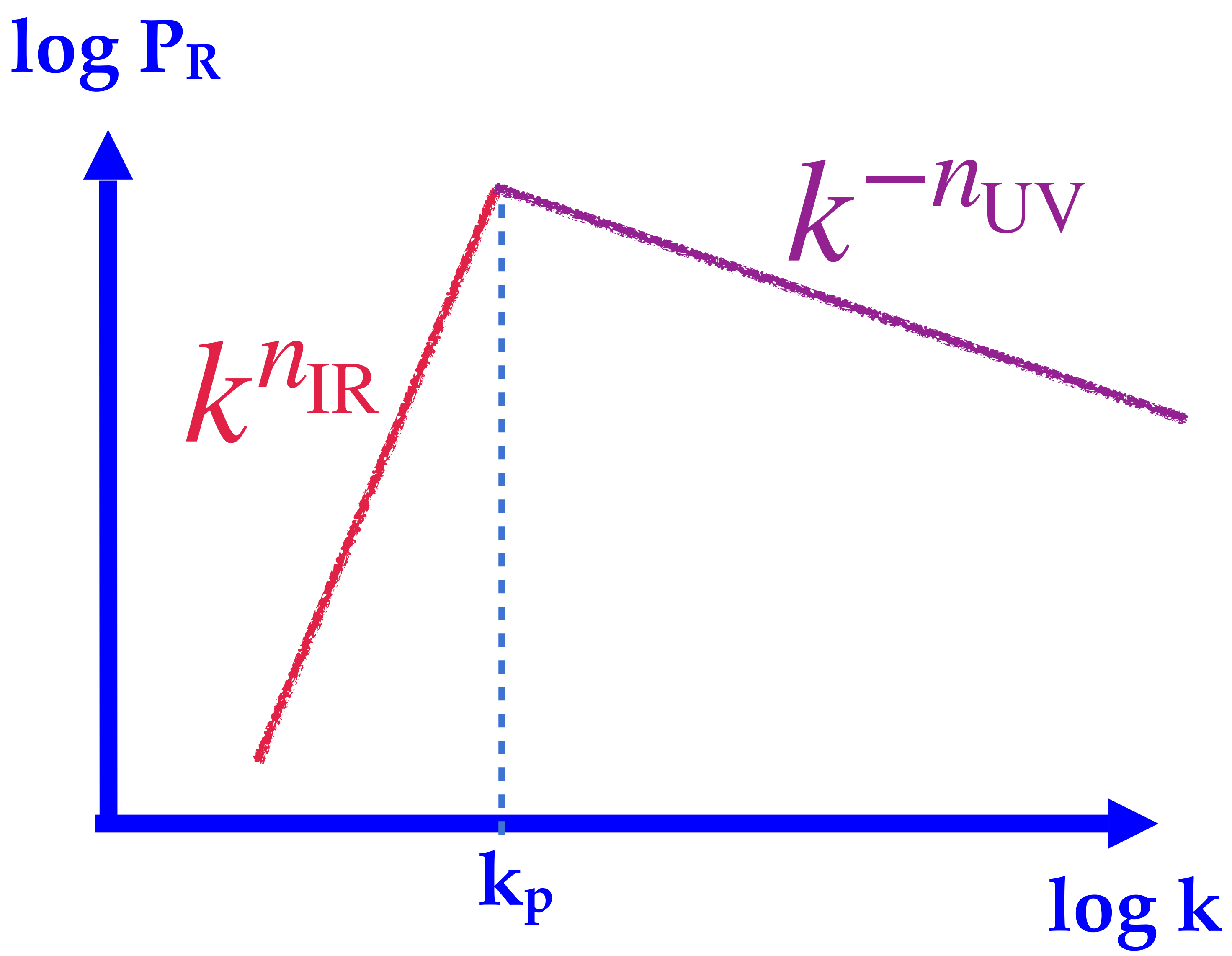
$$b = \frac{1 - 3w}{1 + 3w}$$

$$\Delta = \begin{cases} 2n_{UV} & n_{UV} + b < 4 \\ 4 + n_{UV} & n_{UV} + b > 4 \end{cases}$$



Double broken-power law

Induced GWs spectrum



Broken-power law

$n_{UV} \propto f_{NL}$

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$w \neq 1/3$

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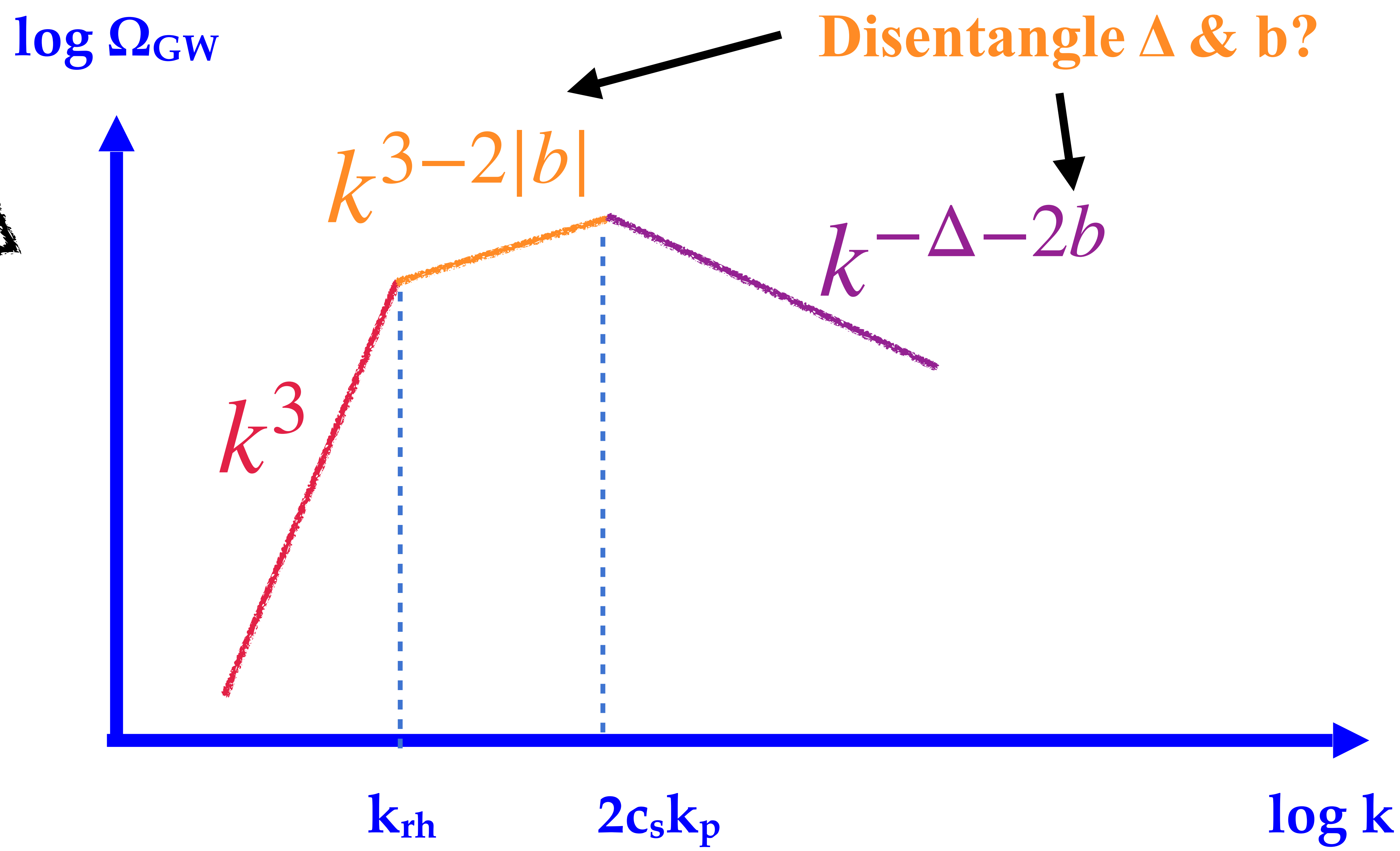


Double broken-power law

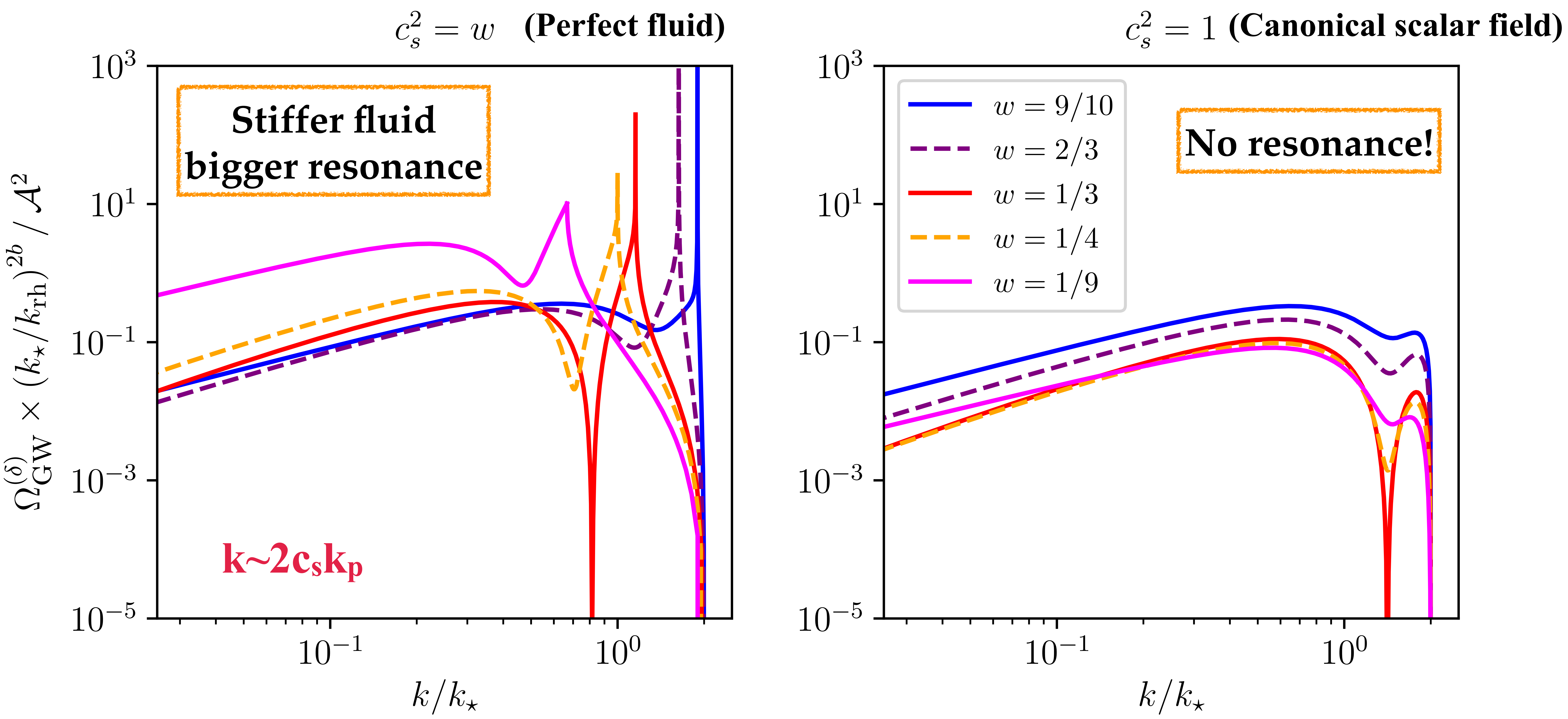
2nd example:

Very sharp peak

Sharp peaks

[GD, 1912.05583]
[GD, S.Pi, M.Sasaki, 2005.12314]

Infinitely sharp peak: $\mathcal{P}_\mathcal{R}(k) = \mathcal{A}_\mathcal{R}\delta(\ln(k/k_*))$

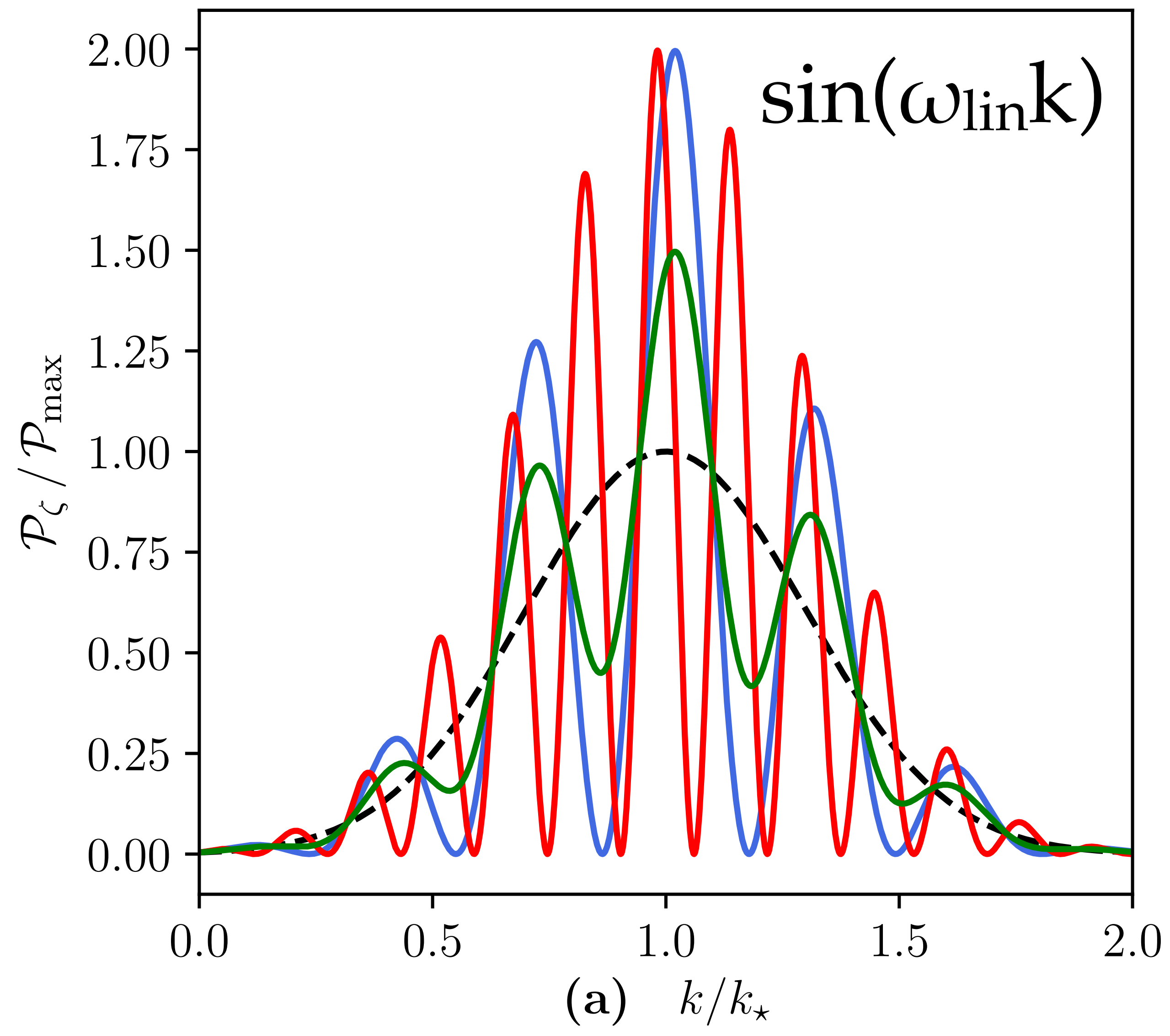


3rd example:

Large oscillatory modulations

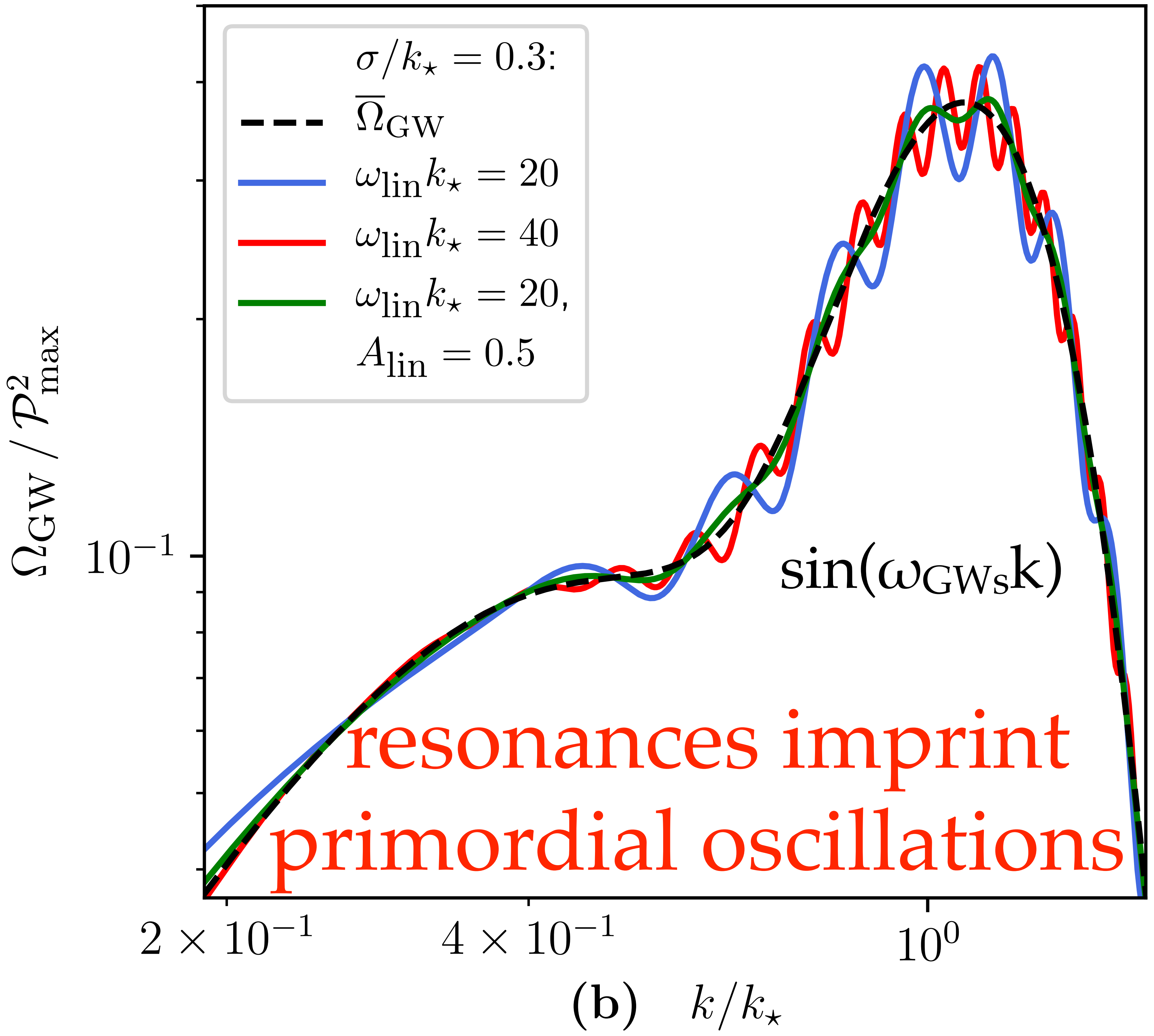
Oscillatory features

[L.Witkowski, GD, J.Fumagalli, S.Renaux-Petel, 2110.09480]
[J.Fumagalli, S.Renaux-Petel, L.Witkowski, 2004.08369, 2012.02761, 2105.06481]
[Cai+1901.10152]



$c_s^2 = w = 1/3$

$c_s^2 = 1$; $w = 1/3$

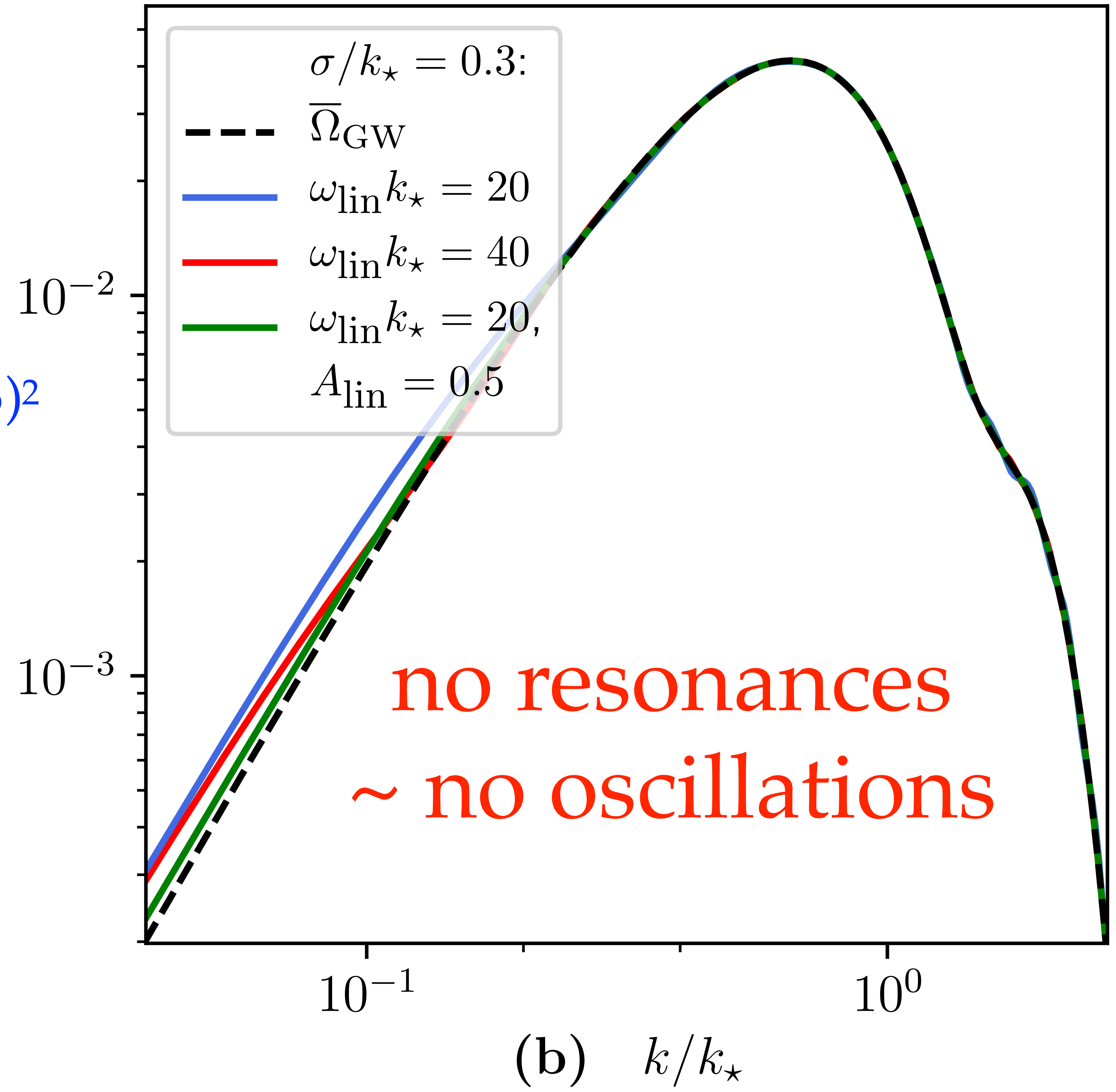


$$(\partial_t^2 + 3H\partial_t - \Delta) h_{ij} \sim \widehat{TT}_{ij}^{\ ab} (\partial_a \Phi \partial_b \Phi)$$

~ (Dirac deltas comb)²

$\omega_{\rm GWs} = \omega_{\rm lin} / c_s$

$\omega_{\rm GWs}$ **depends on** c_s**!**



Oscillatory features

[L.Witkowski, GD, J.Fumagalli, S.Renaux-Petel, 2110.09480]
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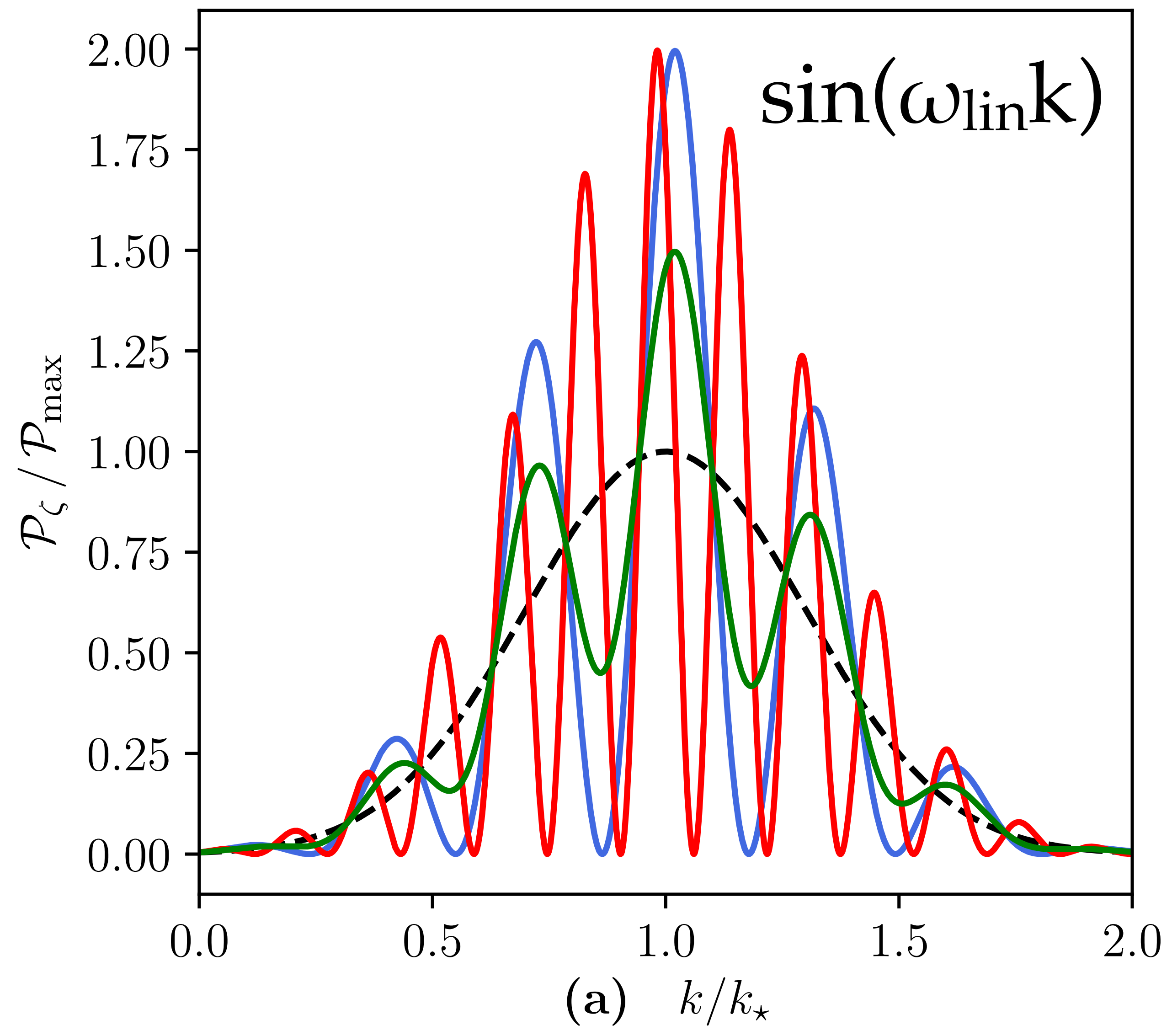
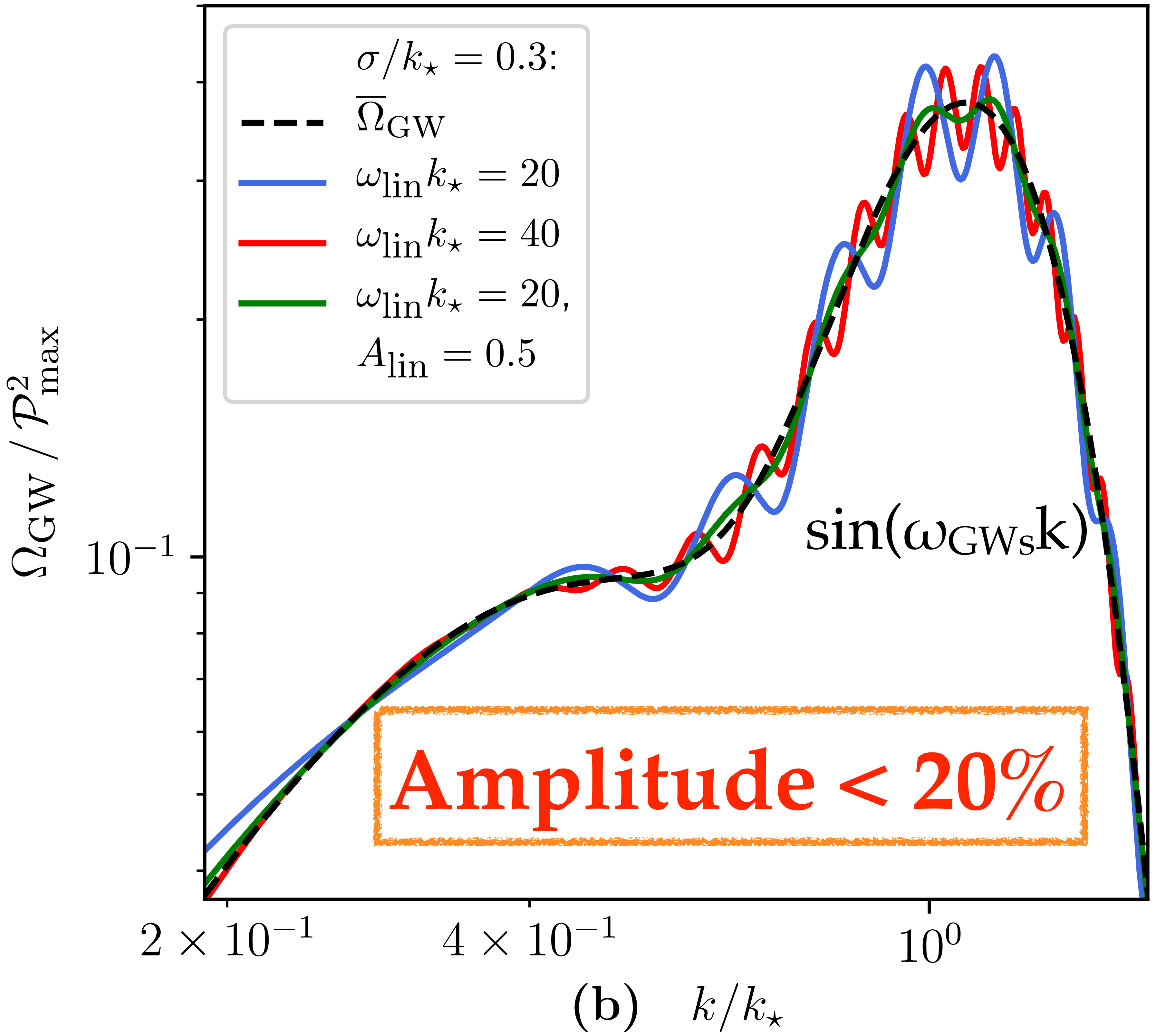


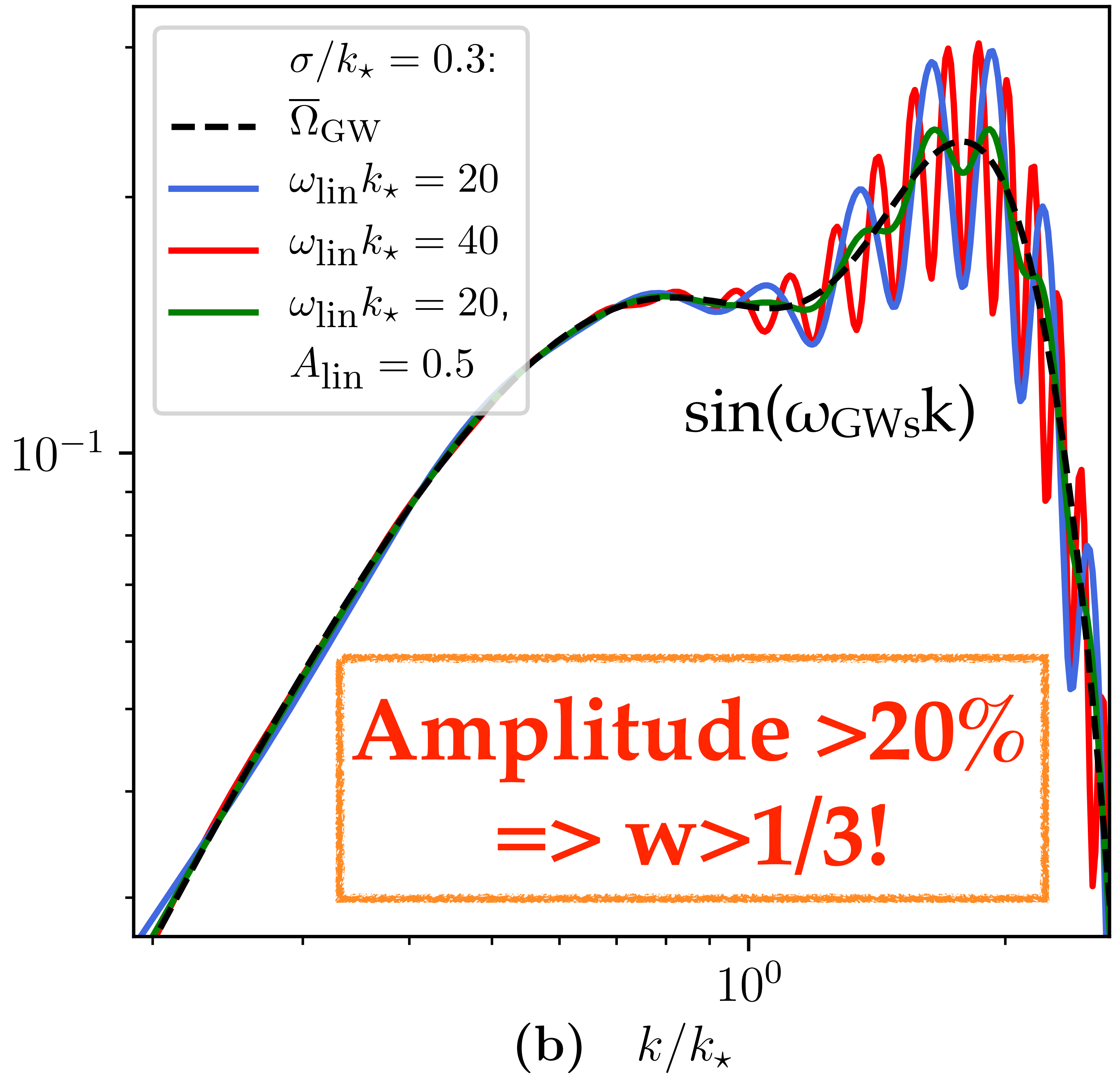
c_s^2=w=1/3



Disentangle ω_{lin} & w?

$\omega_{GWs}=\omega_{lin}/c_s$

c_s^2=w=9/10

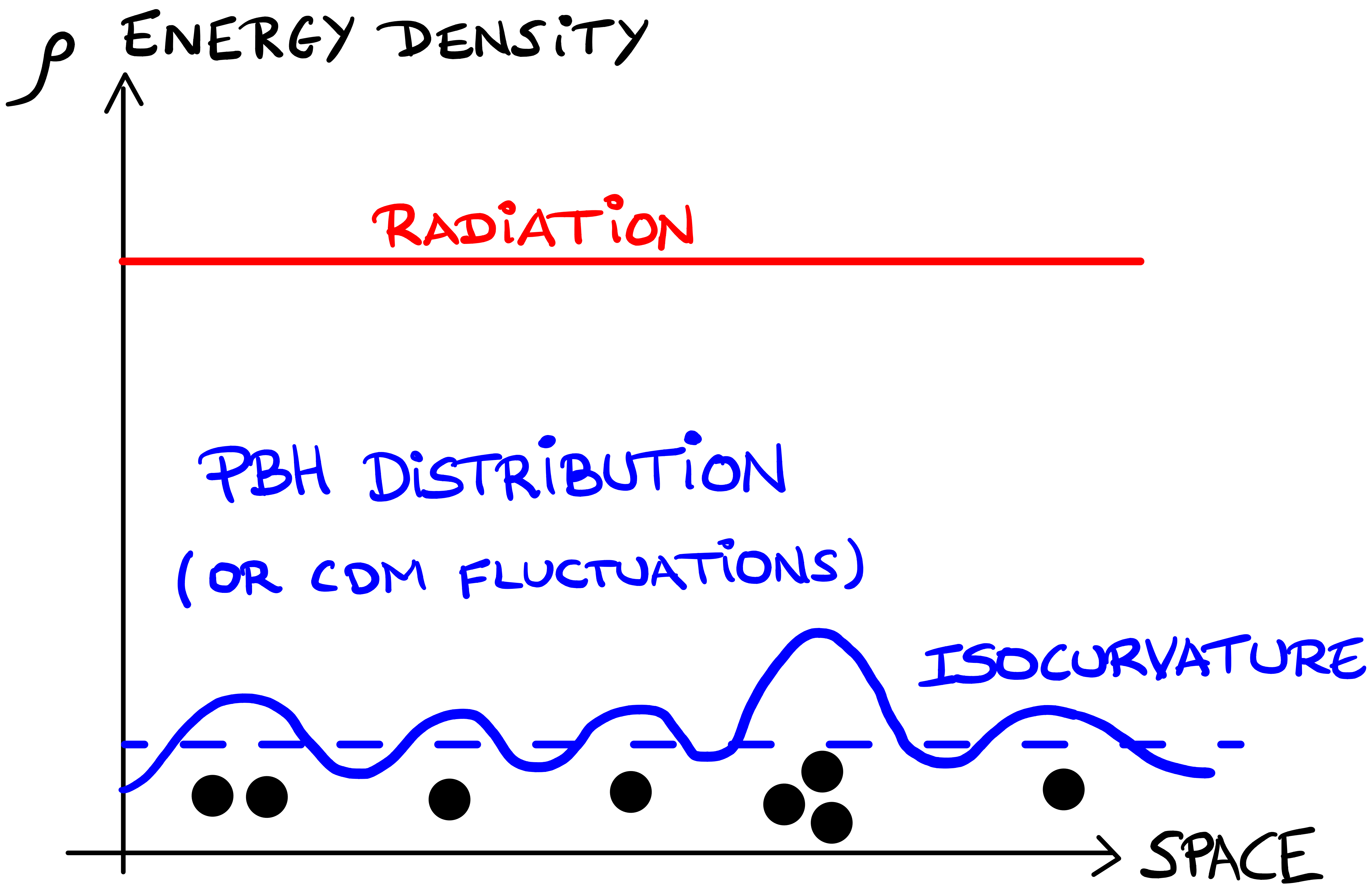


Induced GWs and early isocurvature fluctuations

ISOCURVATURE EVOLUTION

[T.Papanikolaou, V.Vennin, D.Langlois, 2010.11573]

[GD, C.Lin, M.Sasaki, 2012.08151]



Short explanation:

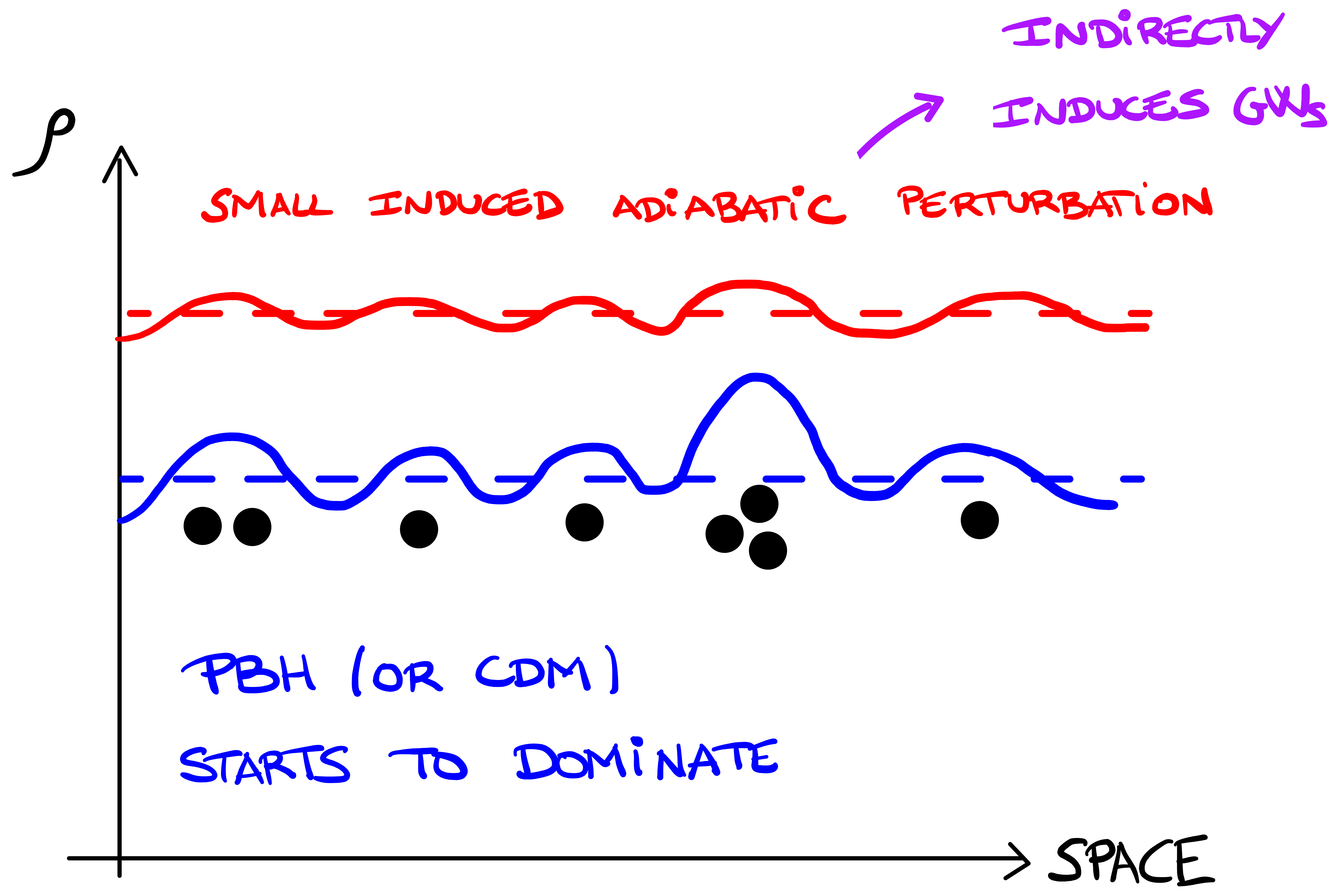
Initial conditions
for a universe after PBH formation

The (inhomogeneous) PBH distribution
give rise to density fluctuations

These fluctuations must be isocurvature
as they form by the collapse
of homogeneous radiation

The evolution of isocurvature onwards also applies to CDM isocurvature!

ISOCURVATURE EVOLUTION



The later a mode enters the horizon,
the larger the curvature fluctuation induced

Short explanation:

PBH (or CDM) energy density gains relative weight

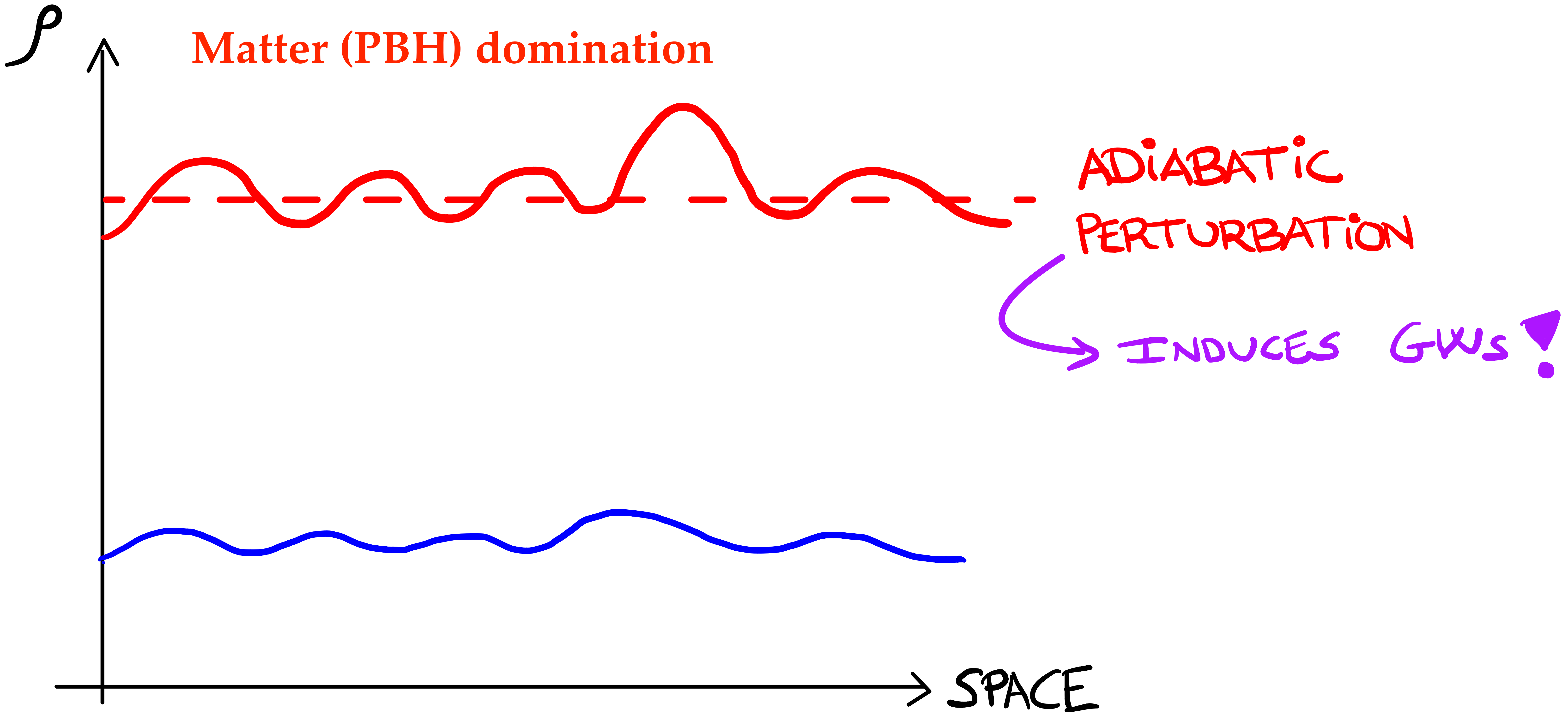
Isocurvature induces curvature fluctuations

Curvature fluctuations induce GWs

Relevant component for large CDM isocurvature!

1st possibility: GWs by CDM isocurvature

ISOCURVATURE EVOLUTION



Short explanation:

PBH (or CDM) dominates the energy density

Curvature fluctuations induced GWs

Relevant component for PBH dominated universe!

GWs induced in an early matter era get greatly enhanced for sudden reheating!

[Inomata+1904.12878 ,1904.12879]

Like PBH evaporation!

2nd possibility: GWs by PBH isocurvature

1st example:

GWs by CDM Isocurvature

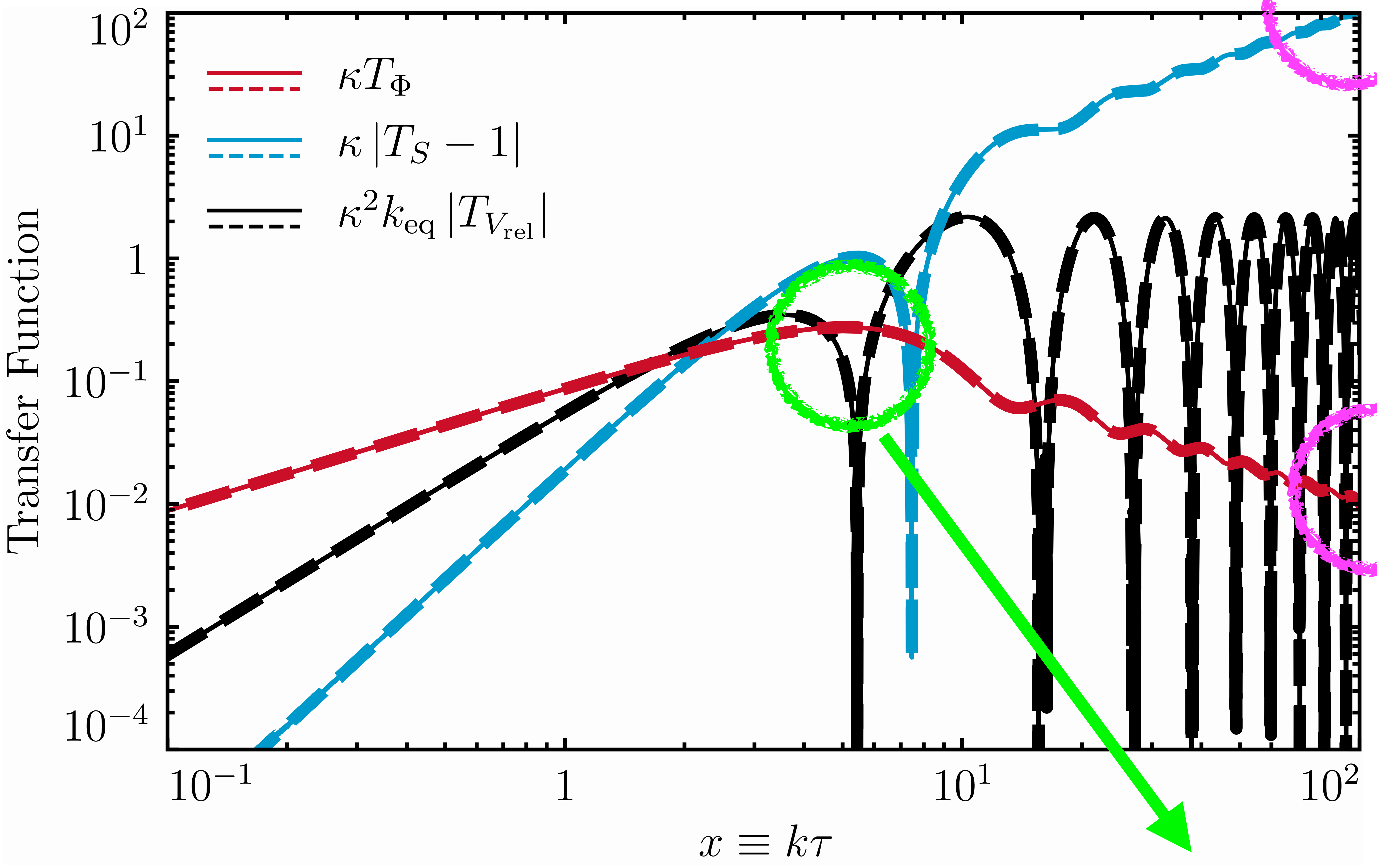
GWs by CDM isocurvature

Evolution of scalar modes (Φ sources induced GWs)

May hit the non-linear regime and form PBHs!

[S.Passaglia, M.Sasaki, 2109.12824]

$\kappa = k/k_{eq}$



But Φ already decayed!

Most of generation of induced GWs at horizon crossing.

$$(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{\ ab}(\partial_a\Phi\partial_b\Phi)$$

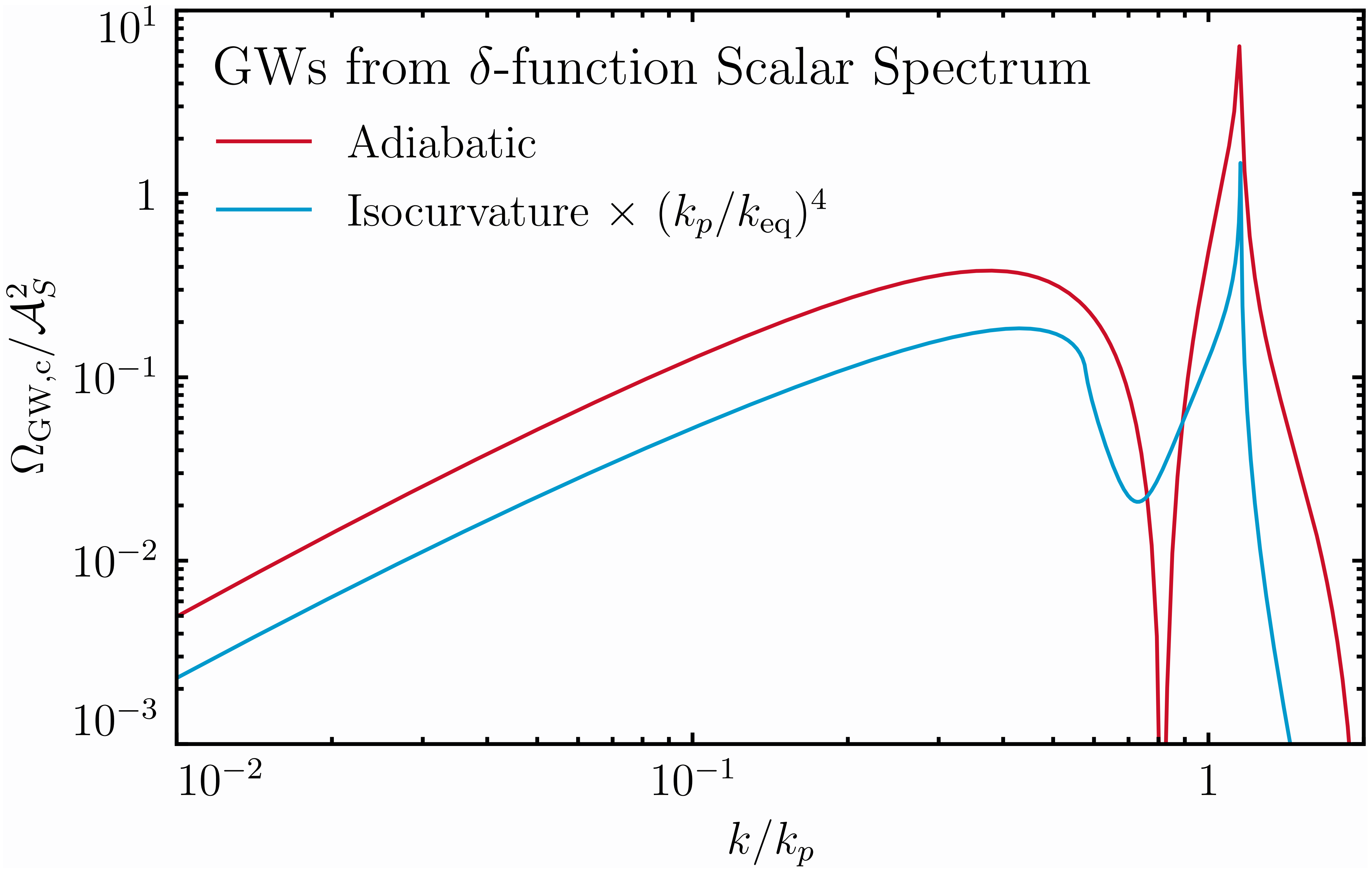
GWs by CDM isocurvature

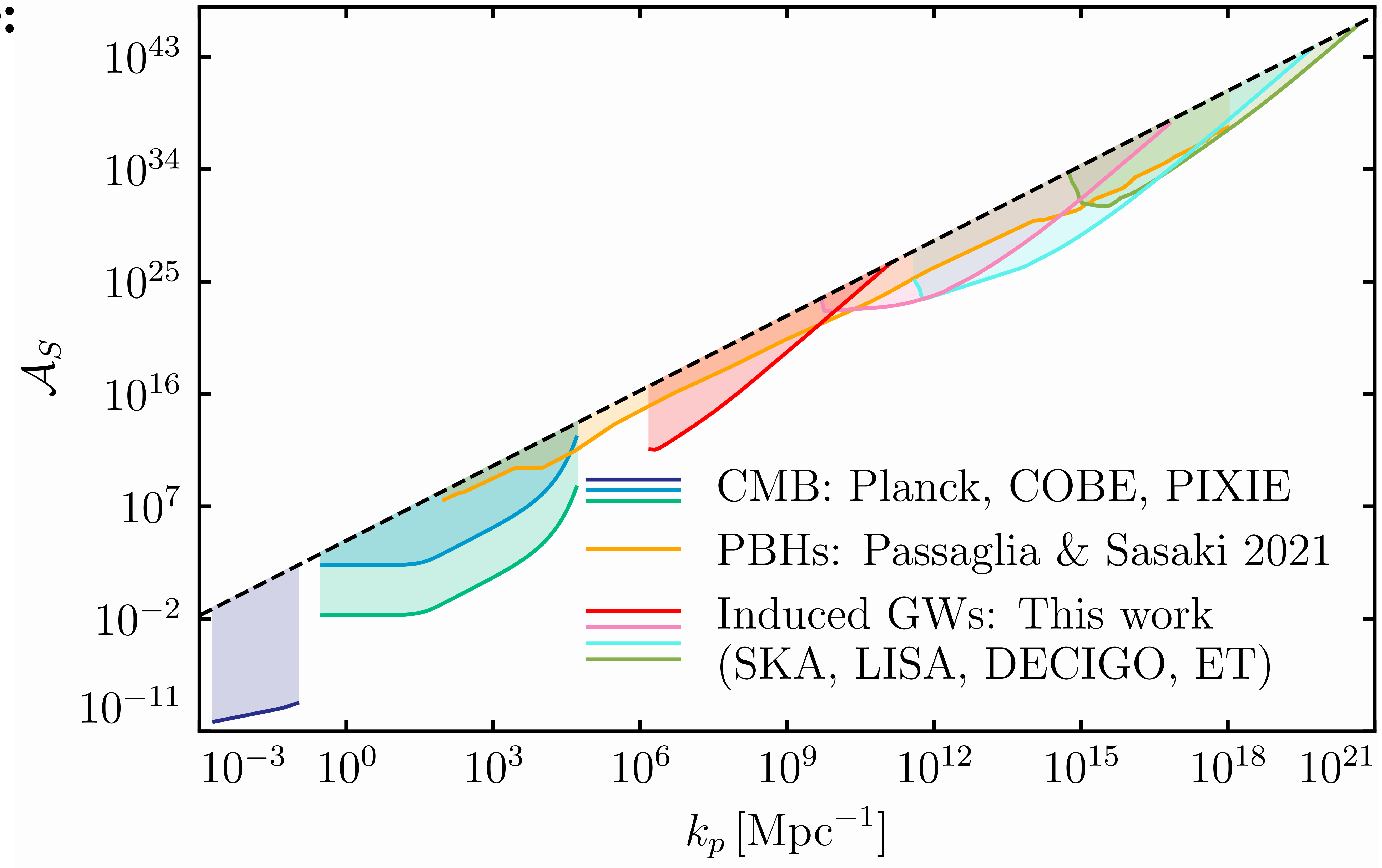
Large enough CDM isocurvature leads to a detectable GW signal!

Analytical kernels in:

[GD, S.Passaglia, S.Renaux-Petel, 2112.10163]

Spectral shape different from the adiabatic one:





GWs by CDM isocurvature (future work)

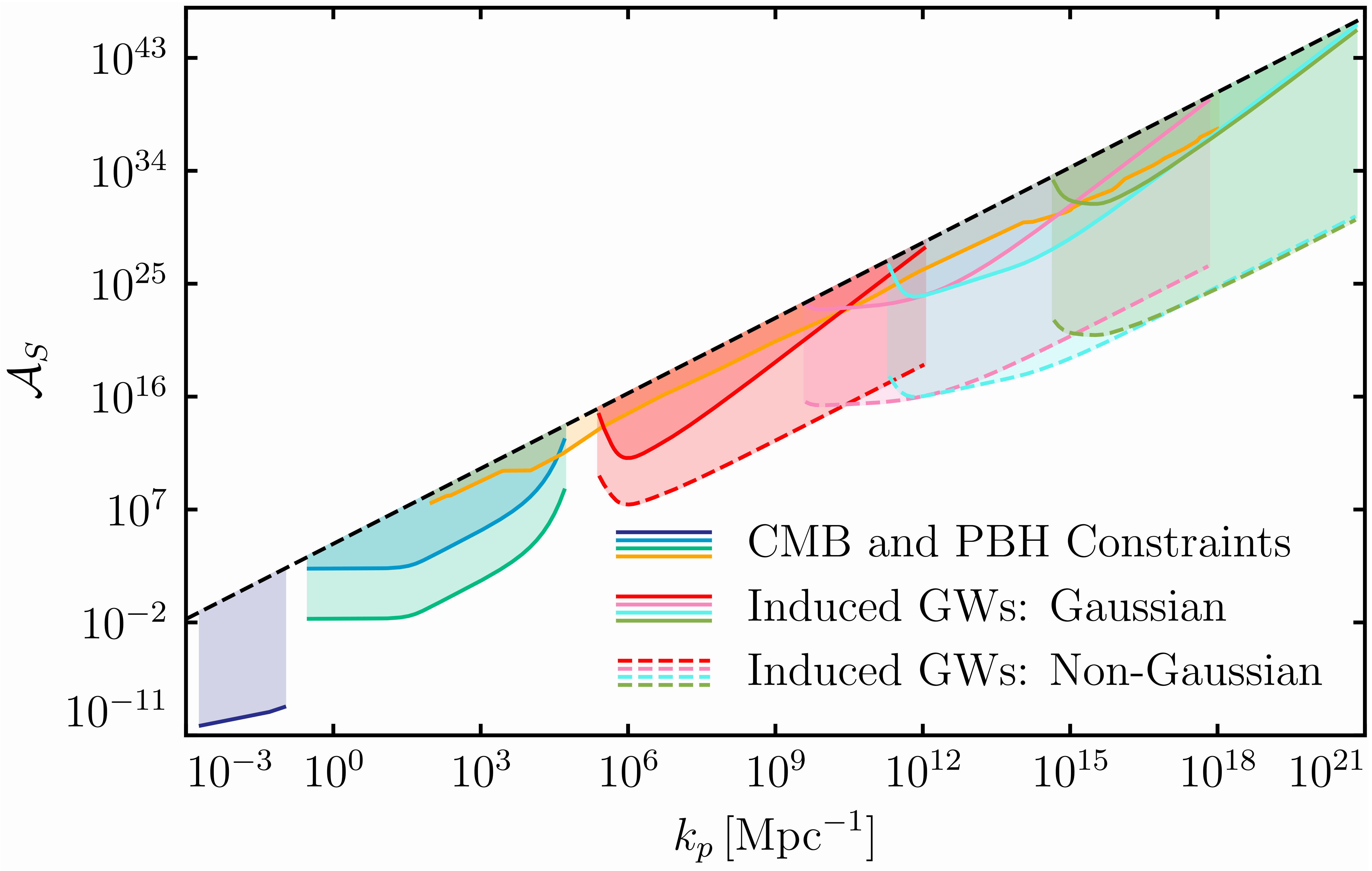
Large initial isocurvature probability distribution

$$S = \frac{\rho_{cdm}(x) - \bar{\rho}_{cdm}}{\bar{\rho}_{cdm}} \gg 1$$

must be HIGHLY SKEWED (HIGHLY NON-GAUSSIAN)

Early (heuristic) estimates:

(Skewness, long tail, enhances GW production)



2nd example:

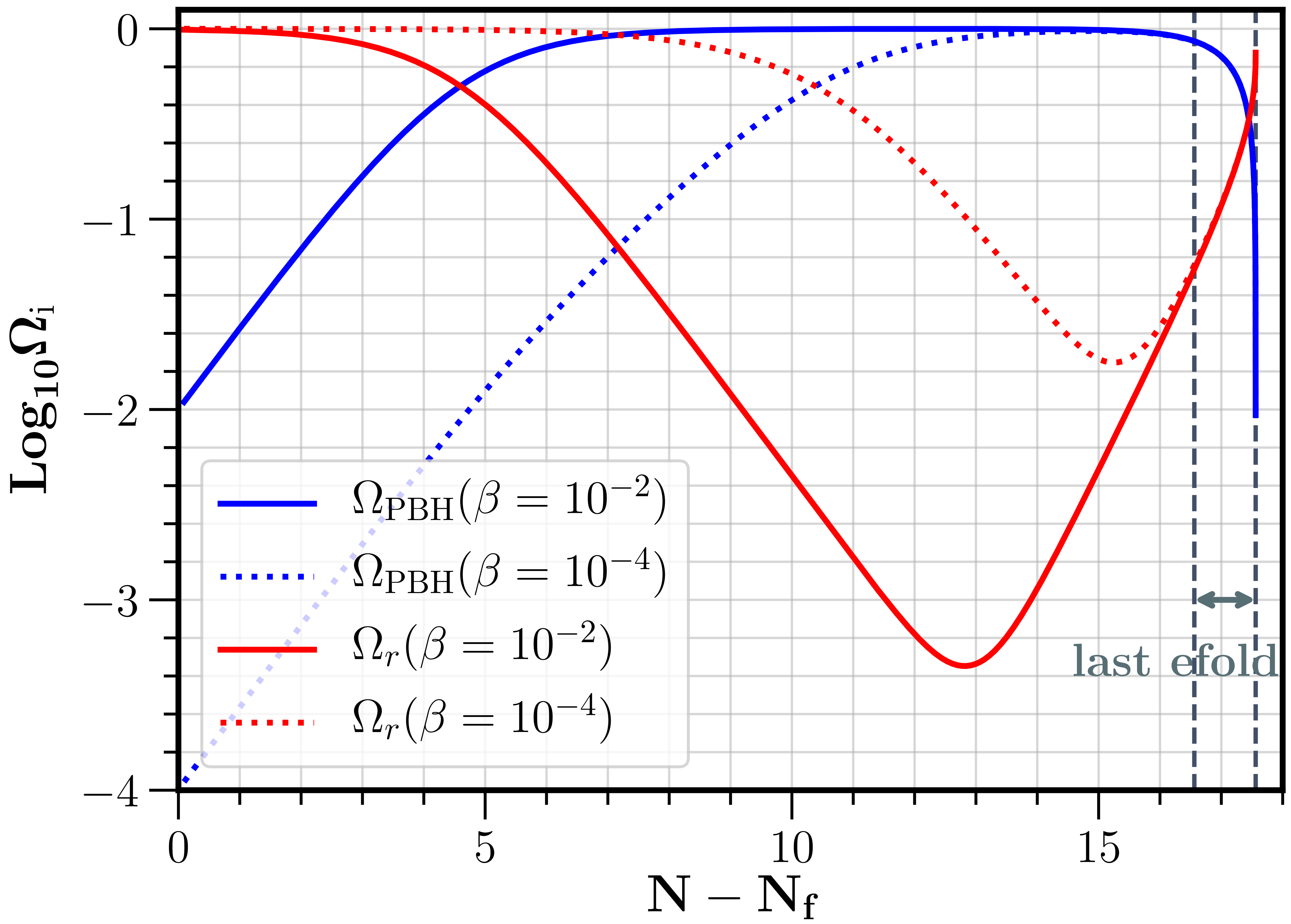
GWs by PBH density fluctuations

GWs from PBH density fluctuations

[K.Inomata+2003.10455]
[T.Papanikolaou+ 2010.11573]
[GD+2012.08151]

Sudden evaporation of all PBHs, creates huge velocity fluctuations in the radiation fluid

and a loud GW signal! $(\partial_t^2 + 3H\partial_t - \Delta)h_{ij} \sim \widehat{TT}_{ij}^{ab}(\partial_a V \partial_b V)$ $V \propto \Phi'/\mathscr{H}$



There also induced GWs during the PBH dominated era [T.Papanikolaou+ 2010.11573]

GWs from PBH density fluctuations

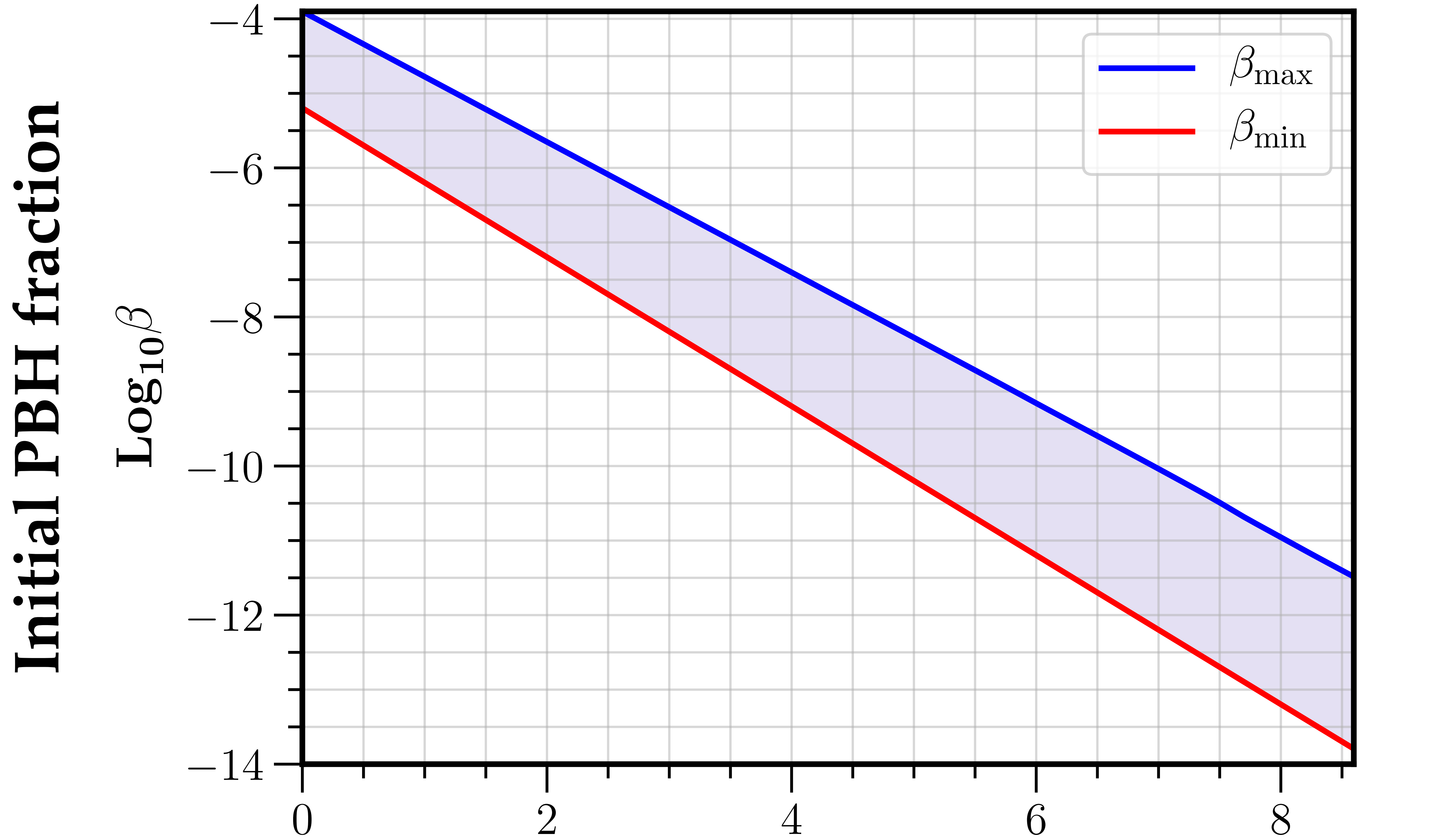
[K.Inomata+2003.10455]
[T.Papanikolaou+ 2010.11573]
[GD+2012.08151]

Sudden evaporation of all PBHs, creates huge velocity fluctuations in the radiation fluid

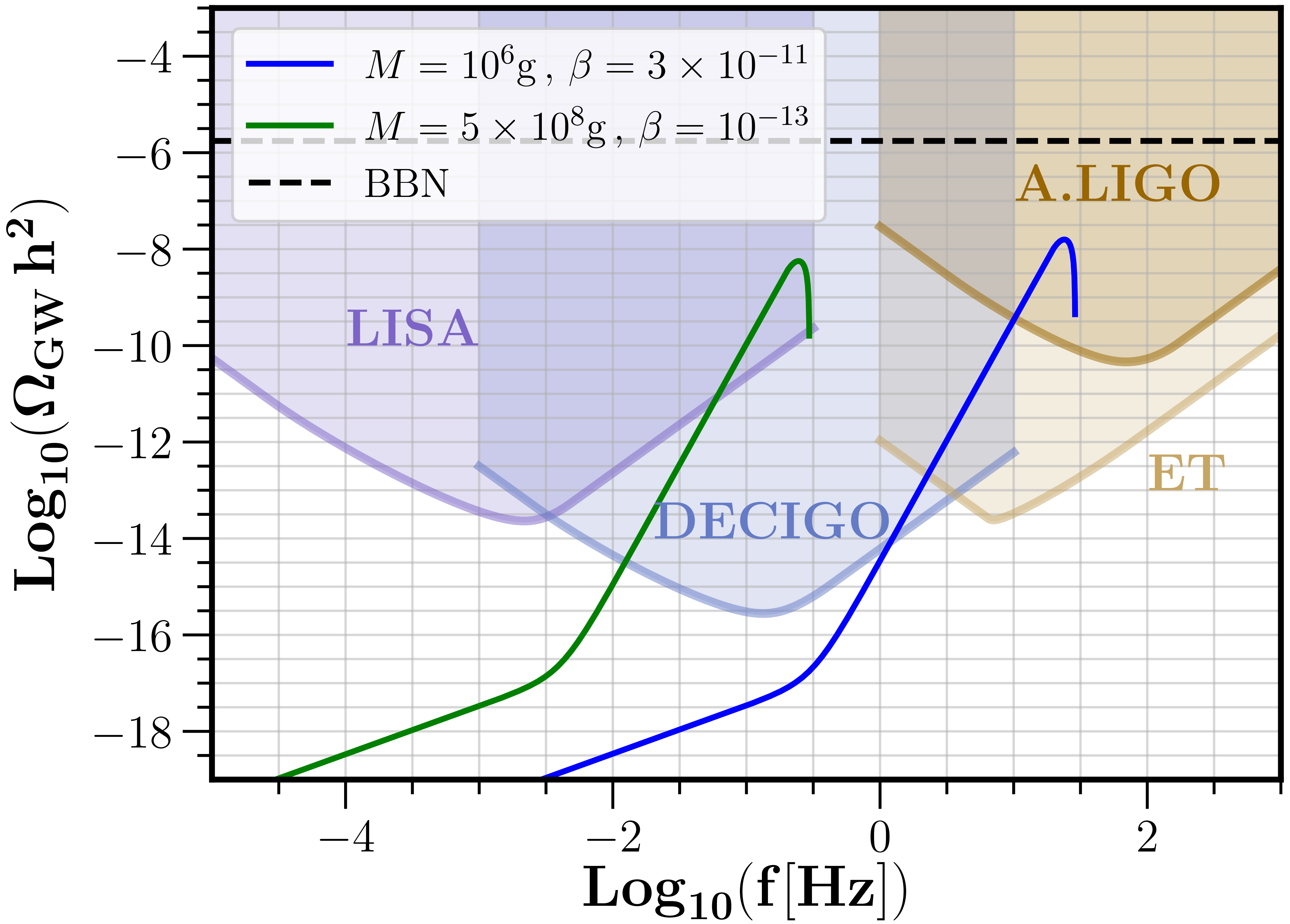
and a loud GW signal!

We can use it to test the PBH dominated universe:

(If PBH spin, we can combine info on Neff [GD+2105.06816])



[GD+2012.08151]



GWs from PBH density fluctuations (future work)

Most of PBH density fluctuations reach the non-linear regime during PBH domination

$$0.5 < \delta < 90 \left(M_{\rm PBH}/10^4 {\rm g}\right)^{1/6}$$

This means that there will be BH mergers.

However, production of GWs will not stop and we still have Φ<<1

(so our estimate might be on the right track in orders of magnitude)

How the system behaves in the non-linear regime with PBH evaporation needs numerical work.

(maybe it generates turbulences? Kozaczuk et al. 2108.12475)

Summary

Induced GWs (together with PBHs) are a unique probe of:

- **Physics of inflation**

  - Primordial spectrum — **Unique for small scales**
  - Early CDM isocurvature — **New probe of isocurv.**
  - Gaussian vs non-Gaussian

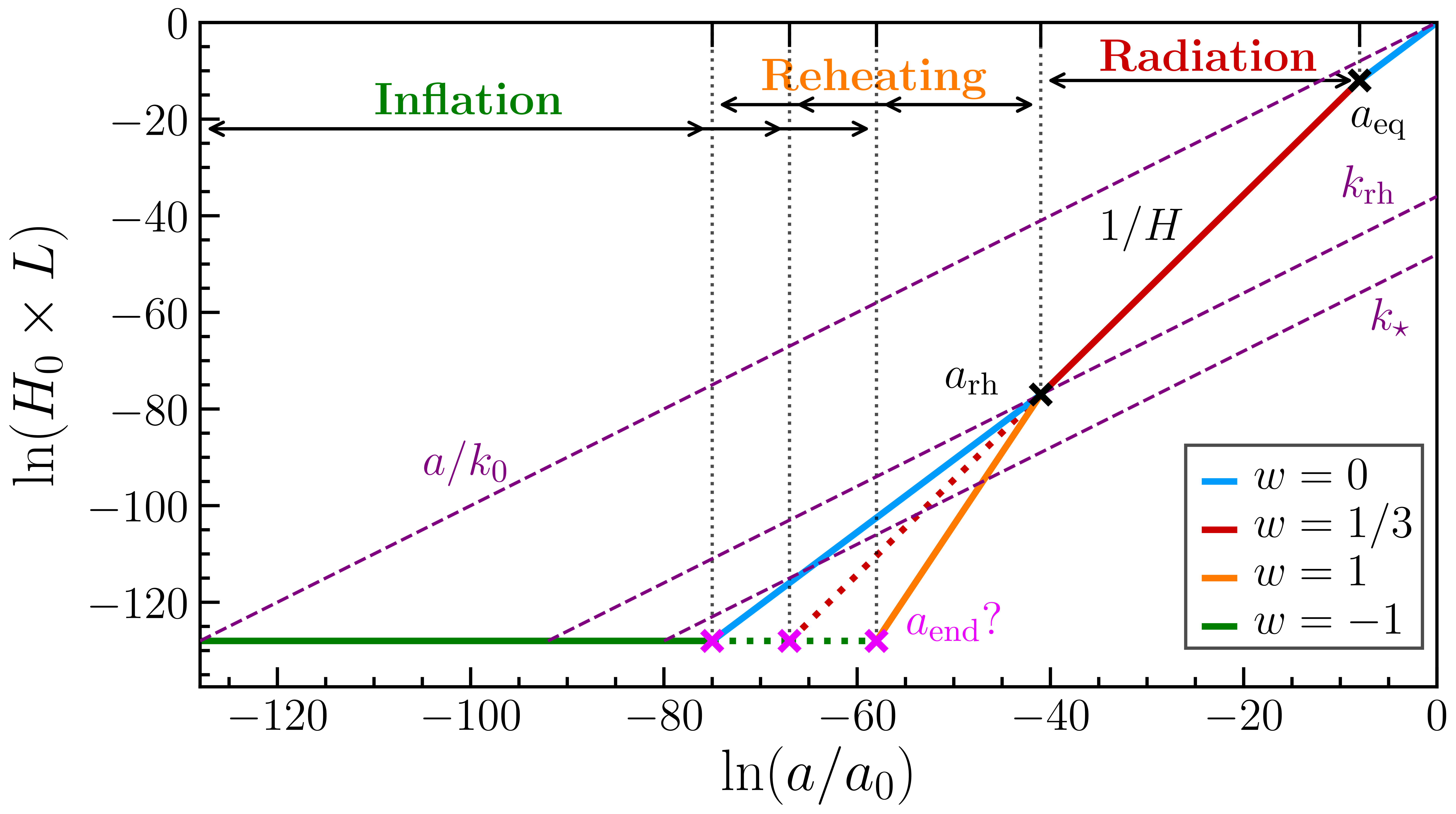
We need to know both!

- **Physics after inflation (before BBN)**

  - Equation of state — **IR broken power-law**
  - Sound speed of fluctuations — **Resonant peak**
  - PBH dominated universe — **Constraints on β**

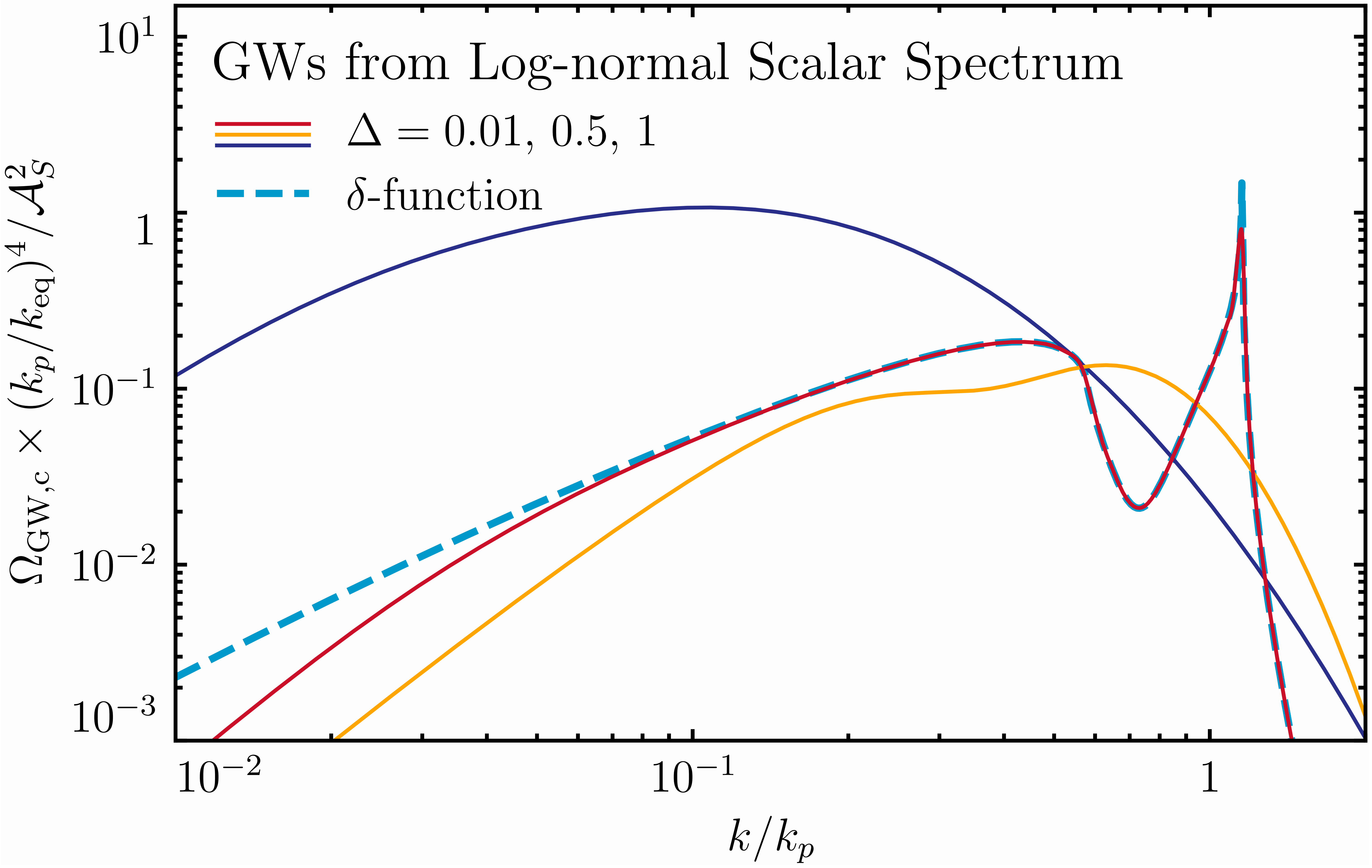
≋ The End ≋

The unknown expansion history

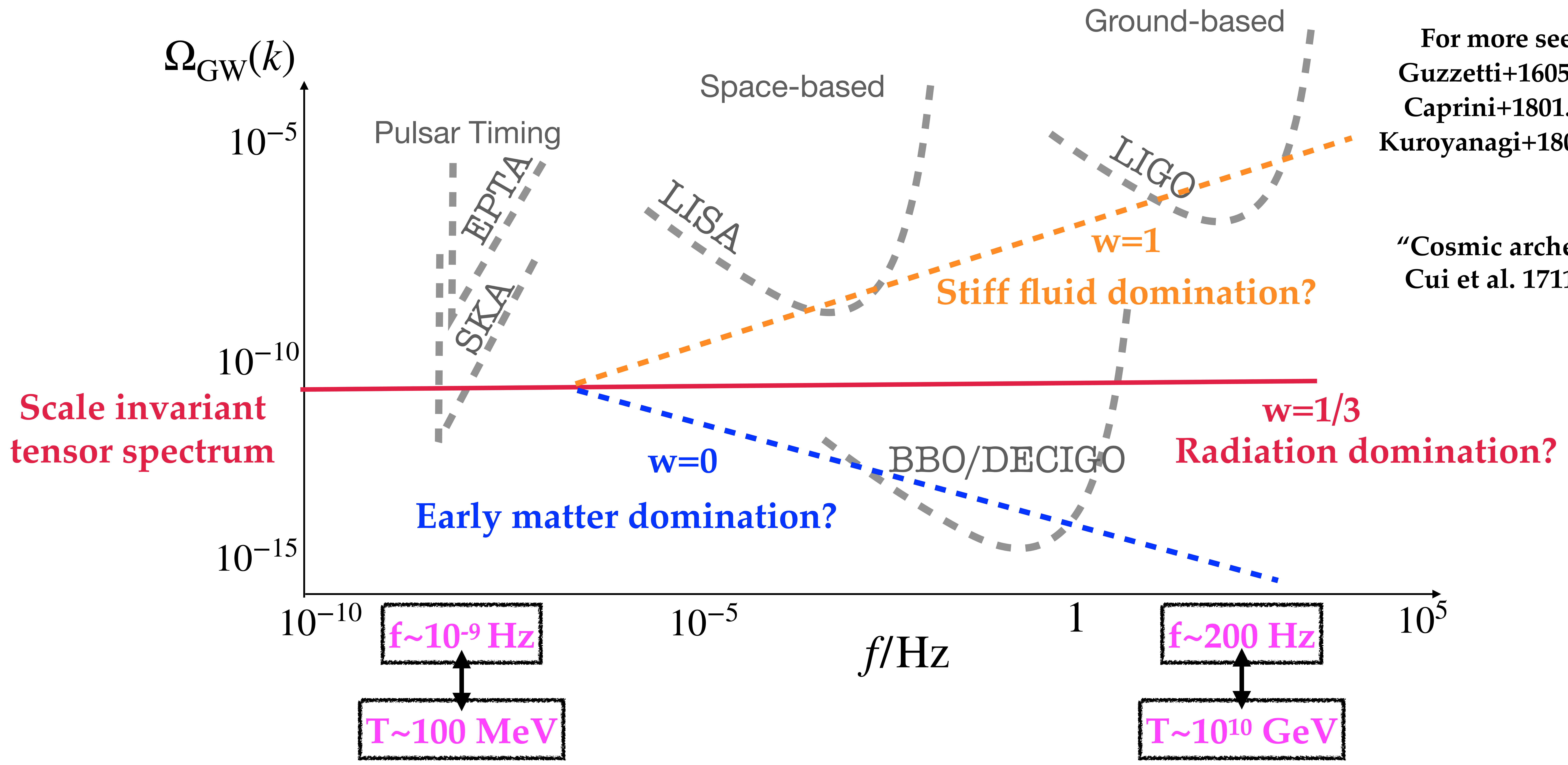


GWs by CDM Isocurvature

The peak follows effective curvature perturbation spectrum: $P_{\text{curv}} \sim k^{-2} P_{\text{iso}}$



Expansion history of the universe?



For more see e.g.
Guzzetti+1605.01615
Caprini+1801.04268
Kuroyanagi+1807.00786

“Cosmic archeology”
Cui et al. 1711.03104